Chapter 15
Application Quantile-Based Risk Measures in Sector Portfolio Analysis—Warsaw Stock Exchange Approach

Grażyna Trzpiot

Abstract The measurement of financial risk has been one of the main goals of the investors as well as actuaries and insurance practitioners. Measuring the risk of a financial portfolio involves firstly estimating the loss distribution of the portfolio, next computing chosen risk measure. In the resent study, the robustness of risk measurement procedures and their sensitivity into point out for the dataset in present. The results show a gap between the subadditivity and robustness of risk measurement procedures. We apply into analyses alternative risk measurement procedures that possess the robustness property. The quantile-based risk measures have been applied in sector portfolio analysis for the dataset from Warsaw Stock Exchange.

Keywords Risk measurement · Value-at-risk · Expected shortfall · Robustness

JEL Classification G11 · C19

15.1 Introduction

The main aim of quantitative modeling in finance is to quantify the risk, especially the risk of financial portfolios. The Basel Committee guidelines for risk-based requirements for regulatory capital, and frequent use of Value-at-Risk, had created related risk measurement methodologies and methodologies for measuring of the risk of financial portfolios [1–3, 8]. Generally, in theoretical approach to risk measurement, a risk measure is represented as an assignment to each random payoff a number (a measure of risk). The goal in most of theoretical approach has been on the properties of defined maps and requirements for the risk measurement procedure to be coherent, in a static or dynamic setting. Usually, in real applications, the probability distribution is unknown and should be estimated from (historical) data, which means as part of the risk measurement procedure. Thus, in practice, measuring the risk of

G. Trzpiot (✉)
University of Economics in Katowice, Katowice, Poland
e-mail: grazyna.trzpiot@ue.katowice.pl

W. Tarczyński and K. Nermend (eds.), *Effective Investments on Capital Markets*, Springer Proceedings in Business and Economics,
https://doi.org/10.1007/978-3-030-21274-2_15

a financial portfolio involves two steps: estimating the loss distribution of the portfolio from available observations and computing a risk measure that we chosen for measures the risk of this loss distribution. Estimation method connected with these procedure errors on the portfolio loss distribution can have an important impact on the final results on risk measures [9, 10].

The main goal of this paper is an application of the two-parameter quantile-based risk measure to sector portfolio analysis for data from Warsaw Stock Exchange. Next, the evaluation of the empirical results points out the existing gap between the subadditivity and robustness of risk measurement procedures.

15.2 Estimation of Risk Measures

The payoff of a portfolio over a specified horizon may be represented as a random variable $X \in L \subset L^1(\Omega, F, P)$, where negative values are assumed to be a convex cone containing all constants. A risk measure on L is a map $\rho : L \to R$ assigning to each $X \in L$, a number representing measure of risk. Artzner et al. [3] defined the axioms of coherent risk measures.

Below we list some properties for risk measures: for $X; Y \in L$,

(a) Monotonicity: $\rho(X) \leq \rho(Y)$ if $X \leq Y$;
(b) Cash invariance: $\rho(X + c) = \rho(X) + c$ for any $c \in R$;
(c) Positive homogeneity: $\rho(\lambda X) = \lambda \rho(X)$ for any $\lambda > 0$;
(d) Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$;
(e) Law determination: $\rho(X) = \rho(Y)$ if X and Y have the same distribution.

Definition 1 A monetary risk measure satisfying conditions (a) and (b), and a coherent risk measure is a risk measure satisfying (a)–(d).

For a set $C \subset D \subset L^1$ representing the set of admissible (robust) return distributions, we can notice according to the literature [4] the condition of qualitative robustness of a risk estimator and use it to examine the robustness of the chosen for application risk estimators.

Definition 2 A risk estimator $\hat{\rho}$ is C-robust at F (the empirical return distributions) if, for any $\varepsilon > 0$, there exist $\delta > 0$ and $n_0 \geq 1$ such that, for all $G \in C$, $d(G, F) \leq \delta$ where d is the Lévy distance.[2]

When $C = D$, then we have situation not interesting in econometric or financial applications[3] since requiring robustness against all perturbations of the distributions F that means it is restrictive and excludes estimators with the lower break point such as the sample mean. In application, first we have to estimate the return distribution F of the portfolio from available data and then apply the risk measure ρ to this

[1] $D \subset L$ is the convex set of cumulative distribution functions (cdf) on R.

[2] Huber [12].

