



# Examination of the KAWAI CLD Method for Fatigue Life Prediction of Composites

Yael Buimovich<sup>(✉)</sup> and Dvir Elmalich

Fatigue and Damage Tolerance Department, Aviation Group,  
Israel Aerospace Industries, Ben-Gurion International Airport,  
70100 Lod, Israel  
ymotola@iaai.co.il

**Abstract.** The Kawai Constant Life Diagram (CLD) method was examined for fatigue life prediction of composite materials. Static and fatigue tests were carried out for open-hole coupon specimens made of unidirectional carbon/epoxy tapes for examining the applicability of the KAWAI modified constant life model. The Goodman method was examined as well. The prediction of the Kawai model was slightly conservative for R-ratios  $\geq -1$  and un-conservative for R-ratio that equals 10. The Goodman CLD highly overestimated fatigue life for purely tension and tension-compression areas and underestimated fatigue life for R-ratios lower than  $R = -1$ . A modified Kawai model was suggested to overcome the un-conservatism of the Kawai model at the pure compression zone of the CLD.

**Keywords:** Fatigue · Composites · Constant life diagram

## 1 Introduction

The use of advanced composite materials in aircraft primary structures has been significantly increased over the past 30 years. Modern large commercial aircraft are designed with more than 50% composite materials, due to the demand for fuel-efficient, light-weight, and high-stiffness structures that have fatigue durability and corrosion resistance (Raiter 2013; FAA Doc: DOT/FAA/AR-10/6 2011; Rouchon 2009). Furthermore, a comprehensive part of the UAV structures worldwide is made of composite materials.

The main lesson learned from the 30 years of experience in service civil aircraft shows that fatigue aspect is not crucial at the design stage of composite aeronautical structures, due to the fact that their static margins (allowable versus aircraft static load) are usually lower than their fatigue margins (allowable versus aircraft fatigue load), i.e., aircraft fatigue loads were demonstrated to be below the fatigue sensitive threshold of the composite materials (Raiter 2013).

Nevertheless, it is possible that the static margins of a composite component will be higher than the fatigue margins for severe profile of aircraft; for example, a combat UAV, built for firing missiles. It may be emphasized that the UAV market and combat UAV market in particular is expected to be greatly increased in the near future (Forecast International, Analysis 1, 2013).

It is well known that aircraft structures are subjected to complex fatigue loading that accompanies changes in the amplitude, mean, frequency and waveform of stress

cycling during service. Apparently, a large number of fatigue experiments under different kinds of cyclic loading conditions is required to elucidate the effect of loading mode on the composites sensitivity to fatigue; thus, this consumes considerable time and cost. Therefore, a time and cost-saving procedure for identifying the loading mode dependence of composites fatigue strengths is required. This procedure should assure reasonable accuracy on the basis of a minimal amount of test data.

It has been shown that the use of the linear Goodman curve for composites does not reflect the actual behavior of the material (Boerstra 2007; Harris 2003; Philippidis and Vassilopoulos 2004; Kawai and Koizumi 2007; Kawai and Teranuma 2012). Hence, several methods were suggested for fatigue life prediction of composite materials over the past years (Boerstra 2007; Harris 2003; Philippidis and Vassilopoulos 2004; Kawai and Koizumi 2007; Kawai and Teranuma 2012). Philippidis and Vassilopoulos (2004) presented a piecewise linear CFL diagrams for GFRP laminates. In this work, several R-ratios are considered. At each R-ratio the CFL values are determined from the S-N curve. The number of cycles at every two consecutive values or R ratio is connected with straight lines. Harris (2003) developed semi-empirical equation based on fatigue test data for carbon fibers reinforced plastics (CFRP) and glass fibers reinforced plastics (GFRP) laminates. The equation includes several parameters that have to be calibrated based on empirical results of at least three R ratios. Boerstra (2007) proposed alternative formulation that uses random fatigue data in which the R ratio is not a parameter in the model. The model includes five parameters that must be defined from a multi-objective optimization. Boerstra (2007) did not specify the extent of the experimental data that is required or the effect of the data base on the prediction of the CFL diagrams. The studies mentioned above (Boerstra 2007; Harris 2003; Philippidis and Vassilopoulos 2004) have one major disadvantage due to the relatively vast data that is required, which consumes a great amount of tests. Kawai and Koizumi (2007) and Kawai and Teranuma (2012) proposed a nonlinear CFL diagrams for several laminations that are based on one critical R-ratio and demonstrated this model on several layouts of carbon/epoxy laminates. In this method, fatigue tests are carried out at one critical R-ratio, defined as the ratio of the Ultimate Compression Strength (UCS) over Ultimate Tensile Strength (UTS) of the material. This critical ratio is called  $\chi$ . This method is a time and cost-saving procedure that provides reliable data.

