ComMute—Towards a Computational Musical Theory of Everything

Guerino Mazzola(✉)

School of Music, University of Minnesota,
2106 Fourth Street South, Minneapolis, MN 55455, USA
mazzola@umn.edu
http://www.encyclospace.org

Abstract. This paper draws future perspectives of music as a comprising cultural achievement of humans. We discuss the role of music for mathematics and physics from Pythagoras to String Theory, its global human presence, transcending specific fields of knowledge in its synthetical force that unifies distant fields of knowledge and action in the concrete and abstract realms.

Keywords: Theory of Everything · Future music theory · String theory

1 The Physical Theory of Everything (ToE)

Before we discuss the idea of a musical Theory of Everything (ToE), we recall the physical ToE and its motivations. This theory claims the integration of the four fundamental physical force types: electromagnetic, weak, strong, and gravitation. The electromagnetic and weak are already united, and called "electro-weak". The still hypothetical integration of the electro-weak force with the strong force is called GUT: Great Unification Theory. Integration means that all these forces are special cases of a fundamental force, which splits into the four forces by a breaking of structural symmetries when energies are below a threshold.

Let us stress that even the reduction to those four forces is everything but evident. For example, would you guess that the mechanical forces of, say, a hammer hitting a nail, are the same type as the forces of adhesion of a glue? Or the forces of chemical reactions? Or the force of sunlight tanning your skin? They all pertain to the electromagnetic force type. The physical sciences and their outlets in chemistry have achieved an incredible reduction of the apparent variety of force types. This means that the surface of physical actions does not prevent a deep theory from unifying superficial diversities. We should keep in mind this fact when stepping over to the musical realm.

It is remarkable that "Everything" in physics relates to physical forces, but not to psychological or symbolic realities. This restriction is significant since it avoids any physicalist totalitarism. Physics does not claim a total explanation

M. Montiel et al. (Eds.): MCM 2019, LNAI 11502, pp. 21–30, 2019.
https://doi.org/10.1007/978-3-030-21392-3_2

of this world, physics deals with the outer nature and has never tried to reduce psychological or symbolic realities to physics. Quite the opposite: Prominent theoretical physicists, such as Roger Penrose (Fig. 1), argue that the innermost physical ontology might rely on mathematics. He also argues in [15] that physics has not yet included the psychological realm of the human mind in its basic conceptual architecture.

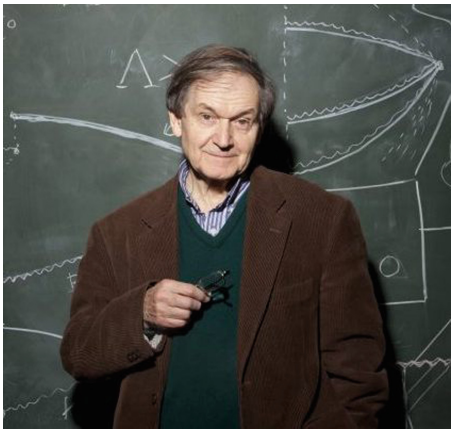

Fig. 1. Theoretical physicist Roger Penrose.

Despite the success of simplifying physical force types to three (electro-weak, strong, and gravitation), it is not clear why the ToE and even the weaker GUT should work. Physicists seem to believe in an ultimate unification, probably because of some monotheist paradigm: There is only one innermost, well yes: divine, entity that shapes the universe. The success gives them enough motivation to work in this direction with an impressive shared social, organizational,[1] and economic effort called "Big Science", see [4].

2 Why Would We Think About a Musical ToE?

In view of the fundamental role of music in all cultures,[2] it is not astonishing to think of a unified view of music in the spirit of the physical ToE:

Could it be that the variety of musical expressivity is the unfolding of a unique fundamental "force field"?

Before we delve into this hypothesis of musical unification, we should understand that the wording ComMute, meaning *Computational Music Theory of*

[1] The Internet was invented by Tim Berners-Lee at the CERN to coordinate nuclear research efforts globally.

[2] Even where music is virtually forbidden, with the Taliban, for example, its force is recognized, and that is why it is forbidden, sad irony.

Everything, would not, similar to ToE, include strictly everything. This is also why we add the adjective "computational". The idea is to think of a music theory that is computational; other theories might exist, but they are not addressed here.

