Chapter 12
Transitions Toward Digital Resources: Change, Invariance, and Orchestration

Paul Drijvers, Verônica Gitirana, John Monaghan, Samet Okumus, Sylvaine Besnier, Cerenus Pfeiffer, Christian Mercat, Amanda Thomas, Danilo Christo, Franck Bellemain, Eleonora Faggiano, José Orozco-Santiago, Mdutshekelwa Ndlovu, Marianne van Dijke-Droogers, Rogério da Silva Ignácio, Osama Swidan, Pedro Lealdino Filho, Rafael Marinho de Albuquerque, Said Hadjerrouit, Tuğçe Kozaklı Ülger, Anders Støle Fidje, Elisabete Cunha, Freddy Yesid Villamizar Araque, Gael Nongni, Sonia Igliori, Elena Naftaliev, Giorgos Psycharis, Tiphaine Carton, Charlotte Krog Skott, Jorge Gaona, Rosilângela Lucena, José Vieira do Nascimento Júnior, Ricardo Tibúrcio, and Anderson Rodrigues

Abstract This chapter reports on the work of Working Group 4 and focuses on the integration of digital resources into mathematics teaching and learning practices. There are five central sections, focusing on, instrumental genesis, instrumental orchestration, the documentational approach to didactics, digital resources and teacher education, and the design of learning environments with the use of digital resources. A range of constructs and theoretical approaches are covered in these five

P. Drijvers · M. van Dijke-Droogers
Freudenthal Institute, Utrecht University, Utrecht, The Netherlands

V. Gitirana (✉) · R. Lucena
CAA - Núcleo de Formação Docente, Federal University of Pernambuco, Caruaru, PE, Brazil
e-mail: veronica.gitirana@gmail.com

J. Monaghan
Agder University, Kristiansand, Norway

University of Leeds, Leeds, UK

S. Okumus
Recep Tayyip Erdogan University, Rize, Turkey

S. Besnier
CREAD: Center of Research on Education, Learning and Didactic, University Rennes 2, Rennes, France

C. Pfeiffer
Cerenus Pfeiffer, Stellenbosch University, Stellenbosch, South Africa

C. Mercat · P. Lealdino Filho
S2HEP (EA4148), IREM, Claude Bernard Lyon 1 University, Lyon, France

L. Trouche et al. (eds.), *The ‘Resource’ Approach to Mathematics Education*, Advances in Mathematics Education,
https://doi.org/10.1007/978-3-030-20393-1_12

sections, and the opening section comments on construct validity and issues in "networking" theoretical frameworks. The chapter can be viewed as a literature review which surveys past and present (at the time of writing) scholarship with an eye to possible future research. The chapter is extensive in several dimensions: a large range of digital resources and applications are considered; the subjects using digital resources are not just teachers but also students, student teachers and student teacher

A. Thomas
University of Nebraska-Lincoln, Lincoln, NE, USA

D. Christo · S. Igliori
PUC/SP – Pontifíce Catholic University of São Paulo, São Paulo, Brazil

F. Bellemain · R. da Silva Ignácio · R. Marinho de Albuquerque · R. Tibúrcio · A. Rodrigues
Federal University of Pernambuco, Recife, Brazil

E. Faggiano
University of Bari Aldo Moro, Bari, Italy

J. Orozco-Santiago · F. Yesid Villamizar Araque
Cinvestav-IPN, Centre for Research and Advanced Studies, Mexico City, Mexico

M. Ndlovu
University of Johannesburg, Johannesburg, South Africa

O. Swidan
Ben-Gurion University of the Negev, Beer Sheva, Israel

S. Hadjerrouit · A. S. Fidje
Agder University, Kristiansand, Norway

T. Kozaklı Ülger
Bursa Uludağ University, Bursa, Turkey

E. Cunha
Instituto Politécnico de Viana do Castelo, Viana do castelo, Portugal

G. Nongni
University of Laval, Quebec City, Canada

E. Naftaliev
Achva Academic College, Arugot, Israel

G. Psycharis
National and Kapodistrian University of Athens, Athens, Greece

T. Carton
Paris 8 University, Saint-Denis, France

C. Krog Skott
University College Copenhagen, Copenhagen, Denmark

J. Gaona
Universidad Academia Humanismo Cristiano, Santiago, Chile

J. Vieira do Nascimento Júnior
State University of Feira de Santana, Feira de Santana, Brazil

educators. Issues raised in the sections include individual and collective use of resources, the adaptation of these resources for specific learning goals and to prepare (pre- and in-service) teachers for the use of digital resources.

Keywords Digital resources · instrumental genesis · instrumental orchestration · documentational approach to didactics · teacher education · design of learning environments

12.1 Introduction

Paul Drijvers, Verônica Gitirana, John Monaghan and Samet Okumus

This chapter reports on the work of Working Group 4 (WG4), which had the title of this chapter. This introduction to the chapter describes the original remit of WG4, outlines the range of papers accepted, describes and comments on the formation of five thematic subgroups formed during the conference, and comments on constructs and theoretical frameworks referred to in these thematic sub-groups.

> Digital resources have become an important part of teachers' and students' resource systems. the integration of digital resources into teaching and learning practices, however, raises many questions to teachers and educators.
> How to choose appropriate resources from the myriad of available options?
> How to adapt these resources to the specific learning goals at stake?
> How to orchestrate the students' use of the digital resources?
> What do student resource systems look like?
> How to prepare pre- and in-service teachers for these challenging tasks?
> Which role can digital resources play in assessment?
> Which opportunities do they offer for new learning formats, such as blended learning and flipped classrooms?
> How do classroom experiences inform the (re)design of a digital resource?
> What are the options for personalized learning in adaptive environments?

The remit of WG4:

In this working group, some of these issues will be addressed from theoretical perspectives, including instrumental genesis, instrumental orchestration and documentational genesis.

WG4 was the only Working Group to focus on digital resources and the only one to include a focus on students' use of the digital resources; it is hardly surprising, then, that it was the biggest Working Group – 25 papers and 2 posters. The papers can be found in the conference proceedings. The titles, below, give a flavor of the issues discussed in WG4 at the conference:

- A proposal of instrumental orchestration to integrate the teaching of physics and mathematics.
- Instrumental meta-orchestration for teacher education.
- Orchestrations at kindergarten: articulation between manipulatives and digital resources.
- Orchestrating the use of student-produced videos in mathematics teaching.
- Pre-service mathematics teachers' investigation of the constraints of mathematical tools.
- Transition from a paper–pencil to a technology-enriched teaching environment: A teacher use of technology and resource selection.
- An examination of teacher-generated definitions of digital instructional materials in mathematics.
- Teachers' intervention to foster inquiry-based learning in a dynamic technological environment.
- TPACK addressed by trainee teacher educators' documentation work.
- The birth of the documentary system of mathematics pre-service teachers in a supervised internship with the creation of a digital textbook chapter.
- Planning of the teaching of the standard deviation using digital documentary resources.
- LEMATEC Studium: A support resource for teaching mathematics.
- Using an app to collect data on students' use of resources for learning mathematics.
- Analysis of the use of resources on internet by pre-service mathematics teachers.
- From sample to population: A hypothetical learning trajectory for informal statistical inference.
- Teaching and learning of function transformations in a GeoGebra-focused learning environment.
- Creation of innovative teaching situation through instrumental genesis to maximize teaching specific content: Acid–base chemical balance.
- A proposal of instrumental orchestration to introduce eigenvalues and eigenvectors in a first course of linear algebra for engineering students.
- Teaching computational thinking in class: A case for unplugged scenario.
- A computational support for the documentational work mathematics teachers documentational work in EFII.
- From digital "bricolage" to the start of collective work: What influences do secondary teachers non-formal digital practices have on their documentation work?
- Digital resources: Origami folding instructions as lever to mobilize geometric concepts to solve problems.
- Exploring teachers' design processes with different curriculum programs.
- Prospective teachers' interactions with interactive diagrams: Semiotic tools, challenges and new paths.
- Instructors' decision-making when designing resources: The case of online assessments.

The Working Groups met in three 2-hour sessions over the conference. At first, it was difficult to see themes through the diversity of approaches and foci, but five themes appeared: instrumental genesis, instrumental orchestration, the documentational approach to didactics, teacher education, and design. We (the WG4 organizers) suggested these themes to the WG4 members and a collective discussion endorsed the themes as representative. Members were asked to pick their theme-group by going to different areas of a large room – everyone went to an area without fuss (a form of "embodied validity" for the five themes). The theme-groups then started discussing their theme: initially how their paper fitted into the theme and then structuring ideas and constructs around the theme. These five theme groups liaised after the conference and produced the next five sections of this chapter. We now move on to constructs and theoretical frameworks.

We first comment on what we mean by "constructs" and "theoretical frameworks." We use the word "construct" for a mental image and name of a phenomenon. "Instrumental genesis" and "instrumental orchestration" are examples of constructs. Zbiek et al. (2007) use constructs "that have specific applications to mathematics, that have an empirical basis, and that help one understand relationships among tool, activity, students, teacher, a curriculum content" (p. 1172). Also, academics may use constructs to talk about general properties of "things" in the real world. Academics should, of course, ensure that the constructs they use are clearly tied to the real world and accurately describe the phenomenon under examination – this is called "construct validity." A "theoretical framework" (or "theory" or "theoretical approach") is a perspective for interpreting reality that usually includes a number of constructs specific to the theory. There are "grand" and "local" theoretical frameworks: Piaget's (1955) genetic epistemology is a grand theory and radical constructivism, and the theory of didactical situations includes local theories that are aligned with Piaget's grand theory (see Lester 2005). The *documentational approach to didactics* is a local theory, but what, if any, is the grand theory to which it is aligned? "Networking" theoretical frameworks (using a bit of one in another) has occupied the attention of mathematics education academics for several decades (see Kidron et al. 2018); the state of the art with networking theoretical frameworks is that it is often possible (at some level) but must be done with careful attention to detail. We now comment on constructs and theoretical frameworks referred to in these thematic sub-groups.

The principal construct of Sect. 12.2 is instrumental genesis. It is aligned with Rabardel's instrumentation theory and constructs from Vergnaud's (2011) Piagetian approach (e.g., operational invariants). The authors utilize Gibson's construct of affordances in their discussion of instrumental genesis. The principal construct of Sect. 12.3 is instrumental orchestration (IO) (which makes essential use of instrumental genesis). Central constructs of IO are "didactical configurations" and "exploitation modes" and, in later formulations, "didactical performances." There is mention of possible networking with Koehler and Mishra's (2009) *Technological, Pedagogical and Content Knowledge* (TPACK) framework teachers' professional knowledge and Ruthven's (2014) model of *Structuring features of classroom practice*. Besnier and Gueudet's (2016) construct of "chaining orchestrations" (which itself arose from networking IO with *the Anthropological theory of didactics*) is also used. The section ends by employing ideas from Lakoff and Núñez's (2000) embodied cognition perspective. The focus of Sect. 12.4 is the local theory *documentational approach to didactics (DAD)*, which links, obviously, to IO. The section explicitly discusses networking DAD to other theoretical frameworks, for example, *Activity Theory*, the *Joint action theory in didactics* and TPACK (and a variant, the model *Mathematical pedagogical technological knowledge*). A host of emerging construct is considered, for example, "documentational trajectory" and "resource system metamorphosis," among others. Section 12.5 employs constructs introduced in earlier sections but implicitly introduces new theoretical frameworks because this section is essentially concerned with teacher education and how one views teachers, and teaching depends very much on one's theoretical perspective. There was no room in that section to consider possible tensions in some of these perspectives, but we take the opportunity here to mention that Teresa Assude's approach is informed by *the Anthropological theory of didactics* and Kathleen Heid's by *constructivism* and that networking these approaches is problematic. Section 12.6 is concerned with the design of learning environments. As with other sections, it considers various theoretical approaches and employs a number of specialized constructs, but an added element of complexity is that the design of learning environments is not just a meeting of approaches, it is a meeting place of disciplines – computer science and didactics (with ideas and approaches from engineering).

We make these comments on constructs and theoretical frameworks partly as an advanced warning to the reader but partly to remind ourselves to be aware of the importance of construct validity and the difficulty of networking theoretical approaches.

12.2 Instrumental Genesis: A Theoretical Lens to Study Mathematical Activities with Digital Tools

Cerenus Pfeiffer, Danilo Christo, Mdutshekelwa Ndlovu, Said Hadjerrouit and Sonia Igliori

This section focuses on instrumental genesis.[1] For this, we will seek to investigate, in a synthetic way, what instrumental genesis means. One answer to this question was presented by Gueudet (Chap. 2) when she took up the foundation elements of this theory, the distinction between an artifact (a digital artifact for the purpose of this section), a product of human activity designed for human activity and directed by objectives, and an instrument developed by a given subject (Rabardel 1995); the notion of instrument as an artifact + utilization scheme; the notion of scheme with

[1] This section also mentions "documentational genesis." Sect. 12.4 below considers the Documentational Approach to Didactics. The processes governing instrumental genesis and documentational genesis are similar, although the underlying artifacts these processes work on differ.

its four components, the objective of the activity, rules of action, operational invariants, and inferences (Vergnaud 1996); and highlighted two processes behind instrumental genesis – instrumentation and instrumentalization.

12.2.1 Theoretical Approaches to Instrumental Genesis

Drijvers and Trouche (2008) view instrumentalization as the process by which subjects shape the instrument and its use, and instrumentation is the process by which the artifact influences the activity and the thinking of the subjects. Both aspects influence and are influenced by the pedagogical design of the teachers, which gives rise to this genesis. Ratnayake and Thomas (2018) argue that teachers have to adapt digital resources and appropriate them to their practices by shaping and transforming them (instrumentalization and instrumentation). Lagrange and Monaghan (2009) argue that the availability of technology challenges the stability of teaching practices; techniques that are used in "traditional" settings can no longer be applied in a routine-like manner when technology is available. In order to help teachers to benefit from technological resources in everyday mathematics teaching, it is therefore important to have more knowledge about the new teaching techniques that emerge in the technology-rich classroom and how these relate to teachers' views on mathematics education and the role of technology as a teaching resource therein (Drijvers et al. 2010). Drijvers et al. (2013a) also contend that a deep understanding of students' learning processes is a core challenge of research in mathematics education.

The theory of instrumental genesis (TIG) ascribes a major role to artifacts that mediate human activity in carrying out a task (Drijvers et al. 2013a). When the artifact is used to carry out a task, it becomes an instrument (Drijvers and Trouche 2008). Ndlovu et al. (2011) also view instrumentation as the process by which the user of the artifact is mastered by his or her tools or by which the artifact influences the user by allowing him or her to develop activity or utilization schemes within some boundaries. Such limits include constraints, which assist the user in one way and impede in another; enablements, which effectively make the user able to do something; and potentialities, which open up possibilities and affordances that favor particular gestures or movement sequences (see also Noss and Hoyles 1996; Trouche 2004).

The notion of "affordance" is particularly important to the theory of instrumental genesis. The notion was originally proposed by E & J Gibson in the 1950s. Gibson (1977) is an authoritative account and refers to action possibilities, that is, what the user can do with an object. Norman (1988) applied the notion of affordances to digital tools. In this context, affordances refer to the perceived and actual properties of the tool, primarily those fundamental properties that determine just how the tool could possibly be used. Kirchner et al. (2004) developed three levels of affordances for digital tools. Firstly, technological affordances are properties of digital tools that are linked to usability issues. Secondly, educational or pedagogical affordances are properties of tools that act as facilitators of teaching and learning, and, finally, social affordances are properties of tools that act as social facilitators.

Given these considerations, we argue that, within the context of instrumental genesis, the affordances of digital tools are actualized at the technological, didactical/pedagogical and social levels. Technological affordances provide opportunities that facilitate the learning of mathematics, such as ease-of-use, ease-of navigation, accurate and quick completion of mathematical activities, drawing of graphs and functions, etc. Didactical/pedagogical affordances help in building and transforming mathematical expressions that support conceptual understanding of mathematics, such as collecting real data and creating a mathematical model; using a slider to vary a parameter or drag the vertex points of a triangle in geometry software; moving between symbolic, numerical, and graphical representations; simulating mathematical concepts; or exploring regularity, change, etc. Finally, social affordances facilitate group work and discussion, collaborative learning, and students taking greater control over their own learning (see Hadjerrouit 2017).

12.2.2 Papers Presented at the Conference

Taranto et al. (2018) treat a Massive Open Online Course (MOOC) as an artifact, that is, a static set of materials. They claim that when a MOOC module is activated, it dynamically generates a complex structure that is called an ecosystem. The researchers add that the process of transforming an artifact into an instrument is replaced here by the evolution, artifact – ecosystem/instrument.

