Chapter 3
In Search of the Best Proxy for Liquidity in Asset Pricing Studies on the Warsaw Stock Exchange

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Abstract Stock liquidity is unobservable, and thus, its level needs to be approximated. There is a large body of liquidity measures recorded in the existing literature. The main goal of this paper is to investigate which measure is the most appropriate one to measure stock liquidity for the purposes of asset pricing studies on the Warsaw Stock Exchange. To indicate the most appropriate proxy for liquidity, a series of correlation analysis between different liquidity measures and estimation error measures have been applied. Four high-frequency liquidity measures were used as a benchmark for liquidity, and fourteen low-frequency liquidity proxies were examined. The study was conducted on a group of 100 companies listed on the Warsaw Stock Exchange between 2006 and 2016. The ranking of low-frequency proxies for liquidity has been created based on eleven performance dimensions. It shows that the most appropriate liquidity measure on the Warsaw Stock Exchange is that developed by Fong et al. [12], which is a simplification of the zero-return-days measure developed by Lesmond et al. [20]. In addition, two modifications of Amihud's [2] illiquidity are presented as the second and third best-performing ones. To the best of the author's knowledge, this is the first such extensive study of the performance of liquidity measures on the Warsaw Stock Exchange. It examines both existing liquidity measures and some modifications proposed on the basis of the literature overview.

Keywords Stock liquidity · Liquidity measures · Liquidity proxies

3.1 Introduction

Liquidity is viewed as the investor's ability to buy or sell large quantities of an asset quickly, at low cost and without causing adverse price impact [24]. This is an extremely important issue on the stock market, from both practitioners' and academics' perspective. Its relevance comes from its effects on asset pricing (see, e.g.,

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[4]; Amihud [1–3, 24], on portfolio allocation (see, e.g., [14–17, 21, 25] as well as on risk management. Undoubtedly therefore, stock liquidity is of great interest for investors on the capital market. For investors, its measurement is equally important as the stock liquidity itself. For several reasons, accurate measurement of the level of asset liquidity on the market is difficult. This difficulty is mainly due to the elusiveness and multidimensionality of the concept of liquidity. Predominantly, the bid-ask or effective spread is used as a measure of liquidity, as it reflects the cost of immediate trade execution.

To compute some of the liquidity measures, ultra-high-frequency data on order flows is needed. As pointed out in the literature, these are the most appropriate measures to analyse the effect of liquidity on stock returns [13]. It results from the nature of liquidity, which depends on the equilibrium of buy and sell orders that flow into the market. Applying these measures allows us to measure the level of liquidity costs more accurately and thus indirectly infer the level of stock or market liquidity. However, access to ultra-high-frequency data is barely available or even impossible to obtain in some markets, especially emerging ones. Acquiring this type of data is usually expensive [20], and, in addition, it is not available for longer periods of time [1]. It is worth noting the fact that for NYSE and NYSE MKT (formerly AMEX) exact data on order flow is available only for the period after 1983 [8]. In turn, for many other, smaller, less developed and less liquid markets, data on order flow is unavailable or availability is limited (for example in the Polish market).

Due to the difficulties in obtaining data for the calculation of ultra-high-frequency measures, many authors use measures that are less data-demanding, i.e. low-frequency proxies that require data only on a daily frequency. They are an alternative for high-frequency measures or are simply the only option if ultra-high or high-frequency data is not available. Moreover, the use of daily data is far less costly and time-consuming. The use of low-frequency measures also allows us to obtain much longer time series than in measures based on order flow data and intra-daily data [19, 22, 30]. As noted by Goyenko et al. [18], low-frequency measures are just as effective as the measures using higher-frequency data. Fong et al. [12] come to a similar proposition from their research. The popularity and usefulness of low-frequency measures is evidenced by the fact that they have even become commonplace in studies focusing on the US markets for which ultra-high-frequency data is available [5].

The main goal of the paper is to investigate which measure is the most appropriate one to measure stock liquidity for the purposes of asset pricing studies on the Warsaw Stock Exchange. It is considered that liquidity should be measured using the so-called high-frequency measures whose application requires often costly and scarcely available data on order flows. For this reason; for time, computational and cost savings, a number of proxies are used instead, which require more affordable low-frequency data, in particular daily data on prices and volumes. These proxies measure liquidity with varying degrees of accuracy, and the studies carried out so far do not provide clear indications as to which of them is the best one, in particular with regard to emerging markets. Research conducted in this paper includes data from the Warsaw Stock Exchange in the period from 2006 to 2016. In this period, WSE was the biggest and fastest growing emerging market in the region of Central and Eastern

Europe. It can be therefore assumed that the results obtained for the WSE could be widened to the other stock exchanges in the CEE region.

This paper contributes mainly to the literature on the measurement of liquidity in stock markets. Similar studies done so far were carried out for several reasons including the search for the best liquidity measure for international research or testing the usefulness of newly developed liquidity measures. Fong et al. [12] tried to find the best liquidity proxy for global research. They utilised data on 42 securities exchanges from 38 countries around the world in the period from 1996 to 2014. Porcenaluk [26] investigated estimation errors of six different bid-ask spread estimators, namely Roll's [27], Thompson and Waller's [29], Choi et al.'s [9], Chu et al. [10] and two versions of CS estimators. He indicated that the above-mentioned estimators are characterised by large estimation errors on the Polish capital market and also noted that, due to the positive autocorrelation of stock returns, it is reasonable to use estimators based on the price range. In addition to the aforementioned research, one should also recall the paper of Będowska-Sójka [6]. Using the data of 52 companies listed on the WSE in the years 2009–2016, she indicated that the best liquidity proxies on the WSE are Amihud's [2] illiquidity measure, daily price range and CS estimator.

