Chapter 6
Mathematics Teachers as Curriculum Designers: An International Perspective to Develop a Deeper Understanding of the Concept

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Abstract In this chapter, we investigate the notion of "teachers as curriculum designers" from the literature and from six international perspectives. This is done in order to (1) develop a deeper understanding of the concept, and (2) provide an international perspective and illustrations of the different facets of teacher design. Based on this investigation, we could identify different modes of teacher design: from teacher *d*esign activities at micro level (e.g., lesson preparation alone or in small groups), over those at meso level (e.g., *D/d*esigning in collectives of colleagues for the purpose of use by others), to *D*esign at macro level (e.g., involvement in the design of national frameworks by professional design teams for the use of many others). More generally, we claim that the often casually used term of "teacher design" has different meanings in different contexts and that teacher design activities may be for different purposes and for different expected end results. A major distinction is whether the design is more oriented towards the process, or the product. We argue that the most promising form of teacher design might lie at the

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L. Trouche et al. (eds.), *The 'Resource' Approach to Mathematics Education*, Advances in Mathematics Education,
https://doi.org/10.1007/978-3-030-20393-1_6

crossroads between product and process orientation, with connections between the two. This has implications for teacher education and professionalism.

Keywords Mathematics teacher design · Curriculum designer/s · International perspective/s

6.1 Introduction

In previous research (e.g., Margolinas 2014; Pepin et al. 2013; Remillard 2005), mathematics teacher interaction with resources has been discussed. It has become clear that teachers interact with (curriculum) resources in different ways (e.g., adaptation, appropriation), and one of the forms of interaction has been labelled as "design" (e.g., Brown 2009; Pepin et al. 2017a). At the same time, the term "design" is used differently by different educationists, which in turn creates the need for clarity and a better conceptualization: What are the dimensions of "teacher design? What does it entail (e.g., compared to the teacher as a "user" of materials)? A common "language" is needed that we can share when discussing mathematics "teacher design."

At the "Res(s)ources 2018" conference (at the French Institute of Education (IFÉ) in Lyon in May 2018), international scholars (the authors of the paper) were invited to participate in the panel discussion "Mathematics Teachers as Designers: An International Perspective," to provide illustrative examples of "teacher design" in different contexts. The aim was to explore the notion of "teachers as designers" in different international contexts, in order to develop a deeper understanding of the concept. After the conference, (1) a literature review was conducted on the notion of the "teacher as curriculum designer" and (2) the participants were invited to contribute their respective "cases" in writing. These two sets of sources formed the basis for our investigation and analyses, which in turn helped us to re-conceptualize "teacher design" and provide illustrations of its different facets in different international contexts.

In this conceptual chapter (and after a short introduction), we report, first, on a review of the literature with respect to three notions (teacher design and teacher design capacity; modes of teacher design work; and curriculum design). Second, (a) the development of the six international perspectives (and cases) is explained and (b) the findings from the investigation of the cases are presented in a table and discussed. Third, we draw conclusions by reflecting on the findings, attempting to nuance and re-conceptualize teacher design, and we outline implications of the investigation.

6.2 Conceptualizations of the Mathematics Teacher as Designer

In previous curriculum renewals, teachers have often been the “implementers” of the curriculum that was mandated and/or designed by the ministries (and their agencies). For decades, scholarship on factors affecting curriculum implementation has pointed to the importance of involving teachers, to varying degrees, in shaping the learning scenarios and trajectories in their own classrooms (e.g., Ben-Peretz 1990). More recently, in many countries worldwide (e.g., Australia, Canada, China, France, the Netherlands, Scotland, Singapore), a new wave of curriculum renewal has been initiated by the respective ministries and conducted/carried out by their curriculum development agencies. Distinctively, and unlike most earlier curriculum changes, teachers have been more included than before. While the benefits of teacher involvement in curriculum design (albeit not at macro/national level) are acknowledged in the literature (e.g., Priestley et al. 2017), far less is known about shaping that involvement to yield expected benefits. At the same time, recent technological developments (e.g., new web-based curriculum resources) have changed the nature of teacher design work. Teachers design, redesign, customize, and appropriate not only conventional but also digitally enhanced learning materials, curriculum resources, and activities. Moreover, they are often (co-)designers of their own (school and classroom) curriculum and the associated and envisaged student learning trajectories.

In this section, we bring together the research literature under the following three themes: (a) teachers as designers and teacher design capacity; (b) modes of teacher design work/activity; and (c) curriculum design, representations, and structure. This allows us to better frame our view on the different aspects of teacher design and curriculum.

6.2.1 The Notions of Mathematics Teacher Design and Teacher Design Capacity

There has been considerable research carried out in the field of teacher design (e.g., in curriculum studies; instructional design with regard to technology enhanced learning), more recently with regard to how design is conceptualized within teachers’ work and practice (Huizinga 2009; Nieveen and van der Hoeven 2011). Selected studies (within the domain of mathematics education) emphasize the relational aspects of design work (e.g., Pepin et al. 2017a) or teacher curriculum design within the context of educational reform and change (e.g., Trouche et al. 2019), to name but a few.

