# **Sunset Variations of the Bottom Edge of the Ionosphere During the Proton Precipitations on and After 29 September, 1989**



**G. F. Remenets, M. I. Suhovey and V. A. Shishaev**

**Abstract** One of the most powerful solar proton precipitations (SPP), which began on 29 September 1989, was at the same time the longest one. This peculiarity gave to us the possibility to study on the base of VLF data the precipitation effect during a sunset on the dynamics of the electric properties of the lowest part of the ionosphere in the terms of complex reflection coefficient for a sequence of 8 days. The input data for the analysis were the relative variations of amplitudes and variations of phases for 3 VLF monochromatic, ground based signals with radio path North Norway—Kola Peninsula having a moderate length, 885 km. A self-consistent method for solution of an inverse VLF problem was used. The sunset dynamics of electric conductivity of the temporally modified bottom ionosphere was stated for all days of the SPP. The solution of such inverse VLF problem was possible due to the multi frequency measurements for the radio path and the synchronous registration of amplitudes and phases.

**Keywords** Proton precipitation on 29 September 1989 · Sunset effect · Inverse VLF problem · Self-consistent method

## **1 Introduction**

Our paper is devoted to the terrestrial electromagnetic sounding of the lowest part of ionosphere during the natural disturbances. Such investigation may be named as a solution of inverse radio physics problem. Thirty years ago a self-consistent (S-C) method for solution of an inverse VLF problem (SIvlfP) in non-stationary radio path conditions was developed, and it was used for an analysis of 3 different VLF disturbances on 29 September 1989: an ultra-energetic relativistic electron precipitation (04:00–10:00 UT), a sudden ionospheric disturbance due to the Sun

G. F. Remenets  $(\boxtimes) \cdot M$ . I. Suhovey

Saint-Petersberg State University, Saint-Petersburg, Russia e-mail: [g.remenets@spbu.ru](mailto:g.remenets@spbu.ru)

V. A. Shishaev Polar Geoghysical Institute, Apatity, Murmansk Reg., Russia

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X-ray flare at 11 UT and a solar proton precipitation (SPP)—one of the most powerful event of this kind for 70 years [\[1\]](#page-12-0), which began at 12 UT. Its analysis was stopped at 17 UT [\[2\]](#page-12-1), at the maximum of precipitation. The SIvlfP gave the dynamics of the complex refection coefficient with its modulus  $R$  and phase recalculated into the effective height (altitude) *h*. The radio path was purely auroral with moderate length (885 km, North Norway—Kola Peninsular, id est. Aldra station—Apatity town, Murmansk region). The radio signals were monochromatic with frequencies 10.2, 12.1, 13.6 kHz. The effective height *h* for a real waveguide at fixed time is a height of an effective model empty waveguide with the abrupt bottom and top boundaries. This waveguide is effective in the sense that the phase pass of a given propagating mode (a ray in our case) is the same as in the real waveguide.

The peculiarity of the SIvlfP is its independence on the nature of a geophysical disturbance, and this solution does not demand a priori knowledge of the ionosphere parameters before a disturbance. A "ray" (hop) version of theory of the wave propagation in the near ground (terrestrial) waveguide  $[3, 4]$  $[3, 4]$  $[3, 4]$  was used in the solutions pointed above: the diffraction wave above the spherical model of the Earth, 1st and 2nd rays reflecting from a top virtual boundary one and two times correspondingly. We worked in the conditions for which the used radio frequencies are much less the frequency of effective electron collisions with the atmosphere neutrals, therefore, the bottom edge of ionosphere is an inhomogeneous electric conductor depending on the altitude, and the radio waves react on the time changes of electric conductivity.

For the same radio path pointed above the same S-C method [\[2,](#page-12-1) [5,](#page-12-4) [6\]](#page-12-5) was used for analysis of the SPP on 16 February 1984 [\[7\]](#page-12-6) and for 18 sunset variations in 8 August–8 September, 1974, half of which referred to the quiet geophysical conditions [\[8\]](#page-12-7). In the case of SPP which continued according to the VLF data from 9 to 13 UT the effective height *h*(*t*) fell down from 63–67 to 44–47 km at 10 UT. The modulus of the first sky wave (1st ray) slightly changed about 0.6 during the event. Figure [1](#page-1-0) represents these statements, and in Fig. [2](#page-2-0) the comparison of the experimental phase variations with the calculated ones are given.