So far, research done in search of the best proxy for liquidity on the Warsaw Stock Exchange was carried out only to a limited extent. With the aim of indicating the most useful liquidity measure on the Polish stock market, an empirical study on the sample of companies listed on the WSE was carried out. The results of the study indicate that the best proxy for liquidity in asset pricing studies is the measure developed by Fong et al. [12], which is a simplification of the zero-return-days measure developed by Lesmond et al. [20]. In addition, two modifications of Amihud's [2] illiquidity measure are presented as the second and third best-performing ones in asset pricing studies on the WSE.

The rest of this paper is organised as follows. The following section describes the methodological design of the study carried out. Section 3.3 is devoted to the presentation of the empirical results of the study. The final section contains the summary and concluding remarks.

3.2 Design of the Empirical Study

When assessing the usefulness of a measure for measuring the liquidity in a given market, one should pay attention to several important issues. Primarily, it should be assessed if a given measure can be applied for measuring liquidity in this market (applicability); so whether it fits to the organisation of the trade in this market (i.e. whether the assumptions made while constructing the measure are fulfilled), whether the required data is available and how complicated and time-consuming the calculations are. The two latter issues are important mainly from the investors' point of view; in academic research, they have much less importance. The assessment of selected liquidity measures in terms of their applicability on the Warsaw Stock Exchange is presented in the paper of Stereńczak [28].

Next, one should decide whether each given indicator reflects the level of liquidity accurately (the veracity of the measurement), i.e. that it orders the stocks according to their liquidity in line with the reality, and also has minimal error in estimating the real liquidity cost. The former criterion is important mainly in asset pricing research, as in these studies it is important to order stock according to the decreasing or increasing level of liquidity. In turn, the measures characterised by low estimation errors are more useful in research on market efficiency, as in this type of analysis, one should obtain properly calibrated measures of transaction costs [18].

The accuracy of the measurement was assessed using several criteria, which were commonly used in other studies in this field. For the measure to be considered the best measure of liquidity, it had to be characterised by:

(1) the highest average cross-sectional correlation with the benchmark of liquidity,
(2) the highest average Spearman rank cross-sectional correlation with the benchmark of liquidity,
(3) the highest average time-series correlation with the benchmark of liquidity (at single stock level),
(4) the highest time-series correlation with the benchmark of liquidity (at the level of a portfolio containing all stocks),
(5) the highest time-series correlation of first differences with the benchmark of liquidity (at the level of a portfolio containing all stocks),
(6) the highest pooled cross-sectional time-series correlation with the benchmark of liquidity,
(7) the lowest average value of the root-mean-squared error (RMSE hereafter),
(8) the lowest value of the average mean error.

In this paper, to compare low-frequency liquidity measures in terms of the accuracy of measurement, 14 liquidity proxies were chosen and four ultra-high-frequency measures constitute liquidity benchmarks. The selection of liquidity measures was based primarily on the frequency of their use in the research on liquidity in other stock markets. In addition, some of the measures were chosen because of their good applicability on the Polish stock market [28]. A few proxies used in the study are modifications of existing ones, aiming to fit better to the organisation of trading on the Polish stock exchange. In turn, high-frequency measures, serving as a benchmarks of liquidity, were selected based on other studies on the accuracy of liquidity level measurement by various liquidity proxies (among others [6, 12, 18]).

3.2.1 High-Frequency Liquidity Benchmarks

High-frequency liquidity measures are considered as benchmarks of liquidity due to the fact that they measure liquidity *ex ante* and liquidity measured in such a way is useful in investors' decision-making. The first of such measures used in the study is the relative bid-ask spread computed using the following formula:

$$s_t = \frac{p_t^A - p_t^B}{p_t^M} \tag{3.1}$$

where p^A, p^B and p^M reflect, respectively, the best ask price, the best bid price and the midquote, which is the average of the best bid and the best ask prices.

The values of the bid-ask spread were computed for each transaction. If the value of the spread was nonpositive (equal to zero or negative), it was set as missing. A nonpositive value of the spread indicates that the best ask price is lower than or equal to the best bid price, which indicates that the transaction should occur. The monthly bid-ask spread was calculated as the average of the spread for each transaction in a given month. On the Warsaw Stock Exchange, bid-ask spread calculated in this way reflects the level of price concession that the investor should make in order to execute a trade of a volume not exceeding the volume of the best bid or ask order.

In the study, as a liquidity benchmark, the effective spread was also used. It is computed using the following formula:

$$s_t^{\text{eff}} = \frac{\left|p_t - p_t^M\right|}{p_t^M} \tag{3.2}$$

where p is the transaction price.

The values of the effective spread were computed for each transaction and then averaged monthly. Two different averages were used: simple average and the volume-weighted average. The former version will be marked as s^{eff}, and the latter one as $s^{\text{eff,V}}$.

The aforementioned high-frequency liquidity measures allow us to measure the level of price concession that the investor willing to trade has to accept. The values of the bid-ask spread or effective spread, computed using Formulae (3.1) and (3.2), inform us of the costs incurred by the investor, regardless of the transaction volume. The transaction volume has an effect on the liquidity cost; therefore, a measure taking into account the transaction volume was proposed. This measure is given by the following formula:

$$PI_t = \frac{s_t^{\text{eff}}}{Vol_t} \tag{3.3}$$

The measure described by Eq. (3.3) was calculated for each transaction, while its monthly value was calculated as an arithmetic average.