In light of the written above, the Kawai CLD was examined in IAI for fatigue life prediction of composites. The Goodman method was examined as well.

## 2 Kawai CFL Model for Fatigue Life Prediction

Kawai and his co-workers (Kawai and Koizumi 2007). Kawai and Teranuma (2012) developed an asymmetric constant life diagram, designated the an-isomorphic constant fatigue life (CFL) diagram for CFRP materials. The main feature of this formulation is that it can be constructed by using only one experimentally derived S-N curve, which is called the critical S-N curve. The R-ratio of this S-N curve is defined as the ratio of the UCS over UTS of the material. The formulation is based on three main assumptions:

1. The stress amplitude for a given constant value of fatigue life is greatest at the critical stress ratio.

2. The shape of the CFL curves changes progressively from a straight line to a parabola with increasing fatigue life.
3. The diagram is bounded by the static failure envelope that consists of two straight lines connecting the peak point on the critical straight line with the UTS and UCS, respectively.

The CFL formulation depends on the position of the mean stress,  $\sigma_m$ , in the domain  $[\sigma_C, \sigma_T]$  as follows

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \left\{ \begin{array}{ll} \left( \frac{\sigma_m - \sigma_m^\chi}{UTS - \sigma_m^\chi} \right)^{2-\psi_\chi} & UTS \geq \sigma_m \geq \sigma_m^\chi \\ \left( \frac{\sigma_m - \sigma_m^\chi}{UCS - \sigma_m^\chi} \right)^{2-\psi_\chi} & UCS \leq \sigma_m \leq \sigma_m^\chi \end{array} \right\} \quad (1)$$

where,  $\sigma_a^\chi$  and  $\sigma_m^\chi$  are the alternating and mean stress components of the fatigue stress for a given constant value of life,  $N$ , under fatigue loading at the critical stress ratio,  $\chi = UCS/UTS$ . The variable  $\psi_\chi$  denotes the fatigue strength ratio and it is defined as

$$\psi_\chi = \frac{\sigma_{max}^\chi}{\sigma_b} \quad (2)$$

where,  $\sigma_b$  ( $>0$ ) is the reference strength to define the peak of the static failure envelope in the  $(\sigma_m - \sigma_a)$  plane and  $\sigma_{max}^\chi$  is the maximum stress extracted from the S-N curve of the critical R-ratio,  $\chi$ , for a given number of fatigue cycles. Therefore, this normalization guarantees that  $\psi_\chi$  always varies in the range  $[0, 1]$  and the exponents  $(2-\psi_\chi)$  in Eq. (1) are always greater than unity. Subsequently, linear ( $2-\psi_\chi = 1$ ) or parabolic ( $2-\psi_\chi > 1$ ) curves can be obtained from Eq. (1). The critical fatigue strength ratio represents the normalized cyclic stress, and its relation to the number of loading cycles defines the normalized critical S-N curve:

$$2N_f = \frac{2}{K^*} \frac{(1 - \psi_\chi)^a}{\psi_\chi^n} \quad (3)$$

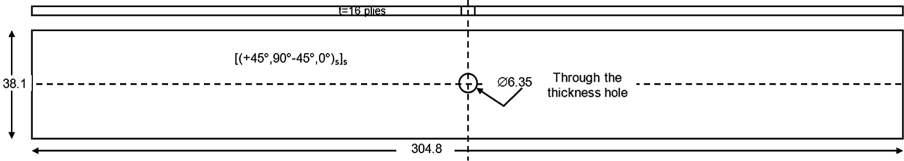
where the constants  $a$ ,  $n$  and  $K^*$  are material constants determined iteratively. The relation between the normalized stress and the number of loading cycles presented in Kawai et al. and shown in Eq. 3 is given implicitly. For most sets of  $a$ ,  $n$  and  $K^*$  the normalized stress cannot be extracted directly from Eq. (3). In these cases, the use of Eq. (2) is more convenient for the determination of  $\psi_\chi$ .

