Chapter 11
Documentation Work, Design Capacity, and Teachers' Expertise in Designing Instruction

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Abstract Teachers use resources in order to support their teaching, to support student learning, and to advance their own pedagogical and content knowledge. Using resources is intrinsically linked to particular knowledge and skills. These are conceptualized within different theoretical frames as competencies, aspects of design capacity, teacher expertise, professional knowledge, or utilization schemes within the instrumentation process. We discuss four different conceptualizations of teachers' work with resources, problems they aim to address, and exemplars of empirical studies in which such conceptualizations are used. We then discuss the affordances, constraints, and blind spots of these frameworks and indicate how they overlap and complement each other.

Keywords Teachers' professional work · Design capacity · Documentation work · Use of resources

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11.1 Introduction

This chapter was developed from the key issues discussed in Working Group 3 during the Re(s)sources 2018 conference in Lyon. The aim of Working Group 3 was to develop a deeper understanding of different theoretical approaches and of how they contribute to insights into teachers' work with resources in empirical studies. This chapter gives a general introduction to four influential frameworks that have been proposed to conceptualize teachers' work with resources, namely, the documentation approach to didactics (Gueudet and Trouche 2009), the framework of components of the teacher–curriculum relationship (Remillard 2005), the design capacity for enactment framework (Brown 2002, 2009), and the socio-didactical tetrahedron (Rezat and Sträßer 2012). To illustrate both, how some of these frameworks are used in conducting empirical research and how such use leads to postulation of new theoretical and analytical constructs, we include and discuss four selected contributions that were submitted to Working Group 3 at the Re(s)sources 2018 conference in Lyon.

11.2 Conceptualizing Teachers' Work with Curriculum Materials and Resources

Sebastian Rezat, Carole Le Henaff, and Jana Visnovska

In their daily work, teachers use resources in order to prepare their lessons, support their teaching, support students' learning, and advance their own pedagogical and content knowledge. A research interest in this particular aspect of teachers' professional work arose in relation to two types of phenomena: 1) implementation of curricular reforms (Remillard 2005) and 2) an ever-increasing offer and diversification of curriculum materials and other teaching resources (Gueudet and Trouche 2009), particularly digital resources[1]. Both are intrinsically linked to change: changes in students' opportunities to learn, in teachers' instructional practices that generate these opportunities, and in the conditions of teachers' daily work and professional development. Following many failed attempts to promote change of students' mathematical learning and/or instructional practice solely through curriculum materials (Keitel et al. 1980; Ball and Cohen 1996; Remillard 2005), teachers' *use* of these materials and their *professional work and development* have become the focus of research attention.

The efforts to understand teacher's use of curriculum materials and other resources, both in the moment and over time, have led to a refined view of teacher's professional work. It is no longer viable to conceptualize teachers as transmitters or

[1] Issues related to digital resources are further developed in Chap. 12 by Drijvers, Gitirana, Monaghan, and Okumus.

mediators of the content and aims of curriculum materials, as a variety of ways in which teachers contribute to designing the opportunities to learn have been documented (Brown 2002). The teachers' design work is usually based on and triggered by their use of multiple resources and has often been described in terms of crafting instruction. Mathematics teachers' planning decisions have been linked to their knowledge and beliefs about mathematics and mathematics education and to the teachers' perceptions of their learners' needs (e.g., Shulman 1986; Yang and Leung 2015).

Various theoretical frames and tools have been developed to conceptualize teacher's work with resources in order to craft instruction (see, for instance, Chap. 5 by Artigue). In the first part of this chapter, we give a short overview of four approaches that are currently used, often concurrently, to conceptualize teachers' work with resources in empirical studies: The documentational approach to didactics (Gueudet and Trouche 2009), the framework of components of the teacher–curriculum relationship (Remillard 2005), the design capacity for enactment framework (Brown 2002, 2009), and the socio-didactical tetrahedron (Rezat and Sträßer 2012). We will compare the different frameworks in terms of the purposes they were developed to advance, and their affordances and constraints with respect to those purposes. Among other issues, we will attend to how these approaches allow for considerations of proactively supporting teachers' professional development. Given the pragmatic importance of change to the work of teaching, different conceptualizations—in our case of relationships of teachers and resources—should not only be explored based on how they allow us to capture teachers' work and document change. It is crucial to also explore to what extent they currently provide tools to proactively inform change.

