

# A Simplified Approach for the Estimation of Settlements of Earth Embankments on Piled Foundations

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**Abstract.** In recent years, the employment of deep foundations as settlement reducers has become increasingly popular in the design of earth embankments over soft soil strata. To further improve the system response, geosynthetic layers are often positioned at the embankment base. Owing to the presence of both piles and geosynthetics, complex interaction mechanisms, transferring stresses towards the piles and reducing those on the soft foundation soil, take place. Although these mechanisms are governed by the relative stiffness of the various elements constituting this system (piles, foundation soil, embankment and georeinforcements), the approaches commonly adopted to design these "geostructures" do not explicitly take into consideration the stiffness of the single element as design parameters. Moreover, the effect of the stepwise embankment construction process is often disregarded. As a consequence, the settlements at the top of the embankment cannot quantitatively be estimated.

In this paper a new model for the evaluation of the settlements at the top of the embankment, considering both the deformability and the yielding of the soils, is presented. The model introduces both the geometry and the soil mechanical properties as constitutive parameters, and explicitly puts in relationship the embankment settlements to the embankment height (herein considered as a generalized loading variable), thus allowing the geotechnician to adopt a direct displacement-based design approach.

**Keywords:** Piled embankment · Geosynthetics · Displacement based design · Constitutive modelling

## 1 Introduction

Piled foundations, either in case of rigid piles (e.g. concrete) or deformable piles (e.g. di Prisco and Galli 2011, 2013), are largely employed as settlement reducers for earth embankments on soft soil strata. In many cases, to further improve this system response, geosynthetics are also positioned at the embankment base. Despite the welldocumented effectiveness of this technique, the complex interaction mechanisms, transferring vertical loads toward the piles, are not fully understood. The approaches suggested in the most used design standards (e.g. EBGEO 2010; BS8006-1 2010) to analyse the response of this "geostructure", based on the concept of "arching effect" (Marston and Anderson 1913; Terzaghi 1943), allow to estimate the stresses applied to the pile and to the geosynthetic layer. However, since these approaches completely disregard the influence of the stiffness on the single element of the system response, the assessment of the embankment settlements is not possible.

In di Prisco et al. (2019) a new model to interpret the mechanical response of an unreinforced embankment on piled foundations was proposed. This model is a constitutive law putting in relationship the increment in the embankment height, the generalized loading variable, with the increment of both average and differential settlements at the embankment top. In this paper, the authors intend to extend the model to the case of embankments reinforced by means of planar reinforcement layers (geosynthetic).

The paper is structured as it follows: in Sect. 2 the numerical model is presented, whereas in Sect. 3 the numerical results are discussed. Finally, in Sect. 4 the constitutive relationship is briefly presented and is compared with numerical results.

### 2 Numerical Model

The construction of embankments on piled foundations is a three-dimensional problem. However, in case of large embankments (i.e. the transversal direction of the embankment is large with respect to its height as is sketched in Fig. 1a), in the preliminary design stages flanks effects may be disregarded. Therefore, only one axisymmetric cylindrical cell may be considered (Fig. 1a). This cell is composed of one pile of diameter d and length l, the surrounding soil and the portion of the embankment of thickness h directly above them. The diameter of the whole cell (s) can preliminarily be assumed to be coincident with the pile spacing.



Fig. 1. (a) Considered geometry and (b) spatial discretization.

Owing to the axial symmetry, only one quarter of this unit cell has been modelled by means of code FLAC3D (Itasca Consulting Group). The spatial discretization employed is reported in Fig. 2, and it consists of 16800 elements. To assess the reliability of the numerical results, the size of the elements was parametrically optimized by the authors (the results are hereafter omitted for the sake of brevity). Normal displacements are not allowed to both the lateral boundaries and the base of the domain.