We take as a starting point a very broad notion of design that includes the processes of appropriating and/or actually developing specific resources for teaching or learning. Recent mathematics education research reveals how individual teachers select, adapt, appropriate, combine, or redesign different curriculum resources (e.g., Pepin et al. 2013) for their personal use and enact the different curriculum elements in their teaching practice (e.g., Remillard 2005). However, depending on the context in which the design work takes place, we also need to consider "larger design," that is, when mathematics teachers are part of national design teams for the renewal of the national curriculum. Hence, we contend that these two notions of teacher design may lie on a continuum. For the moment, we understand "teacher design" as including both (at each end of the spectrum) and as work that involves the interaction between individual and collective capacities and environmental conditions/support (Pepin et al. 2017a; Priestley et al. 2017).

The following quote by Priestley et al. (2017) serves as a point of departure for how teacher curriculum design is leaning on teacher agency:

"The main distinctive factor is that agency [and teacher design work] involves intentionality, the capacity to formulate possibilities for action, active consideration of such possibilities and the exercise of choice. But it also includes the causative properties of contextual factors – social and material structures and cultural forms that influence human behavior – which is why, as mentioned, a full understanding of agency must consider how individual capacity interplays with contextual factors." (Priestley et al. 2017, p. 23).

In an earlier study, we reviewed literature on teacher expertise and teacher design in mathematics education and curriculum studies, to develop a refined understanding of teacher design capacity (Pepin et al. 2017a). In that study teacher design capacity was defined to include the following components:

- An orientation, a goal, or point/s of reference for the design:
  - To know the "status quo" (e.g., what do students know, which problems they do have in terms of misconceptions), as well as what teachers are aiming for in terms of their mathematical-didactical design.
  - To understand the larger (e.g., national curriculum guidelines) and the smaller picture (e.g., learning trajectory for a particular mathematical topic) of their design with respect to the curriculum (e.g., a task/activity; a lesson; a lesson sequence, e.g., for a particular grade).
  - To discern where it fits in the short (i.e., for a lesson cycle) as well as the long term (i.e., connecting topic areas across grades).
- A set of design principles, which must be firm but flexible: a teacher needs a set of "universals" for the design, or principles, which are evidence-informed (e.g., from own practice, or based upon research) and supported by justification for their choices. We call those robust principles. At the same time, these principles must be flexible enough, i.e., didactically flexible, to adapt to new challenges and contexts, so that the teacher's frame of reference can grow and expand, perhaps cover new areas, or differentiate/validate within the existing frame.

- "Design-in-action" type of implicit understandings, reflections, and realizations: a teacher needs to be able to generate relationships or informed potential lines of action, which are often not observable and which develop in the course of instruction.

This "definition" has only partly been helpful, as it provides an ambitious, idealized image of teacher design capacity. The reality is often quite different, and this raises questions about its possible use in practice.

Hence, starting by using the term "design" more broadly (e.g., to include the individual and the "larger" design), we propose the following dimensions as parts of the notion of "teacher design":

- Intentionality dimension: deliberate, goal-directed mental activity/thinking, definition of a clear goal (probably due to an actual "problem"/rationale).
- (Degree of) Novelty dimension: positioned on the continuum between (on one extreme) slight adaptations of current practices, to (at the other extreme) developing a new curriculum resource (e.g., textbook) or scheme of work from scratch
- Approach dimension: strategies, styles, design approaches.
- Time (duration) dimension: depending on the context, on a time continuum between hourly design session/s, to a long-term professional development design activity.
- Individual/collaborative ("teaming") dimension: from individual teacher design (in school, or at home) to professional teacher design teams.
- Audience/use dimension: for the/one teacher's own teaching; for all mathematics teachers in the school (site-specific design); for the whole regional/national teaching staff (generic design).
- Context dimension:
  - Design space/environment: at home, school, or Internet.
  - Resources: resources and tools available in the national/school context and used for the design.

We will refer to this analytical frame for the analysis of our cases. In theoretical terms, it leads us to explore the nature of teacher design work and its different modes.

6.2.2 Modes of Teacher Design Work

Teacher design work can vary in character and take on different forms. Teachers often work alone, or they work in teams; they may take on various roles: that of redesigner of existing materials and activities, or as co-designers, for example. Without claiming to be exhaustive, in this section we first review the literature (e.g., in mathematics education; instructional science; technology-enhanced learning sciences), where we mainly found two modes: *teachers as designers in terms of customization for own teaching* and *teachers as participatory designers (in small local* *or large national teams).* Second, we reflect "across" the two modes pointing to particular affordances and constraints. Moreover, we contend that, at least theoretically, there are four different modes of teacher design work.

6.2.2.1 Teacher Customization for Their Own Teaching

Teachers can contribute to the curriculum in different ways: they may be enactors of the curriculum, that is, they implement ready-made materials (and more or less align with the designers' intentions). Equally, they may use ready-made materials "creatively": they use given materials to try out new activities and improvise in the moments of enactment. Another way would be to redesign mathematics curriculum materials via making small, systematic changes or adjustments, mostly based on their earlier experiences in class (e.g., Remillard 2005). Even when they use the same curriculum materials (e.g., textbooks), they frequently adapt these to accommodate the varied needs of their students. At times this is also done when teaching in class (e.g., "design-in-use," Pepin et al. 2013), as in-the-moment decision/s. Indeed, it appears that materials that yield to teachers' modifications better respond to the changing needs of the classroom, and to its constraints and resources.

It is known that teachers redesign curriculum materials for various reasons: to better align them with their teaching goals or styles, to respond to different students' needs or different classroom situations (e.g., Brown and Edelson 2003). Their customizations may serve to align materials to changing content standards, or to add details that address their students' or local communities' interests, or to adapt the level of challenge to suit individual abilities, to name but a few (Matuk et al. 2015). There are of course often also practicality-related concerns, which hinder or support teachers' (re-) design of curriculum materials (see framework of Ruthven 2014).