3.2.2 Low-Frequency Liquidity Proxies

The first of low-frequency liquidity measures analysed is the Amihud's [2] illiquidity measure, which seems to be the most frequently applied liquidity measure in studies on the relationship between liquidity and stock returns. This measure is given by the following formula [2]:

$$\mathrm{ILLIQ}_{im} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{|r_{imt}|}{\mathrm{Vol}_{imt}} \tag{3.4}$$

where D_{im} denotes the number of days with available data for stock i in month m, r is the stock return, and Vol denotes the respective dollar volume denominated in thousands of PLN. To include the stock in the sample, it was required that the data needed to compute the Amihud measure would be available for at least 15 days of each month of the study period.

Due to various weaknesses of the above measure, pointed out among others by Tobek [30], some modifications were introduced. These modifications consist of changing the numerator and/or the denominator, or changing the frequency of its calculation. The first proposed modification subtracts the market return from the stock return in the numerator of the measure:

$$\mathrm{ILLIQ}_{im}^{E} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{|r_{imt} - r_{Mmt}|}{\mathrm{Vol}_{imt}} \tag{3.5}$$

where r_M denotes the market return, approximated by the daily percentage change in the value of the Warsaw Stock Index WIG. Such treatment allows us to eliminate stock price changes resulting from general market movements and eliminates zero estimates for less liquid stocks. It is expected that values of $\mathrm{ILLIQ^E}$ will be higher than values of ILLIQ for less liquid stocks and lower than values of ILLIQ for more liquid ones.

Another modification is aimed at eliminating the underestimation of the short-term pressure of demand or supply on stock prices, which disappears before the end of trading day and is not reflected in the daily rate of return. Such modification consists in replacing the rate of return in Eq. (3.4) with the log of a daily price range:

$$\mathrm{ILLIQ}_{im}^{R} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{\left|\ln\left(p_{imt}^{H} / p_{imt}^{L}\right)\right|}{Vol_{imt}} \tag{3.6}$$

where p^H and p^L denote, respectively, the highest and the lowest stock price observed in day t.

The next two modifications were made in order to take into account the specifics of the schedule and the organisation of the trading session. On the WSE, after closing call, there is additional phase of trading, namely “trading at last”, during which transactions are executed at a fixed price, regardless of the volume of these transactions. So that the measure should consist of adequate components, the trading volume in the denominator should be lowered by the trading volume from the “trading at last” phase of a session. Next, the rate of return in the numerator was decreased by the market return. These two modifications are given by the following formulae:

$$\mathrm{ILLIQ}_{im}^{D} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{|r_{imt}|}{\mathrm{Vol}_{imt} - \mathrm{Vol}_{imt}^{D}} \tag{3.7}$$

$$\mathrm{ILLIQ}_{im}^{ED} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{|r_{imt} - r_{Mmt}|}{\mathrm{Vol}_{imt} - \mathrm{Vol}_{imt}^{D}} \tag{3.8}$$

where Vol^D denotes the trading volume from the "trading at last" phase of a session.

The last proposed modification of Amihud's illiquidity measure concerns the frequency of measurement. Instead of a daily basis, the measurement was made at one-minute intervals. This measure was averaged for the whole month and does not include the "trading at last" phase. Hereafter, it will be marked as $\mathrm{ILLIQ}^{\mathrm{I}}$.

Pástor and Stambaugh [24] developed another liquidity measure—γ. It is supposed to measure the price impact of trading volume, similarly as Amihud's [2] illiquidity measure. However, it is estimated using the following OLS regression [24]:

$$r_{i,d+1,m}^{e} = \theta_{im} + \phi_{im} r_{idm} + \gamma_{im} sign\left(r_{idm}^{e}\right) \mathrm{Vol}_{idm} + \varepsilon_{i,d+1,m} \tag{3.9}$$

Apart from the price impact measures, the study also analysed measures of transaction costs belonging to the group of measures based on the rates of return, price range or zero-return days. Roll's estimator is given by the formula [27]:

$$s_{\mathrm{Roll}} = 2\sqrt{-\mathrm{cov}(\Delta p_t; \Delta p_{t-1})} \tag{3.10}$$

Computing Roll's estimator for each of the 132 months of the study period for each of the companies included in the sample was not possible due to the existing positive values of the autocovariance of the rates of return. Existence of the positive autocovariances makes it impossible to compute the square root in Eq. (3.10), because its value does not belong to a set of real numbers, but to a set of complex numbers. Therefore, in order to eliminate this inconvenience, one author's modification and two modifications existing in the literature were applied. Each of these modifications is a departure from the original Roll's model; therefore, each will be examined in terms of accuracy of measurement on the Warsaw Stock Exchange.