The four frameworks presented in this section relate differently to notions that have long been present in describing the phenomena of teacher learning, including teachers' knowledge, beliefs, practices, perspectives, and expertise. Furthermore, they highlight different phases or levels of the curriculum (e.g., written curriculum, planned curriculum, enacted curriculum, attained curriculum) resulting from teachers' use of curriculum materials and resources.

In describing the different frameworks, we will relate the notions and concepts used in the frameworks to the more general notions of teachers' practices, knowledge, beliefs, and expertise as well as clarify their relation to the level of curriculum considered.

11.2.1 Documentational Approach to Didactics

In their seminal article, Gueudet and Trouche (2009) introduce the documentational approach to didactics (DAD) as a theoretical approach, which aims at understanding teachers' professional development through the lens of their professional use of resources. The object of study is *teachers' documentation work*, which Gueudet and Trouche (2009) generally describe as "looking for resources, selecting/designing

mathematical tasks, planning their succession and the associated time management, etc." (p. 199). Thus, teachers' documentation work is associated with teacher's design of the planned curriculum. Gueudet further elaborates this notion in Chap. 2.

Although the development of the approach was initiated by the growing availability of digital resources, the very notion of resources within DAD comprises traditional print and digital resources as well as material and nonmaterial resources, such as discussions with colleagues (cf. Adler 2000), when these are used by teachers to re-source their work. However, studies that aim at a better understanding of teachers' adoption of new (and especially digital) resources into their set of previously used resources remain prominent within DAD. This is because adoption process generates conditions under which teachers' documentation work can become more accessible to be studied. In addition, understanding interrelation and interaction of various "new" and "old" resources during adoption is pragmatically significant to supporting teacher change.

According to Gueudet and Trouche (2009), the main premises of the DAD include that (a) changes in teachers' use of resources reflect an important aspect of teachers' professional development, (b) understanding teachers' work with resources at any point in time entails understanding an important aspect of teachers' expertise, and (c) change of professional practice and change of professional knowledge or beliefs are connected.

The DAD builds on the instrumental approach according to Rabardel (2002). At the heart of the instrumental approach lies the distinction between the artifact[2] and the instrument. While an artifact is a material or symbolic object, the instrument is a "composite [psychological] entity made up of an artifact component (an artifact, a fraction of an artifact or a set of artifacts) and a scheme component (one or more utilization schemes, often linked to more general action schemes)" (Rabardel 2002, p. 86). Thus, the instrument links the artifact to individual cognitive representations related to the use of the artifact for a restricted class of situations, referred to as utilization schemes.

The DAD draws a parallel distinction between resources and documents. By replacing the term "artifact" with the plural term "resources," the DAD underlines the importance of understanding the use of a resource in the wider context of a set of resources. While this dimension is inherent in the instrumental approach itself (Rabardel's notion of the instrument also relates to a set of artifacts), DAD proponents viewed it as particularly important to highlight the multiplicity of resources in teachers' work. Just like the instrument, a document then consists of a set of resources and related utilization schemes for a particular class of situations.

Teachers' knowledge is incorporated in the notion of scheme, conceptualized within DAD according to Vergnaud (1998). Vergnaud describes operational invariants as the essential components of schemes, because they represent the knowledge incorporated in schemes. While Vergnaud conceptualizes mathematical knowledge with theorems- and concepts-in-action, Rezat (2011) suggested the notion

[2] From the two synonymous spellings artifact and artefact, we regularly use artifact. The only exception is if the other spelling is used in direct quote.

beliefs-in-action when referring to the knowledge incorporated in schemes related to the use of resources.

The terminological choice of "resources" and "documents" aimed to more seamlessly align with teachers' intuitive interpretations than the original terminology of artifacts and instruments. Nevertheless, it would be unwise to rely on the words alone for their meaning: "documents" are not necessarily material (as the term would suggest), but are instead a psychological entity—like the instrument in the sense of Rabardel. The processes, in which documents are developed, are referred to as documentational genesis.

