Seismic Soil-Pile-Structure Interaction. Theoretical Results and Observations on Pile Group Effects

Roberto Cairo(✉), Giampaolo Francese, Rossana Moraca, and Federica Aloe

Department of Civil Engineering, University of Calabria, Ponte P. Bucci, 87036 Rende, Cosenza, Italy
rcairo@unical.it

Abstract. In this paper the fundamental features of the seismic soil-structure interaction on the response of pile-supported structures are investigated. A stiffness matrix approach is proposed and employed to analyze the dynamic behavior of single pile and pile groups embedded in homogeneous and heterogeneous soil deposit. The superposition theorem is then used to evaluate the response of the structure, modelled as a single degree of freedom oscillator. Different configurations of both piles and structures are considered. In particular, the aim of the present study is to explore the role of the dynamic properties of the soil and pile group effects on the dynamic response of the structure. The performed analyses permit to define reliable seismic design criteria of piled-structures.

Keywords: Piles · Structure · Dynamic stiffness · Damping · Interaction

1 Introduction

The design of all structures founded on piles requires pile-soil-pile interaction effects to be accounted for (CEN 2005; NTC 2018). Under dynamic loadings, wave propagation phenomena occur, which make soil-structure interaction very complex and, in general, strongly dependent on frequency. The behavior of pile groups becomes indeed very different from that of a single pile, and the group efficiency (i.e. the ratio of the group stiffness to the sum of the individual pile stiffness) may exceed unity depending on several conditions (Dente 1999). In the serviceability state, the simplest approach to solve pile-soil-pile interaction is based on the interaction factors developed by Poulos (1968) for static loads and later extended by Kaynia and Kausel (1982) to the dynamic case. Conversely, direct analysis of pile groups can be performed by numerical methods such as FEM (Wolf and von Arx 1978) and BEM (Kaynia and Kausel 1982). From a practical point of view, a better understanding of the key features of the problem can be acquired using the superposition theorem (Kausel and Roësset 1974), which decomposes the analysis into two step: in the first, the *foundation input motion,* or FIM, is determined, without the presence of the mass of the structure; if pile cap is rigid, the structure itself is not required. This phase is called *kinematic interaction*.

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In the second (named *inertial interaction*), the response of the structure resting on frequency-dependent springs and dashpots (*impedance functions*) and subjected to the FIM is computed.

In this paper, a boundary element approach based on wave propagation is presented. It makes use of the dynamic stiffness matrices derived by Kausel and Roësset (1981) and represents an extension of the procedure proposed by Cairo et al. (2005) for the analysis of pile groups subjected to vertical harmonic loads, and Cairo and Dente (2007) for the seismic response of single piles. Particular attention has been reserved to pile group effects on the dynamic response of the structure.

2 Method of Analysis

The system studied refers to a group of vertical piles embedded in a layered soil. Each soil layer is modelled as an elastic material of Young's modulus E_s, Poisson's ratio ν_s, damping ratio β_s and mass density ρ_s; each pile is considered to be an elastic cylinder of length L, diameter d, Young's modulus E_p and mass density ρ_p. The soil-pile system is assumed to be under steady-state conditions, therefore any time-dependent variable is in general expressed as a complex quantity multiplied by the factor $e^{i\omega t}$, being i the imaginary unit and ω the circular frequency of the motion. In the following, this factor will be omitted for the sake of brevity, since it is shared by all time-dependent variables.

The problem can be resolved in two systems: (1) the piles that are subjected to the external (horizontal and rotational) loads or to shear waves propagating form an underlying bedrock, and soil-pile horizontal interaction forces; (2) the soil deposit that is acted on by the interaction forces at the pile-soil interfaces. These two systems are first considered separately and then reassembled enforcing equilibrium and compatibility at the pile-soil interfaces, under the hypothesis of perfect bonding between piles and soil. It is assumed that no interaction through the soil takes place due to the rocking of the singular pile. In addition, the influence of the soil vertical displacements on the pile response is neglected. Each pile is subdivided by a finite number of one-dimensional elements, and the soil is replaced by a horizontally layered continuum.