First of the modifications, introduced by Goyenko et al. [18], is about replacing the positive autocovariances of returns with zeros. In such a situation, if a positive autocovariance of returns exists, it is assumed that transaction costs are equal to zero. Formally, this modification is calculated as follows [18]:

$$\mathrm{Roll}^{0} = \begin{Bmatrix} 2\sqrt{-\mathrm{cov}(\Delta p_t; \Delta p_{t-1})}, & \text{if } \mathrm{cov}(\Delta p_t; \Delta p_{t-1}) < 0 \\ 0, & \text{if } \mathrm{cov}(\Delta p_t; \Delta p_{t-1}) \geq 0 \end{Bmatrix} \tag{3.11}$$

Another modification aimed to eliminate the inconveniences related to positive autocovariances of returns and omitting the necessity of extracting the square root from a negative number consists of replacing the value opposite to the autocovariance with its absolute value, i.e.

$$\mathrm{Roll}^{\mathrm{Abs}} = 2\sqrt{|\mathrm{cov}(\Delta p_t;\, \Delta p_{t-1})|} \tag{3.12}$$

A similar solution was introduced by Olbryś [23]. It consists in multiplying the expression on the right side of Eq. (3.12) by the inverse of the sign of the autocovariance of returns. Such a solution has quite a serious disadvantage; namely, if positive autocovariances of returns appear in some periods, it is assumed that the effective spread was negative in these periods. As a rule, spread should not take values less than zero. A modification introduced by Olbryś can therefore be used to approximate the level of liquidity, but not to estimate the transaction costs as a part of liquidity costs. This modification is given by the formula [23]:

$$\mathrm{Roll}^{\mathrm{Olb}} = -2\,\mathrm{sgn}\big[\mathrm{cov}(\Delta p_t;\, \Delta p_{t-1})\big]\sqrt{|\mathrm{cov}(\Delta p_t;\, \Delta p_{t-1})|} \tag{3.13}$$

As the only measure based on the price range, the spread estimator of Corwin and Schultz [11] was analysed. It is given by the formula:

$$HL = \frac{2(\mathrm{e}^{\alpha} - 1)}{1 + \mathrm{e}^{\alpha}} \tag{3.14}$$

where

$$\begin{aligned} \alpha &= \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}} \\ \beta &= \sum_{j=0}^{1}\left[\ln\left(\frac{p_{t+j}^{H}}{p_{t+j}^{L}}\right)\right]^2 \\ \gamma &= \left[\ln\left(\frac{p_{t,t+1}^{H}}{p_{t,t+1}^{L}}\right)\right]^2 \end{aligned} \tag{3.15}$$

For each day in a given month, the values of α, β, γ and the expression HL in line with Eqs. (3.14) and (3.15) were calculated. Next, regarding the authors' guidelines, negative values of the expression HL were replaced with zeros [11]. It allows us to eliminate the possibility of obtaining negative values of the effective spread, which is calculated as the average value of HLs in a given month.

Another analysed measure is included in the group of measures based on zero-return days. It is based on the limited dependent variable (LDV) model of the relationship between the observed (r) and "true" (r^*) rate of return. The parameters of the model are estimated with the maximum likelihood method, and the log of likelihood function has the following form [20]:

$$\sum_{t\in U_1}\left\{-\frac{1}{2}\ln\left(2\pi\sigma^2\right)-\frac{1}{2\sigma^2}(r_t+\alpha_1-\beta r_{mt})^2\right\}+ \\ +\sum_{t\in U_2}\left\{-\frac{1}{2}\ln\left(2\pi\sigma^2\right)-\frac{1}{2\sigma^2}(r_t+\alpha_2-\beta r_{mt})^2\right\}+ \\ +\sum_{t\in U_0}\ln\left(\phi\left(\frac{\alpha_2-\beta r_{mt}}{\sigma}\right)-\phi\left(\frac{\alpha_1-\beta r_{mt}}{\sigma}\right)\right)\rightarrow \max \tag{3.17}$$

where α_{1j} and α_{2j} denote the thresholds for transactions on negative and positive information, respectively.

Spread is then calculated in the following way [20]:

$$\text{LOT}=\alpha_2-\alpha_1 \tag{3.18}$$

In their paper, Lesmond et al. [20] assumed that U_0 are the days in which the stock return was equal to zero ($r_t=0$); the first region (U_1) are days with negative market return and nonzero stock return ($r_t\neq 0$, $r_{mt}<0$); the last region (U_2) are days with a positive market return and nonzero stock return ($r_t\neq 0$, $r_{mt}>0$). Goyenko et al. [18] proposed a slightly different division: region U_0 are days with the stock return equal to zero ($r_t=0$); region U_1 are days with negative ($r_t<0$), and region U_2— with positive ($r_t>0$) stock return. According to Zhao and Wang [31], the method proposed by Lesmond et al. generates large biases that cannot be eliminated even by enlarging the research sample, which indicates the inconsistency of this estimator. In turn, the Goyenko et al. method is more effective and econometrically correct [31]. In order to indicate the best low-frequency liquidity proxy for the Polish stock exchange, both versions of LOT estimator were analysed in the study. The original one will be hereafter denoted as LOT-M, and the version of Goyenko et al. [18] will be denoted as LOT-Y.

Based on the same assumptions as the LOT estimator, but less computationally demanding is the measure introduced by Fong et al. [12]. It does not require the maximisation of the likelihood function, but it uses the cumulative distribution of a standardised normal distribution. This measure is given by the formula [12]:

$$\text{FHT}_m=2\sigma_m\phi^{-1}\left[\frac{1+\text{Zero}_m}{2}\right] \tag{3.19}$$

where $Zero_m$ denotes the proportion of zero-return days in the month m, σ is the standard deviation of daily stock returns, and $\Phi(\cdot)$ is the cumulative distribution of standardised normal distribution.