In addition to these processes, DAD provides means to capture the processes of teachers' collective work with resources (Pepin et al. 2013). Building on Wenger's (1998) notion of communities of practice, DAD conceptualizes teachers' collective work with resources as comprising changes both in teachers' participation in collective practices of the group (community genesis) and in teachers' views, production, and uses of various resources (community documentational genesis), highlighting the duality between teachers' participation and documentation (Gueudet and Trouche 2012a).

Within the instrumental approach, the social dimension was inherent in the notion of utilization schemes. DAD similarly attends to social dimension of teachers' professional development and brings to the center of attention the immediate social circumstances in which teachers' work with relevant artifacts takes place. Including social and human resources, and acknowledging their fluid nature, motivates DAD to explicitly study the process of community genesis. Through this, social and human resources are seen to generate the very conditions within which the instrumental genesis, conceived here as collective documentational genesis, takes place.

Since the first introduction of DAD, a variety of analytical notions focusing on particular aspects of teachers' work with resources have been proposed. Documentational experience, documentational trajectory, documentation-working mate, and documentational expertise are but a few examples (Rezat et al. 2018). In Sect. 11.5, Wang characterizes documentation expertise in detail and demarcates this notion from other kindred notions such as pedagogical design capacity (Brown 2002) and teacher design capacity (Pepin et al. 2017). These analytical notions in the realm of DAD enrich its core by drawing particular attention to the development of documents over time and also adding the dimension of quality in terms of expertise. While such additions are consistent with DAD's aim of providing a comprehensive framework for analyzing teachers' work with resources, they also suggest that it is yet not clear which analytical tools are likely to become the most relevant or consequential for the endeavors of meaningfully understanding and supporting teachers' work.

11.2.2 Components of Teacher–Curriculum Relationship

Based on an extensive literature review, Remillard (2005) synthesized key constructs of the teacher–curriculum relationship, proposing a framework for characterizing and studying teachers' interactions with curriculum materials. Part of the goal was to understand the impact that the different conceptualizations of constructs such as "curriculum use" had on knowledge in the field and on classroom teaching and learning.

The main constructs of the framework are the teacher, the designed (or written) curriculum, the participatory relationship between teacher and curriculum, the resulting curriculum planned by the teacher, and the curriculum enacted in the classroom. For the teacher, Remillard (2005) highlights resources, perspectives, and stances that the teacher brings to the participatory relationship. These include pedagogical content knowledge, subject matter knowledge, beliefs/goals/experience, pedagogical design capacity, perception of and stance toward curriculum, perceptions of students, tolerance for discomfort, and identity. She also points out that teacher's perception of and stance towards curriculum materials and the teacher's professional identity may critically shape teachers' interactions with curriculum materials and still need to be further investigated.

Related to the designed (written) curriculum, Remillard distinguishes aspects of the objectively given structure (e.g., representations of concepts, material objects and representations, representations of tasks, structures, voice, look) and how these are perceived by the user in terms of subjective schemes.

This framework shares a number of considerations with those of the DAD. While the designed curriculum is conceived as a resource in the sense of DAD, the participatory teacher–curriculum relationship is consistent with the dialectic process of documentational genesis, in that both the teacher and the resource are changed in the process of their interactions. On the other hand, planned and enacted curricula are documents (rather than resources) in sense of DAD, as these terms capture resources in use. However, while Remillard acknowledges that the planned and the enacted curriculum needs to be distinguished due to different contextual factors, DAD does not differentiate between documents on different levels of curriculum use.

The differences in the two frameworks are due to, primarily, differences in their purpose. While DAD aims to understand teachers' work and growth via understanding changes in their resource systems, Remillard's framework is motivated by exploring teachers' interactions with specific resources across different stages of the instructional process. Given the smaller grain of analysis, this framework might be better suited to informing back the resource design or deriving means for teacher support when they are learning to work with new designed resources.

11.2.3 Design Capacity for Enactment

Like Remillard, Brown (2002, 2009) offers a theoretical framework for considering the relationship between curriculum materials and teacher practice. However, his design capacity for enactment framework (DCE-framework) "is rooted in the notion that all teaching involves a process of design in which teachers use curriculum materials in unique ways as they craft instructional episodes" (Brown 2009, p. 18). Similar to the instrumental approach (and as distinct from the DAD), curriculum materials are conceptualized as artifacts (and not resources) within an activity theory (Vygotsky 1978; Wertsch 1998) perspective. Accordingly, they are viewed as mediational means, which afford and constrain human activity.