[3] If $C = D$, we have qualitative robustness called asymptotic robustness as outlined [12].

Table 15.1 Behavior of sensitivity functions for some risk estimators

Risk estimator	Dependence in z of S(t)
Historical VaR	Bounded
Gaussian ML for VaR	Quadratic
Laplace ML for VaR	Linear
Historical expected shortfall	Linear
Gaussian ML for expected shortfall	Quadratic
Laplace ML for expected shortfall	Linear

Source Cont et al. [4]

distribution. As the estimation of the loss distribution $F(X)$, we can use an empirical distribution from a historical or simulated sample (e.g., Monte Carlo) or a parametric form whose parameters are estimated from available data. Coherent measures as ES_α has a non-robust historical estimator [4]. In this paper, authors proposed a robust family of risk estimators by modifying its definition.

Definition 3 Consider $0 < \alpha_1 < \alpha_2 < 1$; we can notice the robust risk measure

$$\frac{1}{\alpha_2 - \alpha_1} \int_{\alpha_1}^{\alpha_2} \mathrm{VaR}_u(F)\mathrm{d}u \tag{15.1}$$

and, respectively, a discrete version of the above risk measure:

$$\frac{1}{k} \sum_{j=1}^{k} \mathrm{VaR}_{u_j}(F), 0 < u_1 < \cdots < u_k < 1 \tag{15.2}$$

We will call ρ_{eff} the effective risk measure[4] associated with the risk estimator $\hat{\rho}$. In order to quantify the degree of robustness of a risk estimator, we can notice the concept of the sensitivity function [4]. The function $S(t; F)$ measures the sensitivity of the risk estimator to the addition of a new data point in a large sample.

Definition 4 (*sensitivity function of a risk estimator*) The sensitivity function of a risk estimator defined as a function of distribution F belongs to set of all effective risk measure distribution (D_{eff}) is the function defined by

$$S(t) - S(t; F) = \lim_{\varepsilon \to 0^+} \frac{\rho_{\mathrm{eff}}(\varepsilon\delta_t + (1-\varepsilon)F) - \rho_{\mathrm{eff}}(F)}{\varepsilon} \tag{15.3}$$

for any $t \in R$ such that the limit exists. See Table 15.1.

[4]In other words, while ρ is the risk measure, we are interested in computing.

15.3 Two-Parameter Quantile-Based Risk Measures

The one-parameter families of risk measures, VaR and ES, can be noticed as a more general two-parameter family of risk measures, called the Range-Value-at-Risk (RVaR). The family of RVaR was introduced in Cont et al. [4].[5] This transformation into RVaR was used by Embrechts et al. [6] to understand properties and comparative advantages of risk measures. It helps on application RVaR as the underlying risk measures in the real problem. Measures as VaR, ES, and RVaR can be represented as average quantiles of a random variable.

Definition 5 For $X \in L$, the RVaR at level $(\alpha;\beta) \in R_+^2$ is defined as

$$\mathrm{RVaR}_{\alpha,\beta} = \begin{cases} \dfrac{1}{\beta} \displaystyle\int_{\alpha}^{\alpha+\beta} \mathrm{VaR}_u(F)\mathrm{d}u & \text{if } \beta > 0 \\ \mathrm{VaR}_\alpha(F) & \text{if } \beta = 0 \end{cases} \tag{15.4}$$

The family of risk measures Range-Value-at-Risk (RVaR) is the family of the truncated average quantiles of a random variable. $\mathrm{RVaR}_{\alpha;\beta}$ is continuous with respect to convergence in distribution (weak convergence). RVaR belongs to the large family of distortion risk measures.[6]

Definition 6 For α, β; $\in [0; 1)$ and $\alpha + \beta < 1$, $\mathrm{RVaR}_{\alpha;\beta}$ belongs to the class of distortion risk measures, that is, risk measures ρ_h of the Stieltjes integral form

$$\rho_h(X) = \int_0^1 \mathrm{VaR}_\alpha(X)\mathrm{d}h(\alpha) \tag{15.5}$$

for some non-decreasing and left-continuous function h: $[0; 1] \to [0; 1]$ satisfying $h(0) = 0$ and $h(1) = 1$, such that the above integral is properly defined. Here h is called a distortion function.