After determining the critical S-N curve by fitting to the available fatigue data, the CFL diagram can be constructed based on the static strengths, UTS and UCS, and the reference S-N relationship.

### 3 Test Description

Static and fatigue tests were carried out for open-hole coupon specimens made of unidirectional carbon/epoxy tapes for examination of the applicability of the KAWAI constant life model. A quasi-isotropic lay-up of intermediate modulus unidirectional

(UD) Carbon/epoxy tapes was examined. The lamination sequence,  $[(+45^\circ, 90^\circ, -45^\circ, 0^\circ)_s]_s$ , is balanced and symmetrically stacked. The geometry of the specimens was in accordance to ASTM standard for open-hole tests (ASTM D5766, ASTM D6484). The specimen is illustrated in Fig. 1.



**Fig. 1.** IM carbon UD/ER450 open-hole tension and compression specimen for quasi-isotropic lamination

A total of 56 composite specimens were tested. The tests included static compression and tension strength to obtain the critical R ratio. Fatigue tests included five R ratios; i.e., 0.5, 0.1,  $\chi$ , -1 and -10. Most R-levels consisted of five levels of stresses (with two tests at each stress level). A picture of the tested specimen at the loading apparatus is given in Fig. 2.



**Fig. 2.** The tested specimen at the loading apparatus

### 4 Analysis of Test Results

The critical R-ratio obtained from the static tests is  $\chi = -0.5$ . Hence, the Kawai CLD was built according to fatigue tests results for  $R = \chi$ . The predicted Kawai CFL and test results of fatigue tests are shown in Fig. 3. Note that the values of the mean and amplitude stress in Fig. 3 are normalized with respect to the reference stress  $\sigma_b$ . A relatively good agreement was obtained between the predicted and experimental results, with the exception of  $R = -10$ . The prediction of the Kawai model was slightly conservative for R-ratios  $\geq -1$  and un-conservative for R-ratio that equals 10.

The Goodman CLD was also compared to test results and is illustrated in Fig. 4. Note that the values of the mean and amplitude stress in Fig. 4 are normalized with respect to the reference stress  $\sigma_b$ . It may be observed in Fig. 4, that the Goodman CLD highly overestimated fatigue life for purely tension and tension-compression areas and underestimated fatigue life for R-ratios lower than  $R = -1$ .

SN curves were built for each R-ratio for analyzing the precision of the Kawai and Goodman constant life diagrams. The SN curves built for  $R = 0.1, 0.5, -0.5$ , and  $-1$  are given in Fig. 5 through Fig. 8, respectively, in terms of the normalized maximum stress. The SN curve built for  $R = 10$  is given in Fig. 9 in terms of the normalized minimum stress. In the curves at Fig. 5 through Fig. 9, both Kawai and Goodman curves are shown, as well as the experimental results for the R-ratio. Note that in Fig. 8, experimental results are compared only to the Kawai curve, since the Goodman curve is built according to test results of  $R = -1$ . In Fig. 7, experimental results are compared