The reinforced concrete pile is modelled as a linear elastic cylindrical inclusion, the mechanical behaviour of both the foundation and the embankment soil is modelled by means of elastic-perfectly plastic constitutive relationships. The failure condition is given by the Mohr-Coulomb criterion and the flow rule is assumed to be non-associated. Smooth interface elements are introduced between pile and foundation soil.

Geosynthetic layer positioned to reinforce the embankment base is modelled by means of isotropic elastic membrane elements characterized by an axial stiffness J and by a nil flexural stiffness.

The embankment construction process is subdivided in single stages, each one corresponding to the positioning of 25 cm of compacted granular material. At each stage, a new layer of elements is added on the top of the embankment: the geometry of the spatial domain thus progressively evolves.

#### **3** Numerical Results

In this section the results relative to one reference geometry (d = 0.5 m, s = 1.5 m and l = 5 m) are discussed. The values of the constitutive parameters employed are enlisted in Table 1. A complete discussion of the influence of the geometrical and mechanical properties is hereafter omitted for the sake of brevity.

	Unit weight	Young	Poisson's ratio	Friction	Dilatancy angle
	[kN/m <sup>3</sup> ]	modulus	[-]	angle	[°]
		[MPa]		[°]	
Column	25	30000	0.3	-	-
Embankment	18	10	0.3	30	0
Foundation soil	18	1	0.3	40	0

Table 1. Mechanical parameters of the pile, of the embankment and of the foundation soil.

By following the approach proposed in di Prisco et al. (2019), the mechanical response of the system can be described by means of the following non-dimensional variables, representing the normalised stresses acting on the column and on the foundation soil ( $\sum_c$  and  $\sum_f$ , respectively), and a set of normalized settlement variables ( $U_{b,c}$ ,  $U_{b,f}$ ,  $U_{t,c}$ ,  $U_{t,f}$ ):

$$\sum_{c} = \frac{\sigma_{c}}{\gamma d} \text{ and } \sum_{f} = \frac{\sigma_{f}}{\gamma d}$$
(1a)

$$U_{b,c} = \frac{u_{b,c}}{d} \frac{E_{oed,f}/l}{\gamma} \text{ and } U_{b,f} = \frac{u_{b,f}}{d} \frac{E_{oed,f}/l}{\gamma}$$
(1b)

$$U_{t,c} = \frac{u_{t,c}}{d} \frac{E_{oed,f}/l}{\gamma} \text{ and } U_{t,f} = \frac{u_{t,f}}{d} \frac{E_{oed,f}/l}{\gamma}$$
(1c)

$$U_{t,diff} = U_{t,f} - U_{t,c} \text{ and } U_{t,av} = \frac{U_{t,f}(S^2 - 1) + U_{t,c}}{S^2},$$
 (1d)

where  $\sigma_c$  and  $\sigma_f$  are the vertical average stresses at the base of the embankment, acting at the top of the column (subscript *c*) and on the foundation soil (subscript *f*), respectively.  $u_{b,c}$  and  $u_{b,f}$  are the average displacements at the base of the embankment,  $u_{t,c}$  and  $u_{t,f}$  the average displacements at the top of the embankment.  $U_{t,diff}$  and  $U_{t,av}$  are defined as the non-dimensional differential and average settlements at the top of the embankment, respectively. The normalized pile spacing S = s/d and the normalized embankment height H = h/d are also defined, whereas  $E_{oed,f}$  and  $\gamma$  are the oedometric modulus of the foundation soil and the embankment soil unit weight, respectively.

In Fig. 2 the numerical results relative to the case of an unreinforced embankment (hereafter indicated as J = 0) are reported. In particular, the evolution of  $\Sigma_f$  and  $\Sigma_c$  (Fig. 2a),  $U_{b,f}$  and  $U_{b,c}$  (Fig. 2b),  $U_{t,f}$  and  $U_{t,c}$  (Fig. 2c) and  $U_{t,diff}$  and  $U_{t,av}$  (Fig. 2d) with H is plotted. The dash-lines in Fig. 2 correspond to the ideal case in which pile and foundation soil stiffness are coincident (geostatic stress distribution).