6.2.2.2 Teachers as Participatory Designers/Partners in Task Design

Teacher involvement in curriculum (e.g., mathematical tasks and activities) design has a long tradition, in particular in mathematics education, where teachers have been designing mathematical tasks (e.g., Instituts de Recherche sur l'Enseignement des Mathématiques network in France, see Trouche (2016); Sésamath association in France, see Gueudet et al. (2016); see also Pepin and Jones (2016)). However, although it is said to foster implementation of curriculum reforms, teachers often encounter various problems while designing, related to conditions set for the design process, and they often lack the knowledge and skills needed to enact design processes.

The expertise required to enact curriculum design has been described by various scholars (e.g., Huizinga 2009, Nieveen and Van der Hoeven 2011). They use different labels to describe elements of the same concept, including curriculum design competencies (e.g., Huizinga 2009), instructional design competencies (e.g., Richey et al. 2001), and design expertise (Huizinga 2009). For Huizinga (2009), design expertise consists of three aspects (curriculum design expertise; subject matter knowledge; pedagogical content knowledge), and it includes analysis, design, development, implementation, and evaluation skills.

In collaborative design, teachers often create new or adapt existing materials in teams, because they are intrinsically interested in designing curriculum materials, or to comply with the intentions of the curriculum designers and with the realities of their context. Often, external experts are involved in the process, and they are expected to provide the team with recent, up-to-date insights, for example, concerning the underlying rationale for particular curriculum changes, or in terms of recent research outcomes related to the intended design. The collaborative process is said to provide opportunities for teachers, for example, to reflect on intentions of a particular reform, and to develop materials that correspond to their needs within the reform context (Voogt et al. 2015). The interaction with peers and experts is expected to deepen and challenge (1) teachers' beliefs, (2) their practice, and (3) their goals for student learning (Borko 2004). These three points link to the main activities of a Teaching Research Group (TRG), a format for teacher design institutionally established in China.

Yuan and Li (2015) report on particular practices in teams:

> During a typical [collaborative] activity, two or more teachers teach a common topic to different groups of students with distinct lesson designs, while their fellow teacher participants observe each of these lessons. After all lessons are completed, all teachers involved gather to discuss the lesson designs and classroom teaching practices, make comments and suggestions for future revisions and improvements. (p. 568)

Borko (2004) also argues that in order for collaborative curricular design processes to have the potential to contribute to teacher learning, these must be well-scaffolded. In addition, the curricular materials resulting from the design process must be based on recent knowledge of good practice and considered by teachers to be usable in their contexts (Penuel et al. 2007).

6.2.2.3 Reflecting "Across" the Two Modes

Across the various modes of teacher design work, individuals and teams work differently to inform both the processes and the products of design. While teachers sometimes design in "multiple expert teams" (e.g., university-based mathematicians; mathematics teacher educators; mathematics school teachers – see Jackson et al. 2015; Penuel et al. 2007), teacher design work is often small scale and close to practice. According to the literature, it typically involves (a) critical reflection on and redesign of one's personal practice, which teachers find insightful (e.g., Pepin et al. 2017a); (b) adaptation based on research evidence (e.g., Cobb and Jackson 2015) (this typically plays a very modest role in a teacher's design work, unless s/he works with teacher design teams with external (research) support); and/or (c) team design within one organization/school (e.g., Yang 2009).

Moreover, there are various reasons why teachers may become involved in design work. First, they want to design artifacts that can provide resources tailored for use in specific classrooms with particular learners, in order to improve their student learning (Pepin et al. 2017b), hence closely related to their daily instructional/pedagogical practices. Second, teachers often engage in design to adapt to curriculum reforms (Trouche et al. 2019). Third, teacher involvement in the design of (innovative) products may be sought, by external agencies (e.g., education ministries), to increase their practicality. Fourth, teachers may value engagement in particular design work, as it is likely to yield increased ownership and commitment for implementation (e.g., Cviko et al. 2014). Finally, teacher design work can provide a rich, authentic, and practical context for teacher learning and professional development about mathematics, curriculum materials, and/or technology suitable for a particular content (e.g., Koehler and Mishra 2005).

Research (e.g., Cobb and Jackson 2015) has shown that support and external expertise are likely to be beneficial to both the processes and the products of teacher design, especially when focused on how to structure work in teams, on substantive vision, and on process guidance. Moreover, to establish and maintain the substantive focus for design, a shared vision is essential (Gueudet et al. 2013). Conversations about vision and goals stimulate teachers to apply their didactical knowledge, especially when tackling new topics (Gueudet et al. 2016). Research has demonstrated that high-quality process support, in addition to substantive support, is crucial for design success (Jackson et al. 2015).

Looking across the modes reported in the literature, teacher design can be seen in (at least) two dimensions: (1) individual/collaborative ("teaming") dimension, from working alone (single) to working in a collective, and (2) "use" dimension – from "for own use" (for his/her teaching; site-specific) to use by others (generic) (see Table 6.1). This alerted us to suggest that there are potentially (and theoretically) four different modes of teacher design – we denoted these with d-esign, d/D--esign, D/d-esign, and D-esign, where, for each dimension, d is a marker of narrower scope and D of broader scope; and in combining dimensions signal situations where the scope of use/teaming differs.