The sunset variations are represented in Fig. [3,](#page-2-1) [\[8\]](#page-12-7).



<span id="page-1-0"></span>**Fig. 1** The dynamics of the modulus of the reflection coefficient of the 1st sky wave (1st hop) from above  $R(t, \psi(h))$  and the effective altitude  $h(t)$  of reflection for the beginning stage (left panel) and for the restoration stage (right panel) [\[7\]](#page-12-6). The angle of the wave incidence  $\psi(h)$  depends on *h*(*t*) according to the geometrical relations for spherical waveguide model and is greater than 1.4 rad, Three curves for each magnitude estimate the accuracy of the SIvlfP method applied to the 16 February, 1984 data

<span id="page-2-0"></span>

<span id="page-2-1"></span>**Fig. 3** The variations of amplitude (12.1 kHz), top panel, in relative units, and of the corresponding effective heights h(t) km, bottom panel, at sunset for the polar radio path North Norway, (Aldra)–Kola peninsula, (Apatity), [\[8\]](#page-12-7). The numbers of the curves correspond to different dates of August and September. On the left of the panel the moderately disturbed dates are collected, and on the right—the quiet days are represented

The pointed results were gotten due to the several frequency amplitude and phase VLF measurements, due to applicability of a supposition about the prolong homogeneity of the radio path at every point of the disturbance time and due to sliding falling of the rays on the top virtual boundary with the altitude *h*.

There are no more works of such kind with the inverse VLF problem solution on the base of monochromatic signal measurements. But there are two unique and beautiful works in which the impulse VLF sounding was used [\[9,](#page-12-8) [10\]](#page-12-9). In the first one the virtual height of the impulse reflections from above at the Berd station (Antarctic Continent) were compared with the solar proton fluxes ( $E > 10$  MeV) which were measured on the board of hand maid Earth's satellite "Explorer-41" [\[10\]](#page-12-9). In the conditions of practically normal falling of an impulse on the disturbed ionosphere (the distance between an impulse source and a receiver was equal 20 km) the virtual height did not fall down under 60 km, with the maximum values 77–80 km while 25–29 September, 1969. About ten years after the inverse VLF problem for the pointed impulse sounding was solved [\[10\]](#page-12-9) and the corresponding  $N_e(z)$  for 6 days of polar night from 27 September to 2 October, 1978 were represented on Fig. [4,](#page-3-0) which we reproduced, the review [\[11\]](#page-12-10). The  $N_e(z)$  profiles (the continuous curves) of this Fig. [4](#page-3-0) are practically identical at first two days: on 27 and 28 September. On the following 4 days the profile lifted at 2–3 km per day.

<span id="page-3-0"></span>

#### **2 A Problem and Its Solution**

We have reported about all science sources devoted to the VLF sounding of the polar ionosphere disturbed by the SPPs and the sunset effect independently. Now we pass to our problem about the same VLF sounding but in the conditions when two causes of ionization are working simultaneously: the SPP and the sunset for the radio path. The SPP on 29 September, 1989 continued not less than 10 days, Fig. [5,](#page-4-0) [\[12\]](#page-12-11). In the present report we characterize the electric conductivity of the disturbed low polar ionosphere by 2 parameters: by an effective height (effective altitude of reflection)  $h(t)$  and by a value of reflection coefficient modulus  $R(t, \psi(h))$  for the first ionosphere ray in a model of effective waveguide with  $h(t)$ . The argument of complex  $R(t, \psi(h))$ is equal to  $\pi$  for sliding angles  $\psi$  of ray incidence ( $\psi > 1.4$  rad.) and for a middle working frequency 12.1 kHz. Our problem is to get the time functions  $R(t, \psi(h))$  and  $h(t)$  according to 6 input experimental VLF functions: 3 amplitudes  $A_i(t)$  an 3 phase variations  $\phi_i(t)$  caused by the geophysical disturbances,  $i = 1, 2, 3$ ; the radio path and the working frequencies are pointed in the Introduction. Our analysis is limited by the sunset hours (15:30–18:30 UT). For this time the most "bad" approximation is done: at every time moment a model waveguide is supposed to be homogeneous along the radio path. Our estimation has shown that the corresponding systematic error is not greater than the apparatus phase error of the experimental data used (1  $\mu$ s).