The aim of the framework is to understand "how the features of the materials interact with the capacities that teachers bring to the interaction" (Brown 2009, p. 26). Brown considers several features of the curriculum materials to be a resource in teacher–tool interactions, including (a) physical objects and representations of physical objects, (b) representations of tasks (procedures), and (c) representations of concepts (domain representations). The resources that the teacher brings into these interactions include teacher's knowledge (subject matter knowledge, pedagogical content knowledge), skills, goals, and beliefs.

According to Brown, these factors are a starting point for identifying and situating the aspects that influence the teacher–tool relationship, but are not exhaustive. In particular, Brown points to the fact that the teachers' abilities to use curriculum materials in productive ways in order to craft instruction are not only a matter of the resources that the teacher and the tool bring to the interaction, but in addition a matter of an ability to perceive "the affordances of the materials and making decisions about how to use them to craft instructional episodes that achieve her goals" (Brown 2009, p. 29). Therefore, he introduces his widely referenced notion of *pedagogical design capacity* (PDC), "defined as a teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" (Brown 2009, p. 29). The design capacity for enactment framework and the notion of pedagogical design capacity are mutually related. According to Brown (2002) the DCE-framework describes the resources and, thus, the *what* that influences teachers' interaction with curriculum materials, while PDC accounts for the *how* these resources are used. As he points out, this differentiation is crucial, because he documented that teachers with similar resources can possess very different capacities to mobilize these resources in order to design instruction.

11.2.4 Socio-Didactical Tetrahedron

Rezat and Sträßer (2012) introduce the didactical tetrahedron as a model of the didactical situation as a whole. They take the classical didactical triangle, which models the relation between teacher, student, and the (mathematical) object, as a

starting point. Adopting an activity theoretical perspective, they argue that artifacts need to be considered as a fourth constituent of the didactical situation, because of their ability to afford and constrain activity and thus have structuring effects on the whole system. Each face of the resulting tetrahedron, except the classical didactical triangle, can be regarded as an individual activity system in which artifacts serve as mediational means. However, the model draws particular attention to the interaction and interrelatedness of these activity systems and the need to view the didactical situation as a systemic whole.

By referring to the didactical triangle, the authors consider the didactical situation in the classroom in the first place. Nonetheless, Rezat (2009) argues that the model is also applicable to the preparation work of the teacher. In this case, students are part of the system as the (imagined) subject the teacher's activity is directed at.

In order to include social and institutional influences on teaching and learning mathematics and classroom interaction, Rezat and Sträßer (2012) expanded the didactical tetrahedron based on Engeström's model of the activity system (Engeström 1987). The resulting *socio-didactical tetrahedron* includes the societal and institutional dimensions of *rules*, *communities*, and *division of labor* for both teachers and students. The students belong to the community of their peers, their family, and maybe their tutors. The teacher's community in the narrow sense is constituted by his/her colleagues and—in a wider sense—by the *noosphere*, which Chevallard describes as "the 'sphere' of those who 'think' about teaching. Crudely put, it consists of all those persons who share an interest in the teaching system, and who 'act out' their impulses in some way or another" (Chevallard 1992, p. 216).

The members of the institution, e.g., the school principal, shape the community shared by students and teachers (and they also shape mathematics instruction). The system of rules of the students and of the teacher, respectively, is constituted by rules and norms about being a teacher and teaching or about being a student and learning, respectively. The division of labor within the model relates to the relevance of mathematics in society and the public image of mathematics.

In activity theory, the social, societal, and institutional dimension is mirrored or apparent in the interaction of the user and the artifact and thus can be conceived as resources in the sense of DAD. The socio-didactical tetrahedron (SDT) draws particular attention to their influence on the activity. It provides a structure for some of the societal and structural resources, which influence activity, and thus allows for a more differentiated view on the interaction of these resources within the activity.