Table 6.1 Two dimensions of teacher design

Use/teaming	Working/designing alone	Working/designing in a team
Designing for own use/teaching	A teacher designing on his/her own for his/her own teaching (e.g., lesson preparation at home)	Teachers designing in a team (e.g., of colleagues in same school) for their own teaching
	d	D/d
Designing for use by others	Teachers designing on their own/alone for use by others (e.g., expert teachers/professional designers)	Teams of teachers/experts designing for use by others (e.g., teams of professional designers)
	d/D	D

6.2.3 Curriculum Levels, Representations, and the Spider Web

In most international contexts, the "curriculum" is seen as a "plan for learning," and each country's National Curriculum provides its plan for what the country values their students/pupils to learn. At the same time, it is important to note that the curriculum "works" at specific curriculum levels, in particular contexts, and it has particular representations.

First, in terms of curriculum levels, it is noteworthy that, at different curriculum levels, particular "products" may be identified (Van den Akker 2003) (Table 6.2).

These levels are important for our analytical/conceptual frame, in particular the meso and micro level, as these are closest to the teachers' work. At the same time, the "higher" curriculum levels affect the "lower" ones, as they set the context for the work of teachers. For example, the national curriculum and national examination programs, at macro level, are part of the context in which teachers work and design in/for their teaching (at meso and micro level). Another example is textbooks: textbook authors typically take the macro frameworks (including innovations and reforms) into account when producing textbooks. It is also worth noting that curriculum products vary greatly in their nature and scope, also depending on the audience. Examples are textbooks, in some contexts approved and used nationwide, as compared to lesson/teaching plans which are typically site-specific and used by one or several teachers for their own practice.

Second, the curriculum can be represented in different forms. Curriculum research (e.g., Goodlad 1979; van den Akker 2003) typically distinguishes between the following curriculum representations (Table 6.3).

The division into six representations is especially useful for our analysis of the processes (and the outcomes) of curriculum innovations (e.g., the French case). For our purpose of investigating "teacher design" of the curriculum, this distinction of forms emphasizes the different layers of the curriculum concept and demonstrates the often-substantial discrepancies between the various forms.

Table 6.2 Curriculum levels and curriculum products (Thijs and Van den Akker 2009, p. 9)

Level	Description	Examples
Supra	International	Common European framework of references for languages
Macro	System, national	Core objectives, attainment levels
		Examination programs
Meso	School, institute	School program
		Educational program
Micro	Classroom, teacher	Teaching plan, instructional materials
		Module, course
		Textbooks
Nano	Pupil, individual	Personal plan for learning
		Individual course of learning

Table 6.3 Curriculum representations (Thijs and Van den Akker 2009, p.10)

Intended	Ideal	Vision (rationale or basic philosophy underlying a curriculum)
	Formal/ Written	Intentions as specified in curriculum documents and/or materials
Implemented	Perceived	Curriculum as interpreted by its users (especially teachers)
	Operational	Actual process of teaching and learning (also: Curriculum-in-action)
Attained	Experiential	Learning experiences as perceived by learners
	Learned	Resulting learning outcomes of learners

Third, curriculum theory often uses the so-called curricular spider web (van den Akker 2003) to denote the close connection between aims and content of learning and the other aspects (e.g., assessment, resources, teacher role) of the curriculum (as the plan for learning). The core and the nine threads of the spider web refer to the ten parts of a curriculum, each concerning an aspect of learning and the learning program for pupils (see Table 6.4).

At the same time, visualizing the relationship between the various aspects as a spider web also indicates the fragility of the relationships: if a teacher designs taking only assessment into consideration (and neglecting the other aspects), it is likely that the web is pulled into one direction and may possibly break; hence, the plan for learning will most likely lack consistency and coherence.

In a previous paper, we had amended these 10 questions for our purpose, to investigate "teacher design" (see Pepin et al. 2017a, b) from a curriculum perspective (Table 6.5).

This frame has been helpful for comparing teacher design, in particular when we compared collective and individual aspects of teacher design, and which audience the design was aimed at.

Of course, teacher curriculum design/innovation can start with any component. Traditionally, the learning content has received the most attention. However, over the past decades, new insights about learning mathematics, and about resources beneficial for learning mathematics, have provided sources of inspiration for innovative practices. It is known that textbooks have been a significant component of the curriculum (and "stirrer") for a long time (e.g., they provided guidance for teachers), and recent opportunities provided by digitalization offer new impulses for innovations. The time factor is a classical object of curriculum discussions: How is the always scarce amount of time distributed across domains and learning tasks? In our quest for teacher design, it raises the question of how much time teachers get to design their own curriculum.

It is clear that the relevance of the ten components varies for the five curriculum levels mentioned earlier. For our purpose, that is, the study of "teacher design," the micro and meso levels (see Table 6.2) are clearly the most relevant ones. At the same time, all other levels, and in particular the macro and nano levels, clearly play a role for teachers as designers. In addition, the consistency and coherence between objectives and content on the one hand, and pedagogical considerations, assessment, and

Table 6.4 Curriculum components in question form (Thijs and van den Akker 2009, pp. 11/12)

Component	Core question
Rationale	Why are they learning?
Aims and objectives	Towards which goals are they learning?
Content	What are they learning?
Learning activities	How are they learning?
Teacher role	How is the teacher facilitating their learning?
Materials and resources	With what are they learning?
Grouping	With whom are they learning?
Location	Where are they learning?
Time	When are they learning?
Assessment	How is their learning assessed?