For α, β; $\in [0; 1)$ and $\alpha + \beta < 1$, the distortion function of $\mathrm{RVaR}_{\alpha;\beta}$ (X) is given by

$$h^{(\alpha,\beta)}(t) = \begin{cases} \min\left\{ I_{\{t>\alpha\}} \dfrac{t-\alpha}{\beta}, 1 \right\} & \text{if } \beta > 0 \\ I_{\{t>\alpha\}} & \text{if } \beta = 0 \end{cases} \quad t \in [0, 1] \tag{15.6}$$

For application on real data, especially in portfolio analysis, the important are some of the relationship between the individual RVaR and the aggregate RVaR.

[5] For any $0 < \alpha_1 < \alpha_2 < 1$, we can notice as $\beta = \alpha_2 - \alpha_1$.

[6] Kusuoka [15], Song and Yan [17], Dhaene et al. [5], Grigorova [11], Wang et al. [18].

Theorem 1 [6]

For any $X_1, \ldots, X_n \in L$ and any $\alpha_1, \ldots, \alpha_n; \beta_1, \ldots, \beta_n > 0$, we have

$$\mathrm{RVaR}_{\sum_{i=1}^{n}\alpha_i, \forall_{i=1}^{n}\beta_i}\left(\sum_{i=1}^{n} X_i\right) \leq \sum_{i=1}^{n} \mathrm{RVaR}_{\alpha_i\beta_i}(X_i) \tag{15.7}$$

By setting $\alpha_1 = \cdots = \alpha_n = 0$ and $\beta_1 = \cdots = \beta_n$, Theorem 15.1 reduces to the classic subadditivity of ES. By setting $\beta_1 = \cdots = \beta_n = 0$, we obtain the following inequality for VaR.

For any $X_1, \ldots, X_n \in L$ and any $\alpha_1, \ldots, \alpha_n > 0$, we have

$$\mathrm{VaR}_{\sum_{i=1}^{n}\alpha_i}\left(\sum_{i=1}^{n} X_i\right) \leq \sum_{i=1}^{n} \mathrm{VaR}_{\alpha_i}(X_i) \tag{15.8}$$

For $n = 2$, we can notice

$$\mathrm{RVaR}_{\alpha_1+\alpha_2,\beta_1\vee\beta_2}(X_1 + X_2) \leq \mathrm{RVaR}_{\alpha_1,\beta_1}(X_1) + \mathrm{RVaR}_{\alpha_2\beta_2}(X_2) \tag{15.9}$$

for all $X_1; X_2 \in L; \alpha_1; \alpha_2; \beta_1; \beta_2 \in R_+$. This subadditivity involves a combination of the summation of the random variables $X_1, \ldots, X_n \in L$ and the summation of the parameters $(\alpha_1; \beta_1), \ldots, (\alpha_n; \beta_n) \in R_+^2$ with respect to the two-dimensional additive operation (+; v). Note that v-operation is known as the tropical addition in the max-plus algebra [16].

In portfolio analysis, we had to find the optimal allocations for the corresponding aggregate risk value for the RVaR family of risk measures. The following result proved by Embrechts et al. [6] can solve this problem.

Definition 7 The inf-convolution of n risk measures $\rho_1, \ldots, \rho_n$ is a risk measure noticed as

$$\square_{i=1}^{n}\rho_i(X) := \inf\left\{\sum_{i=1}^{n} \rho_i(X_i) : (X_1, \ldots, X_n) \in A_n(X)\right\} \tag{15.10}$$

That is, the inf-convolution of n risk measures is the infimum over *aggregate risk* values for all possible allocations.

Definition 8 For risk measures $\rho_1, \ldots, \rho_n$ and $X \in L$:

(i) An n-tuple $(X_1, \ldots, X_n) \in A_n(X)$ is called an optimal allocation of X if

$$\Box_{i=1}^{n}\rho_i(X_i) = \sum_{i=1}^{n}\rho_i(X_i) \tag{15.11}$$

(ii) An n-tuple $(X_1, \ldots, X_n) \in A_n(X)$ is called a *Pareto-optimal allocation* of X if for any $(Y_1, \ldots, Y_n) \in A_n(X)$ satisfying $\rho_i(Y_i) \le \rho_i(X_i)$ for all $i = 1; \ldots, n$, we have $\rho_i(Y_i) = \rho_i(X_i)$ for all $i = 1; \ldots, n$.

Now we can notice the theorem that has been used in portfolio optimization.