3.2.3 Data

To calculate all liquidity measures, both data on order flow and daily and intra-daily data on transactions were needed. Data on order flow was delivered by the Warsaw Stock Exchange and covered the period from 1 January 2006 to 31 December 2016. The database contains information on five best bid and five best ask orders before each transaction. In addition, it contains data on transaction volume, price and time. Data on order flow was used to compute ultra-high-frequency liquidity benchmarks. Data on transactions with a one-minute frequency is from the bossa.pl service. They were used to obtain trading volume from the "trading at last" phase and the intra-daily version of Amihud's illiquidity measure. Thus, intra-daily quotations were useful to compute $ILLIQ^{D}$, $ILLIQ^{ED}$ and $ILLIQ^{I}$, which are the modifications of Amihud's measure.

Daily quotations, needed to calculate the remaining part of low-frequency liquidity proxies, originate from the GPWInfoStrefa service. Quotations were corrected by corporate actions, i.e. dividend payouts, subscription rights issuances, splits and reverse splits. This required the creation of a database of these actions, which was created by the author on the basis of the information contained in companies' card, WSE annals, the WSE session archives, Official Quotations of Warsaw Stock Exchange (Ceduła GPW) and GPWInfoStrefa service. The calculations were mainly carried out with the use of a spreadsheet together with the use of Visual Basic for Applications, while R programming was used to compute LOT measures. Operationalisation of the data on order flow required the use of the filtrating programme.

3.3 Empirical Results

3.3.1 Coherence of Liquidity Proxies with Liquidity Benchmarks

The coherence of liquidity proxies with liquidity benchmarks is assessed based on the six measures of correlation. Tables 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6 show the values of the correlations of low-frequency liquidity proxies with high-frequency liquidity benchmarks. The highest correlations are highlighted in bold. The presented values are averaged for the whole period. The highest average cross-sectional correlation with bid-ask spread, effective spread and volume-weighted effective spread has the FHT measure. The LOT measures are characterised by slightly lower correlation, both in the original version (LOT-M) and in the modified one (LOT-Y). Aforementioned measures are also well correlated with the PI measure, which reflects the cost-per-dollar-volume. However, the intra-daily version of Amihud measure ($ILLIQ^{I}$) is best correlated with the PI measure.

It is worth noting that the correlations of ILLIQ and HL measures with volume-weighted effective spread are similar to those presented by Będowska-Sójka [6]. In

Table 3.1 Average cross-sectional correlation of liquidity proxies with liquidity benchmarks

Measure	s	s^{eff}	$s^{\text{eff,V}}$	PI
ILLIQ	0.4397	0.4047	0.3757	0.2782
ILLIQ^{E}	0.4311	0.3946	0.3637	0.2711
ILLIQ^{R}	0.5374	0.4984	0.4582	0.4092
ILLIQ^{D}	0.4021	0.3668	0.3383	0.2373
ILLIQ^{ED}	0.3607	0.3290	0.3010	0.2121
ILLIQ^{I}	0.4547	0.4405	0.4121	**0.7227**
P-S	0.4840	0.4625	0.4334	0.3605
Roll^{0}	0.4451	0.4426	0.4289	0.3708
Roll^{Abs}	0.3902	0.3878	0.3848	0.3324
Roll^{Olb}	0.3803	0.3618	0.3487	0.3013
HL	0.3198	0.3228	0.3100	0.3928
LOT-M	0.5720	0.5505	0.5369	0.4496
LOT-Y	0.5812	0.5526	0.5344	0.4697
FHT	**0.6102**	**0.5823**	**0.5639**	0.4838

Table 3.2 Average cross-sectional Spearman rank correlation of liquidity proxies with liquidity benchmarks

Measure	s	s^{eff}	$s^{\text{eff,V}}$	PI
ILLIQ	0.8806	0.8706	0.8153	0.7296
ILLIQ^{E}	0.8797	0.8674	0.8119	0.7198
ILLIQ^{R}	**0.8937**	**0.8858**	**0.8303**	0.7788
ILLIQ^{D}	0.8758	0.8642	0.8084	0.7169
ILLIQ^{ED}	0.8727	0.8593	0.8031	0.7053
ILLIQ^{I}	0.7509	0.7574	0.7203	**0.9430**
P-S	0.6928	0.7014	0.6865	0.7099
Roll^{0}	−0.0300	−0.0163	−0.0093	0.0103
Roll^{Abs}	0.2140	0.2290	0.2307	0.2467
Roll^{Olb}	0.2343	0.2305	0.2140	0.2001
HL	0.1872	0.2118	0.2149	0.3520
LOT-M	0.5132	0.5163	0.5072	0.5188
LOT-Y	0.5077	0.5010	0.4860	0.4911
FHT	0.5318	0.5256	0.5076	0.5219

Table 3.3 Average time-series correlation of liquidity proxies with liquidity benchmarks (single stock level)

Measure	s	s^{eff}	$s^{eff,V}$	PI
ILLIQ	0.4073	0.3486	0.2943	0.3802
ILLIQ^{E}	0.4121	0.3519	0.3068	0.3969
ILLIQ^{R}	0.4695	0.3984	0.3539	0.4581
ILLIQ^{D}	0.3457	0.2911	0.2563	0.3139
ILLIQ^{ED}	0.2946	0.2497	0.2330	0.2539
ILLIQ^{I}	**0.5606**	**0.4692**	**0.3749**	**0.6370**
P-S	0.3353	0.2940	0.2659	0.3297
Roll^{0}	0.2545	0.2411	0.1977	0.1479
Roll^{Abs}	0.2109	0.1989	0.1773	0.1006
Roll^{Olb}	0.1470	0.1367	0.1147	0.1280
HL	0.2202	0.2046	0.1766	0.1151
LOT-M	0.3294	0.2906	0.2791	0.1903
L1OT-Y	0.3657	0.3090	0.2883	0.2365
FHT	0.4087	0.3471	0.3264	0.2609