Table 6.5 Teacher design components in question form (Pepin et al. 2017a, pp. 801–802)

Component	Core question
Rationale	Why are teachers designing? – e.g., dissatisfaction with textbook; to become less dependent on the textbook; to make teaching more varied
Aims and objectives	What are their aims and goals? – e.g., to prepare a series of exemplary lessons for particular topic areas
Audience	What is the audience? – e.g., fellow teachers; teachers nationwide; students
Content	What are they designing? – e.g., lessons; assessment questions
Activities	How are they designing? – e.g., design approaches; sequences; strategies; styles
Materials and resources	What are the resources and tools used for the design? – e.g., resources used
Grouping	With whom are they designing? – e.g., in a group; individually; team membership
Location	Where are they designing? – e.g., in school; on the internet – The design environment
Time	When are they designing? – e.g., how long does the design take
Assessment	How is the design evaluated? – e.g., expert appraisal; peer appraisal; observation/interviews of/with users; assessing learning results

the nature of resources used, on the other hand, are of great importance at these levels. At school and classroom levels, nearly all components play a role. Here, overall consistency is of crucial importance for successful and sustainable implementation of innovative designs.

6.3 International Perspectives

In this section, we (a) report on the development of the six international "cases" and (b) present and discuss the findings from the investigation of the cases.

6.3.1 The Development of the Cases

The overall purpose of our investigation was to identify different facets of the commonly used constructs of "teacher design" or "teachers as curriculum designers" in order to develop a more nuanced understanding of the concept. To structure our panel discussion, and subsequently our cases, we took into consideration our knowledge of previous mathematics education research on teacher design. Making the investigation feasible, we opted for the three (what we judged as) most important questions/lines, in order to investigate the phenomenon "mathematics teacher as curriculum designer" in different international contexts. The panelists were given the following questions:

- Why are teacher design activities relevant? Why would they design?
- What would teachers design? What are the most interesting/challenging design tasks?
- How would teachers design? What sorts of design approaches would they use, and under which conditions?

In practical terms, in the first stage each panelist (separately) wrote up "his/her case" (broadly structured by the three questions) related to their experiences in their contexts, in addition to a description of each context. As it turned out, the cases included important additional information. In the second stage, the first author (alerted by the previous findings from the literature) sent the table she had produced (based on the individual case stories) to the case authors for validation, together with the first version of the paper. Subsequently, case authors amended and validated their cases. For an overview, we have collected the findings in Table 6.6.

More precisely, the cases were analyzed based on our knowledge of

1. The notion of "teachers as curriculum designers," which included both curriculum design theory/research (e.g., Nieveen and van der Hoeven 2011) and mathematics education research (e.g., Pepin et al. 2017a, b), and our previously stated two dimensions of "teacher design" (see Table 6.1).
2. The different modes of teacher design work (see Table 6.1).
3. Curriculum design, more precisely, the spider web (e.g., van den Akker 2003; Thijs and van den Akker 2009).

We note here that the previous identification of modes (see Table 6.1) was theoretical, whereas the analysis allowed to identify only three of the four modes: D, D/d, and d could be identified (see next section).

6.3.2 Discussion of Findings from Cases

In this section, we present our findings (based on the cross-case analyses – see Table 6.6), and we distinguish between four main claims.

Table 6.6 Findings from the cases

Country		Context	Why	What	How
France	D	IREM groups across France; Sésamath	Support of innovations/reform/ renewal of curriculum	Production of resources for teaching and teacher education	Didactical engineering in collectives (teachers, teacher educators, university mathematicians, didacticians)
				Tasks and didactic situations	
	d	Teachers in schools	Translate the National Curriculum into teaching (resources) in schools adaptation	Development of curriculum progressions	Teachers' personal work
					Lesson preparation in teacher groups/collectives (thematic groups)
					Building on a variety of resources
England	D	"National regulation" of curriculum and assessment specifications by government agencies	"School improvement" through setting national standards aimed at establishing sound provision and raising performance in system-wide national and international assessments	Official documentation setting out national specifications for curriculum coverage and learning progression	Official documentation prepared by government agencies, with input from working groups
		"Free market" in curricular resources developed by multiple commercial and non-commercial organizations and individuals		Curricular resources (including complete curriculum schemes) promoted nationally by various organizations	Curricular resources developed by multiple commercial and non-commercial organizations and individuals
	d	"Curriculum resourcing" by teachers in school departments, working collectively and individually	"Local customization" aimed at assembling curricular resources well suited to staff and students of school, and addressing issues of "school improvement"	Mandated production of "schemes of work" for the school/ department to create a system of classroom resources for "curriculum coverage"	Collective and/or individual creation of teaching sequences for "schemes of work" through selection, and adaptation of classroom resources from a variety of sources

(continued)

Table 6.6 (continued)