We shall compare quantitatively the daily variations of solar proton fluxes, which began on September 29, 1989, 12 UT, with the sunset changes of our output parameters *R*(*t*) and *h*(*t*) for a completely auroral radio path Aldra–Apatity which we pointed above. For realization of this purpose we have used the satellite data [\[12\]](#page-12-11), the experimental VLF data of the Polar Geophysical Institute—RAS, Apatity, Murmansk



<span id="page-4-0"></span>**Fig. 5** Satellite data on the proton fluxes with different energies (>1 MeV, >10 MeV, >50 MeV and >100 MeV) for the period from 28 September (09.28) to 6 October (10.06), 1989, [\[12\]](#page-12-11). The event began on 29 Sept. (09.29), and to 5 Oct (10.05). the proton fluxes were greatly weakened. On 6th Oct. the fluxes of electrons with energy less than 50 MeV increased again significantly. This figure was plotted by us according to the data [\[13\]](#page-12-12)

region and the S-C method of SIvlfP [\[2\]](#page-12-1). This investigation is a natural continuation of the analysis which was begun in this pointed work.

The VLF amplitude and phase variations corresponding to the proton precipitations of Fig. [5](#page-4-0) are represented by the data in the Fig. [6.](#page-5-0) We used this data in our analysis only partly, id est. we analyzed the sunset hours 15:00–19:00 for every date. In Fig. [7](#page-6-0) such data on 28 and 29 September are represented.

We solved the inverse problem and established the following: (i) The presented graphs in Figs. [8](#page-6-1) and [9](#page-7-0) give an error estimate of the method by comparing the analysis of sunset VLF variations for the positive direction of time (solid lines) with the analysis for the negative direction of time (dashed lines). Each of these analyses, based on the solution of the inverse VLF problem by S-C method and which did not use any geophysical data, was fulfilled independently. (ii) With the change of proton flux density in the period from September 29 to October 2, 1989 the daily variation of the effective height  $h(t)$  at sunset changed from 5 km on September to 10 km on October and the reflection coefficient of the first ionospheric ray at a sunset was practically constant. In this date period the value of effective altitude changed from 47 to 54 km at 15:00 UT, from 52 to 64 km at 19:00 UT. The value of reflection coefficient modulus has changed from 0.8 to 0.6, id est. to the value of 16 February 1984, Fig. [1.](#page-1-0) (iii) From October 3 to October 5, the proton flux density did not yet come back to its undisturbed value but weakened significantly.

In a real wave guide there is none ray mode as for high frequencies, because of a wave length value being comparable with the altitude scale of inhomogeneity of electric conductivity. But it was shown numerically several decades ago that an effective height  $h(t)$  was placed inside the altitude layer of electric conductivity which was significant for the radio wave reflection. Roughly speaking the *h* is an analog of



<span id="page-5-0"></span>**Fig. 6** The relative amplitude (top panel) and phase (bottom panel) variations for three frequencies: 10.2 (read curve), 12.1 (green) and 13.6 (blue) kHz. The time of registration: 28 September (09.28)–6 October (10.06), 1989. The amplitudes are at relative units and the phases are given in microseconds



<span id="page-6-0"></span>**Fig. 7** The time fragments of the VLF variations before (the left half) and on (the right half) the solar proton precipitation 29.09.1989. These fragments correspond to the sunset hour for the radio path



<span id="page-6-1"></span>**Fig. 8** The computation results: Dynamics of the effective height *h*(*t*) at sunset. The variations of effective height *h* at sunset (15:00–19:00 UT) for a week of measurements having begun with an absence of proton precipitations (28 September) and having continued with solar proton precipitation for the following days. The continuous curves—is an analysis in the positive direction of time. The dashed curves—is an analysis in the opposite direction



<span id="page-7-0"></span>15 16 17 18 1915 16 17 18 1915 16 17 18 1915 16 17 18 1915 16 17 18 1915 16 17 18 1915 16 17 17 18 1915 16 17 18 1915 16 17 18 19

**Fig. 9** The computation results: dynamics of the modulus of the reflection coefficient *R*(*t*) from above at sunset, 15:00–19:00 UT, for the week of measurements having begun with an absence of the proton precipitation (28 September) and having continued with solar proton precipitation for the following days. The continuous curves—is an analysis in the positive direction of time. The dashed curves—is an analysis in the opposite direction

the weight center for a solid. The variations of *h* indicate on the altitude variations of all electric conductivity altitude range which determines the VLF reflection effect.