In a similar vein, Ratnayake and Thomas (2018) analyze the process of designing tasks using the structure of documentational genesis and identify a series of items in the set of resources employed by the research communities of teachers. These include artifacts such as the criteria for designing rich tasks, the three-point framework for lesson planning, delivery and review and an exemplary task, GeoGebra, students' worksheet, and an A-level syllabus. The tasks before and after an intervention were evaluated using the Rich Task Framework, which comprises 12 factors including the appropriateness of the tasks for the instrumental genesis of the student. They claim that groups that freely shared ideas were more flexible in their use of digital technology than others, seeking and incorporating appropriate digital technology techniques into the tasks to help students understand mathematical concepts. This, they claim, allowed them to improve their personal instrumental genesis by learning new techniques and follow-up schemes; this evidences a development in professional instrumental genesis. Overall, this research suggests that there is merit in encouraging teachers to design digital technology tasks by working collaboratively in small groups provided specific support is given to the professional development of teachers to assist them. In turn, there may be beneficial effects in the broader documentary and instrumental geneses.

In Lucena et al. (2018), the notion of instrumental genesis appears in the scope of IO, when metaphorically they say that an orchestra in general can be recognized as an instrumental grouping comprising a conductor and instrumentalists, their instruments and scores, all well arranged in a space for the purpose of performing a piece of music. This concept of IO aims to model the practice of the teacher to sustain the instrumental genesis of students in rich mathematical learning. They cite Rabardel (1995) to argue that instrumental genesis is a transformation of an artifact by the action of someone, transforming it into an instrument while the subject goes through the process of instrumentation integrating it into their practice. The transformation of the artifact into an instrument is not characteristic of the structure of the tool but of the schemes that the subject develops to integrate it. They go on to say that, from their perspective of students' instrumental genesis, two concepts are fundamental for the orchestrating teacher: the concept of scheme and the concept of situation (that does not assume here the meaning of didactic situation but the meaning of task). The idea here is that any complex situation can be analyzed as a combination of tasks, each with its own nature, and difficulties are important to know.

Orozco et al. (2018) inform us that the integration and the use of new technologies in mathematics education have had an impact, but in many cases, this impact is anarchic; the digital age induces change to the access of information and construction of knowledge, among other actions by human beings. It is a fact that these new and sophisticated tools do not immediately become efficient instruments of teaching-learning. The instrumental approach (Guin and Trouche 1999) is a structure that allows one to take into account the role of technology in learning and teaching mathematics, in which the role of the teacher in this structure is fundamental, since s/he is responsible for the instrumental genesis of students, carried out by means of orchestrations (Drijvers et al. 2010).

In the paper by Igliori and Almeida (2018), instrumental genesis is implicit, since it is present in the production of the teacher's documentation. The paper presents a web tool, built for the purpose of providing digital resources, which favor the instrumentalization of the user teacher. The construction steps, from the digital objects to the teaching of mathematics at the elementary school level, can still be used as support in the work of the instrumentalization of its students. The process of instrumentalization is the first step of instrumental genesis.

Pfeiffer and Ndlovu (2018) describe their research carried out with students in a bridging program at a South African university participating in a qualitative study with TIG as a theoretical framework. This exploratory study investigates which instrumentation processes are dominant in a GeoGebra-enhanced mathematics learning environment to support students to develop an understanding of concepts in function transformations and circle geometry. The instrumentation process in this study was thus how GeoGebra shaped the thinking of the students and how it helped them to understand concepts. The instrumentalization process, in turn, was how the students used GeoGebra on their own as a tool, for example, to validate their answers and test their conjectures. During in-depth and focus-group interviews in Pfeiffer's (2017) study, students were asked if GeoGebra had helped them with certain concepts. Most of them affirmed that GeoGebra use had indeed helped them to better understand function transformation and circle geometry. The following responses concern perceived affordances of GeoGebra.

- "I knew from last year that if you reflected a graph about the y-axis, the x-values change sign (from positive to negative or negative to positive). We were just doing it mechanically, but with GeoGebra, this year, I could see what is going on (visual affordance) and it made sense."
- "I could see the signs. I understand now better why the sign changes if g(x) is reflected in x-axis ….. then I know the negative sign has to stand in front of g(x) and it also meant that the new graph is h(x)."
- "With all the different circle geometry theorems, you could see which angles are equal to each other or different segments. I could see them."
- "It helped. Specially to see them visually (visual affordances). Like the chords, the angle subtended by the same chord to show that they are equal."

These responses suggest that the students acquired "physical and logico-mathematical" knowledge of function transformations. GeoGebra use also afforded the students an opportunity to link visual graphic representations to the algebraic representations of the same concepts.

The visual affordances of GeoGebra identified by students are as follows:

- GeoGebra acted as a tool to visualize the transformations, the utilization scheme of changing sliders, and gave them an enactive sense of what the parameter in the equations mean: that the change in the equation transforms the original function; the instrumented action scheme of typing the transformation notation gave them visual understanding of how the horizontal translation, reflection in *x*- and *y*-axis, occurs; and that the reflection in the $y = x$ means the inverse graph of a function.
- The utilization scheme of changing the colors of the different resultant graphs helped students compare them to the original graph, resulting in better understanding the nature of the "shifts."
- GeoGebra use gave students a better understanding of sketching the inverse graph of an exponential function because they came to know it as a mirror image of the exponential graph. It enabled them to use the instrumented action scheme of sketching the graph by using critical points.
- GeoGebra use also helped with the understanding (instrumental genesis) of theorems in circle geometry – for example, showing which angles are equal and which angles are subtended by the same chord or arcs. Responses showed how GeoGebra shaped the thinking of the students and how it helped them to understand and visualize theorems.

With regard to the instrumentalization process, responses of the students showed how GeoGebra was independently utilized as a tool to validate their answers and test their conjectures. Observations showed how the students discussed and analyzed the properties of a GeoGebra applet and conjectured what the transformation of the function should be. They tested and validated their conjectures by dragging sliders in the applet. The students, therefore, had an opportunity to make and validate conclusions about the type of transformation on the basis of intuition or experience obtained through GeoGebra. Observations also showed how students acquired or discovered physical knowledge of function transformations and circle geometry.

We now turn to visualization, the ability to use and reflect upon pictures, graphs, animations, images, and diagrams on paper or with digital tools with the purpose of communicating information, thinking about and advancing understandings (Arcavi 2003). Visualization tools are becoming important in mathematics education.

Two papers emphasize the role of visualization tools for teaching and learning mathematics. Barbosa and Vale (2018) highlight the potential of visual solutions and strategies to promote mathematical learning. Even though the term "instrumental genesis" is not explicitly mentioned in the article, there are clear indications of instrumentation and instrumentalization processes. The authors present two examples of tasks and their visual solutions. The first one is related to the area of the area of rhombus using a visual figure with colors (a square) as an artifact with four midpoints of each side of the square. In the process of instrumentation, the students shaped the artifact using their own mathematical knowledge to find a visual solution to the problem, while the artifact enabled the students to produce the solutions within its constraints (instrumentalization). The second example involved the manipulation of rational numbers, equations, and proportionality. The students produced many solutions including a solution obtained by visualization. Similar to the rhombus task, instrumentation and instrumentalization processes were at work in this case, too. The paper points to the affordances of visualizations to achieve more efficient solutions, since these provide additional strategies. The social affordances of the tool are also emphasized, since it allowed students to discuss their strategies.

The second paper on visualization tools, Martinez et al. (2018), describes a case of instrumental genesis at a Mexican university, where teachers use digital technology to give feedback on their teaching practice in order to move from instrumentation to instrumentalization and orchestration processes. The intention is to establish the importance of the digital tools as supports for didactical activities and mediators of mathematical knowledge in classrooms. This can be characterized as the teacher's instrumental genesis, where the processes of instrumentation and instrumentalization are intertwined, involving the planning of the class session, selection, distribution, and management of the artifacts with their affordances and constraints; giving rise to a scheme of use by identifying the features of the artifacts, the subjects of the activity, and their knowledge; and thus providing a form of IO.

The final paper we consider in this section demonstrating the usefulness of the theory of instrumental genesis is the paper on teaching computational thinking (CT) in classroom environments (Lealdino Filho and Mercat 2018). Even though the process of instrumental genesis is not explicitly mentioned in this article, there are clear indications of instrumentation and instrumentalization processes, affordances, and constraints of the artifacts as well. As an example of CT, the article presents a binary magic trick using five cards with numbers. The task consists of asking the student to choose a secret number between 1 and 31, showing her/him each card one after another to decide whether the card contains the secret number. In terms of instrumentation, the work consists of understanding the binary magic trick and writing an algorithm which performs it. The algorithm is the artifact that is shaped by the student using his/her knowledge of CT and mathematics. The algorithm itself has its own rules with affordances and constraints that must be followed by the students in order to return the same result independently of who performs the steps. This is the instrumentalization process. The article shows the possibility of using CT to design an algorithm without the use of digital tools. Implementing the algorithm on a computer follows basically the same logic, but it requires understanding the programming language in order to create an algorithm for the computer to yield the solution to be achieved. In addition, instrumental geneses that use programming languages as artifacts that mediate between the student and the task have an element of creativity in order to solve the problem. Programming is an iterative process where it is common for a program not to work as expected and thus cannot yield the correct answer the first time it is performed, in contrast to conventional digital tools such as GeoGebra, for example. The search for a better or more efficient solution can be achieved in various ways such as testing the program with different data and strategies, discussions with fellow students and the teacher, or conducting a search on the Web for alternative solutions, etc. The programming process provides affordances and constraints at the technological, pedagogical/didactical, and social level and creates interactions that facilitate the emergence of varied utilization schemes for the students. The combination of technological, pedagogical, and social elements, in addition to the creativity element of the programming process has huge impact on students' instrumental genesis and the schemes they develop when using CT and mathematics.

We conclude this section by noting that although much has been built on the notion of instrumental genesis (e.g., instrumental orchestration and documentational genesis), there is still much to learn about instrumental genesis itself.

12.3 Revisiting Instrumental Orchestration: Past Findings and Future Perspectives

Paul Drijvers, Sylvaine Besnier, José Orozco-Santiago, Tuğçe Kozaklı Ülger and Freddy Yesid Villamizar Araque

Soon after instrumental genesis was recognized as a key process in exploiting the potential of digital technology in mathematics education, it was acknowledged that teachers play a crucial role in enhancing this process. Instrumental orchestration arose an answer to the question of how to foster students' instrumental genesis. Even though the focus may have shifted toward teachers practices in terms of the DAD since then, this chapter revisits IO and identifies five future perspectives of this notion, to further extend its value for mathematics educations, and for teacher training in particular: (1) a shift toward student-centered orchestrations, (2) extending the repertoire of orchestrations, (3) chaining orchestrations, (4) didactical performance, and (5) teachers' and students' gestures.

12.3.1 Past Findings

As shown in Sect. 12.2, the notion of instrumental genesis was an important step ahead in research on the use of digital tools in mathematics education. It acknowledged the subtlety and the complexity of turning artifacts into (parts of) instruments through the joint development of techniques for using a particular tool for a particular task, and the corresponding insights to understand the mathematics involved. Soon, the crucial role of teachers in this process was recognized. The question was what teachers can do to foster this co-emergence of techniques and schemes, i.e., to create appropriate environments to make instrumental genesis happen. This is where the notion of IO came into play.

An instrumental orchestration was defined by Trouche (2004) as the teacher's intentional and systematic organization and use of the various artifacts available in a learning environment in a given mathematical task situation to guide students' instrumental genesis. An IO consists of two layers, a didactical configuration and an exploitation mode. A didactical configuration is an arrangement of artifacts in the environment or, in other words, a configuration of the teaching setting and the artifacts involved in it. Through the didactical configuration, the teacher "sets the scene" for instrumental genesis. An exploitation mode is the way the teacher wants to exploit a didactical configuration for the benefit of the didactical intentions. It is the expected way in which the didactical configuration can be exploited for the targeted instrumental genesis. As a paradigmatic example of an IO, Trouche (2004) presented the "Sherpa orchestration," in which a student uses an artifact in front of the class, thus allowing the teacher to guide the use, the students to react to that and the Sherpa student (and, through her/him, the class) to get feedback on the techniques *in use*.

This notion of IO soon received attention. Assude (2007) introduced the notion of instrumental integration, including initiation, exploration, reinforcement, and symbiosis (see also Hollebrands and Okumus 2018). Also, it was pointed out that, in spite of the somewhat formal word "orchestration," the teacher in this model should not be considered a conductor of a symphony orchestra but, rather, a jazz band leader who prepares a global partition but also is open to improvisation and interpretation (Drijvers and Trouche 2008; Trouche and Drijvers 2010).

To do justice to the multiple *ad hoc* decisions that teachers take in split seconds while teaching, the IO model was expanded with a third layer called didactical performance (Drijvers et al. 2010). The didactical performance refers to all (bounded) choices made on the fly with respect to how to actually perform in the chosen didactical configuration and exploitation mode: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals. Figure 12.1 depicts the three IO layers.

Since its early years, the notion of IO has widened its scope. Its relationships with other models for teacher behavior and teacher knowledge have been investigated. For example, Tabach (2011, 2013) and Drijvers et al. (2013b) combined and contrasted the IO approach with the TPACK model on teachers' professional knowledge. The

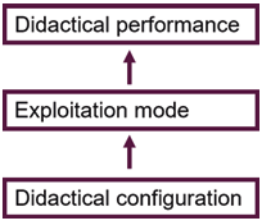

Fig. 12.1 The three-layer model of an IO

two lenses showed to be complementary and together provided a richer view on teachers' practices in ICT-rich classrooms. Also, the relationships with Ruthven's model of Structuring features of classroom practice (SFCP) framework have been explored (Bozkurt and Ruthven 2017; Ruthven 2014). In particular, the instrumental orchestration shows resemblance with the Activity Structure notion in the SFCP framework. To explore another connection, Trouche and Drijvers (2014) investigated the relation between instrumental orchestration and the notion of webbing. Whereas webbing focuses on the construction of a web of connected mathematical ideas, instrumental orchestration stresses the situation that invites this process. A further focus on teachers' practices with respect to designing, using and arranging resources has been developed under the name of the documentational approach to didactics, which is elaborated in Sect. 12.4. As far as student level and age are concerned, the work on instrumental orchestration originally focused on the upper secondary level, but since then, it has been widened, as far as kindergarten level (Besnier 2018; Carlsen et al. 2016).

If we look back at these developments, how well did IO do over the previous 15 years? It did lead to the acknowledgement that the way in which teachers foster instrumental genesis is a key issue. In addition to this, some orchestration types have been identified. In spite of the widening scope described above, however, we wonder if IO really had the impact that it might have had. Our view is that its potential has not yet been fully exploited, if we take into account the limited number of publications on this topic on the one hand, and the increasing role of digital tools in mathematics education on the other. The agenda for this section, therefore, is to revitalize the notion of IO. To do so, we outline five future perspectives that we consider promising and address below: (1) a shift toward student-centered orchestrations, (2) extending the repertoire of orchestrations, (3) chaining orchestrations, (4) didactical performance, and (5) teachers' and students' gestures.

12.3.2 Future Perspectives

12.3.2.1 A Shift Toward Student-Centered Orchestrations

When digital technology became more common in mathematics education, it was hoped that it would offer opportunities for students' ownership of their learning and that it would provide a "context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe" (Papert and Harel 1991, p. 1). In line with this view, one might be tempted to expect new types of student-centered orchestrations to emerge, which invite students engage in mathematics through creating mathematical objects.

Findings so far, however, seem to show a dominance of teacher-centered orchestrations. Drijvers et al. (2010) quote teachers privileging teacher-centered orchestrations such as Technical demo because they feel more in control of the situation, compared to student-centered orchestrations. This reminds of the experiences in the UK, where the large-scale introduction of interactive whiteboards in the UK led to traditional teacher-centered teaching practices: "the mere introduction of such technologies is insufficient to promote greater interactivity in the classroom, and indeed, that use may have had detrimental effects" (Rudd 2007, p. 2).

As another example of teachers preferring teacher-centered orchestrations, Kozaklı Ülger and Tapan Broutin (2018) described a study on one mathematics teacher's integration of technology in her course. Compared to her lessons, which usually were traditional, new orchestrations were observed in her technology-enriched lessons, and she implemented various orchestration types in the teaching process: Explain-the-screen, Discuss-the-screen, Link-screen-board and Not-use-tech (Drijvers et al. 2010). However, this did not prevent the teaching process from being teacher-centered. In spite of tablets with GeoGebra being available, the teacher hardly used them and stuck to whole-class teaching. This preference for teacher-centered orchestrations may have different reasons. The first reason is that students lack the skills of using software, in this study GeoGebra and that the teacher does not want to spend precious teaching time to make them more experienced. The second reason is the lack of technological-pedagogical knowledge and experience by the teacher. Consequently, she might feel losing control if much is left to the students' initiative. For example, students might come up with solutions, strategies and questions that are beyond the teacher's knowledge and experience.