Moreover, it is also, similar to ToE, not intended to subsume all realities, the physical, psychological, or symbolic. Nevertheless, recall that the musical idea has been a driving force in the development of physics and astronomy from Pythagoras to String Theory. It is not evident in how far a ComMute would connect to ToE, but one should keep in mind these deep relationships, especially when a composer wants to justify his/her overall motivation.

The physical ToE is a precise hypothesis, the unification of all physical force types, independently of how this would work. For a ComMute the analogy to physics is problematic. Music does not share the simple idea of forces which are embodied by quanta.[3] However, the core idea of Mazzola's modulation model (see Fig. 2) was an exact analogy to physical force quanta, musical symmetries being the analogy of forces, while modulation quanta (sets of pitches) are the analogy of physical quanta. But this is a special situation that cannot for the time being be generalized to general modulations.

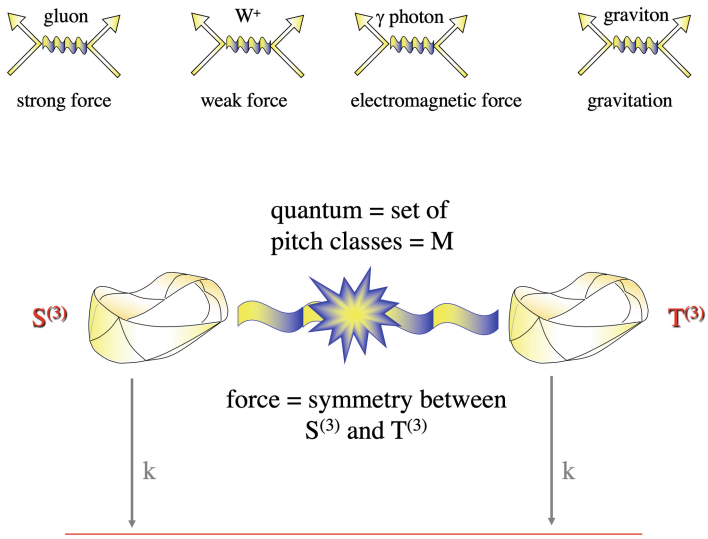

Fig. 2. Physical quanta and musical quantum M for a modulation from tonality S to tonality T; k is the cadence.

Another type of forces was used in the mathematical model of counterpoint (see [2]). Here, the dichotomy of consonances and dissonance was deformed by a symmetry in the role of an elastic force acting on the set of intervals.

[3] These are photons for the electromagnetic force, W and Z bosons for the weak force, gluons for strong force, and hypothetical gravitons for gravitation.

The idea of forces in music (theory) is not new, see [7]. Already Schoenberg used the metaphor of erotic (!) forces to explain harmonic tension in [17]. But there is no such universally acclaimed theoretical architecture as in physics, forces may show up (metaphorically or litteraly), but music has a number of very different structural paradigms, such as local-global duality, geometric ideas (the harmonic Moebius strip, for example), or gestural and topological approaches.

What are arguments against such a strong ComMute hypothesis? To begin with, the individual creativity of a musical composer or improviser seems to forbid any "universalist" background. This is the heritage of the Renaissance movement, which opposes to the Pythagorean "world formula" in an irreducible individual genealogy. This is however not stringent since a painter, for example, may create a deeply individual work of art with colors being completely described by the electromagnetic force that defines light and its action within the human eye. And on a higher level of structural abstraction, the variety of musical transformations can be described by a huge mathematical group, typically ranging within the cardinality[4] of 10^{40}. This is all comprised within a very clear and unified conceptual architecture of theory, but such a virtually infinite number guarantees an unlimited variety of individual utterances.

Another argument against ComMute would be the suspicion that this hypothesis is a consequence of the Western (Christian) colonialist mentality. Are we trying to unite all musical cultures under a big equalizing umbrella? And thereby destroying unsurmountable differences? This is a delicate question since we already have the example of a destructive reduction when transcribing Arab Maqam music to Western notation—all the essential pitchbend effects are eliminated. Other examples of the same type are abundant. But such a translational pathology may be eliminated by a more diligent conceptualization. For example, the language of denotators as described in [8, Ch. 6] can describe musical objects much better than the traditional Western score. This means that this question could be answered by extending the given language to a state where some differences would be taken care of.