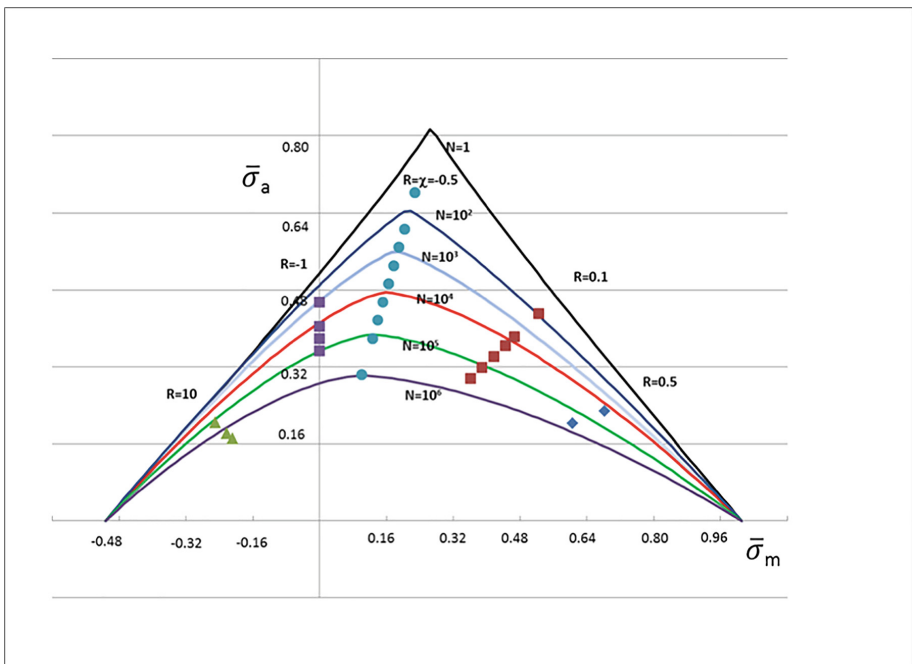


Fig. 3. Kawai Constant Fatigue Life Diagram for unidirectional carbon/epoxy laminates

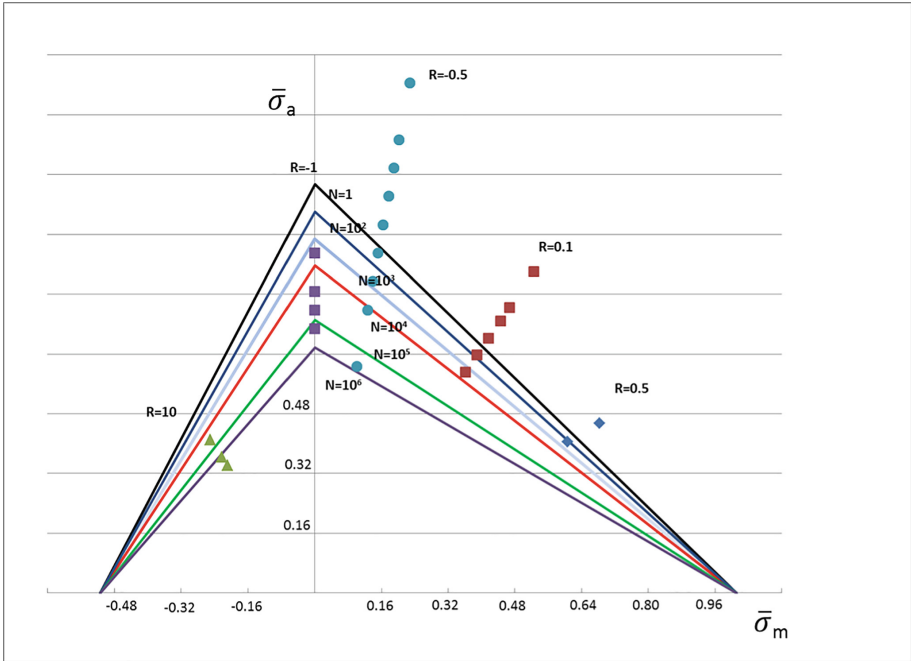


Fig. 4. Goodman Constant Fatigue Life Diagram for unidirectional carbon/epoxy laminates