To make students take full benefit of the potential digital technology offers, it might be good to use more student-oriented orchestrations. To be capable of doing so, teachers should feel the confidence on their own technical skills, trust their student learning capacities with respect to using digital tools, and dare to be out of control and to deal with unexpected situations. How pre- and in-service mathematics teachers can acquire these skills and how they can make a shift toward student-centered orchestrations is a research question that deserves more attention.

12.3.2.2 Extending the Repertoire of Orchestrations

In the literature, a small number of orchestrations have been identified. After Trouche's (2004) paradigmatic Sherpa orchestration, the collection of IOs remained very limited until the publications by Drijvers et al. (2010, 2013b). This resulted in the identification of classes of whole-class and individual orchestrations, ranging from being more teacher-centered to more student-centered (see Fig. 12.2). Since then, other researchers used this typology as a point of departure to identify additional IOs or describe variations (Tabach 2011, 2013).

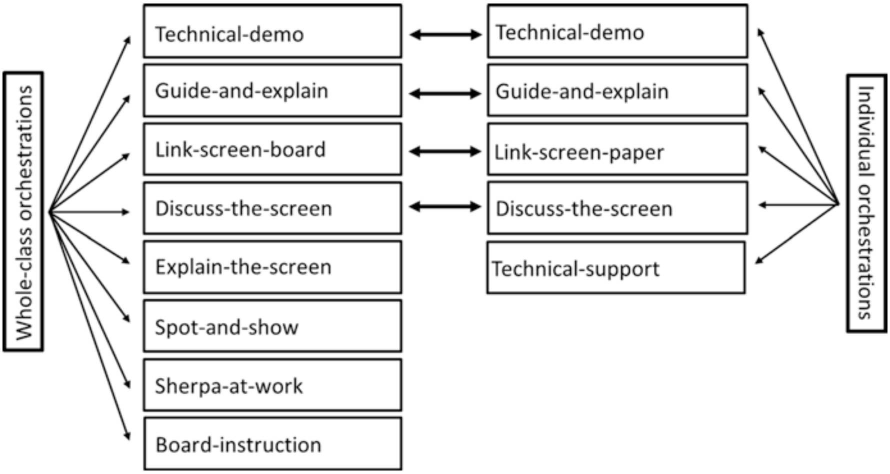

Fig. 12.2 Whole-class and individual orchestrations. (From Drijvers et al. 2013b, p. 998)

The question is, however, how context specific this limited repertoire is, how general are the orchestration types, and how do they depend on the digital tools *in use*, the mathematical topic, the teachers' views on teaching, and other possible factors? Also, we expect the repertoire to need further extension, for example, in the light of the increasing diversity of digital tools that came into play, such as MOOCs, flipped classroom tools, etc. For example, Orozco et al. (2018) study instrumental orchestrations in the case of university-level courses in linear algebra and the topic of eigenvalues and eigenvectors in particular. Digital tools include computer algebra systems and dynamic geometry software, and the results might shed light on possibly new orchestrations in this context.

In short, many questions on the repertoire of IOs are waiting to be answered. How general is the set of IOs identified so far? Do we need a more comprehensive taxonomy of orchestrations? How exactly is the relationship between the IO and the targeted instrumentation schemes? These questions are high on the future research agenda in our field.

12.3.2.3 Chaining Orchestrations

So far, the focus within IO research has been on isolated orchestrations. Hardly any attention is paid to integrating them into instructional sequences. How can teachers sequence orchestrations into productive chains? Are there specific chains that form natural sequences, like IO trajectories? Even if this idea was present in the early years of instrumental orchestration (e.g., see Trouche 2004), it has not been further elaborated so far.

In addressing these questions, an interesting approach could be to first identify the teachers' goals while setting up a classroom organization. To characterize such an organization, Besnier (2016) developed a link between moments of study (Chevallard 2002) and the notion of orchestration. Chevallard considered that "whatever the concrete path of the study, certain types of situations are almost necessarily present during the study" (Chevallard 2002, p. 11).[2] These types of situations are called moments of study. Chevallard identified four types of moments, described by Besnier (2016) as follows: designing and implementing introduction and discovery moments; designing and implementing learning and training moments; designing and implementing synthesis moments; and designing and implementing evaluation moments. While studying IO in Kindergarten, Besnier (2018) observed an orchestration linked to the design and implementation of a moment of synthesis, to support discussions between pupils about the procedures they used for solving a mathematical task. This orchestration was called "the manipulatives and software duo" and was considered a variant of the "link screen board" orchestration already identified in secondary school (Drijvers et al. 2010). We consider these two orchestrations as a part of a continuum, which starts with prior orchestrations that give the students the opportunity to experience moments of introduction and discovery and to experiment moments of learning and training.

Besnier and Gueudet (2016) identified specific chains of orchestrations within the same lesson. Orchestrations took place successively but also simultaneously. With regard to successive orchestrations, the authors observed, in a moment of introduction and discovery, a chain of three types of orchestrations, "discuss the screen," "explain the screen," and "Sherpa at work," and note teachers combining teacher-centered and student-centered orchestrations for the same goal. In this chain, the teacher leaves more or less room for the students' experience or actions. When should students be given more control? When should the teacher take over? In connection with these questions, this manipulation of orchestration chains, by the teacher and for the benefit of students' learning, seems to require dexterity and expertise from the teacher.

As for the simultaneous orchestrations, we observed orchestrations such as "accompanied use" and "peer work" carried out simultaneously during learning and training moments. The teacher's expertise in choosing a particular orchestration targeted at specific students and simultaneously managing several orchestrations seemed crucial here, to do justice to the differences between students.

In spite of this example, much remains unknown about the ways in which IOs may be chained and connected. This is an important topic to investigate in more detail and to address in pre-service and in-service teacher training.

12.3.2.4 Didactical Performance

As shown in Fig. 12.1, the IO model distinguishes three levels: a didactical configuration (the setting), an exploitation mode (the way in which the teacher intends to use this setting), and a didactical performance (the way in which the teacher actually

[2] Our translation.

carries out the teaching, including unforeseen events and follow-up decisions). So far, research has mainly focused on the didactical configurations and exploitation modes. It has hardly addressed the latter phase of didactical performance, which in the end might be decisive in the IO's effect. How do teachers take their decisions, and how can they be empowered to do so in a fruitful way?

Villamizar et al. (2018) studied the teacher's didactical performance in a high school course that integrated mathematics, physics and digital technology using the Cuvima model (Cuevas et al. 2017). The objective was to promote insight into both sciences based on the modeling of a physical phenomenon. One of the didactical configurations included printed guides, a projection room and tablets, with an app for video analysis and dynamic geometry. In groups of three, the students investigated the physical phenomenon of conservation of energy in the free fall of a ball.

The teacher's exploitation mode was guided by the four phases of the Cuvima model: experimentation of a physical phenomenon (use of guides and tablets), modeling by digital device (use of apps in the tablets), and conceptual analysis in physics and mathematics (use of didactic guides, projector, and blackboard), in which the teacher used Link-screen-board and Discuss-the-screen orchestrations (Drijvers et al. 2010). The teacher's didactical performance was evident during the discussion of the results, in which the teacher pointed out that the experimental data were imprecise. To improve data collection, a student proposed to add new artifacts as pointers to the tablet (USB, On-The-Go and mouse); in response to this, the teacher assigned this student the role of *Sherpa-student* (Trouche 2004). This decision clearly illustrates the importance of the didactical performance.

To summarize, the "proof of the pudding" of an IO to an important extent depends on the teacher's didactical performance. Consequently, it is highly relevant to know more about effective didactical performance and about the ways in which pre- and in-service teachers can further develop their skills on this point.

12.3.2.5 Teachers' and Students' Gestures

As part of the didactical performance, teachers use gestures while teaching. Students gesture as well while using digital tools. What is the relationship between the type of gestures and the techniques invited in the IO? Is there a relationship between the gestures, seen from an embodied perspective, and the techniques in use?

Notions on embodiment (Lakoff and Núñez 2000) stress that cognition, even in the domain of mathematics, is rooted in bodily experiences, which take place in interaction with the world. Sensori-motor schemes, in this view, might form a foundation for instrumentation schemes that are formed through instrumental genesis. However, research on IO seems to have neglected the embodiment and gesture perspective, and, in fact, one might wonder how to incorporate this view in the integration of digital tools in mathematics education. For example, Kozaklı Ülger and Tapan Broutin (2018) showed that even in technology-enriched lessons, teachers may prefer typical teacher gestures, such as tracing out a curve in the air, to using technological resources.

In short, further research is needed to investigate how IOs can take into account the bodily experiences in which mathematical experiences are rooted. How can we use digital technology to overcome the limitation of just neglecting embodiment? What is the relationship between mathematical concepts, body and the material activity with instruments? Recent developments in this field suggest promising relationships between the use of digital tools, gesture, and embodiment (e.g., see Ferrara and Sinclair 2016), but much is to be explored in more detail in this field.

12.3.3 Conclusion

This reflection on the past and the future of the notion of IO, on the one hand, shows its potential: it is widely acknowledged that teachers play a crucial role in enhancing the process of instrumental genesis, and that appropriate support to students is a subtle matter. The three-layer IO model may help teachers become aware of this subtlety and to develop their skills in exploiting the affordances of digital technology in their mathematics classes. For example, the notion of didactical performance highlights the flexibility that IOs need, to allow on-the-fly adaptations by the teacher. As such, the notion of IO is considered an answer to the question of how to foster students' instrumental genesis.

On the other hand, the increasing role of digital technology in mathematics education and the wide variety of digital tools makes us feel the IO model has not yet been fully exploited. We recommend further research in the five directions outlined above, to further develop IO as both a theoretical and a practical framework but also to better align it with current trends in mathematics education, including foci on student-centered learning and on the importance of gestures and embodiment as foundations of mathematical knowledge.

12.4 Perspectives of the Documentational Approach to Didactics with Regard to Transitions Toward Digital Resources

Sylvaine Besnier, Verônica Gitirana, Rogério da Silva Ignácio, Rafael Marinho de Albuquerque, Gael Nongni, Giorgos Psycharis, Charlotte Krog Skott and José Vieira do Nascimento Júnior

The roots of the DAD (Gueudet and Trouche 2008) are interrelated with a transition of research interest from resources used by teachers and/or teacher educators to digital resources. The increasing development of the DAD (Trouche et al. 2018), however, points to its potential to obtain deeper understanding of teachers' practice with resources, digital or not. A basic assumption of DAD is that the multiplicity of the digital resources (including applets and e-textbooks) offers increased opportunities for teachers to design their lessons and modify teaching approaches traditionally adopted in the classroom. At the same time, new digital means such as e-textbooks, offering new potential structures to the teacher and new interactions with the users, influences teachers' work at the level of both design and professional development. Also, the study of collective design work taking place in diverse contexts and communities raises the question of collective documentational genesis.

As in the evolution of other theoretical approaches, its use as a framework implies the identification of gaps that lead to new developments within its own theoretical construction. These advances are often strongly demarcated by characteristics of the object or context analyzed. For example, Rocha (2018) introduces the notions of "documentational experience" and "documentational trajectory" as theoretical and methodological tools to analyze teachers' documentation processes over long periods of time.

In this section, we discuss some perspectives of DAD appearing in research on the transition toward digital resources. We address two main questions in this chapter:

– What are the perspectives under which DAD has been used to study to support teachers'/teacher educators' effective transition toward digital resources?
– How is DAD influenced by (and how does DAD influence) these perspectives in terms of networking, extensions, and new areas of research?

This section is structured in the six subsections: DAD, connections with IO, networking of DAD and other theories in the transition toward digital resources, individual and collective documentation work in the transition toward digital resources, the development of DAD in relation to pre-service teachers, using the reflective methodology of DAD to support teachers' meta-cognitive reflections on their practices, and DAD and the design of digital resources. The section closes with final remarks.

12.4.1 DAD, Connections with Instrumental Orchestration

As discussed in the previous section, the framework of IO (Trouche 2004) was created to allow exploration of the ways by which teachers create systematic and intentional arrangements of artifacts and persons at the classroom to facilitate learners' instrumental genesis. Trouche (2005) and Drijvers and Trouche (2008) argue that it is not enough to adapt classical mathematical situations, but teachers must design new situations considering the affordances and constraints of the technologies. Designing new situations partly based on digital resources requires the development of specific skills and knowledge of the teacher.[3] Several research studies that are

[3] Concerning the question of knowledge and skills mobilized in a broader perspective of resource use (not only with digital resources), reference can be made to the work of Working Group 3:

based on DAD explore the complexity of teachers' practices in relation to the use of digital resources. To understand this complex work, these studies focus on the skills, knowledge, and expertise of teachers in the context of the design and implementation of situations involving digital resources (e.g., Psycharis and Kalogeria 2018; Ratnayake and Thomas 2018). This work also looks at the (bounded) choices made by teachers and factors that may explain these (bounded) choices. To take these issues into account, we note that explicit links between DAD and IO have been developed as part of research that considers teachers' practice toward the integration of digital technology.

Kozaklı Ülger and Tapan Broutin (2018) use DAD to understand teachers' (bounded) choices in classroom planning in rich digital environments, using DAD and IO in a complementary way. To understand teachers' practice in a technological environment, it considers IO to analyze teachers' actions in a technologically enriched environment, and DAD to determine teachers' (bounded) choices of resource and changes in this process. This case study looks at the practices of a teacher for whom designing and implementing instruction in a technological environment is new. The study highlights an important aspect of this work as "weaving" (Billington 2009) as a common occurrence in observations. They observed teachers' movement with the available tools. Three tools were used: board, computer + screen, and body movements. During the lesson, digital tools were intentionally used, while in spontaneous situations, teachers used wooden boards or gestures.

The complementarity of the two frameworks is also explored in Besnier (2018), who uses DAD to study aspects of teachers' documentation of their process of orchestrating classroom lessons for teaching numbers at kindergarten (4- and 5-year-old pupils) with digital and analogic materials. The research focus is on the teachers' adaptations of resources as well as on their classroom orchestrations. For Besnier, orchestration is considered as part of the document developed by the teacher. Orchestration corresponds to the recombined resources, and the action rules part of the document. In this context orchestrations are the emergent part of the scheme. In this research, Besnier identified a variant of the "link-screen-board orchestration" (Drijvers et al. 2010), called "the manipulatives and software duo" orchestration. This orchestration and its implementation are linked in the case of the teacher to professional knowledge related to importance of verbalization and peer exchanges. To allow pupils to discover the procedures, they must experiment in the technological environment and discuss this experimentation with each other. Besnier argues that it is necessary for the teacher to create a new resource and to implement an orchestration and to make a link between manipulatives and software. The changes in orchestrations observable in classrooms are then considered as the mark of changes in the teachers' resource systems. They reflect on changes in teachers' knowledge.

Considering the importance of IO within teacher documentation, Lucena (2018) and Lucena et al. (2018) propose the notion of "instrumental meta-orchestration" to

---

"Instrumentation, skills, design capacity, expertise"; see Chap. 4 of this book, "Documentation Work, Design Capacity, and Teachers' Expertise in Designing Instruction."

promote teacher reflection about IO with regard to their documentational genesis in integrating digital resources. They work within a composition of IO, sometimes sequenced, sometimes overlapping, focusing on meta-situations, which allow teachers to reflect on the notion of IO.

12.4.2 Networking of DAD and Other Theories in the Transition Toward Digital Resources

The assumptions and challenges, reinforced with existing research work based on DAD over the last decade, suggest advantages of using additional theoretical lens to study phenomena related to teachers' and/or teacher educators' (TEs) documentation work (DW) including the use of digital resources. We now consider attempts to connect DAD with other theoretical frameworks and constructs. One strand of this research targets elaboration and refinement of theoretical terms traditionally used to describe mathematics teachers' work inside and outside the classroom such as "mathematics teacher design" and "mathematics teacher design capacity." Networking of DAD with Brown's (2009) theory of "teachers as designers" is based on the common perception of teacher interaction with curriculum resources by the two theories as a participatory two-way process of mutual adaptation (Pepin et al. 2017). This research is anchored in the French Sésamath association for the design of a grade 10 e-textbook and a European funded project targeting inquiry-based learning in mathematics and science (PRIMAS). The study leads to a new definition of "teacher design capacity" as comprising (1) an orientation or goal, (2) a set of design principles (called robust principles) that are evidence-informed (e.g., from own practice) and supported by justification for their (bounded) choices, and (3) "Reflection-in-action" type of implicit understanding developed in the course of instruction ("design-in-use"). This definition is used to investigate design capacity development stemming from teachers' transformation of digital curriculum resources to (re-)design instruction and work with/in collectives.