Table 3.4 Time-series correlation of liquidity proxies with liquidity benchmarks (portfolio level)

Measure	s	s^{eff}	$s^{eff,V}$	PI
ILLIQ	0.6948	0.4948	0.6725	0.4484
ILLIQ^{E}	0.7083	0.4989	0.6847	0.3931
ILLIQ^{R}	0.7013	0.4882	0.6726	0.4501
ILLIQ^{D}	0.7197	0.5211	0.7087	0.3424
ILLIQ^{ED}	0.4020	0.2836	0.4017	0.1108
ILLIQ^{I}	0.8434	0.6216	0.8090	0.2943
P-S	0.6228	0.4761	0.6202	0.4569
Roll^{0}	0.6562	0.4725	0.6143	0.3994
Roll^{Abs}	0.4114	0.3119	0.3912	0.1813
Roll^{Olb}	0.4754	0.3511	0.4449	0.3630
HL	0.1822	0.1345	0.1689	−0.1281
LOT-M	0.3838	0.2812	0.3617	0.1815
LOT-Y	0.8279	0.5696	0.8051	**0.6353**
FHT	**0.9373**	**0.6348**	**0.9016**	0.6121

Table 3.5 Time-series correlation of first differences liquidity proxies with liquidity benchmarks (portfolio level)

Measure	s	s^{eff}	$s^{\text{eff,V}}$	PI
ILLIQ	0.1971	0.0098	0.1215	0.2468
ILLIQ^{E}	0.2174	0.0085	0.1060	**0.2780**
ILLIQ^{R}	0.2143	0.0438	0.1297	0.2036
ILLIQ^{D}	0.2699	0.0295	0.1815	0.2142
ILLIQ^{ED}	0.1849	0.0114	0.1094	0.0937
ILLIQ^{I}	0.4525	0.0445	0.3269	0.1659
P-S	0.1578	0.1014	0.1758	0.2371
Roll^{0}	0.4444	0.1623	0.4133	0.1500
Roll^{Abs}	**0.5283**	**0.1951**	**0.5273**	0.0824
Roll^{Olb}	−0.0887	0.0181	−0.1279	0.0886
HL	0.4678	0.0653	0.4685	−0.0404
LOT-M	−0.0602	0.0018	−0.0376	−0.0980
LOT-Y	0.1265	−0.0230	0.0699	0.1180
FHT	0.4798	−0.0042	0.3508	0.2225

Table 3.6 Pooled cross-sectional time-series correlation of liquidity proxies with liquidity benchmarks

Measure	s	s^{eff}	$s^{\text{eff,V}}$	PI
ILLIQ	0.4474	0.2345	0.3928	0.3699
ILLIQ^{E}	0.4435	0.2325	0.3906	0.3518
ILLIQ^{R}	0.4430	0.2287	0.3886	0.4882
ILLIQ^{D}	0.3563	0.1865	0.3161	0.2306
ILLIQ^{ED}	0.1078	0.0577	0.0955	0.0670
ILLIQ^{I}	0.4510	0.2388	0.4111	0.2729
P-S	0.4301	0.2264	0.3903	0.3108
Roll^{0}	0.8248	0.4357	0.7835	0.4736
Roll^{Abs}	0.7498	0.4034	0.7019	0.4225
Roll^{Olb}	0.6222	0.3378	0.5775	0.3197
HL	0.4392	0.2394	0.4311	0.1772
LOT-M	0.3909	0.2134	0.3632	0.1568
LOT-Y	0.8174	0.4410	0.7501	0.5260
FHT	**0.9367**	**0.5058**	**0.8717**	**0.5711**

her sample, Amihud's ILLIQ was correlated with spread at 0.3523 (0.3757 in this study), and the Corwin and Schultz estimator (HL) was correlated with spread at 0.3378 (0.3100 in the present study). However, the correlations for the two other measures differ quite significantly from each other. Roll's estimator (marked here as $Roll^0$) in Będowska-Sójka's [6] study was correlated with volume-weighted effective spread at 0.1279, and the LOT measure (the original version, i.e. LOT-M) was correlated at 0.2739. Corresponding correlations in the present study are equal to 0.4289 and 0.5369. Discrepancies are probably the result of differences in the research sample.

The analysis of cross-sectional correlation of liquidity measures is complemented by the cross-sectional Spearman rank correlation. The highest rank correlation with the measures of spread in the whole study period has $ILLIQ^R$ measure. Slightly lower values of the coefficients have the other versions of Amihud's measure, except for $ILLIQ^I$. The latter one is in turn best correlated with the PI measure, similar to the case of Pearson correlation.

Taking into account correlation over time, the $ILLIQ^I$ measure is most strongly linked with the high-frequency liquidity measures at the level of a single stock. Subsequently, these measures are $ILLIQ^R$, $ILLIQ^E$, FHT, ILLIQ and LOT-Y. Also the remaining versions of Amihud's measure are characterised by high time-series correlation with liquidity benchmarks.