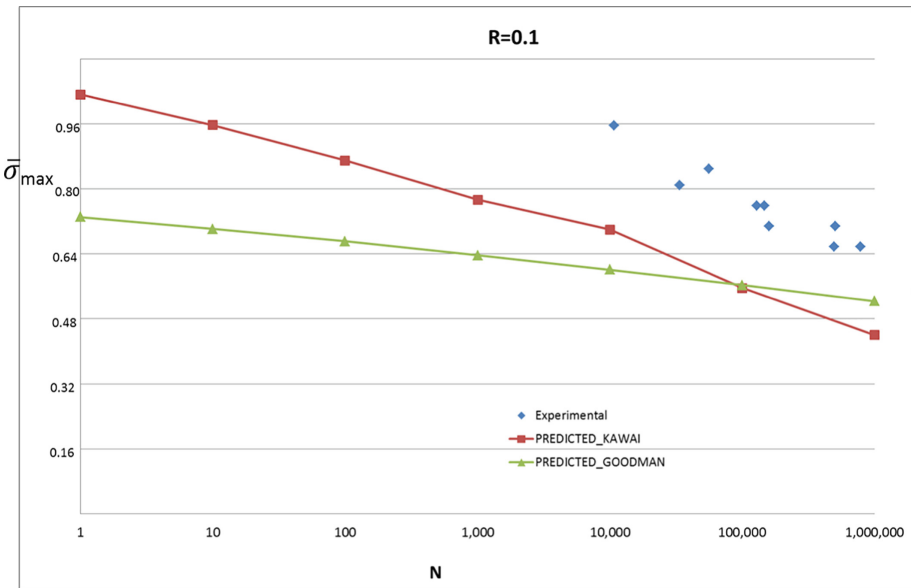


Fig. 5. SN curve for R = 0.1 for unidirectional carbon/epoxy laminates

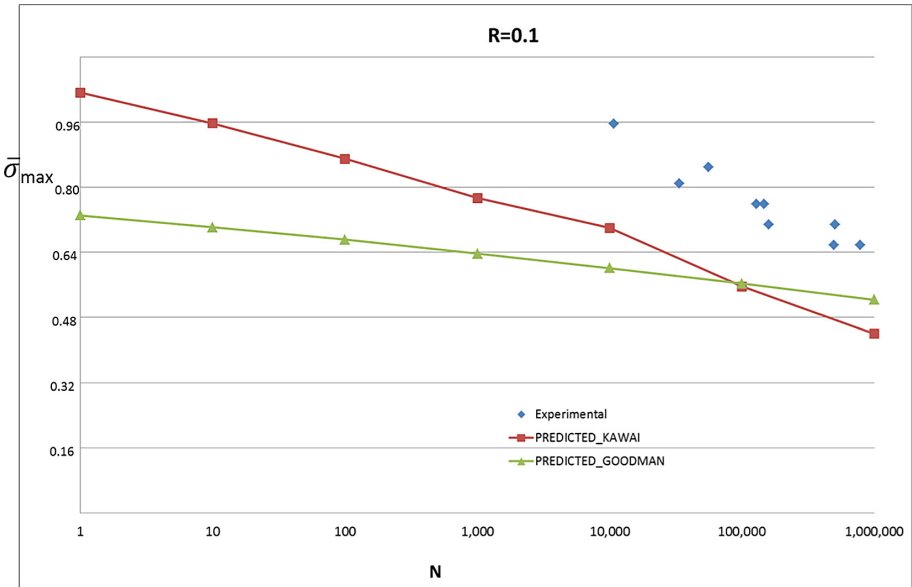