Another case of networking, between DAD and Cultural-Historical Activity Theory (CHAT), was triggered by the need to investigate design processes in teacher collectives working on the development of e-textbooks (Gueudet et al. 2016). The authors study the activity system of a community of teachers working in the context of a teacher association (Sésamath) for about 4 years to design/redesign a chapter (functions) of an e-textbook. At the micro-level, DAD allowed the researchers to capture the evolution of resources and rules shared by the community. At the macro-level, CHAT helps them to understand different types of collective geneses that result from tensions in the system indicating a change of the object of the activity at different moments: from designing a "toolkit" for mathematics teachers to interactive exercises and, finally, to a more "classical e-textbook." However, the authors do not provide a theoretical explanation of the term collective geneses. Similarly, Essonnier and Trgalová (2018) connect DAD with Engeström's (1987) activity theory and Fischer's (2001) concept of community-of-interest, as described later in this chapter.

Another strand of studies concerns networking of DAD to theoretical frameworks focusing on aspects of teachers'/TEs' knowledge. Psycharis and Kalogeria (2018) network DAD and the TPACK framework (Mishra and Koehler 2006) to study trainee TEs' DW in technology enhanced mathematics. They investigate which TPACK forms of knowledge targeted by trainee TEs in their documents and which operational invariants are related to these forms of knowledge. The analysis reveals one type of documents emphasizing the T aspect of TPACK (instructive) and two types of documents emphasizing the P aspect of TPACK (explanatory, facilitative). Operational invariants underlying trainee TEs' DW are directly linked to the trainees' teaching practice as well as to their epistemologies concerning the role of technology in the teaching and learning of mathematics and the ways they conceive trainee teachers ("as students"/"of students"). Ratnayake and Thomas (2018) connect the DAD with the theoretical model of *Mathematical Pedagogical Technology Knowledge* (MPTK) (Thomas and Hong 2005) to study what factors influence secondary mathematics teachers' development and implementation of digital technology algebra tasks. Although knowledge is not explicitly considered as a resource in DAD, MPTK includes an extension of the concept of resources to embrace aspects of Schoenfeld's (2010) decision-making theory which includes teacher's knowledge as a primary resource.

Another aspect of networking concerns connections between DAD and frameworks used to study teachers' DW in different subject fields. For instance, Messaoui (2018) connects DAD and *Personal Information Management* (Jones 2007) to study the operational invariants underlying the scheme of how a teacher classifies a new resource in her/his resource system. The analysis, based on the observation of teachers' classification of resources in using computers, reveals operational invariants related to didactic knowledge (e.g., type of activity, teaching grade) as well as knowledge linked to digital literacy (e.g., create a file, drag and drop a folder). Another example is the study of Jameau and Le Hénaff (2018) who combine DAD and the *Joint action theory in didactics* (Sensevy 2011) to explore how a science teacher uses digital resources (e.g., videos) for her *Content and Language Integrated Learning* lessons to support language and science learning.

12.4.3 Individual and Collective Documentation Work in the Transition Toward Digital Resources

In their seminal article introducing DAD, Gueudet and Trouche (2009a) emphasize teachers' involvement in professional collectives as one out of three fundamental factors of the theory. Despite this early emphasis on the collective dimension, they do not theoretically detail it further. Rather, they describe teachers' DW as highly personal, as it results from their professional, social, and personal background. It is thus interesting to see how this distinction or interplay between collective and individual DW is treated in the ongoing development of DAD.

The need for further development of this interplay is spurred by the evolution of digital technologies offering both new opportunities for learning formats for teachers, teacher educators, etc., and new forms of collaboration (e.g., e-mail communication, designing and sharing of resources on platforms, and noninstitutional digital spaces). Such new formats and forms are the primary focus in our selection of papers from both inside and outside the *Re(s)source 2018 International Conference*.

Gueudet and Trouche (2011) and Gueudet et al. (2012) investigate an innovative, online teacher-training program in France (Pairform@nce) designed to sustain ICT integration but from two different perspectives, teachers and online teacher educators. Both papers focus on the teachers' collective DW and provide empirical evidence of professional development in terms of documentational genesis. However, in this early stage in the development of DAD, the conception of the interplay between the individual and collective in DW is rather vague. In recognition of this, Gueudet and Trouche (2011) suggest further developments of this interplay: "What is the 'common part' of the individual documents generated by a collective work? To what extent is it possible to speak of a common knowledge coming from a community documentation genesis?" (p. 410).

More recently, Carton (2018a) and Essonnier and Trgalová (2018) investigate entirely new digital forms of teacher collaboration. Carton (2018a) studies how teachers use non-institutional digital spaces to enrich their DW using an early definition of "the social" by Gueudet and Trouche (2008). The paper provides empirical evidence that these spaces offer favorable settings for collective work. Essonnier and Trgalová (2018) study the influence of designers' resource systems and knowledge on their (bounded) choices when collaboratively designing a c-book (c for creative) in the MC^2-project (Mathematical Creativity Squared Project – http://mc2-project.eu/) by supplementing DAD with activity theory (Engeström 1987) and the concept of Community-of-interest (Fischer 2001). Networking DAD with other approaches, the authors argue, provides a more coherent theoretical conceptualization of the collaborative design, where they foreground the designers' joint enterprise and social interactions by viewing the collaborative design "as a collective DG (documentational genesis), starting from a resource or a set of resources contributed to the joint enterprise by the designers and resulting in a c-book resource" (Essonnier and Trgalová 2018, p. 62).

The work of other professionals rather than teachers also inspired and promoted developments of DAD. For example, Kieran et al. (2013) extend the framework to the collective activity of design researchers (i.e., the authors), contributing convincing analytical interpretations of the interplay between individual and collective documentational genesis. Focusing on "the team's documentational genesis" (p. 1048), they analyze what they call a "taken-as-shared genesis" by using a dual perspective. An individual perspective: "a document relates directly to the cognitive structures of those who have been involved in its design" (p. 1047), combined with a social perspective: "Each round of the process (of genesis) encouraged the sharing of individual IOs (Operational invariants) (and associated ARs (action rules)), so that eventually the final version of the (…) document came to be based on a shared set of IOs" (p. 1049). However, the authors do not theoretically elaborate these concepts and processes to provide a coherent taken-as-shared approach.

Hence, in recent developments of DAD, we see promising theoretical and analytical proposals of how to interpret the interplay between individual and collective DW in the transition to digital resources, either by linking DAD to other theoretical approaches or by extending the framework beyond primary and lower secondary teachers' work. Despite this, there is a need for further theoretical elaboration of the interplay to provide more accurate answers to the requests mentioned by Gueudet and Trouche (2011).

12.4.4 The Development of DAD in Relation to Pre-service Teachers Exploiting Digital Resources

In the first publications introducing DAD (Gueudet and Trouche 2008, 2009a, b, 2010), and even in more recent ones (Besnier 2016), the research considers teachers in the middle of their careers. Prieur (2016) includes teachers at the beginning of their careers in his investigations of teachers' documentational genesis (the heart of DAD). Nonetheless, further studies of teachers' documentational genesis are needed. Indeed, the elements of a scheme's development often take place during initial teacher training.

Nongni and DeBlois (2018) discuss documentational genesis in the transition of pre-service teachers to becoming teachers and their "epistemological stances" when planning lessons. Leroyer (2018) investigates the influence of these stances (Bailleul and Thémines 2013) on the interactions between teachers and their resources. According to them, the teacher can adopt three epistemological stances: the ancient pupil, the university student, and the teacher (DeBlois 2012). Nongni and DeBlois (2018) observe the influence of these epistemological stances on the documentational genesis, in part on the use patterns and arrangement variables. They also observe the influence of pre-service teacher's documentational genesis on epistemological stances, in particular how documentational genesis allows the transition of pre-service teachers to becoming teachers and their epistemological stances when they are interested in students' understanding. Nongni and DeBlois (2017, 2018) also orient the documentational genesis toward the arrangement variables, artifacts and didactic variables, when studying how the pre-service teachers exploit digital resources. They posit a reciprocal influence among these variables that could provide a framework for understanding the documentational genesis of pre-service teachers with regard to digital resources, by observing these epistemological stances (DeBlois 2012). The epistemological stances adopted by the pre-service teachers can then be used to understand pre-service teachers' development in their anticipation of activities while planning their teaching.

Assis et al. (2018) also investigate pre-service teachers' activities, within a *documentational trajectory* (Rocha 2018), and go toward the understanding of what they call a *resource metamorphosis*, from the "resource to study" toward the "resource to teach." When studying an early-career teacher's resource system, they analyze the pre-service mathematics teacher training to consider how they structure their resource systems. The concept of *resource system metamorphosis* helps them understand the transition from a system of study-oriented resources to a teaching-oriented resource system. Their study presents the activities of two teachers who transpose between two classes of different situations: one structured to perform mathematical tasks using Dynamic Geometry and another to create tasks for students to learn mathematics using Dynamic Geometry. The results suggest that pre-service teachers rely on their study-oriented resources, including textbooks to develop their *teaching-oriented resource system*, which includes dynamic geometry tasks.

Ignácio et al. (2018) focus on a pre-service teacher who is developing a supervised internship project that involves two cycles of the production and use of a digital textbook chapter on the role teaching. The analysis of the production of this material shows that, in addition to the visible adaptations of printed textbook parts for the digital medium, the pre-service teacher mobilized a vast system of resources previously developed. The analysis provides evidence that the pre-service teacher has developed professional knowledge related to the development and use of digital resources for the teaching of functions.

12.4.5 Using a Reflective Methodology of DAD to Support Teachers' Metacognitive Reflections on Their Practices

The term "reflective investigation methodology" was introduced in the context of DAD to study teachers' DW (Pepin et al. 2013). Enlarging the term beyond methods of data collection and analysis, Ignácio et al. (2018) use it to organize a teacher education program for pre-service teachers involving design, use, reflection and validation of an e-textbook chapter.

Reflecting about one's own documentation process also appeared as an important tool for action research. Nascimento Jr. et al. (2018) use DAD, networked to other theories, to analyze and modify their own actions while designing and experimenting with innovative lessons integrating digital technology for university science teaching and learning. Among other aspects, their own documentational genesis was analyzed considering the analysis of students' instrumentation. Conventional and innovative digital resources interacted and played relevant roles in the process. Drawing attention to their own experiences, they acknowledge how little they can control the outcomes of such interactions.

DAD research which involves a self-reflective methodology allows the subject to focus on his/her own documentation, documentational genesis, systems of document, and documentation work, in particular, on how one creates his/her own "individual schemes of use" (Gueudet and Trouche 2009a, p. 204). For example, Nascimento Jr. et al. (2018) argue that an attempt to adapt multiple materials (including traditional textbooks, e-books, online familiar and unfamiliar materials making use of large databases) demands that teachers require not only design capacity to prepare lessons, but also expertise and decision-making skills. DAD can help teacher educators to be aware of these needs, as is discussed in (Males et al. 2018, p. 207) with regard to emerging methodology "What do teachers attend to in curriculum materials?"

Thomas and Edson (2018) also focus on the need to consider meta-cognitive processes when analyzing teachers' documentation work. They examine teachers' conceptions of digital instructional resource as a way to understand how digital resources impact on teachers' work. They contrast teachers' definition of theoretical terms. As regards DAD, they consider resource and document from the teachers' perspective (i.e., who designs, selects, and implements resources). They show that while defining the term, the teachers "tended not to distinguish between the resource and the genesis through which it becomes a document" (p. 343). Thus, they argue, teachers' DW may also occur in the meta-cognitive process of defining "what counts" as digital instructional materials in a more general sense.

12.4.6 DAD and the Design of Digital Resources

DAD has been used to analyze not only teachers' work but also the work of other professionals (researchers, software designers, artists, etc.) involved in the process of designing digital resources. It has also been used to interpret, mainly for classifying, actions and principles related to the design of digital resources, as well as to design curricular digital resource for their effective use by teachers.

Essonnier and Trgalová (2018) consider the DAD as a tool to identify the designer's resource system and its influence in choosing digital resources, which is consonance with the motivation of Bellemain et al. (2018), for using the DAD to identify and establish requirements for a web environment to support teachers' DW and the design of digital resources. Indeed, they develop the idea of a web document, based on the DAD concept of a document: software composed of other software programs or digital components, that is, a set of digital resources and utilization schemes designed by a teacher for a specific teaching aim. They propose a classification for such resources (static, dynamic and active) depending on the kind of content displayed and/or interaction made possible with these digital documents. For them, an activity is interpreted as a web document activity, since teachers organize both the activity to be done by students and students' actions in the activity, generating a new document. Design issues are considered further in Sect. 12.6.

12.4.7 Final Remarks

DAD has its origins within the digital resources integration problem, and its evolution within digital scenarios brings into the approach new needs and new concepts such as *documentational expertise*, *documentational trajectory*, and *metamorphosis of the resource system.* These new concepts and tools now comprise part of the framework. In its origin, the networking of theories, sometimes articulated and sometimes contrasted, has led to new networks that bring new issues into DAD discourse, especially within teachers' transition toward digital resource systems.

The potential of networking between IO and DAD is especially important in research. It sheds light on how to support teachers' use of digital resources and, at the same time, the effects on their documentation as well as the correlation between teachers' documentation and teachers' choice of resources. This dialectic leads us to consider both frameworks for understanding teachers' effective use of digital resources and also goes toward an extension of IO into an instrumental meta-orchestration framework to teachers' education toward using IO as a support to design this use. The use of DAD in teacher education goes even further, with extensions of DAD examining the beginning of the documentation process within initial teacher training, as well as extending the idea of resource systems to pre-service teachers; a reflective methodology is important in dealing with teachers' initial education. This elicitation of the characteristics of teachers' documentation can also be used to improve one's own practice in action research. Characteristics of teachers' work on the web also lead to perspectives of analyzing collective documentation and individual documentation within the collective work. The research considered in this section suggests the need for more investigations and greater precision regarding the collective documentation approach. The continuous evolution of research using DAD to support teachers' effective transition toward digital resource also leads to the emergence of new concepts and research tools for improving DAD.

12.5 Digital Resources and Teacher Education

Samet Okumus, Amanda Thomas, Eleonora Faggiano, Osama Swidan, Elisabete Cunha, Elena Naftaliev and Rosilângela Lucena

Mathematics teachers are the principal actors who are responsible for planning and enacting school mathematical activity; and their enactment of technology into the mathematics classroom is influenced by a wide range of factors (Assude et al. 2010; Zbiek and Hollebrands 2008). The factors of technology integration are like a jigsaw puzzle in that each component must be supported and merged into another for a successful implementation of technology into practice. A missing, or weakly connected, piece in this jigsaw puzzle may impede or impoverish the use of technology.

One factor that influences the enactment of technology in mathematics classrooms is teachers' knowledge. For instance, the TPACK framework (Koehler and Mishra 2009) describes aspects of teacher knowledge that interact to influence how teachers integrate technology for the teaching of content (i.e., mathematics). Assude et al. (2010) categorize some of the factors that influence mathematics teachers' utilization of technology into four components: "the social, political, economic, and cultural level, the mathematical and epistemological level, the school and institutional level, the classroom and didactical level" (p. 406). Heid (2008) stresses the importance of how teachers use technology and how their educational beliefs affect educational settings and student learning with technology, since students are likely to use technology in the way that their teacher designed curriculum. She draws from research studies to highlight that teachers who have constructivist teaching beliefs are more likely to integrate technology into curriculum and allow students to be explorers through the use of technology.

12.5.1 Teaching Activity Prior to a Mathematics Lesson

Artzt et al. (2015) characterize mathematics teaching activity in three stages: decisions teachers make before, during, and after a lesson. Teachers' activity prior to a mathematics lesson includes lesson planning and considering the affordances and constraints of tools and resources to be integrated in the lesson. During the lesson, teachers' work focuses on monitoring and regulating. Evaluating and revising are the main mathematics teaching activities after a lesson.

12.5.1.1 Lesson Planning

Artigue (2002) identifies four key dimensions in technology-enhanced mathematics learning: the mathematics, the teacher, the learner, and the tool. These dimensions also apply to the documentation work in which teachers engage prior to teaching a lesson (Gueudet and Trouche 2009a). Along with the four key dimensions, Zbiek and Hollebrands (2008) identify external factors that must be considered when planning for technology-enhanced mathematics teaching, including ready access to technology and support staff, technology training and professional development, time constraints, logistical constraints, technology and device availability, and the availability of curriculum materials that capitalize on technology. The relationship between curriculum materials and technology resources is particularly salient to consider as teachers engage in documentation work prior to teaching mathematics lessons because the nature of this work requires teachers to select and integrate curriculum materials with technology and other resources.