The dynamic equilibrium equation for the pile group can be expressed in matrix form as

$$\left(\boldsymbol{K}_{pG} - \omega^2 \boldsymbol{M}_{pG}\right)\boldsymbol{u}_{pG} = \boldsymbol{P}_G + \boldsymbol{P}_{pG} \tag{1}$$

where $\boldsymbol{K}_{pG}$ and $\boldsymbol{M}_{pG}$ indicate, respectively, the global stiffness matrix and mass matrix of the pile group; $\boldsymbol{u}_{pG}$ is the vector containing the horizontal 'total' displacements and rocking components of the nodes of the pile elements; $\boldsymbol{P}_G$ is the vector of the loads applied to the head of the piles; $\boldsymbol{P}_{pG}$ the vector containing the horizontal pile-soil interaction forces acting on the piles. The stiffness and mass matrices of the piles may be determined using standard techniques of the structural analysis (Bhatti 2005).

When the soil is interested by seismic wave propagation, the displacement and stress fields are decomposed into two parts (Banerjee and Butterfield 1981): the free-field and the scattered field. The free-field represents the wave motion that occurs in the soil in the absence of the piles, whereas the scattered motion consists of the waves diffracted from the surface of the piles and propagating towards infinity. In this case, the following dynamic equilibrium equation can be written:

$$\boldsymbol{K}_{sG}\left(\boldsymbol{u}_{sG} - \boldsymbol{u}_{sG}^{f}\right) = \boldsymbol{P}_{sG} - \boldsymbol{P}_{sG}^{f} \tag{2}$$

in which $\boldsymbol{K}_{sG}$ is the dynamic stiffness matrix of the soil; $\boldsymbol{u}_{sG}$ the vector containing the horizontal total displacements of the soil in correspondence with the nodes of the piles; $\boldsymbol{u}_{sG}^{f}$ the free-field soil displacement vector; $\boldsymbol{P}_{sG}$ and $\boldsymbol{P}_{sG}^{f}$ are the interaction forces acting on the soil in the total system and the free-field, respectively. Because of the equilibrium of the interaction forces and compatibility of the horizontal displacements at the soil-pile interface, Eq. (1) becomes

$$\left(\boldsymbol{K}_{pG} + \boldsymbol{K}_{sG} - \omega^2 \boldsymbol{M}_{pG}\right)\boldsymbol{u}_{pG} = \boldsymbol{P}_{G} + \boldsymbol{Q}_{sG}^{f} \tag{3}$$

with

$$\boldsymbol{Q}_{sG}^{f} = \boldsymbol{K}_{sG}\boldsymbol{u}_{sG}^{f} - \boldsymbol{P}_{sG}^{f} \tag{4}$$

It should be noted that the stiffness matrix of the soil is augmented with zeros appropriately to be added to the stiffness matrix of the piles. Accounting for the presence of the pile cap, assumed to be rigid and not in contact with the soil, the dynamic response of the pile group can be thus evaluated, once the free-field soil response $\boldsymbol{u}_{sG}^{f}$ and the dynamic stiffness matrix $\boldsymbol{K}_{sG}$ of the soil have been computed. To this end, the stiffness matrix approach originally developed by Kausel and Roësset (1981) is used. By assembling the stiffness matrices of the soil layers and the underlying halfspace furnished by the authors, the following set of equations can be solved

$$\boldsymbol{KU} = \boldsymbol{R} \tag{5}$$

where $\boldsymbol{K}$ is the global stiffness matrix of the soil; $\boldsymbol{U}$ the vector of the corresponding displacements; $\boldsymbol{R}$ is the vector containing the external loads applied at the interfaces of the layers. The method works in the wave number domain, consequently it first requires that the loads are expanded spatially in harmonic components using the Hankel transform, and then the transformed displacements $\boldsymbol{U}$ are obtained as a discrete function of the wave number. Once the transformed displacements are derived for all the harmonic components considered, the actual displacement functions at a given location within the soil can be calculated using the inverse Hankel transform. Within this framework, both the free-field soil response $\boldsymbol{u}_{sG}^{f}$ (Cairo and Dente 2007) and the soil stiffness matrix $\boldsymbol{K}_{sG}$ (Cairo et al. 2005) are determined. Details of the present approach can be found elsewhere (Francese 2017).