At the portfolio level, in which liquidity was calculated as a simple average of liquidity of all stock; the best correlated with spread measures turns out to be the FHT measure. The correlations of $ILLIQ^I$ and LOT-Y, as well as the other versions of Amihud's measure, are slightly weaker.

Changes in the liquidity of the portfolio are best reflected by $Roll^{Abs}$ and $ILLIQ^E$ measures. The former one reflects the changes in the spread measures well, while the latter one shows the changes in the level of the PI measure. The FHT measure also has the highest coefficients of pooled cross-sectional time-series correlation with three measures of the spread and the PI measure. Slightly lower values of these coefficients can be observed in the case of $Roll^0$ and LOT-Y measures.

3.3.2 Estimation Errors

Two measures of estimation errors are utilised in the study: namely, the root-mean-squared error of estimation and mean error of estimation. Estimation errors of liquidity measures are presented in Tables 3.7 and 3.8. The lowest values of the error are highlighted in bold. The root-mean-squared errors of estimation (RMSE) of a given measure should be analysed jointly with the values of mean errors of estimation. Working solely on the basis of the value of RMSE, one cannot discern if this is a result of a different order of magnitude than the benchmark, or if it is an effect of the inaccurate reflection of the level of liquidity. The lower the value of RMSE and the lower the absolute value of mean error, the better the liquidity proxy reflects the liquidity costs measured with high-frequency measures.

Table 3.7 Root-mean-squared errors of estimations

Measure	s	s^{eff}	$s^{eff,V}$	PI
ILLIQ	1.3027	2.1941	2.526	139.215
ILLIQ^E	1.4845	2.6643	2.8012	179.908
ILLIQ^R	1.1197	1.5563	1.6204	54.202
ILLIQ^D	1.7268	3.1990	3.3936	944.037
ILLIQ^{ED}	8.1118	16.6453	18.4906	6074.38
ILLIQ^I	13.0104	26.8263	29.0302	1008.02
P-S	0.9972	**0.9945**	**0.9944**	**2.4536**
Roll^0	2.7818	4.8853	5.0162	7849.58
Roll^{Abs}	3.9266	6.9200	7.0432	11,646.5
Roll^{Olb}	4.5317	7.5442	7.6663	11,646.9
HL	1.3237	2.3958	2.4281	4947.61
LOT-M	3.4049	6.6718	6.9283	8594.85
LOT-Y	1.4923	2.9064	3.1636	2514.15
FHT	**0.7773**	1.2854	1.3795	1822.95

Table 3.8 Mean errors of estimations

Measure	s	s^{eff}	$s^{eff,V}$	PI
ILLIQ	−0.7627	−0.4955	−0.4572	42.389
ILLIQ^E	−0.6645	0.2932	−0.2342	58.226
ILLIQ^R	−0.8613	−0.7082	−0.6854	25.246
ILLIQ^D	−0.6067	**−0.1741**	**−0.1084**	156.434
ILLIQ^{ED}	0.3830	1.8806	2.1583	802.12
ILLIQ^I	4.5892	10.2146	10.8545	850.37
P-S	−0.9972	−0.9943	−0.9941	**−0.0676**
Roll^0	0.5425	1.8041	1.8915	2443.7
Roll^{Abs}	1.8185	4.0733	4.1814	4515.8
Roll^{Olb}	−0.73334	−0.4650	−0.3984	371.52
HL	0.1942	1.1491	1.1905	1929.3
LOT-M	1.7578	4.1401	4.2955	3584.4
LOT-Y	**−0.0507**	0.8096	0.8901	985.04
FHT	−0.3429	0.2490	0.2998	739.31

According to both RMSE and the mean error of estimation, the best proxy for the PI measure is Pástor–Stambaugh's γ. The mean error of estimation is −0.0676, which means that the values of P-S measure are on average 6.76% lower than the corresponding values of PI measure. Pástor–Stambaugh's measure is also characterised by relatively low values of RMSE when reflecting the measures of a spread. However, taking into account the mean error of estimation, such value is due to the fact that it has a much smaller order of magnitude and gives estimates on average 99.5% lower than the values of the spread. Therefore, despite being characterised by the lowest RMSEs in the estimation of the effective spread and the volume-weighted effective spread, the P-S measure cannot be considered as a good reflection of the level of liquidity costs. In this case, when considering the RMSE, the best proxy for the spread measures (also the bid-ask spread) is the FHT measure.

Summarising the results of the study of the accuracy of the measurement of liquidity by low-frequency liquidity proxies, one may indicate a few proxies that are characterised by good correlation with benchmarks and low estimation errors. As the best ones, one should indicate the following measures: FHT, $ILLIQ^I$, $ILLIQ^R$, LOT-Y and P-S. The two latter proxies measure liquidity well mainly due to the low estimation errors.

3.3.3 Ranking of Liquidity Measures

When summarising the considerations regarding the usefulness of individual measures to proxy for liquidity on the Polish stock market, one should compare its applicability with the accuracy with which they measure liquidity. As mentioned, the applicability of selected liquidity measures is presented and assessed in the paper of Stereńczak [28]. Table 3.9 contains the comparison of low-frequency liquidity measures, which is basically a ranking of liquidity proxies. Similar ranking is done (among others) by Bleaney and Li [7]. Each measure was evaluated in terms of its applicability and accuracy of measurement. Within each criterion, 0–4 points were awarded. Points within the scope of the applicability of the measure have been granted to take care that the number of points awarded reflects the most objective assessment as possible. Measures correlated with the benchmark at less than 0.2 received 0 points; a correlation between 0.2 and 0.4 was awarded with 1 point; between 0.4 and 0.6—2 points; between 0.6 and 0.8—3 points; above 0.8—4 points. The measure which was the best in terms of estimation errors received 4 points, two more measures—3 points, measures in places 4 to 7—2 points, measures in places 8 to 12—1 point, and the rest—0 points.