Having gotten the values of the effective height at different sunset time with the SPP we represented in the Table [1](#page-8-0) the sunset effect on the electric ionospheric conductivity in a term of *h* for different stages (days) of the SPP. It is important to stress on the point that the values before and after the sunset (the values of *h* in the second and third columns) were determined independently on any geophysical suppositions about the electron profile in the ionosphere. Deviation from the monotony decreasing of the proton flux with the proton energy less than 50 MeV on 6 October, Fig. [5,](#page-4-0) declared itself at the deviation from monotone restoration of the  $h(t)$  and  $R(t)$ , as it is seen due to Figs. [8](#page-6-1) and [9](#page-7-0) and Table [1.](#page-8-0)

The most interesting result, as it seems to us, is a sunset effect for the effective height *h*(*t*) on 29 and 30 September when the proton fluxes were extreme. That means that although the SPP was one of the most powerful, the electric conductivity which appeared below 50 km did not isolate the upper ionospheric D-layer from the VLF penetration. We see that the radio waves penetrated at the altitudes where the electron concentration of ionosphere was controlled by the Sun zenith angle, were experienced the partial reflections from the altitude inhomogeneity, may be, not monotonous and came to the receiver creating the effect pointed.

In Table [2](#page-8-1) we represent the approximate correspondence between the values of *h* for different days of the EPP event and a minimal value of the proton energy *E* with which it, precipitating from above, may achieve the altitude *h* in the Earth's atmosphere according to the energy estimate [\[14\]](#page-12-13).

The last point of our analysis is a question about the accuracy of our results. As the VLF magnitude variations are significantly larger than the apparatus errors (the signal/noise ratio is larger than 10 and the phase error about  $0.5 \mu s$ ) the main troubles of our procedure were the following: (i) while our deterministic analysis from one time point to another point several approximations of the theory [\[2,](#page-12-1) [15,](#page-13-0) [16\]](#page-13-1) were used, (ii) practically there were none statistic data, (iii) the over fullness of the input experimental data (3 amplitude and 3 phase VLF input data against 4 unknown Sunset Variations of the Bottom Edge of the Ionosphere ... 327

<span id="page-8-0"></span>**Table 1** The values of effective attitude *h* for different dates at the case of fully lighted (day) and fully shadowed (night) radio path and the values of differences between the pointed values for the same date

Date	$h$ , km at day-time	$h$ , km at night-time	$h$ day-night variations, km	The notes
28.09.1989	63	76	13	Without the proton precip.
29.09.1989	48	52	6	The first day of <b>SPP</b>
30.09.1989	47	53	6	The second day of SPP
01.10.1989	52	63	11	The third day of <b>SPP</b>
02.10.1989	55	64	9	The fourth day of <b>SPP</b>
03.10.1989	55	67	12	The fifth day of <b>SPP</b>
04.10.1989	60	71	11	The sixth day of <b>SPP</b>
05.10.1989	57	74	17	The seventh day of SPP
06.10.1989	57	68	11	The eighth day of <b>SPP</b>

<span id="page-8-1"></span>**Table 2** The correspondence for different dates between the effective attitude *h* values and the proton energy *E* values



Such energy is sufficient for a proton penetration from above at altitude *h* according to [\[14\]](#page-12-13)

magnitudes— $\Delta h$ ,  $\Delta R$  for every time step  $\Delta t$  and unknown initial values  $h_0$ ,  $R_0$ ) was used for the  $h_0$  and  $R_0$  determination. The minimization of a discrepancy function relatively these two parameters gave a solution of the inverse VLF problem which had the physics sense. The quality and accuracy of such solutions are represented in Fig. [10—](#page-9-0)the amplitude data, and in Fig. [11—](#page-10-0)the phase data. The dates and the frequencies used are pointed on the figures. This comparison of the experimental data with the calculated ones according to the SIvlfP, Figs. [8](#page-6-1) and [9,](#page-7-0) is quite satisfactory. These results are the analogs of Fig. [2.](#page-2-0)