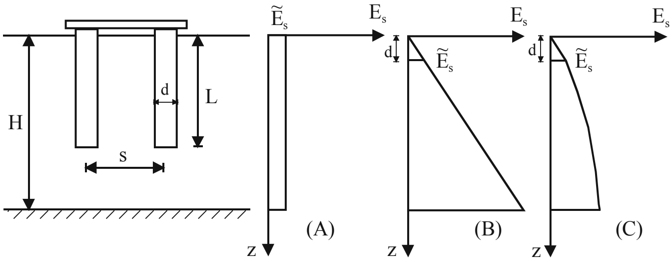

Fig. 1. Soil-pile system considered.

3 Impedance Functions of Piles

In the realm of the superposition theorem, a fundamental step is the evaluation of the dynamic impedances of pile foundations for each mode of vibration. For single pile, very simple expressions are available in the literature (Gazetas 1991). Three characteristic soil profiles are considered: (A) a homogeneous soil stratum resting on a rigid bedrock; (B) a soil stratum with Young's modulus E_s proportional to depth; (C) a parabolic soil profile. In the following, the impedance functions of a single pile (Fig. 1) obtained with the present approach are compared to those calculated with these approximate formulas. For each mode of vibration *j,* the dynamic impedance is expressed as

$$S_j(a_0) = K_j^{st}[k_j(a_0) + ia_0c_j(a_0)] \tag{6}$$

where K_j^{st} is the static stiffness of the pile, k_j and c_j are the dynamic stiffness and the damping coefficients, respectively; $a_0 = \omega d/V_S$ the dimensionless frequency, being V_S the shear wave velocity of the soil. The analyses refer to a single free-head pile with L = 20 m, d = 0.6 m, E_p= 2.5 · 10^7 kN/m^2, and ρ_p= 2.5 Mg/m^3. The thickness of the soil layer is H = 30 m; the properties of the soil are: ν_s= 0.4, ρ_s= 1.9 Mg/m^3, and β_s= 0.10. A reference shear wave velocity $\tilde{V}_S$ of 200 m/s is assumed; the shear wave velocity of the bedrock is 1200 m/s. Under these assumptions, a flexible pile with

$$\frac{L}{d}\left(\frac{E_p}{E_s}\right)^{-(0.20\div0.25)} > 2 \tag{7}$$

for which the simple formulas are valid, is considered.

For the sake of brevity, only the comparison for a homogeneous soil layer is shown (Fig. 2). The dynamic stiffness and damping coefficients are plotted versus the dimensionless frequency for horizontal, rocking and coupled horizontal-rocking vibrations.

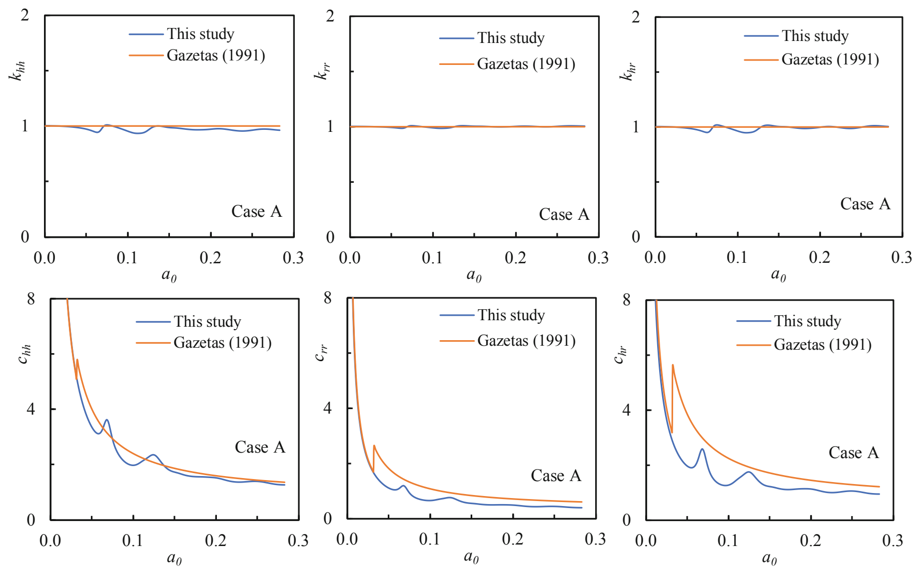

Fig. 2. Dynamic stiffness and damping coefficients of a single pile in a homogeneous soil layer.