With respect to the epistemological stance of curriculum materials, Choppin (2018) finds that teachers who engage in lesson design work with new types of curriculum materials tend to exhibit practices aligned with the curriculum programs to which they are most accustomed. Thus, when teachers plan lessons that integrate emerging technology resources with existing curriculum materials, prior practices and habituation to curriculum resource should be considered. Teachers' documentation work during the planning phase of lessons also includes the identification of potential resources that could enhance the mathematical and didactical goals of the lesson. In defining what constitutes digital instructional materials, findings from Thomas and Edson (2018) suggest that teachers consider not only the resource itself but how, where, and by whom it might be used. That is, teachers consider resources in relation to the context and learners, as well as their potential use for the lesson.

Zbiek and Hollebrands (2008) note that "teachers' conceptions, beliefs, knowledge, and use of technology seemed to influence the activities they created for their students who were using technology to learn mathematics" (p. 310). According to Farrell's (1996) findings, technology affects teachers' selections of tasks and activity types in mathematics curriculum. Based on her research study, Farrell (1996) observes that teachers are more likely to prefer using activities that require investigation and group work when they use technology. In addition, with the use of technology, teachers adopt tasks requiring more problem solving and higher level thinking (Farrell 1996). Such preferences could be considered as a result of shifts of teachers' and students' roles when technology is integrated. When planning to teach mathematics with technology, teachers should consider the resources available to them, the affordances and constraints associated with those resources (Kennewell 2001) and, more importantly, how those resources relate to the lesson's mathematical and didactical goals (Artigue 2002).

12.5.1.2 Affordances and Constraints of Tools

In recent years, digital tools and online resources have become increasingly accessible for teachers. When teachers incorporate technologies into mathematics lessons, "they may also utilize activities and examples from curricula that use technology. Finally, they include representations and strategies specific to technology" (Hollebrands et al. 2016, p. 273). However, as Dick and Burrill (2016) emphasize, "realizing the unique benefits of dynamic interactive mathematics technologies to enhance students' conceptual learning of mathematics depends heavily on teachers having the skills and knowledge necessary to make sound judgments in choosing and using these technologies in the classroom" (p. 43).

Mathematical tools have different affordances and constraints for mathematical learning. An affordance is considered by means of what an environment offers the agent who uses the tool (Gibson 1977) and "a constraint of an environment is related to affordance in as much as it specifies what the environment does not afford" (Monaghan 2016, p. 168). Okumus and Ipek (2018) emphasize that teachers should be able to identify not only the affordances but also constraints of the tools. In Okumus and Ipek's (2018) study, pre-service mathematics teachers work out the Triangle Inequality Theorem with hands-on manipulatives and digital tools, and identify tools that do not "stay true to the mathematics" (Dick and Burrill 2016,

p. 29). Tools may give rise to misconceptions or obstacles for students, and teachers have important roles in identifying them (Okumus and Ipek 2018).

Dick and Burrill (2016) use design principles that may assist teachers in choosing digital tools. For example, according to the *Sandbox Principle*, "technology-based environments should be constrained to minimize the change that students inadvertently escape or get lost in irrelevant aspects of the technology" (p. 29). A constraint in the design of a digital tool may also give an opportunity for mathematical learning, rescuing students from irrelevant aspects of the technology (Dick and Burrill 2016; Naftaliev and Yerushalmy 2017).

Naftaliev and Yerushalmy (2017) design interactive diagrams that are "relatively small unit(s) of interactive text in e-textbooks or another materials" (p. 154) with built-in constraints. Students generate different representations using the tool and attempt to overcome the built-in constraints of interactive diagrams by modifying the given representations or constructing new ones. According to the researchers, constraints of interactive diagrams play an important role in mathematical investigation. In this sense, Naftaliev and Yerushalmy's (2017) use of constraints of interactive diagrams seems to align with Kennewell's (2001) perspective who asserted:

> The affordances are the attributes of the setting which provide potential for action; the constraints are the conditions and relationships amongst attributes which provide structure and guidance for the course of actions… Constraints are not the opposite of affordances; they are complementary, and equally necessary for activity to take place (p. 106).

Dick and Burrill (2016) claim that constraints of digital tools are helpful in directing students to think mathematically and "serve to support student attention and focus on the mathematical implications of the actions they take on the mathematical objects in the environment" (p. 30). Some researchers argue that constraints of digital tools can give students more opportunities for mathematical learning than hands-on manipulatives (Dick and Burrill 2016; Kaput 1995). According to Kaput (1995), "most physical actions on physical manipulatives do not leave a trace sufficiently complete to reconstitute the actions that produced them" (Kaput 1995, p. 167). Dick and Burrill (2016) claim that hands-on manipulatives do not have any constraints and "can be arranged in ways that are mathematically nonsensical" (p. 30). On the other hand, digital tools can be constrained, which allow for removing irrelevant aspects of the technology. The built-in constraints can assist students with focusing on relevant aspects of technology that are linked to mathematics.

In recent years, several researchers have used duos of artifacts: pairs of hands-on manipulatives and digital tools that support one another (Faggiano et al. 2016; Maschietto and Soury-Lavergne 2013; Voltolini 2018). In Cunha's (2018) study, students follow written directions and fold papers during the origami activities. Then, they are asked to reproduce the required construction steps using a dynamic geometry program. The researcher states that the activity enables students to explore the mathematical relationships between hands-on manipulatives and the digital tool and stimulates students to produce representations to accomplish the origami task.

In Maschietto and Soury-Lavergne's (2013) research study, primary school students produce turning gestures using a gear train of five wheels (Pascaline) to perform arithmetic operations. Based on the feedback from the pascaline students used, Maschietto and Soury-Lavergne design a new artifact (e-pascaline) that is the pascaline digital counterpart. The design of the e-pascaline is influenced by student-produced signs that emerge during the use of the pascaline, and the semiotic potential of the e-pascaline is promoted by the continuity (similar usages) and discontinuity (different usages) between the two artifacts. Design decisions for the duo of the artifacts are made with regard to didactic goals, so that using one artifact adds value to the other. The researchers stressed "that the pascaline also has added value compared with the e-pascaline, which explains why one cannot be substituted for the other" (Maschietto and Soury-Lavergne 2013, p. 969).

In this line, Voltolini (2018) proposes a duo, combining digital and pen-and-paper environments, through triangle-construction tasks, taking into attention the links between the two, highlighting the continuity and discontinuities of the duo of artifacts to promote "the evolution of pupil knowledge" (p. 87). Also, Faggiano et al. (2016) investigate the synergic use of manipulative and digital artifacts (passing from one to the other) to construct and conceptualize axial symmetry and its properties, trying to understand how this synergic action is developed so that each task improves the learning of the others.

12.5.2 Teaching Activity During a Mathematics Lesson

According to Artzt et al. (2015), the main tasks of mathematics teachers during lesson implementation are *monitoring* and *regulating*. When the teacher monitors students, he or she "observes, listens to, and elicits participation of students on an ongoing basis to assess student learning and disposition toward mathematics" (p. 87). Regulation refers to in-the-moment lesson adjustments, "teachers must be flexible and able to modify their lessons based on their formative assessment of the students" (p. 75).

12.5.2.1 Monitoring

Researchers have characterized teachers' utilization of digital tools in technology-enhanced mathematics learning with a focus on how they position technology with regard to mathematics and students. For example, Drijvers et al. (2010) observe three teachers' dynamic algebra java applets integration into the mathematics classroom with a focus on how they orchestrate the whole-class discussions. The results indicate that each teacher's focus differs. The first teacher, who focuses on students' learning using technology, utilizes student-centered orchestrations. On the other hand, the second teacher, who focuses on conventional representations of mathematics, associates technology with representations. The third teacher, whose focus is technology, gives technology directions and utilizes teacher-centered orchestrations.

Swidan et al. (2018) identify the orchestration processes of teachers who aim to promote inquiry-based learning in a classroom setting where students collaborate in small groups and use digital resources. The researchers pay special attention to the ways the teachers use the digital resources to boost inquiry-based learning. While teachers are monitoring students, they stand beside students without intervening for a while. This passive teacher action is noted as necessary but insufficient to boost the inquiry processes of students. After a short passive intervention, observing what students are doing, asking students about their exploration processes and requiring them to provide a short summary of their reasoning are found to be helpful in focusing the teachers' attention on the learning objects.

Erfjord (2011) examines three mathematics teachers' utilization of a dynamic geometry program and how teachers organize conditions for instrumental genesis (e.g., organization of students' work, central focus of lessons, etc.). Classroom activities include drawing, constructing geometric figures, and working on parallel and perpendicular mathematical objects using technological and non-technological tools. Two of the teachers focus on technical aspects of the technology and instrumentalization-related tasks (e.g., making constructions that did not mess up). On the other hand, the teacher whose focus is instrumentation (e.g., have students discuss different methods of constructions) utilizes student-centered orchestrations. Tabach (2011) associates a mathematics teacher's orchestration of digital tools with her technological pedagogical knowledge. The researcher reports that the mathematics teacher utilizes more student-centered utilizations over time as her technological pedagogical content knowledge changes.

12.5.2.2 Regulating

Research indicates that teachers regulate their instruction by making *ad hoc* decisions due to feedback from students and factors such as time shortages (Artzt et al. 2015; Cayton et al. 2017; Drijvers et al. 2010). Stockero and Van Zoest (2013) emphasize *pivotal teaching moments* that may prompt teachers to regulate their lessons. They define pivotal teaching moments as "instance(s) in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding" (Stockero and Van Zoest 2013, p. 127). Cayton et al. (2017) identify pivotal teaching moments in technology-rich geometry classrooms. They find that a teacher who utilizes student-centered approaches pursues students' thinking and extends their mathematical thinking by asking follow-up questions in response to pivotal teaching moments. Leung and Bolite-Frant (2015) emphasize mathematics teachers' regulating of instruction as opening a *pedagogical space* when they use a digital tool with *discrepancy potential*. According to the researchers:

> The discrepancy potential of a tool is a pedagogical space generated by (i) feedback due to the nature of the tool or design of the task that possibly deviates from the intended mathe-

matical concept or (ii) uncertainty created due to the nature of the tool or design of the task that requires the tool users to make decisions (p. 212).

On the other hand, teachers may not manage to capitalize on the discrepancy potential of tools. For example, Ruthven et al. (2008) find that one mathematics teacher conceals anomalous situations of dynamic geometry software and makes changes to the lesson on the fly. However, teachers' mathematical knowledge and familiarity with the technology is an important factor for such *ad hoc* decisions.

12.5.3 Teaching Activity After a Lesson

According to Artzt et al. (2015), as a post-active stage of teaching, teachers should be able to evaluate and revise their lessons using "information from evaluations of student learning and instructional practices" (p. 88) that should develop students' mathematical thinking better than their earlier plans. Self-reflectivity may contribute in capturing the important changes that digital resources bring to the teachers' practice. In this line of thought, researchers emphasize that pre-service and in-service teachers should be supported in building up reflective competencies or in becoming *reflective practitioners* (Atkinson 2012; Jaworski 2014).

12.5.3.1 Evaluating

Self-reflectivity is not usually a spontaneous practice and requires motivation. The impact of teachers' beliefs about the role of digital resources in teaching and learning of mathematics plays an important role in technology integration, and "the greatest challenge for professional development aimed at effectively using dynamic interactive mathematics technologies: moving the teachers' tool perspective to one supporting *student* investigation and exploration" (Dick and Burrill 2016, p. 46). Analysis and design of mathematics tasks, the exploration of overarching ideas linked to mathematical contents and the analysis of videotaped classroom situations may enhance teachers' instruction (e.g., Scherrer and Stein 2013). However, as Barth-Cohen et al. (2018) point out, videotaping of classroom discourse remains a challenging and understudied tool.

Lucena et al. (2018) use IO to develop teacher capabilities in integrating digital resource in classroom and propose a new framework, *instrumental meta-orchestration*, that embraces theory and practice. According to the researchers, "an instrumental meta-orchestration is a systematic and intentional design of artifacts and human beings, in an environment of formation by an agent, to execute a meta-situation of formation which aims to guide teachers in their instrumental genesis about the theoretical model of instrumental orchestration" (Lucena et al. 2018, p. 300). A sequence of orchestrations is integrated, sequenced and imbricated to enable theoretical reflection on the practice of IO. Instrumental meta-orchestration requires active involvement in observation, analysis of discourse using a theoretical lens, and also promotes reflection on different aspects (e.g., content, theory, and practice), particularly when a digital tool integration is utilized.

12.5.3.2 Revising

Several researchers have developed frameworks to assist teachers, teacher educators or curriculum writers in evaluating, creating and refining tasks that support students' thinking (Naftaliev 2018; Scherrer and Stein 2013; Sherman et al. 2017; Trocki and Hollebrands 2018). These frameworks bring theories into practices as teachers revise and (re)create their tasks/lessons embracing a critical lens. For example, Sherman et al. (2017) combine two fine-grained frameworks for pre-service teachers: cognitive demands of tasks (high-level vs low-level) (Stein and Smith 1998) and the roles of technology in using these tasks (amplifier vs. reorganizer) (Pea 1985). The researchers find that pre-service teachers most often create high-level tasks that may support students' thinking. Furthermore, they most often use technology as a reorganizer in which "technology has the capability to transform students' activity, supporting a shift in students' mathematical thinking to something that would be difficult or impossible to achieve without it" (Sherman and Cayton 2015, p. 307).

Naftaliev (2018) examines pre-service teachers' interactions with interactive curriculum materials. The study uses a semiotic framework for analyzing the pedagogical functionality of interactive materials (Naftaliev and Yerushalmy 2017). Naftaliev's (2018) study includes five interaction stages. Pre-service teachers first develop intended curriculum with interactive materials, then analyze classroom scenarios where interactive materials are enacted. In the third stage, pre-service teachers build upon their experiences to develop comic representations of scenarios about classes engaged with the interactive materials. The comics are developed in LessonSketch (Herbst et al. 2011), a media-rich environment that "allows creating experiences around classroom scenarios performed with cartoon characters in the form of a slide show" (Naftaliev 2018, p. 305). During the fourth stage, teachers engage in learning mathematics units with interactive materials and reflect on their own processes of learning. In the last stage, the pre-service teachers design their own unit for mathematics teaching and learning with interactive materials and presented an episode of a classroom scenario in which the class is engaged with the units. The semiotic framework for pedagogical functionality of interactive materials and the five-stage procedure enable facilitating the pre-service teachers' design processes, to share, to discuss, and to modify their decisions.

12.5.4 Concluding Remarks

Adequately incorporating technology in the mathematics classroom may be a battle many teachers encounter. Some may only use it sparingly, while others do not use it at all. This, almost certainly, stems from the traditional nature in which teachers have learned and subsequently teach. With the use of technology, teachers may find an increase in classroom discourse as a positive outcome. Applets, computer software, calculators, and other forms of technology may allow students to think more conceptually while offering multiple representations quickly. As a result, students may be able to have more focused discussion about why or how something works, rather than just accept one way of doing something. With technology use, questioning strategies may also change (Zbiek and Hollebrands 2008). The use of technology can increase the questions that can be asked about a given situation and even heighten the demand of questions. However, "technology itself is not a panacea that will remedy students' difficulties as they learn mathematics. Rather, it is teachers' decisions about how, when, and where to use technology that determine whether its use will enhance or hinder students' understandings of mathematics" (Hollebrands and Zbiek 2004, p. 259).

12.6 The Design of Learning Environments with the Use of Digital Resources

Christian Mercat, Franck Bellemain, Marianne van Dijke-Droogers, Pedro Lealdino Filho, Anders Støle Fidje, Tiphaine Carton, Jorge Gaona, Ricardo Tibúrcio and Anderson Rodrigues

The use of digital resources in learning environments, designed and used in a wide variety of ways, is growing. In this context, the discussion of the effectiveness of a designed resource for stimulating learning is an important debate, requiring research in this design process. In this chapter, we will discuss two approaches to gain more information about how to design digital resources: (1) design *for use* and (2) design *in use*. After explaining this difference, we describe how this distinction can shed light on different approaches to digital resources design for learning. Digital resources used in any given didactic situation may range over many different types of resources, and encompassing this complexity in a single theoretical framework is challenging. Hypothetical Learning Trajectories (HLT) (Simon 1995; Simon and Tzur 2004) is a means that can help structure the context and use of a design. IO and the DAD can function as pivotal theoretical constructs to observe teachers' designs. These two approaches can guide the two forms of designs, we have introduced, involving their collective aspects in the life-cycle of a resource, going through diverse disseminations, appropriations, uses and redesigns. These redesigns, ultimately addressed to the students, happen in a variety of contexts ranging from horizontal (socially creative and collaborative group work) to more vertical situations in a one-to-many dissemination from a "guru" to her followers or informal numerical spaces such as blogs or social media.

12.6.1 Design for and in Use

There are various approaches to design, with regard to the use and design of (digital) resources. In this part of the text, we elaborate on "design *for use*" (1) and "design *in use*" (2).