Theorem 2 [6]
For any $X_1, \ldots, X_n \in L$ and any $\alpha_1, \ldots, \alpha_n; \beta_1, \ldots, \beta_n > 0$, we have

$$\Box_{i=1}^{n}\mathrm{RVaR}_{\alpha_i\beta_i}(X) = \mathrm{RVaR}_{\sum_{i=1}^{n}\alpha_i, \forall_{i=1}^{n}\beta_i}(X) \tag{15.12}$$

Now we can notice portfolio risk in the presence of uncertainty of distribution F by using the resulting aggregate risk value. The assumption is that the distribution of the total risk $X \in L$ is misspecified. That in general implies problems for estimation using VaR as a risk measures but not for RVaR or ES. This relates to the issue of the robustness of VaR and RVaR, for a relevant discussion on robustness properties for risk measures.[7] Instead of the robustness of the risk measures themselves, we can write the robustness of the optimal allocation.

Theorem 3 [6]
For risk measures $\mathrm{RVaR}_{\alpha_1\beta_1}, \ldots, \mathrm{RVaR}_{\alpha_n\beta_n}$, $(\alpha_i; \beta_i) \in [0; 1)$, $\alpha_i + \beta_i > 0$, $i = 1, \ldots, n$, $\sum_{i=1}^{n}\alpha_i + \forall_{i=1}^{n}\beta_i < 1$ and a doubly continuous random variable $X \in L$:

(i) There exists an L^1-robust optimal allocation of X if and only if $\beta_1, \ldots, \beta_n > 0$.
(ii) If X is bounded, then there exists an L^∞-robust optimal allocation of X if and only if $\beta_1, \ldots, \beta_n > 0$.

Assuming $\sum_{i=1}^{n}\alpha_i + \forall_{i=1}^{n}\beta_i < 1$, a Pareto-optimal allocation for any $X_1, \ldots, X_n \in L$ can be constructed explicitly as in Theorem 15.2, with the aggregate risk value

$$\sum_{i=1}^{n}\mathrm{RVaR}_{\alpha_i\beta_i}(X) = \mathrm{RVaR}_{\sum_{i=1}^{n}\alpha_i, \forall_{i=1}^{n}\beta_i}(X) \tag{15.13}$$

15.4 Application RVaR in Sector Portfolio Analysis

Sector portfolio analysis was dedicated to food sector. WIG food is a sector index listed on the Warsaw Stock Exchange, containing companies that participate in the

[7] Cont et al. [4], Kou et al. [13], Krätschmer et al. [14], and Embrechts et al. [7].

Table 15.2 Parameters of rate-of-return distribution

	ASTARTA	COLIAN	GOBARTO	HELIO	IMCOMPA
R	0.001	0.000	0.001	0.003	0.002
V	0.000	0.000	0.000	0.001	0.000
S	0.019	0.013	0.022	0.032	0.016
Skewness	0.512	0.571	−1.838	1.966	0.362
Kurtosis	1.892	4.344	46.205	10.51	3.385
	INDYKPOL	KANIA	OTMU	MILKILAND	MBWS
R	0.000	0.000	−0.001	0.001	−0.001
V	0.000	0.000	0.001	0.001	0.001
S	0.022	0.019	0.025	0.034	0.023
Skewness	0.931	0.327	0.819	1.200	−1.407
Kurtosis	6.676	1.406	6.553	6.158	10.87
	MILKILAND	MBWS	OTMU	PEPEES	WAWEL
R	0.001	−0.001	−0.001	0.003	0.000
V	0.001	0.001	0.001	0.001	0.000
S	0.034	0.023	0.025	0.028	0.017
Skewness	1.200	−1.407	0.819	2.243	0.159
Kurtosis	6.158	10.87	6.553	10.941	7.036

Source Main calculation

WIG index. The base date for the WIG food index was set as December 31, 1998. The subindex is characterized by the same methodology with the main WIG index. This means that it is an income index, and when calculating it, you should take into account both the prices of the shares it contains, as well as the right to collect and the income from dividends. The WIG food index consists of 23 companies, of which 15 were selected for analysis, which brings together 86.37% of the total shares in the portfolio and almost total shares in the market, as their sum amounts to 97.80%. The surveyed period from February 18, 1 to February 19, 2018, consisted of 503 observations of closing prices for each of the companies.

The use of risk measures requires examining the types of rates of return distributions. The consistency of the distribution of rates of return with the hypothetical distribution was checked, which is necessary when using quantile risk measures. For this purpose, the Kolmogorov–Smirnov test was used, with the help of which the hypothesis on the compatibility of the distributions of rates of return with both normal and lognormal distribution was verified. In each case, the significance of the test is less than the assumed level of significance of 0.05. This means that distributions of the rates of return are consistent with the normal or lognormal distribution. Next, the parameters of rate-of-return distribution for each of the companies were calculated (Table 15.2), especially the third central moment, which is a measure of the asymmetry of the observed rates of return.