As can be observed, the accordance between the present approach and Gazetas (1991) is quite satisfactory with respect to the dynamic stiffness coefficients, whereas the damping coefficients calculated with the approximate formulas are in general overestimated above the fundamental frequency of the soil deposit. Actually, these expressions present a sharp discontinuity in correspondence with this frequency, that represents a 'cut-off' frequency under which no radiation damping develops. The same trend has been found for the other two soil profiles.

As known, the dynamic stiffness of the single pile does not vary sensitively with frequency. Thus, the stiffness of the pile can be described by its static value and calculated with the expressions also reported in Eurocode 8 (CEN 2005). As a matter of fact, the impedance function of a single pile can be easily computed using its static stiffness and the damping ratio β_s of the soil in the low to intermediate frequency range for design purposes.

The effects of soil profile on the impedance functions are portrayed in Fig. 3. It can be noted that the soil stiffness profile determines a great influence only in the damping factor. In the case of a homogeneous soil, the major damping occurs at the medium and high frequencies, at least. Moreover, the damping coefficient results considerably higher in the horizontal motion than the rocking mode.

When the piles are arranged in group, the impedance functions may be quite different from those of the single pile, as a function of the number of the piles, pile spacing, and the relatively soil-pile stiffness.

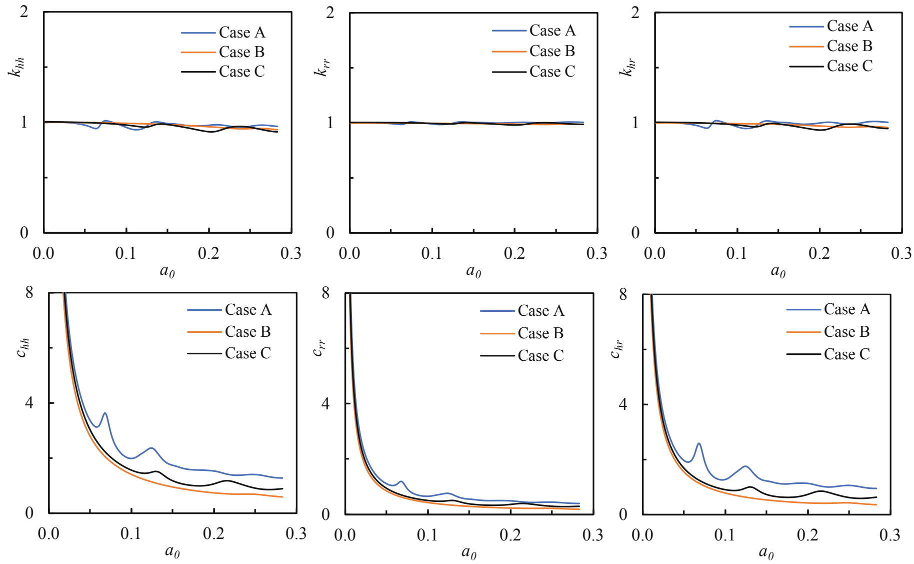

Fig. 3. Dynamic stiffness and damping coefficients of a single pile for three soil profiles (Fig. 1).

Figure 4 displays the dynamic stiffness and damping coefficients for two groups of 2 × 2 and 3 × 3 piles with pile spacing $s/d = 5$, embedded in a homogeneous soil layer (case A). The frequency range studied has been enlarged for convenience. The solution for single pile is also reported. The impedance factors are normalized with the sum of the static stiffness of the single pile. For the rocking mode of vibration, the vertical stiffness has been ignored in the present work.

As can be seen, the horizontal dynamic stiffness of the group varies with frequency. In the low frequency range, the group stiffness is smaller than the sum of the static stiffness of the individual pile as in the static case. At higher frequencies, pile-to-pile interaction effects become more important and the stiffness group factor exceeds unity. For the rocking group stiffness, a fairly smooth variation with frequency is observed, as a result of a small interaction occurring. The horizontal damping factor also shows a significant oscillation with a_0, except for very high frequencies. Moreover, damping factor increases with the number of piles in the low to intermediate frequency range.