Country		Context	Why	What	How
Japan	D/d	Lesson study and open lesson	Process of designing is seen as teacher professional development, to better understand the National Curriculum and curriculum materials (e.g., approved textbooks)	Preparing a series of lessons including one lesson demonstrated to the colleagues	Individual and collective work with colleagues (from own and/or neighboring schools) (e.g., for analysis of textbook and teacher guide – This process is named "kyōzai-kenkyū")
			Improvement of teaching as a school mission	Preparing lesson plan	Analysis, exploration of variety of resources: Internet, workbook, textbook
			Implementation of renewed curriculum		
	d	Individual teachers (or groups of two teachers, e.g., for "team teaching")	Lesson/teaching preparation	Appropriation of curriculum progression and teaching materials	Individual work (e.g., for "kyōzai-kenkyū")
				"Bansho" and questions	Reliance on textbook and teacher guide
China	D/d	Structured system from top down composed of – TROs (teaching research office): They play the role of giving administrative and professional guidance; they often work with teams of professional researchers, and play an important role in bridging the gap between teaching theories and instructional practice. – Same-school TRGs (teaching research Group – Teachers in same school/year group)	Study classroom teaching	Lesson plans and progressions	TROs are powerful middlemen for conducting school-based teaching research activities
			Increase quality of education	Content	
			Implement curriculum reforms	Teaching method/s	
			Work out lesson plans (also progression over a particular period) and teaching methods (one approved textbook)	Problems/activities	In TRGs (and working with expert teachers), in particular TGRs/groups which include school teachers, expert teachers in school and researchers from universities
			Design for teaching and learning is conceived as training for teachers, so that they become "researchers of classroom teaching"	Action research focused on practical teaching problems	

	D/d and d	Individual teachers (e.g., in preparation for TRG work)	Sharing teaching resources collected by individual teachers	The major work of the LPG (lesson preparation group) is about arranging each unit of mathematics content, preparing lesson, and sharing teaching resources collected by individual teachers	Teachers from the same grade and same subject are encouraged to build a LPG (lesson preparation group), which is led by an experienced teacher
		Teachers from the same grade and same subject are encouraged to build a LPG (lesson preparation group) which is led by an experienced teacher			
Brazil	D	A national project of textbooks to evaluate and write a guide for textbooks for state school teachers, coordinated by the national ministry of education involving researchers and teachers in the evaluation process	To support teaching in and amend the materials for the different regions	To evaluate textbooks and design meta-resources	In teams (consisting of teachers, teacher educators and sometimes researchers) who meet online and in person
	D/d and d	Collective of teacher educators and individual teachers	Reflection and amendment of materials	Adapting existing materials/ textbooks	Individual and in collectives (within a school or neighboring schools)
			To cope with diversity	Re-creating activities with respect to social context/cultural context	
			Authoring of materials for teachers' own development		
Netherlands	D	SLO- Centre for Curriculum Development (for all school subject areas)	Support curriculum reforms	(proposals for) curriculum frameworks at national scale	Consult/co-design with experts, teacher educators, teachers (in interaction with)
		Variety of textbook publishers	Curriculum innovations	Textbook production	
				Exemplary materials (within those frameworks) to inspire/ stimulate teachers and textbook publishers	
	D/d and d	D/d TDTs (teacher design team)	Get good/better pupil attainment and school results	Adapt existing materials for their teaching e.g., ICT in maths; create concept-context materials	Teams of teachers (sometimes also teacher educators) in TDTs
		d individual teachers	Professional development/ learning	Prepare lessons	Individual teachers (at home)

1. Different contexts allowed for different *modes of teacher design*, and these were linked to different design spaces at different curriculum levels. We could identify at least three *design modes* (and they are not hierarchically organized):

   (a) At macro (system, national) level, teams of experts (e.g., in ministries, or expert centers or task forces) worked on the design of a national curriculum, which included not only mathematical content (for each age group) but often also didactical considerations and particularly pertinent activities and tasks (e.g., French context). These teams of experts rarely included classroom teachers (teachers were more often consulted to react on drafts than involved as active designers of national frameworks), and mainly consisted of subject and subject didactics experts and professional curriculum designers. This type of mathematics curriculum design is for generic use, and we have proposed (in Table 6.1) to denote it capital D: "*D-esign.*"

   (b) At meso level (school, institute), different scenarios offered opportunities for teacher design, and at this level classroom teachers were typically involved. For example, in England, mathematics teachers in school departments would work individually or collectively on "curriculum resourcing" for their respective school/department. This included "local customization" aimed at assembling curricular resources well suited to staff and students of their school and at addressing issues of "school improvement." Teachers would typically be expected to produce "schemes of work" for their school's mathematics department, to create a system of classroom resources to "cover" the official curriculum. This would be done, either individually or collectively, by creating teaching sequences for the "schemes of work" through selection and adaptation of classroom resources from a variety of sources, including those published by a host of commercial and non-commercial organizations, as well as others exchanged more informally. Except for sporadic professional development courses (provided by various organizations), there was little further help to "interpret the official curriculum" and to design tasks, activities, and learning lines for students.

   In France, it appeared that regionally established IREM groups, and associations such as Sésamath, bridged the gap between the nationally offered curriculum/innovations (including guidelines to the curriculum) and teaching in schools. These associations of "experts" (part-time teachers/part-time teacher educators, university mathematicians, didacticians) designed resources for mathematics teaching and teacher education, in particular those in support of curriculum innovations. Particular practices and theories were evident, such as "didactical engineering" and "didactic situations," which reflected a common theoretical underpinning and the importance of sharing a common language on (curriculum) design tasks among designers.

   In Japan, Lesson Study at school level (Fernandez and Yoshida, 2004; Isoda et al. 2007; Stigler and Hiebert 1999) included the processes of designing a series of lessons and of teaching one of them as an open lesson, by one teacher. This was done for the sake of professional development for this teacher (who designed the lessons), as well as for the colleagues who participated in the observation and discussions. The main aim was to better understand the national curriculum (including reforms) and the use of particular materials, as well as to improve the teaching and learning in their school.