Table 15.3 ES values for selected quantiles

ES	ASTARTA	HELIO	IMCOMPANY	KSGAGRO	PEPEES	Portfolio
0.95	0.0454	0.0821	0.0354	0.0813	0.0704	0.0498
0.96	0.0477	0.0897	0.0370	0.0860	0.0774	0.0531
0.97	0.0511	0.0969	0.0394	0.0920	0.0870	0.0574
0.98	0.0537	0.1077	0.0428	0.0984	0.1005	0.0628
0.99	0.0570	0.1334	0.0486	0.1089	0.1259	0.0727

Source Main calculation

Table 15.4 RVaR measurement values for selected value values (α, β)

RVaR	ASTARTA	HELIO	IMCOMPANY	KSGAGRO	PEPEES	Portfolio
(0.95, 0)	0.0368	0.0599	0.0293	0.0652	0.0457	0.0384
(0.96, 0.01)	0.0417	0.0717	0.0312	0.0736	0.0542	0.0433
(0.97, 0.01)	0.0482	0.0786	0.0348	0.0835	0.0675	0.0498
(0.98, 0.01)	0.0537	0.1077	0.0428	0.0984	0.1005	0.0628
(0.99, 0.01)	0.0610	0.1467	0.0511	0.1488	0.1325	0.0793

Source Main calculation

Next the optimal portfolio was built; it has been assumed that the expected rate of return on the portfolio had to be greater than or equal to 0.001. In addition, it was assumed that the number of companies in the portfolio should be in the range from 5 to 7, and therefore, simulations were carried out to finally choose the optimal solution. Wallet received the optimal parameters. The following companies were included in the portfolio companies: IMCOMPANY (43,3%), ASTARTA (28%), PEPEES (14,8%), HELIO (0,77%), and KSGAGRO (0,61%). Portfolio parameters $E(Rp) = 0.001860$, $V(Rp) = 0.000115$, $S(Rp) = 0.010713$.

Following to final step, an assessment of the risk of the designated portfolios with the use of quantitative ES and RVaR risk measures has been made. Consideration of all possible combinations of values (α, β) is a demanding task. In order to secure the capital of the designated portfolio, we save selected results of the application of the downside risk measures presented in the previous points.

Analyzing the obtained results, it can be concluded that the highest levels of capital collateral are determined by the ES measure, followed by RVaR with a fixed value of $\beta = 0.01$. By changing the value of β and a fixed value of α, the lowest levels of capital collateral were obtained. When we used robust estimator, then we uses specific technique on a tail of the distribution of the rate of return. Presented chosen results (Tables 15.3, 15.4 and 15.5) confirm general rules, described in Sect. 15.2. We had to take into account robustness of applied quantiles risk measures (Table 15.6).

In the presented application of the quantile risk measures on the portfolios, we based on the selected sector. Portfolios from a selected sector were analyzed, and the variability of the distribution of the rate of return in the audited period was not so much significant. In the surveyed sector, all returns of the rate of return were

Table 15.5 RVaR measurement values for selected value values (α, β)

RVaR	ASTARTA	HELIO	IMCOMPANY	KSGAGRO	PEPEES	Portfolio
(0.95, 0)	0.0368	0.0599	0.0293	0.0652	0.0457	0.0384
(0.95, 0.01)	0.0399	0.0650	0.0304	0.0699	0.0504	0.0411
(0.95, 0.02)	0.0425	0.0693	0.0321	0.0744	0.0566	0.0441
(0.95, 0.03)	0.0454	0.0821	0.0354	0.0813	0.0704	0.0498
(0.95, 0.04)	0.0454	0.0821	0.0354	0.0813	0.0704	0.0498

Source Main calculation

Table 15.6 RVaR measurement values for selected value values (α, β) for portfolio

ES	Portfolio	RVaR	Portfolio	RVaR	Portfolio
0.95	0.0498	(0.95, 0)	0.0384	(0.95, 0)	0.0384
0.96	0.0531	(0.96, 0.01)	0.0433	(0.95, 0.01)	0.0411
0.97	0.0574	(0.97, 0.01)	0.0498	(0.95, 0.02)	0.0441
0.98	0.0628	(0.98, 0.01)	0.0628	(0.95, 0.03)	0.0498
0.99	0.0727	(0.99, 0.01)	0.0793	(0.95, 0.04)	0.0498

Source Main calculation

characterized by a significant asymmetry, which means volatility in the tail of the returns. These properties can have strong impact on the results.