As the assumed reference value of the Young's modulus $\tilde{E}_s$ of the soil (Fig. 1) increases from $z = d$ with depth (i.e. within the 'active length' of the pile), the dynamic stiffness of the piles increases for soil profiles B and C with respect to case A (not shown here). Besides, damping reduces. This is consistent with the fact that the behavior of the piles depends on the soil properties near the surface (especially for the horizontal mode of vibration). Therefore, in the cases examined, pile-soil-pile interaction effects tend to be less pronounced as the soil becomes harder in the proximity of the ground surface (i.e. variation of the impedance functions with frequency vanishes).

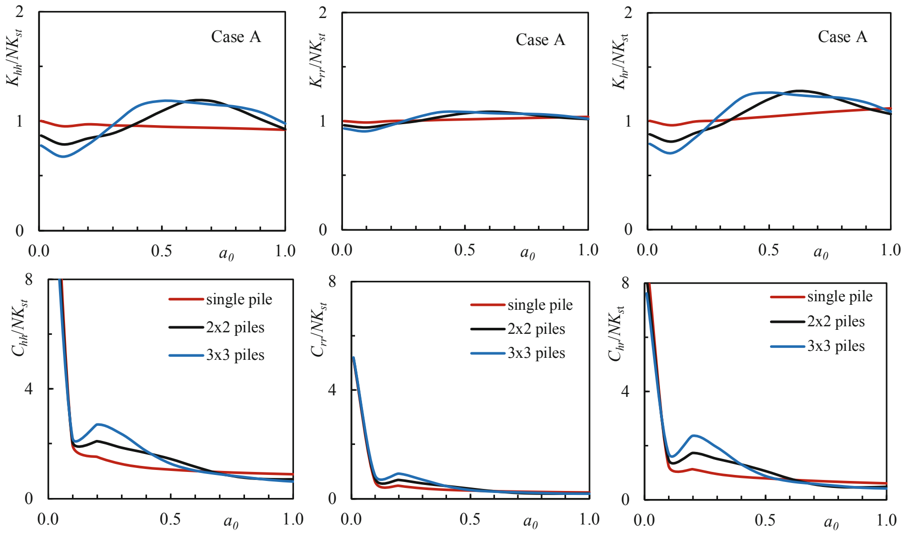

Fig. 4. Dynamic stiffness and damping factors of single pile and pile groups in a homogenous deposit.

4 Response of the Structure

In this section, the inertial interaction between soil and structure is examined and the influence of pile foundation discussed. A key parameter controlling inertial effects of this interaction is

$$\frac{1}{\sigma} = \frac{hf}{V_s} \tag{8}$$

where h and f are the height and the fixed-base frequency of the structure, respectively. In the presence of piles, important inertial interaction effects have been found in the case of structure with $1/\sigma > 0.2$, usually in conjunction with fairly soft superficial soils and marked stiffness increase with depth (Stewart et al. 1999). This quite empirical observation is consistent with the theoretical aforementioned observations regarding the impedance functions of the piles, as a consequence of the dynamic soil-pile interaction.

In order to investigate the dynamic response of structures founded on piles, Rovithis et al. (2009) conducted a robust parametric investigation. For the purposes of this work, three different soil-structure systems studied by the authors are used herein. The structure consists in a single degree of freedom oscillator with height h = 10 m, mass m = 100 Mg, damping ratio ζ = 0.05, and fixed-base frequency f equal to 1.67 Hz (Str1), 5 Hz (Str2), and 8.33 (Str3). A single pile foundation is considered, with L = 30 m, d = 1.5 m, $E_p = 20 \cdot 10^7$ kN/m^2, ρ_p = 2.5 Mg/m^3, embedded in a soil layer with H = 30 m, ν_s = 0.4, ρ_s = 1.7 Mg/m^3, V_s = 200 m/s, and β_s = 0.05.

In the following, the impedance functions of the pile calculated with the present approach are employed. Moreover, the kinematic interaction is solved and the FIM determined in the hypothesis of fixed head pile. The dynamic responses of the three structures are thus computed and illustrated in Fig. 5 in terms of the amplification ratio A versus the frequency f of the motion. In particular, the amplitude of the horizontal displacement U_s of the structure, normalized by the free-field ground motion U_{ff} and the displacement U_o of the pile head, is plotted. By comparison, the fixed-base response U_{fb} of the structure (adimensionalized with the amplitude of the base motion) is displayed. The pile head response U_o/U_{ff} is also shown.