This question is delicate because the argument of an extended language seems to be purely formal, it would not touch the cultural differences of the role of music. For example, the African social role of music is radically different from the Western role, and also different from the Indian role of Raga music, say. This aspect relates to music sociology or psychology. We do not include these aspects in the hypothesis of ComMute. At the time being such an inclusion seems too ambitious and also dangerous for the named reasons. But we shall see below in Sect. 3.3 that on the level of music theory, relations between European counterpoint and Raga music are appearing. Perhaps should one also reflect upon the idea of "fundamentally incomparable cultures" in view of a separation and

[4] The number of affine transformations on the local score of the software [8, Ch. 49] is

$$10'445'260'466'832'483'579'436'191'905'936'640'000 \approx 1.04453 \times 10^{37}.$$

even the famous "clash of civilizations". On the level of human rights the vast majority of cultures have agreed on a shared canon, such as the *habeas corpus* principle. In the movie *Teak Leaves at the Temples* [14], Mazzola argued that the language of gestures might be an approach to a non-divisional understanding of musical cultures.

Let us terminate this introduction with a discussion of the specification "computational" in ComMute. This means two things. First, such a music theory should be covering all that is accessible by mathematical methods and concepts. Second, it should be accessible via computational engines such as computers. This is a restriction of "everything" to "computational everything". It has the same function as in physics: What is not accessible in this way is not physically relevant. It is simply an expression of modesty, one only considers topics that are 'visible' to computation. Of course, as musical research progresses, more and more things may become 'visible'. But it is also risky to restrict one's views to what is actually computable. For example, Stephan Hawking claimed that the concept of a God is superfluous in physics. He did not consider the language and spirit of mathematics as being a *conditio sine qua non* for physics. The computational principle is pre-physical, and Hawking forgot to ask where we get that language from. Radical neuroscientists might argue that mathematics is an artifact of neurons, i.e., a product of physical reality. But any proof there of would use huge mathematical tools, which creates a circular argument, it would not explain anything.

In this sense, ComMute is a methodological limitation and should not be taken as a definition of music, but as a perspective, which can be tested and used to shape concrete progress. Nevertheless, the hypothesis is a very strong one, similar to ToE, or even more tricky because the conceptual landscape in music is less unified than in physics.

3 Some Directions Towards ComMute

In the following sections we want to describe a number of vectors towards such a ComMute, without claiming completeness.

3.1 Harmony and Rhythm

Harmony and rhythm have played very different roles in the history and cultural diversity of music. For example, recent research [13] stresses the fundamental difference in harmony and rhythm between classical European and African music. The complexity of Western harmony corresponds to the complexity of African polyrhythms, and vice versa: the simplicity of Western rhythms corresponds to the simplicity of African harmony. This difference however does not mean that rhythm cannot be dealt with like harmony. In fact, both phenomena deal with periodic sets of events, octave periodicity in harmony vs. time periodicity of rhythm. In mathematical music theory, Mazzola's modulation model is not limited to pitch, but can equally be applied to time, just rotate pitch by 90 degrees

into time. This has been used for rhythmic modulation in the first movement of the composition *Synthesis*, but see [8, Ch. 50.2].

This double periodicity has also been used to investigate periodic structures in pitch or time using the finite Fourier decomposition of periodic functions. The classical theory of Fourier for time functions is a classical theory (partials, Fast Fourier Transform, etc.), but the (octave-)periodic pitch functions have been analyzed only recently by David Lewin, Ian Quinn, and Emmanuel Amiot [1]. This research proves that harmony and rhythm could converge to a unified theory of periodic functions in a number of parameter spaces. And such a unification could eventually create a less diversified perspective of Western and African musical cultures. Of course, the difference between music sociology of Europe and Africa remains untouched: The African music culture is shared by everybody, from childhood to adult life, Africans are musicians, whereas the European musician as a specifically educated person doesn't play that African standard role. This difference may vanish in the future if we learn to think in pitch and time according to the same theoretical and compositional paradigms.

3.2 Gestures for Harmony and Counterpoint

In Mazzola's mathematical music theory, modulation theory and counterpoint were developed using symmetries on pitch class spaces. This was not a topological approach and had its limits. For modulation theory the paring of tonalities was only permitted for tonalities in the same orbit of a symmetry group. Modulation from a major tonality to a gipsy tonality or even a pentatonic one was not conceived. A similar restriction happened to contrapuntal concepts. This model used a finite number of consonances selected by specific symmetries. It would crash if we had to consider an infinity of consonances in a continuous and therefore infinite interval model of counterpoint.