15.5 Conclusion

In this paper, we present that the estimation properties as robustness and sensitivity are important and need to be accounted to the dataset, with the same attention as the coherence properties. An unstable or non-robust risk estimator can be useless in practice, never less it has to be related to a coherent measure of risk. Regulatory risk measures, as VaR and ES, in parametric estimation procedures for VaR and ES lead to non-robust estimators. On the other hand, weighted averages of historical VaR have robust empirical estimators. Historical VaR is a qualitatively robust estimation procedure. The family of RVaR was introduced in the context of robustness properties of risk measures. This family of two-parameter risk measures (RVaR) can be seen as a bridge connecting VaR and ES, which are the two most popular regulatory risk measures. Measures as VaR, ES, and RVaR can be represented as average quantiles of a random variable.

In the recent research, we obtain original results: We establish the level of RVaR as a measurement values for selected value values (α, β) for sector portfolio from

Warsaw Stock Exchange. We can claim that estimation RVaR level as the underlying risk measures in the real problem and connected with these procedures errors is still useful. Working with two-parameter quantile-based risk measures can have strong impact to manage expected levels of capital collateral. Additionally, by using two-parameter quantile-based risk measures, we received a tool for control a chosen part of the tail loss distribution of the portfolio in the estimating process. To conclude we would have to argue that the important impact form empirical results is that RVaR looks as an agile risk measures, which is also robust and coherent. That means RVaR should be an important part of the portfolio risk measurement procedure.

References

1. Acerbi, C.: Coherent measures of risk in everyday market practice. Quant. Finance **7**, 359–364 (2007)
2. Acerbi, C.: Spectral measures of risk: a coherent representation of subjective risk aversion. J. Bank. Finance **26**, 1505–1518 (2002)
3. Artzner, P., Delbaen, F., Eber, J., Heath, D.: Coherent measures of risk. Math. Finance **9**, 203–228 (1999)
4. Cont, R., Deguest, R., Scandolo, G.: Robustness and sensitivity analysis of risk measurement procedures. Quant. Finance **10**(6), 593–606 (2010)
5. Delbaen, F.: Monetary Utility Functions. Osaka University Press, Osaka (2012)
6. Embrechts, P., Liu, H., Wang, R.; Quantile-Based Risk Sharing. Working Paper, 24 Oct 2017
7. Embrechts, P., Wang, B., Wang, R.: Aggregation-robustness and model uncertainty of regulatory risk measures. Finan. Stochast. **19**(4), 763–790 (2015)
8. Föllmer, H., Schied, A.: Convex measures of risk and trading constraints. Finance Stochast. **6**, 429–447 (2002)
9. Gourieroux, C., Liu, W.: Sensitivity analysis of distortion risk measures. Working Paper (2006)
10. Gourieroux, C., Laurent, J., Scaillet, O.: Sensitivity analysis of values at risk. J. Empir. Finance **7**, 225–245 (2000)
11. Grigorova, M.: Stochastic dominance with respect to a capacity and risk measures. Statist. Risk Model. **31**(3–4), 259–295 (2014)
12. Huber, P.: Robust statistics. Wiley, New York (1981)
13. Kou, S., Peng, X., Heyde, C.C.: External risk measures and Basel accords. Math. Oper. Res. **38**(3), 393–417 (2013)
14. Krätschmer, V., Schied, A., Zähle, H.: Comparative and quantitative robustness for law-invariant risk measures. Finance Stochast **18**(2), 271–295 (2014)
15. Kusuoka, S.: On law invariant coherent risk measures. Adv. Math. Econ. **3**, 83–95 (2001)
16. Richter-Gebert, J., Sturmfels, B., Theobald, T.: First steps in tropical geometry. Contemp. Math. **377**, 289–318 (2005)
17. Song, Y., Yan, J.: Risk measures with comonotonic subadditivity or convexity and respecting stochastic orders. Insur. Math. Econ. **45**, 459–465 (2009)
18. Wang, R., Bignozzi, V., Tsakanas, A.: How superadditive can a risk measure be? SIAM J. Financ. Math. **6**, 776–803 (2015)