Fig. 6. SN curve for R = 0.5 for unidirectional carbon/epoxy laminates

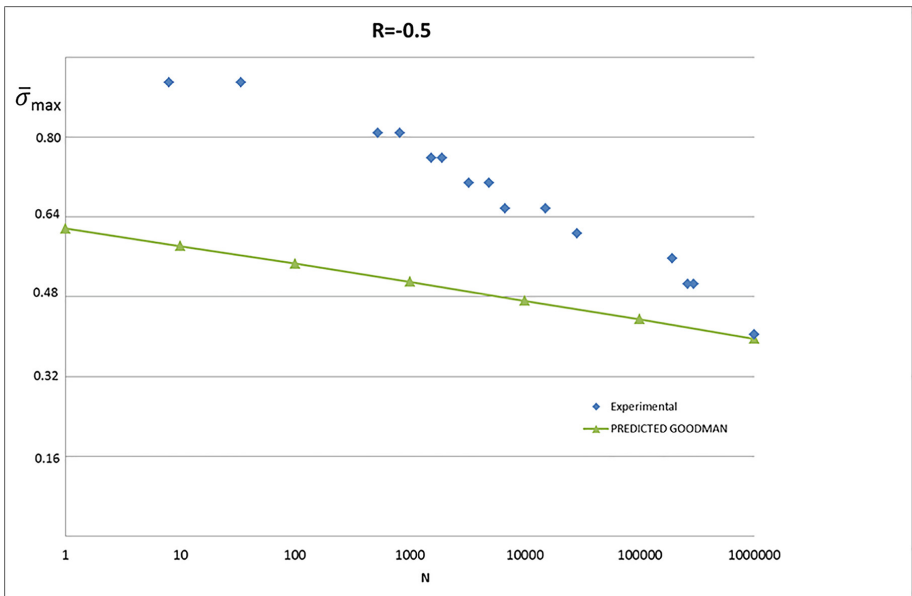
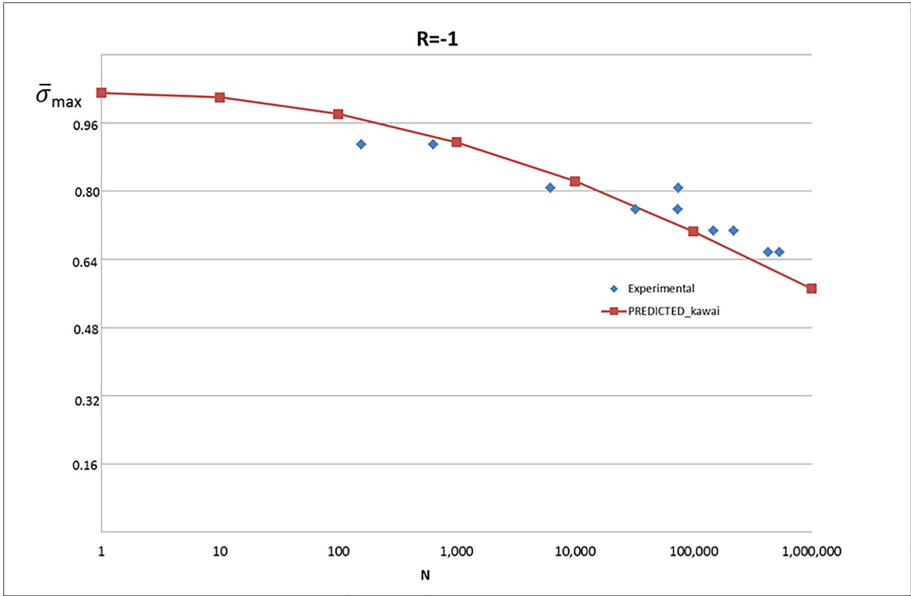
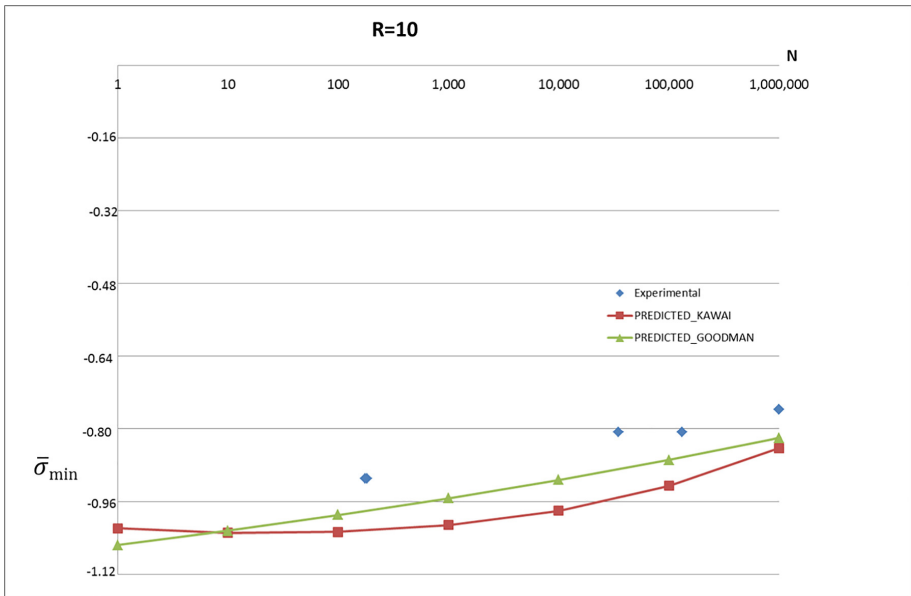