Each criterion has an assigned weight, which reflects its importance in studies on asset pricing. According to Goyenko et al. [18], the most important ones are criteria related to the coherence with high-frequency benchmarks. Therefore, to each correlation criterion, a 10% weight was assigned. Thus, the criteria related to the correlation with liquidity benchmarks are jointly assigned with 60% weight. Both of the criteria referring to estimation errors were assigned weights of 5%, due to

Table 3.9 Comparison of the usefulness of low-frequency liquidity measures

Measure	Fitting to market	Data requirements	Computational efforts	Cross-sectional correlation	Cross-sectional rank correlation	Time-series correlation (single stock level)	Time-series correlation (portfolio level)	Time-series correlation of first differences	Pooled cross-sectional time-series correlation	RMSE	Mean error of estimation	SUM
ILLIQ	2	4	4	1.5	3.8	1.3	2.5	0.3	1.3	2.3	2	2.16
$ILLIQ^{E}$	3	3.5	3.5	1.3	3.8	1.3	2.3	0.5	1.3	2	2.5	2.23
$ILLIQ^{R}$	2.5	3.5	3.5	2	3.8	1.5	2.5	0.5	1.5	2.3	2	**2.29**
$ILLIQ^{D}$	2.5	2	3	1.3	3.8	1	2.3	0.5	0.75	1.3	3	1.89
$ILLIQ^{ED}$	3.5	2	3	1	3.8	1	1.3	0.5	0	0.3	1.3	1.70
$ILLIQ^{I}$	4	2	3	2.3	3.3	2	3	0.8	1.5	0.5	0.3	**2.26**
P-S	3	3	2	1.8	3	1	2.5	0.3	1.3	3.3	1.8	2.08
$Roll^{0}$	1	4	3.5	1.8	0	0.5	2.3	1	2.8	1	1	1.65
$Roll^{Abs}$	1	4	3.5	1	1	0.3	1	1	2.5	0	0	1.40
$Roll^{Olb}$	1	4	3	1	1	0	1.5	0	1.8	0	1.8	1.31
HL	4	3	3	1	0.8	0.5	0	1	1.3	1.8	1.5	1.66
LOT-M	3	3	1	2	2	0.8	0.8	0	0.8	0.8	0	1.46
LOT-Y	3	3	1	2	2	1	3.3	0	2.8	1	2.3	2.06
FHT	3	3	2.5	2.3	2	1.3	3.5	1	3	2.8	2.5	**2.44**

the fact that in the studies on the relationship between the level of liquidity and rates of return, the accuracy of estimating liquidity costs is of secondary importance. Another important criterion is also the fulfilment of the assumptions adopted in the construction of the measure, and hence the conformity to market organisation—this criterion is assigned a weight of 15%. The criterion related to the data requirements is assigned with a weight of 10%. This is not a very important criterion, but when constructing a set of weights, the liquidity measures for the needs of investors were also taken into account, albeit to a lesser extent. The smallest weight has been assigned to the criterion of computational efforts, due to the fact that this criterion is important primarily for investors and is less important in scientific research. The table shows the average number of points obtained when comparing each of the four benchmarks. In the last column, the average number of points awarded under all criteria is given, with specific weights. In the presented ranking, the largest number of points was granted to the FHT measure. The ILLIQ^{R} measure was ranked second. The third place in the ranking taking into account the estimation errors was the intra-day version of Amihud's measure (ILLIQ^{I}).

3.4 Summary and Conclusions

This paper was aimed at indicating the most appropriate proxy for liquidity for the purposes of asset pricing studies on the Warsaw Stock Exchange. The measurement of liquidity is extremely important from the investors' point of view, as well as from the scientific researchers' perspective. Equally as important, measurement of liquidity is difficult, what results from its multidimensional and elusive nature. Using a set of eleven assessment criteria, fourteen liquidity proxies were assessed in terms of their usefulness in measuring liquidity on the Warsaw Stock Exchange. Each criterion was assigned with a specific weight, which was justified by the significance of this criterion in measuring liquidity for the purposes of asset pricing studies. The weights may seem arbitrary, as all interested parties (e.g. investors, researchers) are able to freely modify the presented set of weights such that it will correspond to the assessment of liquidity proxies for other purposes, e.g. studies on market efficiency.

In the presented empirical study, the measure developed by Fong et al. [12]—FHT—was indicated as the most useful in asset pricing studies on the Warsaw Stock Exchange. FHT measure reflects the average price concession that has to be made by an investor in order to trade immediately, regardless of the volume of the transaction. The next two best-performing measures are two modifications of Amihud's [2] illiquidity measure. These modifications are intended to improve the fit of the measure to the market and include: replacing the absolute value of return in the numerator of the measure with the absolute value of the log of the price range (ILLIQ^{R}), and computing the measure with intra-daily frequency (ILLIQ^{I}). The study has its own limitations, the largest of which is the arbitrariness of the weights. Nevertheless, 70% of the final assessment of each proxy for liquidity is a result of objective criteria, i.e. the accuracy of measurement.

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