Our last remark relates to our induced supposition about homogeneity of the model waveguide at every time moment of a sunset. We may report that according



<span id="page-9-0"></span>Fig. 10 Comparison of experimental and calculated amplitudes for 3 frequencies and 3 dates: before, on the first and the 4th day of the proton precipitation (28 September, 1989; 29 September, 1989 and 2 October, 1989). The analysis was done for positive direction of time



<span id="page-10-0"></span>**Fig. 11** Comparison of experimental and calculated phases for 3 frequencies and 3 dates: before, on the first day and the 4th day of the proton precipitation (28 September, 1989; 29 September, 1989 and 2 October, 1989). The analysis was done for positive direction of time

to our calculations, Fig. [12,](#page-11-0) the differences between optical lengths for red curves is equal to  $0.2 \mu s$  and for blue curves is equal to  $0.4 \mu s$ . For this figure the radiuses for two cylinders were 6370 km and  $(6370 + 70)$  km; the distance between a source and a receiver—900 km and in the model of inhomogeneous waveguide (the dashed curves) the difference between the boundary heights—20 km. As the apparatus phase errors were about  $1 \mu s$ , then the pointed variations are not critical when the terms of ray summation are far enough from their anti phase state.



<span id="page-11-0"></span>**Fig. 12** A scheme of the 1st and 2nd hop trajectories for the homogeneous (continuous curves) and inhomogeneous (dashed curves) along the radio path waveguides

### **3 Discussion and Conclusions**

The dynamics of the effective height  $h(t)$  and the reflection coefficient  $R(t)$  at sunset for the auroral radio path were gotten for 8 days of proton precipitations. The minimal vale of  $h(t)$  was equal to 45–47 km. It is pleasant to note that although our analysis is not based on the impulse sounding of the ionosphere bottom (the disturbed D-layer), as it is in the case of Fig. [4](#page-3-0) [\[10\]](#page-12-9), and is "monochromatic" sounding, our new results do not contradict to the old ones and they are supplemental to each other. We have used the "sounding" with 3 radio signal frequencies for the (10–14) kHz range for the "homogeneous" auroral West-East radio path for every time, with magnetic latitude about 64° and moderate length (885 km); for these radio path parameters, as it is evident from Figs. [6](#page-5-0) and [7](#page-6-0) the VLF signal variations for different frequencies, are not similar. An absence of similarity is due to interference between 3 terms, Fig. [12.](#page-11-0) The presence of the second sky wave (the second ray or "hop") with two radio wave reflections from above at every frequency signal increases the solution capability of the inverse VLF problem. Returning to the comparison of the Helm's electron profile results, Fig. [4,](#page-3-0) and our ones, Figs. [8](#page-6-1) and [9](#page-7-0) we see that they having the same physics nature but referring to the different dates (1978 and 1989 years) and to the radio paths sunlit differently. In one case the conditions were stationary, in the other case there was a sunset.

It was shown several year decades ago  $[17]$  that the effective height  $h(t)$  was inside the significant altitude range of the ionosphere relative to a concrete VLF magnitude. For example, if we demand that the accuracy of our calculations have to be not worthier than the accuracy of the experiment than the altitude width of the range is equal to 6, 4 and 5 km for the frequencies 10, 15 and 20 kHz correspondently (for a definite daytime profile  $N_e(z)$ ) [\[18\]](#page-13-3). According to Fig. [4](#page-3-0) the significant altitude range of the profiles *Ne*(*z*) represented is 5–7 km.

The sunset effect was seen at every day, although the proton precipitation on 29 September, 1989 was one of the strongest between the known ones. That means that (i) the bottom part of disturbed D-layer generated by the proton precipitation did not isolate the altitudes of regular D-layer for the VLF waves propagating at them from the ground and (ii) that the significant altitude range in the case was at several times greater than in the undisturbed conditions. The future analysis of the dawn time may be support these statements.

An increase of proton flux with the energies  $(1–50)$  MeV on 6 October significantly influenced the VLF data. The effective height  $h(t)$  changed from 57–73 km on 5 October to 60–68 km on next day, Fig. [8.](#page-6-1) The *h*(*t*) data on 3 and 6 October are practically identical.

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