Fig. 7. SN curve for R = -0.5 for unidirectional carbon/epoxy laminates

only to the Goodman curve, since the Kawai curve is built according to test results of R = -0.5 ( $\chi$ ). It may be obtained in Figs. 5 and 6 that both SN curves are below the experimental results. However, the Goodman curve is much more conservative than the



**Fig. 8.** SN curve for R = -1 for unidirectional carbon/epoxy laminates



**Fig. 9.** SN curve for R = 10 for unidirectional carbon/epoxy laminates

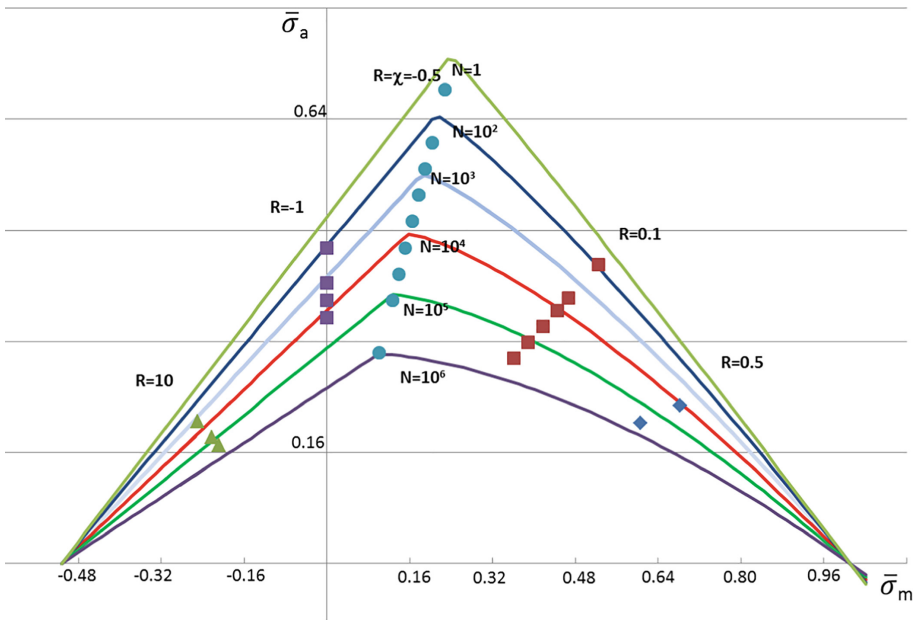


Kawai method in most of the cases. The great conservatism of the Goodman model is also observed in Fig. 7. It may be seen in Fig. 8, that there is a relatively good agreement between the experimental results and the curve built according to the Kawai method for  $R = -1$ . As opposed to the conservative trend of the Kawai method for the pure tension zone and the good agreement obtained for  $R = -1$ , the Kawai SN curve for  $R = 10$  is below the experimental results and hence yields un-conservative prediction (see Fig. 9). It may also be seen in Fig. 9 that the Goodman curve is un-conservative, as compared to the experimental results.

### 5 Modified KAWAI CLD

A modified model was suggested to overcome the un-conservatism of the Kawai model at the pure compression zone. Thus, the constant life curve is built from two parts; i.e. for  $UTS \geq \sigma_m \geq \sigma_m^L$ , constant life diagrams are built according to KAWAI method and for  $UCS \leq \sigma_m \leq \sigma_m^L$ , constant life diagrams are built by a linear line between  $\sigma_a^L$  and UCS. Hence, CFL formulation is determined as follows

$$\begin{aligned}
 -\frac{\sigma_a - \sigma_a^L}{\sigma_a^L} &= \left( \frac{\sigma_m - \sigma_m^L}{UTS - \sigma_m^L} \right)^{2-\psi_\chi} & UTS \geq \sigma_m \geq \sigma_m^L \\
 \sigma_a &= \frac{(\sigma_m - \sigma_c) \cdot \sigma_a^L}{\sigma_m^L - \sigma_c} & UCS \leq \sigma_m \leq \sigma_m^L
 \end{aligned}
 \tag{4}$$



**Fig. 10.** Modified Kawai Constant Fatigue Life Diagram for unidirectional carbon/epoxy laminates

The predicted modified Kawai CFL and test results of fatigue tests are shown in Fig. 3. Note that the values of the mean and amplitude stress in Fig. 10 are normalized with respect to the reference stress  $\sigma_b$ . One may observe in Fig. 10 that the experimental results of  $R = 10$  are within the range of the CLD, as opposed to these values in Fig. 3.