As can be noted, the most significant inertial interaction occurs for Str2 ($1/\sigma$ = 0.25) and Str3 ($1/\sigma$ = 0.42). The peak responses of the coupled soil-structure system result greater than that of the same structure on a rigid base and occur at a lower frequency, corresponding to a more flexible system. Moreover, the maximum amplification ratio U_s/U_{ff} of the structure is attained when the pile-head motion U_o/U_{ff} is amplified (corresponding to the natural frequency of the system), whereas the maximum amplification U_s/U_o takes place at the *pseudo-natural* frequency of the system (Rovithis et al. 2009) in which pile-head displacement U_o/U_{ff} is de-amplified. Conversely, the system Str1 does not show any inertial interaction effect, being $1/\sigma$ = 0.08.

In Fig. 6, the responses of the same structures founded on a 2 × 2 pile group with *s*/*d* = 5 are portrayed. Pile group effects can be observed comparing Figs. 5 and 6. The maximum displacement of the structures Str2 and Str3 is considerably smaller in the latter case, as a consequence of the larger damping factor of the pile foundation. In addition, the dynamic stiffness of the foundation increases so that both the natural and the *pseudo-natural* frequencies of the coupled systems tend to reduce in a less dramatic way with respect to the fixed-base frequency. Definitely, no interaction develops for Str1.

It should be noted that Str2 and Str3 in the soil-structure system present nearly the same natural/pseudo-natural frequencies in the case of a single pile foundation (Fig. 5), whereas very different frequencies of vibration are shown as a function of pile group configuration (Fig. 6).

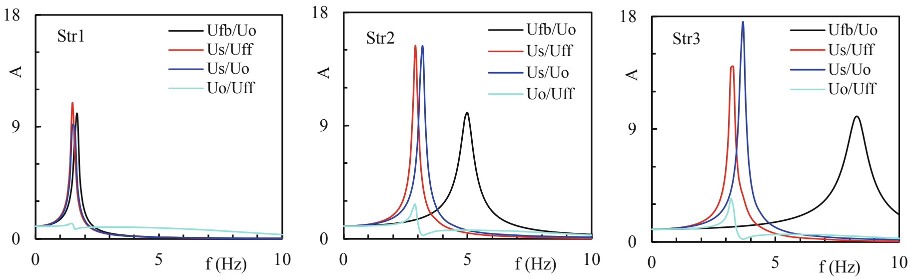

Fig. 5. Amplification ratios of the soil-structure systems for a single pile foundation.

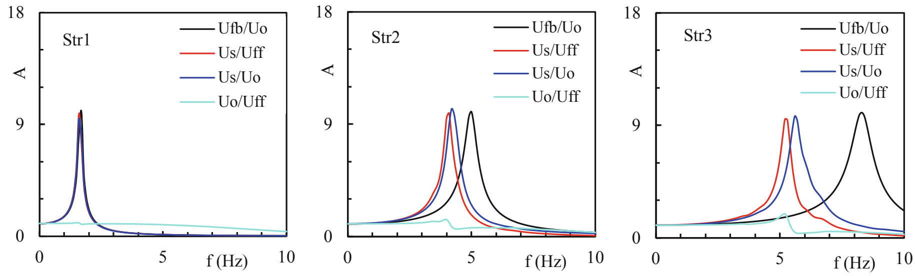

Fig. 6. Amplification ratios of the soil-structure systems for a 2 × 2 pile group (*s*/*d* = 5).

5 Conclusions

In this paper, a stiffness matrix method for the analysis of pile groups under dynamic loadings is presented. The approach can be conveniently used to evaluate the seismic response of the structure taking into account soil-structure interaction phenomena.

The analyses performed show that simplified expressions for the dynamic impedance of piles can be employed only for 'isolated' piles, whereas group effects cannot be neglected, generally. Only in the presence of relatively stiff soils near the ground surface and smooth stiffness contrast in soil profile, pile-soil-pile interaction could be ignored.

It must be noted that these results are valid under the assumptions of linearity for soil and piles and perfect bonding at the pile-soil interfaces.

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