Both restrictions could be solved by a shared new language: musical gestures. Modulation could be remodeled as a gestural deformation of chords instead of a symmetric action [10]. Such a deformation does no longer require the two tonalities living in the same symmetry orbit. Using gestures and their homology theory, counterpoint could be remodeled independently of the number of consonant intervals at stake [2, Ch. 10].

This is an example of gesture theory being a unifier of music theory, an interesting parallel to physics, where string theory is a strong candidate for ToE. Strings are analogues to gestures in music theory, in other words: gestures are the musical analog to strings.

3.3 Counterpoint Worlds for Different Musical Cultures

The mathematical theory of counterpoint has not only embraced Fux tradition, but also opened with its five new worlds connections to Raga music and Scriabin's mystic chord. This theory is on its way to a global counterpoint theory. Connecting counterpoint models with Indian music is a sensational bridge between totally different musical cultures. We should recall here that connections between

Western and Raga music have been investigated by Robert Morris and Chitravina N. Ravikiran under the title of "Melharmony" [16]. This theory[5] "aims to create chords and counterpoints based on the melodic rules of evolved systems across the world." It could happen that our counterpoint worlds and melharmony converge to a new synthesis of two strong theoretical traditions.

3.4 Complex Time for Unification of Mental and Physical Realities in Music

The recent revolution of the time concept in physics was introduced by Stephen Hawking (among others) in order to solve singularity problems of the Big Bang model of the evolution of our universe in the initial moment some 13.8 billion years ago. Hawking's concept of time switches from the real time axis to the plane of complex numbers: Time now has two real coordinates $t = t_{Real} + i.t_{Im}$, it pronounces the real time t_{Real} and the imaginary time t_{Im}. This complex ontology has also been proposed and studied by physicists Bars and Terning [3]. This concept unifies Descartes' *res extensa*, the physical reality, with real time t_{Real}, with *res cogitans*, the thinking, mental reality, with imaginary time t_{Im}. This is a strong step towards a unification of a well-known duality in music: thinking and making, the physical utterance of performance vs. the mental construction in composition, performance, and improvisation. In [12, Ch. 78] were sketched ways of structurally connecting these two realities by means of world sheets, which are completely analogous to world sheets in physical string theory.

The duality of thinking and making in music has been a dividing force between theorists and performers in the Western music world. It is also present in the academic structure where "applied" scholars (teaching instrumental fields) are separated from "academic" scholars (music theory, musicology, education, ethnomusicology, music psychology). Their interaction is reduced to a poor "laissez vivre" and is not based on a shared reality, a fact that complex time theories might eliminate in the future.

3.5 Symbolic and Real Gestures

The idea of complex time has also been applied to connect symbolic gestures on the score to real gestures of the performing artist, see [12, Ch. 78]. This result fosters the unification of thinking and making. Eventually, this should help unify these two aspects of the art of music to a comprising reality.

3.6 Unifying Note Performance and Gestural Performance: Lie Operators

Computational performance theory was strongly developed by the Stockholm group around Sundberg [5]. This approach was concerned with the transformation of notes to sound events. It could be shown by the work of the Zurich group

[5] Citation form Wikipedia.

of Mazzola [11, Ch. 39.7] that important cases of such a performative transformation can be described by classical Lie operators from differential geometry. More precisely, the transition from a given performance stage to a more refined one is described by a Lie operator acting on the given performance vector field. When performance theory was extended to gestural performance, i.e., the transformation of symbolic gestures in the score to physical gestures of a musician, it could be shown that the same Lie operator formalism can be carried over to gestures [12, Ch. 78.2.13]. In other words, *there is now a unified formalism of performance theory for notes and for gestures.*

3.7 Unifying Composition and Improvisation?

It is an open question in how far composition and improvisation could be unified as special cases of a unique still hidden dynamics. We know of many famous composers, such as Beethoven or Mozart, that their compositions were often created from improvisation, see the genesis of Beethoven's Sonata Op. 109 [6], for example. In the Indian tradition of Raga, improvisation and composition are intimately related on the basis of mela scales. Perhaps the combination of flow concepts and gestures, as sketched in the free jazz book [9], could help find a unified understanding of musical creativity in composition and improvisation.