By design *for use*, we refer to studies where a theoretically based design includes conjectures and hypotheses about the way (digital) resources can be used to promote learning in practice: we look at studies that focus on the design of learning environments based on theories, sometimes in combination with teaching skills. In this approach, we focus on the teacher's system of resources and on ways to structure all elements involved in the implementation, such as the necessary and specific educational software engineering, the role of various actors, and the role of instrumentation and instrumentalization.

By "design *in use*," we refer to studies that focus on the way in which learning environments with digital resources are used, particularly through the orchestration of the use, although envisioned a priori by the designers but put into practice by teachers and students. Investigating how this is actually done in practice is a rich source about what works and how it works for further agile design loops, rapidly taking into account actual use. Based on these two approaches, linked in a dialectic way, we can enrich the knowledge about the design of stimulating learning environments with digital resources. Of course, the line between design *for use* and design *in use* is not so clear because the learning environment design already anticipates usage, and the actual use by teachers implies in return adaptations, additions, modifications or in-depth changes of these environments. This ambiguity is related to the teacher's own work, which, for the orchestration of an environment rich in technology, finally develops an activity close to that of an engineer and assistants rather than the usual metaphor of the orchestral conductor where each musician should master his/her own instrument. The difficulty of clearly distinguishing the two designs *for use* and *in use* is also relative to the vocabulary. The verbs we use when talking about the actions of either a computer engineer or a teacher are more or less the same: they both *conceive*, *design*, *elaborate*, *develop*, and *create* resources, but the level of actions is usually different, leading to the design of technical resource for the first and pedagogical resource for the second, all addressed ultimately to the final user, the student. To use a concrete example in order to try to explicit the difference, let us take the case of the use of videos as pedagogical resource. It is a commonly used type of resource, and for the needs of its didactic exploitation, many adjustments can be useful: indexation, selection of extracts, insertion of subtitles and comments, incrustation, etc. The development of interfaces that allow such adjustments is typically a design *for use* issue and their use by the teacher is design *in use*.

The dialectic between design *for use* and design *in use* translates into an articulation between two engineering process with a certain tension between them, on the one hand, the design, founded by theoretical principles, of resources and supports, structures and bindings, for the teacher to use, and, on the other hand, the need for support structuring his actual orchestration, offering some flexibility and documenting the needed adjustments and revisions.

To better highlight this dialectic between design *for use* and design *in use*, and the articulation that it assumes between production-engineering and use-engineering, we can use the elements of IO and associate at some point the design *for use* with a didactical configuration, its elaboration by the engineer and its configuration by the teacher and the design *in use* when the exploitation mode and didactical performance are the primary concerns.

12.6.2 Tackling Complexity

Designing and developing resources and their supports for teachers and their pupils to use are an extremely broad and complex problem; thus even if we limit the study to digital resources, there are many kinds of resources and many ways to use them. A first step to reduce this complexity is suggested by Adler (2000) who proposes a classification of resources as *object* and *action*. In other words, inspired by the IO and the DAD, the classification of resources is based on their own characteristics and their utilization schemes:

– Developed by the teacher for the instrumentation and instrumentalization of these resources.
– Developed by learners when these resources are involved in activities.

Silva (2018) develops this theoretical framework to provide a basis for specifications of a digital system that allows teachers to describe and store resources by integrating them into his/her resource system according to their specific characteristics and utilization schemes. We regard the creation of such systems, for the organization and articulation of existing resources, as having potential to enrich the range of object-action-activity of the teacher.

12.6.3 Designing New Resources for Use

The engineering processes underlying the creation of these resources and supports are various and depend on the kind of resource conceived. Indeed, Tchounikine (2011) argues that we do not implement the same theoretical and methodological principles in, for example, designing a microworld[4] or a supporting environment for collaborative learning.

In the context of design *for use* of digital resources, an important line of research and development is interested in the conception of artifacts that effectively enable the teacher to offer mathematical activities to the learners in a computational environment. Typically, microworlds, simulations or games are considered useful in activities designed to foster mathematical thinking. Common software of choice in school mathematics is dynamic geometry systems, but we are interested as well in more general microworlds and simulations, offering tools which may be:

– Used by the teacher for the orchestration of didactical situations.
– Used by the learner for the exploration and the resolution of problems related to specific mathematical content.

Many other parameters have to be taken into account: the context of design; individual or group use (and, in the latter case, collaborative or cooperative); the context of use; whether *for use* in the presence of the teacher, collectively in synchronized distance learning or individually in asynchronous learning. We focus here on the contexts of a few examples.

Designing a new resource *for use* may be approached from a multidisciplinary, even transdisciplinary, perspective. The design *for use* of new resources can be analyzed with regard to the "transposition informatique" (Balacheff 1994) supported by a prior analysis of the epistemological, cognitive, didactic, and informatic dimensions. *Didactical Informatic Engineering* (Tiburcio and Bellemain 2018), a rereading of the didactic engineering (Artigue 1990) considering the Information Technology (IT) dimension, proposes a systematic, operational and anchored approach in the didactics of the mathematics of the "transposition informatique."

By integrating the IT dimension to the didactic engineering, it is a matter of carefully analyzing the actual contributions of IT to support the mathematical activity of the learners. Thanks to the interfaces and operational capabilities of the computer, Siqueira and Bellemain (2018) are particularly interested in the contribution of dynamic representations and articulations between these representations. Such a resource can create an interactive object that provides feedback on abstract notions it represents. The theoretical and methodological principles used in didactical informatic engineering and the specification (design) of these digital resources are rooted in epistemology, the theory of semiotic registers of representation (Duval 1993), the theory of didactical situations (Brousseau 1997) and the Anthropological Theory of Didactics (Chevallard 2002).

As an example of an implementation of this specific didactic informatic engineering model, we consider the LEMATEC project (www.lematec.net.br), in which design of artifacts allows for the dynamic articulation of various representations of mathematical objects. The mathematical contents addressed by these resources are the notion of function (Function Studium, Bellemain et al. 2016), the conics (Conic Studium, Siqueira and Bellemain 2018), and area and perimeter (Magnitude

[4] See Hoyles & Noss (1992) for an explication of this term.

Studium, Rodrigues et al. 2018). In the context of teaching, the "designing-a-new-resource" open question can become the guiding thread of the teaching of various disciplinary contents with varying focus depending on the specific content approach. In the study of Lealdino Filho and Mercat (2018) on teaching computational thinking in classroom environments, unplugged resources can be used to promote computational thinking, and this activity leads the students to *design* digital resources. Material resources are therefore designed *for use* and digital resources are not the initial teaching resource but the *product* of the activity. This study, in the Computer-Science unplugged framework, elaborates computational thinking competences through implementing a design without an a priori use of computers. An initial step in the convergent thinking phase (Mercat et al. 2017) is describing impressions, beliefs about what is experienced, here a magical trick but generative art and optical illusions in other works. In order to express them, thinking and expressing takes place, iteratively replacing an abstract and subjective construct, by a concrete, objective, and meaningful method which makes any information-processing agent return the expected result. Solving the task of writing an algorithm to perform the magical trick and to solve this particular problem did not need the use of any digital resource. Implementing it on a computer requires further work in order to translate the phenomena into a programming language. Implementing the activity requires versatility and flexibility on the side of the teacher. The possibility exists, of course, to restrict the tools made available to the students and conduct a thorough a priori analysis of the possible implementations that might emerge.

12.6.4 From Resource for Use to Resource in Use

In the perspective of "design *in use*," these artifacts have to be increased with tools that, when used by the teacher, allow their orchestration in her teaching, with minute tweaking and documentation process (Gueudet and Trouche 2008). To continue with the example of dynamic geometry, in addition to tools for editing and manipulation of figures, we find functions for the configuration of menus, the elaboration of a statement, the sharing of a figure at a distance, etc.

The implementation of learning activities using (digital) resources requires a system in which all elements involved in the design and implementation of learning activities are structured and organized. On the designer side, to make this instrumentalization and organization possible requires developing interfaces, supports, guides to instrumentation, as well as, on the teacher's side, robust resources systems, in an IO or in a documentation process. Investigating the design *for use* by the teacher of digital resources is the best way to gather information of how teachers build and use resources and systems of resources.

Brown (2009) investigates how the teacher works as a designer and regulates his/her Pedagogical Design Capacity (PDC). He proposed to draw a parallel between design and teaching, showing that these two activities share common procedures: "Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs and devise strategies – all in the pursuit of their instructional goals." Stating that we should consider "teaching as design," he developed the concept of PDC in order to describe how teachers would interpret and use curriculum materials. Using the example of a middle school science teacher trying to set up a science lab in her class, he defined PDC as a "skill in perceiving the affordances of the materials and making decisions about how to use them to craft instructional episodes that achieve her goals." Pepin et al. (2017) also argued that design could be considered, when applied to teachers, as "designing for teaching." We could therefore say that there is a strong link between teachers' design activities and their DW.

In a design *for use*, HLT (Simon 1995; Simon and Tzur 2004) can structure a priori information on both sides, of the teacher and of the expected users, helping the designers to shape the resources *for use*. During a teaching experiment, an HLT is implemented and tested, gathering data on the use of the resources, leading to a revision of the design. For example, in the study by van Dijke-Droogers et al. (2018), a HLT was designed and a teaching experiment (in the Netherlands) was conducted to evaluate and revise this HLT. The challenge was to invite ninth grade students, inexperienced with sampling, to making informal statistical inferences without the knowledge of the formal probability theory. As educational materials that focus on the development of informal statistical inferences for grade 9 in the Netherlands hardly exist, the materials had to be designed. In the HLT, the students were expected to proceed from a first experience with sampling physical objects, through an understanding of sampling variation and resampling, to reasoning with the simulated empirical sampling distribution. Design guidelines were identified through a literature review, and the possibilities of (digital) resources were explored. The designed eight-step HLT included information about the theoretical background, the learning steps, teaching approach, lesson activities, tools and materials, practical guidelines, expected student behavior, and data collection. For example, in step 6, students investigated what happened if the sample size increased. The hypothesis in this step was that students would understand that the characteristics (e.g., the mean) and the shape of the distribution of a larger sample usually better resemble the underlying population. To conceptualize this idea, students used TinkerPlots (Konold and Miller 2005) to easily and quickly simulate samples of different sizes. A learning activity based on growing samples (Bakker 2004) and the use of TinkerPlots was expected to help students develop aspects of informal inference and argumentative reasoning (Ben-Zvi 2006). Next, the students were asked in step 6 to compare similarities and differences between their simulated sample results and during a whole-class session, to the underlying population. Embedding students' findings in a classroom discussion was expected to enhance their statistical reasoning (Bakker 2004). This HLT was, as a next step, tested in a teaching experiment. The teaching experiment comprised a ten 45-minute lesson series and was piloted in one class with 20 students. The data analysis consisted of verifying whether the designed hypotheses actually occurred. To this end, for each step of the design, the formulated hypotheses were translated into visible student behavior.

12.6.5 Design in Use

When designing *in use*, the investigation can focus on the *instrumentation* of specific resources to enrich and refine the schemes used by the teachers in this instrumentation. There are many ways to observe *orchestration*, that is, to say the way the teacher appropriates these available resources and rely on them to conduct the activity of the students.

For example, Fidje (2018) in his study investigates the way teachers use student-produced video in mathematics teaching. This research aimed to identify and characterize different orchestrations used by a teacher in a mathematical discussion with regard to student-produced videos. Open coding was used to propose a framework adopted from Brown's (2009) degrees of artifact appropriation: offloading (use as is), adapting the resource, and improvising (disregard the resource and enact without specific guidance from the presentation). The findings show that the teacher orchestrated the use of videos in distinctly different ways, capitalizing on the affordances and working around the constraints of the medium. The teacher applied what appeared as a quite fixed framework for every mathematical problem presented in the discussion, first, with a presentation of the problem, followed by an elaboration through a back-and-forth discussion, and ending with a conclusion and connecting the current problem with the succeeding problems. This fixed framework was evident throughout the lesson; a new problem was never presented without a conclusion to the former. Within this framework, a number of orchestrations related to the student-produced videos emerged. Firstly, there were *offloading* orchestrations where the teacher used the videos as they were. The most notable examples of offloading were when the teacher used the videos as an introduction to a problem or as the conclusion to the problem. Secondly, the teacher used *adaptation* orchestration, as he chose to adapt most of the student-produced videos in some way or another. For example, the teacher started the video, paused it, and directed a question to the presenters in the video. Thirdly, the teacher used *hybrid* orchestrations, where students were asked to present something from their video. The teacher used this orchestration to improve the video or to elaborate on the problem addressed. Fourthly, the teacher gradually improved orchestrations. The improving orchestrations were all prompted by the presentations in the videos, even though they were not used to present or elaborate the questions. This study showed how the teacher identified perceived affordances in the different use of the resource in his lesson design, while planning the lesson, culminating in utilization schemes for the set of resources used.

12.6.6 Collaboration as a Way to Optimize Design

Gueudet et al. (2013) reflected upon conditions which were necessary for collective work to happen. They defined this collective work as "teachers working with 'other participants', that is, teachers working with and in teams, communities and networks." They proposed the following criteria: a common working room, "official" working hours, and possibly the intervention of institutions linked to school. After analyzing the DW of two mathematic teachers, their representations and practice of collective work, they came to the conclusion that collective lesson or task preparation was very important for teachers' DW. Nevertheless, they argued that the simple fact of being colleagues – working with the same students or in the same schools were not accurate sufficient criteria to guarantee satisfactory collective work. According to them, collective work and design could develop owing to conditions very similar to that of "communities of practice" (Wenger 1998) – groups of teachers who share a "joint enterprise, a mutual commitment, and a resource repertoire" (Pepin et al. 2013): a "mutual endeavor," that is to say, agreeing to work on resources according to similar objectives; "minding the system," that is to say agreeing on norms of participation and pedagogical actions; and "common forms of addressing and making sense of resources," in other words, allowing shared resources to become collective resources appropriated by the group. Therefore, these conditions are complex to gather as material settings (getting specific time and space, e.g., a common room to work together) are not sufficient for satisfactory collective design to happen. It requires both a sharing of values about teaching and teachers' subject-matter and a sharing of resources. It also requires a particular attention to boundary crossing allowed by brokers, bringing new acceptable techniques and ideas into a community. They enrich the community without disrupting it, allowing for social creativity in the realm of technology enhanced learning, as Essonnier (2018) shows in her PhD thesis.

Carton (2018a) showed that indirect collaboration on non-formal digital shared spaces could foster teachers' Pedagogical Design Capacity, but also that non-formal digital common spaces could offer favorable settings for collective work even though it might lead to individual design. The analysis she carried out showed that networks and platforms that were not originally dedicated to education or linked to school institutions could offer favorable settings for collective work, for instance, small groups of teachers connected through apps (Google Drive, Dropbox, WhatsApp), e-mail correspondence or social media (Facebook groups or pages which are not institutional but linked to subject matters or groups dealing with teachers' professional identity and experiences). These groups appeared to be either defined by precise circumstances (teachers who met during their internship year during their teacher education and wanted to stay in touch), or by teachers who already knew each other personally or professionally or who already met or built an online relation because they shared affinities or a similar status.

Different degrees of collaboration seemed to happen in these non-formal digital spaces: first, each teacher interviewed admitted they consulted, were inspired by, copied, printed, or used colleagues resources available on the Internet, through personal spaces like blogs, websites, social media like Facebook pages, or subject-matter dedicated groups, in order to "see what others do" and to "inspire" oneself, most of the time "without saying thank you." This pedagogical monitoring activity seemed to trigger a documentational genesis (Gueudet and Trouche 2008), starting with a selection of the initial resources owing to a follow-up of the colleagues' work, even if there was no communication between the teachers.

Then, teachers seemed to value these non-formal groups because they felt they could express themselves or ask questions about didactical practices or resources without fear of judgment or assessment, which could be considered as an indirect way of getting feedback about one's resources. Expression of shared trust and goodwill seemed to be two essential criteria to reach the first step of collective work: not only getting teachers to upload their resources but also getting them to express their own "voice" (Remillard 2010, p. 206) or affordances about their resources or their practices. Other connections between participants of these non-formal groups were 1) their desire to develop their resource system alongside their didactical practices in order to avoid routines and to adapt to their students; 2) a feeling of loneliness regarding these interrogations or due to their interest in digital resources among their school team.

Although every teacher who engaged (either actively or indirectly) in digital groups admitted these spaces were a melting pot which fostered their documentary work, they almost never mentioned that actual collective design happened directly within the space where they found the resources. The feedback around posted activities or documents seemed more frequent than the actual reposting of transformed resources. Digital spaces, which constituted small groups bound by close ties (WhatsApp, Dropbox, email correspondence, private mailbox on apps), seemed to be more favorable for collective feedback on resources, as modified resources were exchanged and commented upon, while in bigger groups, especially on social media (Facebook and Twitter) dialogue and interactivity seemed to serve each participant's professional development more than collective work.