**Fig. 2.** Numerical results: evolution with the embankment height of (a) stresses, (b) displacement at the embankment base, (c) displacements at the embankment top and (d) differential and average displacements at the embankment top.

From Fig. 2a it can be derived that during the early stages of the construction process no significant stress redistribution takes place: the numerical results are almost superimposed to the "geostatic" line. By increasing H, the stress redistribution mechanism becomes more and more evident and vertical stresses concentrated within the pile, without completely stopping the increase in  $\Sigma_{f}$ .

In this case, in which the pile is assumed to be quasi-rigid,  $U_{b,c}$  is negligible (Fig. 2b) whereas the  $U_{b,f}$  trend is bilinear with a marked knee.

From a kinematic point of view, a punching mechanism initially develops within the embankment body. For larger values of H a more and more efficient stress redistribution mechanism takes place, thus inducing an increase in both  $U_{t,f}$  and  $U_{t,c}$ . These latter curves are in particular characterized by an upward concavity, due to the progressive increase in the embankment height, inducing an increment in its vertical deformability.

From Fig. 2d we derive that differential and average displacements initially evolve simultaneously, until a critical value ( $H^*$ ) is reached. For  $H > H^*$ , the differential displacements remain constant, whilst the average displacements continuously increase. The value  $H^*$  may be assimilated to the plane of equal settlement (McKelvey 1994; Naughton 2007; McGuire 2011; Filz et al. 2012), i.e. the minimum thickness of the granular stratum necessary to nullifying the increment in differential settlements.

The influence of the geosynthetic stiffness is discussed in Fig. 3 (J = 0, J = 1000 and 3000kN/m). For the sake of brevity, only the influence on the differential (Fig. 3a) and on the average settlements (Fig. 3b) at the embankment top is discussed. As was expected, by increasing the geosynthetic stiffness both differential and average displacements at the embankment top decrease.



**Fig. 3.** Influence of the geosynthetic stiffness on the system response: (a) on the differential displacements and (b) on the average displacements at the embankment top.

### 4 Simplified Approach for the Settlement Estimation

The simplified approach for the embankment settlement estimation defines an incremental relationship between the displacement at the top of the embankment (differential  $\dot{U}_{t,diff}$  and average  $\dot{U}_{t,av}$ ) and the generalized loading variable  $\dot{H}$ :

$$\begin{bmatrix} \dot{U}_{t,diff} \\ \dot{U}_{t,av} \end{bmatrix} = \begin{bmatrix} C_{diff} \\ C_{av} \end{bmatrix} \dot{H}$$
(2)

where  $C_{diff}$  and  $C_{av}$  represent the non-dimensional generalized compliance terms. According to di Prisco et al (2019), both  $C_{diff}$  and  $C_{av}$  can analytically be derived by employing the rheological model of Fig. 4a, where  $H_p$  represents the height of the yielded portion of the embankment transferring stresses toward the pile ("process" height):

$$C_{diff} = C_2 - C_1 - 4k \cdot \tan \phi_{ss}' \left( C_1 + \frac{C_2}{S^2 - 1} \right) H_p,$$
 (3a)

$$C_{av} = \frac{C_1}{S^2} + \frac{C_2(S^2 - 1)}{S^2} - \frac{4k \cdot \tan \phi_{ss}' H_p(C_2 - C_1)}{S^2} + C_3$$
(3b)



**Fig. 4.** (a) rheological model and (b) comparison between the numerical results and the proposed model (d = 0.5, s = 1.5 m, l = 5 m and mechanical properties of Table 1).