4 Imagining Big Science for ComMute

A future music theory that includes Big Science should above all change some of the dominating traditional characteristics of music theory and musicology as follows:

1. *Change "Meditative" into "Operational"!* We argue that it is wiser to try out several variants of a composition on a sequencer than to meditate on a fictitious best solution.
2. *Change "Metaphoric" into "Explicit"!* For example, it is advantageous to look for tempo curves instead of nebulous metaphors of movement.
3. *Change "Ubiquity" into "Topography"!* In fact, it is nonsense to postulate that the musical work has a monolithic omnipresence—and to be deceived if you won't find it that way round. Better look for topographically differentiated traces of what could contribute to a distributed concept of the musical work.
4. *Change "Ontological" into "Semiotic"!* Musicology often tries to find "the true and essential meaning" of something instead of distinguishing between layers of signification which are best described by semiotic categories such as denotation or connotation.
5. *Change "Magic" into "Communicative"!* The old-fashioned and absurd swearing to the genius of a great composer should be replaced by communication of ideas and perspectives.
6. *Change "Transcendence" into "Precision"!* It would be more interesting and scientific to publish detailed analyses of musical works instead of writing all those feuilletonistic books on composers. After all, the tools are ready; the minds should realize this.

This list might sound provocative, but without these changes no Big Science in music will be feasible. Perhaps are we now facing a transformation of music theory that is comparable to the Galilean transformation, which replaced reading Aristotelian philosophical books on physics by computational and experimental methods.

With this in mind, musical creativity will dramatically change its face with Big Science. With the elimination of the outdated genius paradigm, music will be more of a collaborative research with huge technological tools for its globally distributed realization. This collaborative style will also eliminate the strict separation between artist and audience. And the artistic virtuosity in the performance of musical works will become a niche exercise that can be performed better by robots. This does not mean that human instrumental virtuosity will disappear, but it will play the completely different role of a thinking-making research, not of a (however sophisticated) performance of given templates.

References

1. Amiot, E.: Music Through Fourier Space. Springer Series Computational Music Science. Springer, Heidelberg (2016). https://doi.org/10.1007/978-3-319-45581-5
2. Agustín-Aquino, O.A., Junod, J., Mazzola, G.: Computational Counterpoint Worlds. Springer Series Computational Music Science. Springer, Heidelberg (2015). https://doi.org/10.1007/978-3-319-11236-7
3. Bars, I., Terning, J.: Extra Dimensions in Space and Time. Springer, New York (2010). https://doi.org/10.1007/978-0-387-77638-5
4. http://www.cern.ch
5. Friberg, A., Bresin, R., Sundberg, J.: Overview of the KTH rule system for music performance. Adv. Exp. Psychol. **2**, 145–161 (2006). Special issue on Music Performance
6. Kinderman, W.: Artaria 195. University of Illinois Press, Urbana and Chicago (2003)
7. Larson, S.: Musical Forces. Indiana University Press, Bloomington (2012)
8. Mazzola, G., et al.: The Topos of Music—Geometric Logic of Concepts, Theory, and Performance. Birkhäuser, Basel (2002)
9. Mazzola, G., Cherlin, P.B.: Flow, Gesture, and Spaces in Free Jazz-Towards a Theory of Collaboration. Springer Series Computational Music Science. Springer, Heidelberg (2009). https://doi.org/10.1007/978-3-540-92195-0
10. Mazzola, G.: Gestural dynamics in modulation: (towards) a musical string theory. In: Pareyon, G., Pina-Romero, S., Agustin-Aquino, O.A., Lluis-Puebla, E. (eds.) The Musical-Mathematical Mind. Springer Series Computational Music Science, pp. 171–188. Springer, Heidelberg (2017). https://doi.org/10.1007/978-3-319-47337-6_18
11. Mazzola, G.: The Topos of Music II: Performance. Springer Series Computational Music Science. Springer, Heidelberg (2018). https://doi.org/10.1007/978-3-319-64444-8
12. Mazzola, G., et al.: The Topos of Music III: Gestures. Springer Series Computational Music Science. Springer, Heidelberg (2018). https://doi.org/10.1007/978-3-319-64481-3

13. Munyaradzi, G., Zimidzi, W.: Comparison of western music and African music. Creat. Educ. **3**(2), 193–195 (2012)
14. Nugroho, G.: Teak Leaves at the Temples. DVD, Trimax Enterprises Film, New York (2007)
15. Penrose, R.: The Road to Reality. Vintage, London (2002)
16. Ravikiran, C.N.: Robert morris and the concept of melharmony. Perspect. New Music **52**(2), 154–161 (2014)
17. Schönberg, A.: Harmonielehre (1911). Universal Edition, Wien (1966)