In the MC^2 project (Essonnier 2018; Essonnier et al. 2018), a platform, named CoICode, was designed for capturing some of the social interactions regarding the path of an idea, documenting its diverse sources and influences until the final first cycle of a pedagogical resource. The analysis of the produced traces allowed for the characterization of traits in a community that promote social creativity. Of course, the TPACK of its members have to be compatible and complement each other. The context and atmosphere have to be free and trustful enough to allow for a fruitful divergent phase but professional enough to succeed in producing something usable as the conclusion of a convergent phase.

Teachers' collective work is also shaped by and for students, mainly through non-formal interactions. Carton's (2018b) analysis of 24 semi-structured interviews around secondary teachers' creativity showed that teachers described their DW as if it was a kind of "addressed creativity," in the first place addressed to their students. Participants of the study seemed to consider students both as an "audience" and as feedback providers, offering the most direct and genuine assessment teachers could get, which turned into a strong motivation for documentational genesis (Gueudet and Trouche 2009b), or design.

Lastly, the analysis also revealed that most teachers felt that their PDC and skills in crafting pedagogical episodes were mostly underestimated by school institutions. Therefore, some of the interviewees chose to turn to companies (either publishing houses or edtech players) that would "publish" their work – either through textbooks or instructional kits. They seemed to expect a symbolical, financial and professional recognition of their expertise from these partnerships, even though they admitted the deals did not often offer them satisfactory conditions, most of all from a financial point of view. Interestingly, some of the interviewees seemed to implement design habits born from their DW into paid projects, for instance, lessons presented as sheets which were used as models for an instructional kit. A hypothesis which needs further research would be to consider that teachers accept these kinds of partnerships because they throw light upon their PDC and therefore serve their professional development.

12.6.7 From Design in Use to Design for Use

The discussion presented in this chapter shows the richness of the theoretical constructs such as IO and DAD to observe, analyze, systematize and anticipate the activity, individual or collective, of the teacher using digital resources and systems, and this from multiple insights. The first contribution of the works presented is obviously relative to the models by allowing their validation, refinements and evolutions. A second contribution is relative to the conception and development of digital resources, interfaces, supports and systems, which scaffold the engineering-teaching activity undertaken by the teacher.

The realization of resources and platforms founded on theoretical and theoretical reflections is useful for several reasons. The first is that engineering questions theories because it requires tangible operational answers, which can be programed and computed, and this in turn promotes the evolution of the theories. The second is that produced artifacts and platforms provide ways of validating the answers provided by theories. We can consider a theoretical validation by the evaluation of the adequacy between the realization and its specifications. The adequacy in a semi-theoretical setup in laboratory with technologically experienced teachers might differ from practical experimentation by ordinary teachers. The third is that the designed artifacts and platforms are products that enrich teachers' resource systems, participating in their professional development, and infusing theoretical research into society.

A first focus of the research presented concerns on resources and their characterization by their own specificities, by the utilization scheme implemented by the teachers, and by the instrumental geneses implemented by the students. From this first insight, Adler's (2000) systematization of resources (object-action-activity) helps us to better analyze the choice and use of teachers' resources, and provides theoretical and methodological principles to produce the specifications for computer supports for these choices and uses.

A second focus is on the activity of the teacher as an engineer observing and analyzing his activity of preparing his teaching and producing material from digital resources. In particular, the teacher's PDC or HLT can be evaluated.

Although we focus on the conception and implementation of supports for didactic material production for the teacher, many possible orchestrations and articulations of resources can be built since the produced didactic material can be a didactic situation based on problem solving, a list of training exercises, a multimedia presentation of a specific content, a digital textbook, etc. For each of these possible resource orchestrations, specific supports can be provided. Generally, conceiving and implementing supports for the "design *in use*" of resources needs a wide variety of investigations, mostly built on the IO and DAD to understand the way teachers and researchers are selecting, taking decisions, combining and articulating resources, freely or with the support/constraint of platforms, individually or collectively. It has the purpose of several works presented during the Re(s)sources 2018 international conference.

12.7 Conclusion: What Has and Has Not Been Addressed

Paul Drijvers, Verônica Gitirana, John Monaghan and Samet Okumus

The questions in the original remit of WG4 have been unevenly addressed in the Working Group papers and, consequently, in this chapter. Neither the question regarding opportunities for new learning formats, such as blended learning and flipped classrooms, nor the question on what student resource systems look like have been addressed. There has also been little consideration of the role that digital resources play in assessment. The questions on how to choose appropriate resources and how to adapt them to specific learning goals, as well as the question regarding options for personalized learning, have been considered, among other things, in Sects. 12.3 and 12.4. The question on how to prepare pre- and in-service teachers has been addressed in Sect. 12.5.

New foci (or, at least, new takes on existing foci) have been introduced. The relationship between instrumental genesis and affordances is considered in some depth in Sect. 12.2 (and mentioned elsewhere). This is, we feel, an important focus for further work and could link with issues in the design of resources for teaching and for learning. Section 12.3 considers five areas (student-centered orchestrations; extending the repertoire of orchestrations; chaining orchestrations; didactical performance; and gestures) where the model of IO is not fully exploited. This section could/should be used as a springboard for further work in these areas. Section 12.4 raises and partially addresses a number of questions regarding networking the DAD to other theoretical framework, but as we noted in our Introduction, further work needs to be done here. Section 12.5 considers many conceptions of teaching (with digital resources) and advances knowledge in doing so, but further advancement requires networking these conceptions. Section 12.6 helps us appreciate that how learning environments are designed and used in practice and what works (and how it works). Further work in this area includes not just networking theoretical frameworks but networking fields of study (designers and didacticians).

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205–224.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Artigue, M. (1990). Ingénierie didactique. *Recherches en didactique des mathématiques, 9*(3), 281–307.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245–274.

Artzt, A. F., Armour-Thomas, E., Curcio, C., & Gurl, T. J. (2015). *Becoming a reflective mathematics teacher: A guide for observations and self-assessment* (3rd ed.). New York: Routledge.

Assis, C., Gitirana, V., & Trouche, L. (2018). The metamorphosis of resource systems of prospective teacher: From studying to teaching. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), Proceedings of the re(s)sources 2018 international conference (pp. 39–42). Lyon: ENS de Lyon, Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563

Assude, T. (2007). Teacher's practices and degree of ICT integration. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1339–1348). Larnaca: University of Cyprus and ERME.

Assude, T., Buteau, C., & Forgasz, H. (2010). Factors influencing implementation of technology-rich mathematics curriculum and practices. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology – Rethinking the terrain. The 17th ICMI study* (pp. 405–419). New York: Springer.

Atkinson, B. (2012). Rethinking reflection: Teachers' critiques. *The Teacher Educator, 47*, 175–194.

Bailleul, M., & Thémines, J. F. (2013). L'ingénierie de formation : formalisation d'expériences en formation d'enseignants. In A. Vergnioux (Ed.),. *Traité d'ingénierie de la formation L'Harmattan* (pp. 85–112). Paris.

Bakker, A. (2004). *Design research in statistics education: On symbolizing and computer tools*. Utrecht: CD Beta Press.

Balacheff, N. (1994). La transposition informatique. Note sur un nouveau problème pour la didactique. In M. Artigue, R. Gras, C. Laborde, & P. Tavignot (Eds.), *Vingt ans de Didactique des Mathématiques en France* (pp. 364–370). Grenoble: La Pensée Sauvage.

Barbosa, A., & Vale, I. (2018). Math trails: A resource for teaching and learning. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s) sources 2018 international conference* (pp. 183–186). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Barth-Cohen, L. A., Little, A. J., & Abrahamson, D. (2018). Building reflective practices in a pre-service math and science teacher education course that focuses on qualitative video analysis. *Journal of Science Teacher Education, 29*(2), 83–101.

Bellemain, F., Tiburcio, R., Silva, C., & Gitirana, V. (2016). *Function Studium software*. Recife-PE: LEMATEC Research Group.

Bellemain, F., Rodrigues, A., & Rodrigues, A. D. (2018). LEMATEC Studium: A support resource for teaching mathematics. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 255–258). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Ben-Zvi, D. (2006). Scaffolding students' informal inference and argumentation. In A. Rossman & B. Chance (Eds.), *Proceedings of the Seventh International Conference on Teaching Statistics (CD-ROM)*. Voorburg: International Statistical Institute.

Besnier, S. (2016). *Le travail documentaire des professeurs à l'épreuve des ressources technologiques : Le cas de l'enseignement du nombre à l'école maternelle*. PhD. Brest: Université de Bretagne Occidentale, https://tel.archives-ouvertes.fr/tel-01397586/document

Besnier, S. (2018). Orchestrations at kindergarten: Articulation between manipulatives and digital resources. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 259–262). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Besnier, S., & Gueudet, G. (2016). Usages de ressources numériques pour l'enseignement des mathématiques en maternelle : orchestrations et documents. *Perspectivas em Educação Matemática, 9*(21), 978–1003.

Billington, M. (2009). Establishing didactical praxeologies: teachers using digital tools in upper secondary mathematics classrooms. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of European Research in Mathematics Education* (pp. 1330–1339). Lyon, France: ENS de Lyon.

Bozkurt, G., & Ruthven, K. (2017). Classroom-based professional expertise: A mathematics teacher's practice with technology. *Educational Studies in Mathematics, 94*(3), 309–328.

Brousseau, G. (1997). *Theory of didactical situations in mathematics. Didactique des mathématiques, 1970–1990 (edited and translated by N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield). Dordrecht, NL*. Kluwer Academic Publishers.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). New York: Routledge.

Carlsen, M., Erfjord, I., Hundeland, P. S., & Monaghan, J. (2016). Kindergarten teachers' orchestration of mathematical activities afforded by technology: Agency and mediation. *Educational Studies in Mathematics, 93*(1), 1–17.

Carton, T. (2018a). From digital "bricolage" to the start of collective work – What influences do secondary teachers non-formal digital practices have on their documentation work? In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 263–266). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Carton, T. (2018b). *Case study: How teachers' everyday creativity can be associated with an edtech player's strategies*. Présenté au XXIe Congrès de la SFSIC – Création, créativité et médiations, Saint-Denis, 13-15 juin 2018.

Cayton, C., Hollebrands, K., Okumuş, S., & Boehm, E. (2017). Pivotal teaching moments in technology-intensive secondary geometry classrooms. *Journal of Mathematics Teacher Education, 20*(1), 75–100.

Chevallard, Y. (2002). *Organiser l'étude. 1. Structures & fonctions. Actes de la XIe école d'été de didactique des mathématiques* (pp. 3–32). La Pensée Sauvage : Grenoble. http://yves.chevallard.free.fr/spip/spip/IMG/pdf/Organiser_l_etude_1.pdf

Choppin, J. (2018). Exploring teachers' design processes with different curriculum programs. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 267–270). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Cuevas, C. A., Villamizar, F. Y., & Martínez, A. (2017). Aplicaciones de la tecnología digital para activiDADes didácticas que promuevan una mejor comprensión del tono como cualiDAD del sonido para cursos tradicionales de física en el nivel básico. *Enseñanza de las Ciencias, 35*(3), 129–150.

Cunha, E. (2018). Digital resources: Origami folding instructions as lever to mobilize geometric concepts to solve problems. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Ed.), *Proceeedings of the Re(s)sources 2018 International Conference* (pp. 271–274). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563

Deblois, L. (2012). De l'ancien élève à l'enseignant. Quel parecours. In J. Proulx, C. Corriveau, & H. Squalli (Eds.), *Formation mathématique pour l'enseignement des mathématiques* (pp. 313–320). Québec: Presses de l'Université du Québec.

Dick, T., & Burrill, G. (2016). Design and implementation principles for dynamic interactive mathematics technologies. In M. Niess, S. Driskell, & K. Hollebrands (Eds.), *Handbook of research on transforming mathematics teacher education in the digital age* (pp. 23–52). Hershey: IGI Global Publishers.

Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In G. W. Blume & M. K. Heid (Eds.), *Research on technology and the teaching and learning of mathematics. Cases and perspectives* (Vol. 2, pp. 363–392). Charlotte: Information Age.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. P. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics, 75*(2), 213–234.

Drijvers, P., Godino, J. D., Font, V., & Trouche, L. (2013a). One episode, two lenses. *Educational Studies in Mathematics, 82*(1), 23–49.

Drijvers, P., Tacoma, S., Besamusca, A., Doorman, M., & Boon, P. (2013b). Digital resources inviting changes in mid-adopting teachers' practices and orchestrations. *ZDM – Mathematics Education, 45*(7), 987–1001.

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. *Annales de didactique et de sciences cognitives, 5*(1), 37–65.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.

Erfjord, I. (2011). Teachers' initial orchestration of students' dynamic geometry software use: Consequences for students' opportunities to learn mathematics. *Technology, Knowledge and Learning, 16*(1), 35–54.

Essonnier, N. K. (2018). *Étude de la conception collaborative de ressources numériques mathématiques au sein d'une communauté d'intérêt*. PhD. Lyon: Université de Lyon, https://tel.archives-ouvertes.fr/tel-01868226/document

Essonnier, N., & Trgalová, J. (2018). Collaborative design of digital resources: Role of designers' resource systems and professional knowledge. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 61–64). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Essonnier, N., Kynigos, C., Trgalová, J., & Daskolia, M. (2018). Role of context in social creativity for the design of digital resources. In L. Fan, L. Trouche, S. Rezat, C. Qi, & J. Visnovska (Eds.), *Research on mathematics textbooks and Teachers' resources: Advances and issues* (pp. 215–233). Cham: Springer.

Faggiano, E., Montone, A., & Mariotti, M. A. (2016). Creating a synergy between manipulatives and virtual artefacts to conceptualize axial symmetry at Primary School. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedigns of the 40th conference of the international group for the Psychology of Mathematics Education: PME 40* (Vol. 2, pp. 235–242). Szeged: International Group for the Psychology of Mathematics Education.

Farrell, A. (1996). Roles and behaviors in technology-integrated precalculus classrooms. *Journal of Mathematical Behavior, 15*(1), 35–53.

Ferrara, F., & Sinclair, N. (2016). An early algebra approach to pattern generalisation: Actualising the virtual through words, gestures and toilet paper. *Educational Studies in Mathematics, 92*(1), 1–19.

Fidje, A. S. (2018). Orchestrating the use of student-produced videos in mathematics teaching. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 275–278). Lyon: ENS de Lyon. In *Retrieved on November 8th, 2018, at* https://hal.archives-ouvertes.fr/hal-01764563.

Fischer, G. (2001). Communities of interest: Learning through the interaction of multiple knowledge systems. In S. Bjørnestad, R. Moe, A. Mørch, & A. Opdahl (Eds.), *Proceedings of the 24th Information Systems Research Seminar in Scandinavia* (pp. 1–14). Bergen: The University of Bergen

Gibson, J. J. (1977). *The ecological approach to visual perception*. Boston: Houghton Mifflin.

Gueudet, G., & Trouche, L. (2008). Du travail documentaire des enseignants?: Genèses, collectifs, communautés. Le cas des mathématiques. *Education & Didactique, 2*(3), 7–33.

Gueudet, G., & Trouche, L. (2009a). Towards new documentational systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Gueudet, G., & Trouche, L. (2009b). Vers de nouveaux systèmes documentaires des professeurs de mathématiques?). In I. Bloch & F. Conne (Eds.), *Nouvelles perspectives en didactique des mathématiques. Cours de la XIVe école d'été de didactique des mathématiques* (pp. 109–133). Paris: La Pensée Sauvage.

Gueudet, G., & Trouche, L. (2010). Des ressources aux documents, travail du professeur et genèses documentaires. In G. Gueudet & L. Trouche (Eds). *Ressources vives: le travail documentaire des professeurs en mathématiques* (pp. 57–74). Paideia, Rennes: Presses Universitaires de Rennes & INRP.

Gueudet, G., & Trouche, L. (2011). Mathematics teacher education advanced methods: An example in dynamic geometry. *ZDM – Mathematics Education, 43*(3), 399–411.

Gueudet, G., Sacristan, A., Soury-Lavergne, S., & Trouche, L. (2012). Online paths in mathematics teacher training: New resources and new skills for teacher educators. *ZDM – Mathematics Education, 44*(6), 717–731.

Gueudet, G., Pepin, B., & Trouche, L. (2013). Collective work with resources: An essential dimension for teacher documentation. *ZDM – Mathematics Education, 45*(7), 1003–1016.

Gueudet, G., Pepin, B., Sabra, H., & Trouche, L. (2016). Collective design of an e-textbook: Teachers' collective documentation. *Journal of Mathematics Teacher Education, 19*(2), 187–203.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195–227.