In China, similar practices were evident at (1) city and (2) school level: (1) At city level, the Teaching Research Offices (TROs) were powerful middlemen for the conduct of school-based teaching research activities, and they invited teachers from schools to participate in designing series of guidance/support materials (e.g., teaching guidance, standard implementation guidance, examination guidance). (2) At school level, Teaching Research Groups (TRGs) were common. In TRGs, classroom teachers would design curricular/lesson plans and instruction and analyze the quality of particular materials, in short, act as "researchers of classroom teaching." These groups would often include one "expert teacher" who was expected to mediate recent research, and reforms, into the practice of teaching (e.g., Pepin et al. 2017a, b; Yang 2009).

In Brazil, despite a large program of textbook evaluation and distribution, teams of didacticians and teachers felt the need to redesign and amend particular materials, in order to manage the regional diversity of students (e.g., Gitirana et al. 2013; Silva and Lima, 2017).

In the Netherlands, teacher design teams (TDTs) would group teachers and didacticians around particular themes, often themes related to particular/recent curriculum revisions (e.g., reasoning and proof) or persistent problems in mathematics teaching (e.g., integration of ICT, Drijvers et al. 2010).

These kinds of teacher design are characterized by the following: teachers work alongside experts and teacher educators, and the products are also for "generic" use. We have denoted this kind of teacher design by "*D/d*"-*design* (or "*d/D-esign*"), and we found it difficult to distinguish between the two, at least in our cases. For this reason, Table 6.6 only includes D/d. This kind of teacher design appears to offer a bridge between the generic and direct use in class. By "generic" we mean here that the use was broader than strictly own classroom use (in principle available to a wider audience of users) and often included professional development elements (in some cases explicitly).

(c) At micro level (classroom), the individual teacher (or a small group of teachers) designs his/her curriculum, including teaching plan/s, instructional materials, and the curricular progressions/learning lines for his/her students. This was particularly evident in the French context, where individual teachers "designed," that is, adapted and translated the national curriculum in their mathematics teaching in schools, building on a variety of resources. Another example was the Japanese case, where the individual teacher designed their lessons, including setting goals, content, and how to use particular mathematical tasks. The board work ("bansho"; see Yoshida 2005; Tan et al. 2018) and questions for students were also specially designed. The textbook and teacher guide were carefully explored and analyzed for the teaching design – this process was named "kyōzai-kenkyū" (e.g., Watanabe et al. 2008; Fujii 2015). This micro design comes very close to what could always be considered as the "natural" part of teachers' work: lesson preparation. This teacher design has been denoted (in Table 6.1) with a small d: "*d-esign.*"

In several (but not all) of the case contexts rather immediate classroom use was a partial goal, and the immediate and broader goals were closely connected (e.g., Japan, China). In these cases, it would be difficult to distinguish between D/d and d for the design mode. Moreover, it seemed that in some contexts there was hardly any individual teacher design (for individual use); it all took place in teams and for use by all team colleague teachers. For example, in China, Lesson Preparation Groups (LPGs), consisting of a small number of individual teachers, were introduced which could provide in particular novice mathematics teachers with clear structures of mathematical content for each curriculum unit and a concrete plan of how to implement each lesson.

Of course, teacher design does not stop with out-of-class preparation but could also include teacher "design-in-use," that is, the in-the-moment decisions that teachers make in the classroom. As our cases did not provide evidence for such "design-in-use," we have not analyzed this further.

It is worth noting that teacher *d*-esign tasks were, of course, expected of single teachers in all contexts – this was part of their "traditional" responsibilities. However, the degree of guidance varied: from little guidance (e.g., English context) to institutionalized support, e.g., by colleagues (The Netherlands), or colleagues and experienced teachers (France, Japan), or colleagues and "expert teachers" (China).

2. It was interesting to note *that D-esign, D/d-esign* (or *d/D-esign*), and *d-esign* were underpinned by mathematical educational/didactical theory, and theory brought into the designs, but to a greater or lesser extent: In the French context, the knowledge of "didactical engineering" (Artigue 2015) and its main roots in Brousseau's theory of "didactical situations" (Brousseau et al. 2014) seemed to be supporting the design processes, whereas, in the English context Anglo-Saxon/American research-based reform movements (e.g., assessment for learning) including their didactical dimensions, it seemed to be guiding teachers' design efforts. In Japan, the theories and practices of Lesson Study appeared to support the design activities (e.g., Miyakawa and Winsløw 2009), and equally the design activities in TRGs (in China) were supported by theoretically underpinned practical knowledge made "digestible" by experts. In the Brazil example, there has been an emphasis on the French didactic tradition to support teacher curricular design.
3. In terms of *design groups*, they were linked to *design spaces* and would vary in size and participants:

   - A single mathematics teacher, perhaps working with a colleague, in or out of school.

- Several mathematics teachers working together on a particular theme/topic/materials/teaching process at local or regional level (e.g., Teacher Research Groups; Lesson Study; Teacher design teams – TDTs).
- Mixed group of teachers, teacher educators/didacticians, university mathematicians, researchers designing at regional level (e.g., IREM).
- Expert groups designing at institute level (e.g., National Institute for Curriculum Development, often teaming up with other experts, subject didacticians, etc.) or ministry (e.g., national ministries).