SN curve was built for  $R = 10$ , based on the modified Kawai methodology. It is presented in Fig. 11. It may be observed in Fig. 11 that the experimental results are slightly below the predicted curve according to the modified Kawai method for fatigue life above  $10^5$  cycles.

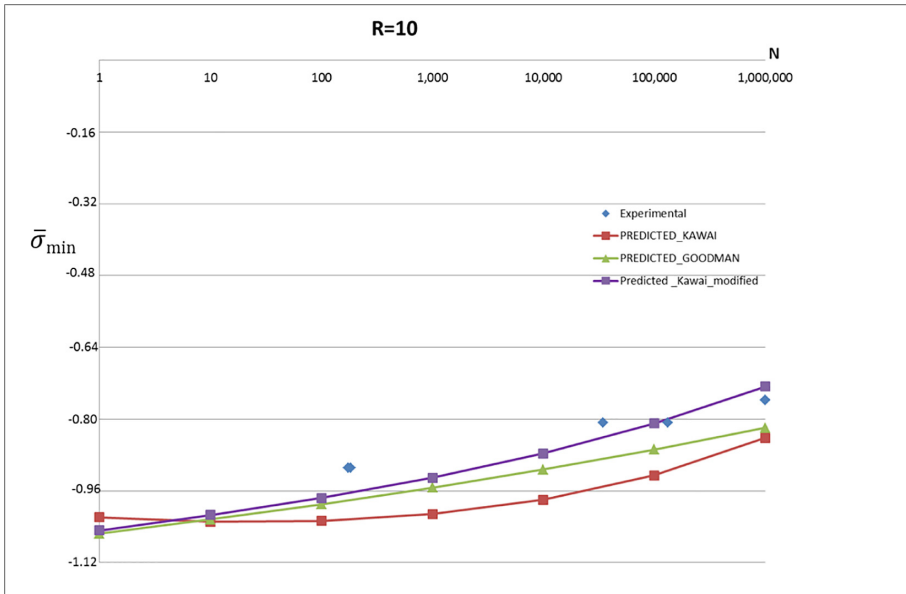


Fig. 11. SN curve for  $R = 10$  for unidirectional carbon/epoxy laminates

## 6 Summary and Conclusions

The applicability of KAWAI CLD method for fatigue life prediction of composites was examined. The Goodman methodology was examined as well. Static tests were carried out first for open-hole coupon specimens made of unidirectional carbon/epoxy tapes for obtaining the critical R-ratio,  $\chi = UCS/UTS$ . This value was determined as  $\chi = -0.5$ . Fatigue tests were conducted at the critical R-ratio for modeling the constant life diagrams, per KAWAI method. In addition, fatigue tests were carried out at different R-ratios for examining the Kawai model.

The prediction of the Kawai model was slightly conservative for R-ratios  $\geq -1$  and un-conservative for R-ratio that equals 10. Whereas, the Goodman CLD highly overestimated fatigue life for purely tension and tension-compression areas and underestimated fatigue life for R-ratios lower than  $R = -1$ .

A modified model was suggested to overcome the un-conservatism of the Kawai model at the pure compression zone. Thus, the constant life curve is built from two parts; i.e. for  $UTS \geq \sigma_m \geq \sigma_m^\chi$ , constant life diagrams are built according to KAWAI method and for  $UCS \leq \sigma_m \leq \sigma_m^\chi$ , constant life diagrams are built by a linear line between  $\sigma_a^\chi$  and UCS.

The predicted curve of the suggested model was compared to the experimental results and was shown as conservative for fatigue life greater than  $10^5$ . Hence, with application of the appropriate scatter factor, the use of the modified Kawai model is considered relatively accurate.

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