where  $C_1$ ,  $C_2$  and  $C_3$  represent the compliances of the elastic springs:

$$C_{1} = C_{1}^{r} + C_{1}^{irr} = \frac{L}{E_{c}} \frac{E_{oed,f}}{L} + \frac{H_{p}}{E_{oed,e}} \frac{E_{oed,f}}{L}$$
(4a)

$$C_2 = C_2^r + C_2^{irr} = \frac{L}{E_{oed,f}} \frac{E_{oed,f}}{L} + \frac{H_p}{E_{oed,e}} \frac{E_{oed,f}}{L}$$
(4b)

$$C_3 = C_3^{irr} = \frac{H - H_p}{E_{oed,e}} \frac{E_{oed,f}}{L}$$
(4c)

being L = l/d,  $E_{oed,e}$  the embankment soil oedometric modulus and  $E_c$  the pile Young modulus. In contrast k,  $\phi'_{ss}$  and  $H_p$  define the behaviour of the plastic slider of Fig. 4a. In particular, k is an average value for the ratio of horizontal and vertical stresses along  $H_p$  and  $\phi'_{ss}$  is the internal friction angle under simple shear conditions (di Prisco and Pisanò 2011). Variable  $H_p$  may be interpreted as the hardening parameter for the model, continuously increasing with H until quantity  $H^*$  is reached:

$$\dot{H}_{p} = f(x) = \begin{cases} \dot{H}, H < H^{*} \\ 0, H \ge H^{*} \end{cases}.$$
(5)

 $H^*$  plays then a fundamental role in the constitutive relationship, since it determines a change in the overall system behaviour: for  $H > H^*$  no further differential settlement increments are observed at the embankment top. The value of  $H^*$  can be analytically derived by imposing the condition  $C_{diff} = 0$ .

The unique parameter not directly related to the geometry of the system and to the material properties is k, that has to be numerically calibrated by following the procedure reported in di Prisco et al. (2019). Herein the following, a k value of 0.83 has been adopted.

A validation of the model has been done by comparing the numerical results of Fig. 2d with the model prediction (Fig. 4b). As is evident, a very satisfactory agreement is obtained.

In case the pile stiffness is significantly larger with respect to the one of the foundation soil, to take into account the presence of the geosynthetic at the embankment base, the  $C_2^r$  is assumed to evolve with  $U_{b,f}$ . In particular,  $C_2^r$  progressively decreases with the non-dimensional displacement  $U_{b,f}$ , since the geosynthetic, working as a membrane, experiences a progressive stiffening for increasing differential displacements. The  $C_2^r$  value to be used to integrate Eqs. 2–5 is obtained step by step, by solving the membrane equations, progressively non-linearly evolving with  $U_{b,f}$ . The membrane equations employed refer to an axisymmetric isotropic linear elastic membrane displacements are zero for both r = D/2 and r = S/2, whereas the vertical displacement is zero only for r = D/2 and the rotation is nil for r = S/2. The complete discussion of this problem, reported in Brignoli (2017), is hereafter omitted for the sake of brevity.



**Fig. 5.** (a) comparison between the differential settlements, (b) comparison between average settlements (d = 0.5, s = 1.5 m, l = 5 m and mechanical properties of Table 1) and (c) evolution of  $C_2^r$  with  $U_{b,f}$  for different J values.

In Fig. 5 numerical results are compared with model predictions in terms of differential and average settlements (Fig. 5a and b). The model is also capable of putting in evidence the stiffening due to the presence of geosynthetic, markedly varying with J (Fig. 5c).

#### 5 Conclusions

In this paper, a non-dimensional constitutive relationship capable of reproducing the drained mechanical response of both unreinforced and reinforced embankments positioned on a piled foundation is presented. The model considers the embankment height as a generalized loading variable and allows the evaluation of both the average and differential displacements at the top of the embankment. From a theoretical point of view, sub-structuring the system gives the capability to clarify the mechanisms developing within the single "substructure". The model results have been validated against 3D numerical analyses data. The agreement is satisfactory and for this reason the Authors are confident in the success of the model to preliminary design pile supported embankments.

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