In terms of curriculum *design spaces*, this ranged from schools (where *d*esign work with colleague teachers took place), over spaces at regional or national organizations or over the web (where *D/d*esign activities took place, e.g., Sésamath, see Gueudet et al. 2016), to ministries and national curriculum institutes (where the *D*esign of national curriculum frameworks and guidelines were situated). However, it is noteworthy that all three modes of design increasingly included classroom teachers' involvement, and the preferred mode of communication, also for practical reasons, was often via the web.

Viewing *design spaces* in a non-literal sense, there appeared to be (at least traditionally) three design spaces: the "national" *D*esign space with its design of the national curriculum and (sometimes approved) textbooks; the "collective" *D/d*esign space with its activities in order to help teachers "digest" curriculum innovations; and the "individual" *d*esign space where individual teachers design their lessons. In recent years, however, a new more dynamic space has been opened for teacher design: the "interactive" *D/d*esign space, which permeates groups and traditional meeting places. In several countries (e.g., France, the Netherlands), *D/design platforms* have been developed where teachers can design with colleagues and experts and with their students.

4. It is also noteworthy that different contexts used, and produced, different resources for the design processes: whereas in England, although *textbooks* continued to be published, they did not seem to be highly valued as curriculum resources (and there was a "free market" in classroom resources developed by multiple commercial and non-commercial organizations and individuals). However, in China and Japan textbooks were highly valued, and they were approved (by the ministry), which made them the main object of and guideline for design. It appeared that in these two contexts teachers (designed and) analyzed textbooks, in order to understand how to align with the national guidelines expressed in the textbook. In the French context teachers appeared to be doing a considerable amount of design work: at classroom preparation/design level with a variety of textbooks and other resources; at Design level in associations (e.g., Sésamath textbooks/resources produced "by teachers for teachers"). In the Dutch context textbooks were used extensively (and "page-by-page") by mathematics teachers in class. However, in the teacher design teams (TDTs), textbooks rarely guided teachers: textbooks seemed to be for the "bread-and-butter" teaching (where hardly any preparation was needed), whereas designing for innovative

teaching seemed to be regarded as “creative.” In Brazil, the textbook has been central for teachers’ *d*esign activities, and teachers “trust” the textbook, as it has been analyzed by ministry specialists committee (comprising of mathematics teachers, mathematics and statistics teacher educators, and mathematicians).

6.4 Conclusions

Our claim is, first, that the term “teacher design” is often used casually, with little understanding of the different facets of “teacher design” and/or demarcation between the levels/characteristics of “teacher design” depending on the context/site, (number of) participants and “teaming,” and the audience and use of the design. We contend that what we term as “teacher design” activities can be regarded to lie on the crossroads between two dimensions: the “teaming” dimension (from working alone (single) to working in a collective) and the use dimension (from own use (for his/her teaching; site-specific) to use by others (generic)) (see Table 6.1). Theoretically, this resulted in four modes of teacher design:

- Teacher *d*esign activities at micro level (e.g., lesson preparation for own teaching).
- Teachers *d/D*esigning on their own/alone for use by others (e.g., expert teachers/professional designers) – we could not identify this mode in our cases, although there were often expert teachers involved in collective design.
- Teacher *D/d*esign activities at meso level (e.g., designing in collectives of colleagues).
- Teacher *D*esign at macro level (e.g., teams of teachers/experts designing for use by others, involvement in the design of national frameworks).

It is important to note that the *d* or *D* should not denote a hierarchical level, diminishing the importance of the d work – perhaps different terms can be found to denote *D, D/d,* and *d* work, or indeed further differentiation could be found.

Second, our international contexts illustrate that teacher design activities may be for different purposes, and for different expected end results. In the Japanese and Chinese contexts, the design activities were (beyond the immediacy of the lesson) for the purpose of teacher professional development, as an effective means of professional learning, in mathematics/subject teacher collectives (supported predominantly by experts). In the Brazilian, Dutch, English, and French contexts, the purpose of teacher design was for designing an artifact or a product. Hence, in one context, the aim was *process-orientated*, while in the other, it was *product-orientated*, and it could be linked to the mode of design characteristic for the context. We contend that the most promising form of teacher design might lie at the crossroads between product and process orientation, where more connections can be found between the two (what are now often two separate worlds).

Third, we claim that such connections could be provided by what we call *digital design platforms*. They have the potential of providing, as affordances, interactions between different dimensions of teacher design. In previous research, we have shown that e-textbooks (Pepin et al. 2016) provide such connections: between teachers and designers and between teacher colleagues (e.g., Sésamath – see Gueudet et al. 2016). More recently, such digital platforms have been created (e.g., in the French and Dutch contexts); we are in the process of analyzing which connections are actually made by such platforms, in other words in which ways they may support teacher design. Such *digital design platforms* are a strong promise for enhancing mathematics teacher design work.

The chapter focuses on one aspect of the recent changes in expectations for teachers' work, often outside the classroom: the teacher as a designer of the curriculum (at different levels). These changes presuppose a new kind of professionalism and imply new/different professional development needs, related to aspects of the design work: while previously teachers "only" had to prepare their own lessons and most of this design was embedded in practice, now they are expected to work in teams and/or design for a wider audience (e.g., colleagues). Seeing curriculum development and the designing of school curricula as a "normal" practice, which is different from teaching in the classroom, opens new ways of seeing teacher professionalism and expertise. Moreover, it has implications for organizing (pre- and in-service) teacher education programs.

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