Hadjerrouit, S. (2017). Assessing the affordances of SimReal+ and their applicability to support the learning of mathematics in teacher education. *Issues in Informing Science and Information Technology Education, 14*, 121–138.

Heid, M., K. (2008). Calculator and computer technology in the K-12 curriculum some observations from a US perspective. In Z. Ususkin, & E. Willmore (Eds.), Mathematics curriculum in Pacific rim countries-China, Japan, Korea, and Singapore: Proceedings of a conference (pp. 293–304). Charlotte, Information Age Publishing.

Herbst, P., Chazan, D., Chen, C., Chieu, V. M., & Weiss, M. (2011). Using comics-based representations of teaching, and technology, to bring practice to teacher education courses. *ZDM – Mathematics Education, 43*(1), 91–103.

Hollebrands, K., & Okumus, S. (2018). Secondary mathematics teachers' instrumental integration in technology-rich geometry classrooms. *Journal of Mathematical Behavior, 49*(1), 82–94.

Hollebrands, K., & Zbiek, R. (2004). Teaching mathematics with technology: An evidence-based road map for the Journey. In R. Rubenstein & G. Bright (Eds.), *Perspectives on the teaching of mathematics: Sixty-sixth yearbook* (pp. 259–270). Reston: National Council of Teachers of Mathematics.

Hollebrands, K., McCulloch, A. W., & Lee, H. S. (2016). Prospective teachers; incorporation of technology in mathematics lesson plans. In M. Niess, S. Driskell, & K. Hollebrands (Eds.), *Handbook of research on transforming mathematics teacher education in the digital age* (pp. 272–292). Hershey: IGI Global.

Hoyles, C., & Noss, R. (1992). A pedagogy for mathematical microworlds. *Educational Studies in Mathematics, 23*(1), 31–57.

Igliori, S. B. C., & Almeida, M. V. (2018). Un support numérique pour le travail de documentation des enseignants de mathématiques de l'EFII (Collège, en France). In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 288–291). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Ignácio, R., Lima, R., & Gitirana, V. (2018). The birth of the documentary system of mathematics pre-service teachers in a supervised internship with the creation of a digital textbook chapter. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 292–295). Lyon: ENS de Lyon. retrieved on November 8th 2018 at https://hal.archives-ouvertes.fr/hal-01764563.

Jameau, A., & Le Hénaff, C. (2018). Resources for science teaching in a foreign language. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 79–82). Lyon: ENS de Lyon. retrieved on November 8th 2018 at https://hal.archives-ouvertes.fr/hal-01764563.

Jaworski, B. (2014). Reflective practicioner in mathematics education. In S. Lerman (Ed.), *Encyclopedia in mathematics education* (pp. 529–532). Dordrecht: Springer.

Jones, W. (2007). Personal information management. *Annual Review of Information Science and Technology, 41*(1), 453–504.

Kaput, J. J. (1995). Overcoming physicality and the eternal present: Cybernetic manipulatives. In R. Sutherland & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education* (pp. 161–177). Berlin: Springer.

Kennewell, S. (2001). Using affordances and constraints to evaluate the use of information and communications technology in teaching and learning. *Journal of Information Technology for Teacher Education, 10*(1–2), 101–116.

Kidron, I., Bosch, M., Monaghan, J., & Palmér, H. (2018). Theoretical perspectives and approaches in mathematics education research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven, (Eds.), *Developing Research in Mathematics Education*: Twenty years of communication, cooperation and collaboration in Europe. London: Routledge.

Kieran, C., Boileau, A., Tanguay, D., & Drijvers, P. (2013). Design researchers' documentational genesis in a study on equivalence of algebraic expressions. *ZDM – Mathematics Education, 45*(7), 1045–1056.

Kirchner, P., Strijbos, J.-W., Kreijns, K., & Beers, B. J. (2004). Designing electronic collaborative learning environments. *Educational Technology Research and Development, 52*(3), 47–66.

Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher education, 9*(1), 60–70.

Konold, C., & Miller, C. D. (2005). *TinkerPlots: Dynamic data exploration (computer software, Version 1.0)*. Emeryville: Key Curriculum Press.

Kozaklı Ülger, T., & Tapan Broutin, M. S. (2018). Transition from a paper-pencil to a technology enriched environment: A teacher's use of technology and resource selection. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s) sources 2018 international conference* (pp. 344–347). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Lagrange, J. B., & Monaghan, J. (2009). On the adoption of a model to interpret teachers' use of technology in mathematics lessons. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.). *Proceedings of the Sixth Congress of European Research in Mathematics Education* (pp. 1605–1614). Lyon: ENS de Lyon.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Lealdino Filho, P., & Mercat, C. (2018). Teaching computational thinking in classroom environments: A case for unplugged scenario. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 296–299). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Leroyer, L. (2018). The capacity to think of transmission of knowledge from learning supports: A proposition of a conceptual model. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceeedings of the re(s)sources 2018 international conference* (pp. 203–206). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM – Mathematics Education, 37*(6), 457–467.

Leung, A., & Bolite-Frant, J. (2015). Designing mathematics tasks: The role of tools. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education*:. *The 22nd ICMI study* (pp. 191–225). Cham: Springer.

Lucena, R. (2018). *Metaorquestração Instrumental: um modelo para repensar a formação teórico-prática de professores de matemática*. Doctoral thesis. Mathematics and Technological Education Pos-graduation Program. Recife-Brazil: UFPE.

Lucena, R., Gitirana, V., & Trouche, L. (2018). Instrumental meta-orchestration for teacher education. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 300–303). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Males, L., Setniker, A., & Dietiker, L. (2018). What do teachers attend to in curriculum materials? In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 207–210). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Martinez, M., Cruz, R., & Soberanes, A. (2018). The mathematical teacher: A case study of instrumental genesis in the UAEM. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 211–214). Lyon: ENS de Lyon. retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Maschietto, M., & Soury-Lavergne, S. (2013). Designing a duo of material and digital artifacts: The pascaline and Cabri Elem e-books in primary school mathematics. *ZDM – Mathematics Education, 45*(7), 959–971.

Mercat, C., Lealdino Filho, P., & El-Demerdash, M. (2017). Creativity and technology in mathematics : From story telling to algorithmic with Op'Art. *Acta Didactica Napocensia, 10*(1), 63–70.

Messaoui, A. (2018). The complex process of classifying resources, an essential component of documentation expertise. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the res(s)ource 2018 international conference* (pp. 83–87). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017–1054.

Monaghan, J. (2016). Developments relevant to the use of tools in mathematics. In J. Monaghan, L. Trouche, & J. M. Borwein (Eds.), *Tools and mathematics: Instruments for learning* (pp. 163–180). New York: Springer.

Naftaliev, E. (2018). Prospective teachers' interactions with interactive diagrams: Semiotic tools, challenges and new paths. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the res(s)ource 2018 international conference* (pp. 304–307). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Naftaliev, E., & Yerushalmy, M. (2017). Design digital tasks: Interactive diagrams as resource and constraint. In A. Leung & A. Baccaglini-Frank (Eds.), *The role and potential of using digital technologies in designing mathematics education tasks* (pp. 153–173). Cham: Springer.

Nascimento, J., Jr., Carvalho, E., & Farias, L. M. (2018). Creation of innovative teaching situation through instrumental genesis to maximize teaching specific content: Acid-base chemical balance. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the res(s)ource 2018 international conference* (pp. 308–311). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Ndlovu, M., Wessels, D., & De Villiers, M. (2011). An instrumental approach to modelling the derivative in sketchpad. *Pythagoras, 32*(2), 1–15.

Nongni, G, & DeBlois, L. (2017). Planification de l'enseignement de l'écart-type en utilisant les ressources documentaires. In A. Adihou, J. Giroux, A. Savard, & K.M. Huy (Eds.). *Données, variabilité et tendance vers le futur. Acte du Colloque du GDM* (pp. 205–2012). Canada, Québec: Université McGill.

Nongni, G., & DeBlois. (2018). Planning of the teaching of the standard deviation using digital documentary resources. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 312–315). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563

Norman, D. A. (1988). *The psychology of everyday things*. New York: Basic Books.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. Dordrecht: Kluwer Academic Publishers.

Okumus, S., & Ipek, A.S. (2018). Pre-service mathematics teachers' investigation of the constraints of mathematical tools. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 316–319). Lyon: ENS de Lyon, retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563

Orozco, J., Cuevas, A., Madrid, H., & Trouche, L. (2018). A proposal of instrumental orchestration to introduce eigenvalues and eigenvectors in a first course of linear algebra for engineering students. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 320–323). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Papert, S., & Harel, I. (1991). Preface, situating constructionism. In I. Harel & S. Papert (Eds.), *Constructionism, research reports and essays, 1985–1990* (p. 1). Norwood: Ablex.

Pea, R. D. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167–182.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teachers' work and interactions: A collective perspective on resources, their use and transformation. *ZDM – Mathematics Education, 45*(7), 929–943.

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM – Mathematics Education, 49*(5), 799–812.

Pfeiffer, C. R. (2017). *A study of the development of mathematical knowledge in a GeoGebra-focused learning environment*. Unpublished PhD dissertation. Stellenbosch: Stellenbosch University.

Pfeiffer, C. R., & Ndlovu, M. (2018). Teaching and learning of function transformations in a GeoGebra-focused learning environment. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)sources 2018 international conference* (pp. 324–327). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Piaget, J. (1955). *The construction of reality in the child*. London: Routledge & Kegan Paul Limited.

Prieur, M. (2016). *La conception codisciplinaire de métaressources comme appui à l'évolution des connaissances des professeurs de sciences. Les connaissances qui guident un travail de préparation pour engager les élèves dans l'élaboration d'hypothèses ou de conjectures*. PhD. Lyon: Université de Lyon, https://hal.archives-ouvertes.fr/tel-01364778v2/document

Psycharis, G., & Kalogeria, E. (2018). TPACK addressed by trainee teacher educators' documentation work. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.),

Proceedings of the re(s)source 2018 international conference (pp. 328–331). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.
Rabardel, P. (1995). *Les hommes et les technologies: Approche cognitive des instruments contemporains*. Paris: Armand Colin.
Ratnayake, I., & Thomas, M. (2018). Documentational genesis during teacher collaborative development of tasks incorporating digital technology. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 219–222). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.
Remillard, J. (2010). Modes d'engagement : comprendre les transactions des professeurs avec les ressources curriculaires en mathématiques. In G. Gueudet, & L. Trouche (Eds.), *Ressources vives: le travail documentaire des professeurs en mathématiques* (pp. 201–216). Rennes/Lyon: INRP/PUR.
Rocha, K. (2018). Uses of online resources and documentational trajectories: The case of Sésamath. In L. Fan, L. Trouche, C. Qi, S. Rezat, J. & Visnovska (Eds.), Research on mathematics textbooks and teachers' resources: Advances and issues. ICME-13 monograph (pp. 235–258). Cham: Springer.
Rodrigues, A., Baltar, P., & Bellemain, F. (2018). Analysis of a Task in three Environments: paper and pencils, manipulative materials and Apprenti Géomètre 2∗. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury–Lavergne, & L. Trouche (Eds.), *Proceedings of the Re(s)source 2018 International Conference* (pp. 223–226). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563
Rudd, T. (2007). *Interactive whiteboards in the classroom*. Bristol: Futurelab Report – IWBs.
Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era* (pp. 373–393). Dordrecht: Springer.
Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice. *Computers and Education, 51*(1), 297–317.
Scherrer, J., & Stein, M. K. (2013). Effects of a coding intervention on what teachers learn to notice during whole–group discussion. *Journal of Mathematics Teacher Education, 16*(2), 105–124.
Schoenfeld, A. H. (2010). *How we think: A theory of goal–oriented decision making and its educational applications*. New York: Routledge.
Sensevy, G. (2011). *Le Sens du Savoir. Éléments pour une théorie de l'action conjointe en didactique*. Bruxelles: De Boeck.
Sherman, M. F., & Cayton, C. (2015). Using appropriate tools strategically for instruction. *Mathematics Teacher, 109*(4), 306–310.
Sherman, M. F., Cayton, C., & Chandler, K. (2017). Supporting PSTs in using appropriate tools strategically: A learning sequence for developing technology tasks that support students' mathematical thinking. *Mathematics Teacher Educator, 5*(2), 122–157.
Silva, A. (2018). *Concepção de um suporte para a elaboração de webdocumentos destinados ao ensino da geometria: o caso das curvas cônicas*. Dissertação do mestrado. Programa de Pós-graduação em Educação Matemática e Tecnológica. UFPE, Recife.
Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.
Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6*(2), 91–104.
Siqueira, J. E. M., & Bellemain, F. (2018). A dynamic multirepresentational resource for conics. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 359–361). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275.

Stockero, S. L., & Van Zoest, L. R. (2013). Characterizing pivotal teaching moments in beginning mathematics teachers' practice. *Journal of Mathematics Teacher Education, 16*(2), 125–147.

Swidan, O., Arzarello, F., & Sabena, C. (2018). Teachers' interventions to foster inquiry-based learning in a dynamic technological environment. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 332–335). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Tabach, M. (2011). A mathematics teacher's practice in a technological environment: A case study analysis using two complementary theories. *Technology, Knowledge and Learning, 16*(3), 247–265.

Tabach, M. (2013). Developing a general framework for instrumental orchestration. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 2744–2753). Ankara: Middle East Technical University and ERME.

Taranto, E., Arzarello, F., & Robutti, O. (2018). MOOC as a resource for teachers' collaboration in educational program. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 167–170). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Tchounikine, P. (2011). *Computer science and educational software design—A resource for multidisciplinary work in technology enhanced learning*. New York: Springer.

Thomas, A., & Edson, A. J. (2018). An examination of teacher-generated definitions of digital instructional materials in mathematics. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 340–343). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Thomas, M. O. J., & Hong, Y. Y. (2005). Teacher factors in integration of graphic calculators into mathematics learning. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of mathematics education* (Vol. 4, pp. 257–264). Melbourne: University of Melbourne.

Tiburcio, R., & Bellemain, F. (2018). Computational engineering, didactical, educational software, software engineering. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 262–264). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Trocki, A., & Hollebrands, K. (2018). The development of a framework for assessing dynamic geometry task quality. *Digital Experiences in Mathematics Education, 4*(2–3), 110–138.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environment: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematics Learning, 9*(3), 281–307.

Trouche, L. (2005). Construction et conduite des instruments dans les apprentissages mathématiques : Nécessité des orchestrations. *Recherches en Didactique des Mathématiques, 25*, 91–138.

Trouche, L., & Drijvers, P. (2010). Handheld technology: Flashback into the future. *ZDM – Mathematics Education, 42*(7), 667–681.

Trouche, L., & Drijvers, P. (2014). Webbing and orchestration; two interrelated views on digital tools in mathematics education. *Teaching Mathematics and its Applications, 33*(3), 193–209.

Trouche, L., Gueudet, G., & Pepin, B. (2018, Online First). Documentational approach to didactics. In S. Lerman (Ed.), *Encyclopedia of mathematics education.* New York: Springer. doi:https://doi.org/10.1007/978-3-319-77487-9_100011-1.

van Dijke-Droogers, M., Drijvers, P., & Bakker, A. (2018). From sample to population: A hypothetical learning trajectory for informal statistical inference. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 348–351). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Vergnaud, G. (1996). *The theory of conceptual fields. In. L.P. Steffe, P.Nesher, P. cobb, G.a. Goldin, & B. Greer (Eds.), Theories of Mathematical Learning* (pp. 210–239). Mahwah: Laurence Erilbaum.

Vergnaud, G. (2011). Au fond de l'action, la conceptualisation. In J. M. Barbier (Ed.), *Savoirs théoriques et savoirs d'action* (pp. 275–292). Paris: Presses Universitaires de France.

Villamizar, F., Cuevas, C., & Martinez, M. (2018). A proposal of instrumental orchestration to integrate the teaching of physics and mathematics. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L. Trouche (Eds.), *Proceedings of the re(s)source 2018 international conference* (pp. 352–355). Lyon: ENS de Lyon. Retrieved on November 8th, 2018, at https://hal.archives-ouvertes.fr/hal-01764563.

Voltolini, A. (2018). Duo of digital and material artefacts dedicated to the learning of geometry at primary school. In L. Ball, P. Drijvers, S. Ladel, H. Siller, M. Tabach, & C. Vale (Eds.), *Uses of technology in primary and secondary mathematics education* (pp. 83–99). Cham: Springer.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Zbiek, R.M., & Hollebrands, K. (2008). A research-informed view of the process of incorporating mathematics technology into classroom practice by inservice and prospective teachers. In M. K. Heid & G. Blume (Eds.), *Research on technology in the learning and teaching of mathematics: Syntheses and perspectives*. Charlotte: Information Age Publishers.

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 1169–1207). Charlotte: Information Age.