11.2.5 Frameworks in Empirical Studies

The remainder of this chapter presents three empirical studies by Sabra and El Hage, Leroyer, and Kim, and a conceptual piece by Wang. What brings the four contributions together is the attempt to characterize, through different theoretical and methodological frameworks, and by exploring various study objects, teachers'

work with resources, and in particular the development of their expertise through their practice as designers, whether they work alone or in groups. They exemplify the wide range of applications of the previously described frameworks and were particularly chosen, because they also propose or exemplify refinements or new analytical tools to the frameworks used.

In the first contribution (Sect. 11.3), Leroyer discusses a framework for analyzing teachers' professional postures produced in the process of documentational genesis and reports on the initial test of this framework in a case study with one French primary teacher. The postures refer to the factors that condition the teacher preparation activity in which interactions between teacher and teaching resources take place. Leroyer has developed a model in which teacher postures and learning supports are linked. The model aims to bring to attention different types of learning goals for students that teachers may prioritize during their preparation work and how these become visible in the products of that work.

In the second contribution (Sect. 11.4), Sabra and El Hage adopt the DAD to address a subject of study that has yet to be explored: how research and teaching interacts in the documentation work of instructors in higher education settings, particularly at a French university. The authors explore how the perceptions of instructors in mathematics and physics, of their research resources, shaped the relation that they maintained between their research activity and their teaching practices. Sabra and El Hage explore broadening the scope of applications of DAD to tertiary settings. They indicate that the collective design of resources in both teaching and research institutions constitutes an important direction for future research which could "elucidate the complex forms of relation between research and teaching in the practices of university teachers."

In the third contribution (Sect. 11.5), Wang elaborates the concept of *documentational expertise* (as an aspect of the DAD) based on a literature review and a series of studies with Chinese high school mathematics teachers. She demarcates this concept from other teacher capacities in the literature and uses it to characterize (1) how teachers develop within collectives, (2) understandings of the knowledge to be taught, (3) how it can be taught, and (4) how teachers' practice can be improved.

Finally, Kim (Sect. 11.6) grounds her work in PDC but also contextualizes it within DAD. She outlines five dimensions of a teacher capacity different from PDC for productive use of existing resources and thus tackles the issue of quality in resource use. Drawing on analyses of elementary teachers' work in the United States, she documents how teachers may not always identify or make use of mathematical affordances present in their resources. In doing so, she argues for the need of nurturing conceptions for productive resource use in teacher education and professional development, and the role of resources in increasing teacher capacity.

Including the following four sections as self-standing contributions allows us to offer additional insights into how different researchers combine and reconcile in their work some of the frameworks that we introduced. We conclude by discussing the use of these frameworks and their affordances and constraints.

11.3 Approaching Knowledge Transmission via Learning Supports: A Conceptual Model[3]

Laurence Leroyer

Teaching can be regarded as a design activity. Pepin et al. (2017) write "we concur with Brown (2009), as we understand his notion of design, to regard 'design' as the practice of designing for teaching, as in lesson preparation (that is design before enactment), as well as in teaching, what we labelled as 'design-in-use' that happens during enactment of the resources /materials" (p. 801).

To prepare the activity supports that will then be introduced to the pupils, the teachers interact with resources as they design for classroom use. They select resources, modify them, and use them. This documentation work "is central to teachers' professional activity" and includes "processes where design and enacting are intertwined" (Gueudet and Trouche 2012b, p. 24).

Faced with a multitude and diversity of resources, it is the teachers' duty to judge the quality of these resources. In a research on teachers' professionalization in initial training, we noticed that students' interest in critically exploring teaching resources keeps decreasing (Leroyer and Bailleul 2017). The question of adequacy of teaching resources is no longer integrated into a didactic reflection that is itself part of the teaching activity. This is even more worrying when we know that learning supports contribute to students' knowledge development. For this reason, it might be important to support teachers to view teaching and particularly documentation work as a design activity and thus engage them in considering quality of resources and developing their design skills.

With Georget (Leroyer and Georget 2017), we have designed a model that allows approaching the teacher trainers' documentation work with a specific focus. This model was used as a tool in trainers' training to develop their design skills and specifically their capacity to think of a training-support approach to knowledge development.

A question arises then: Can this model be adapted for teachers' documentation work? If so, its use could help teachers raise awareness of this work. When incorporated to teachers' training program, it could be used as a tool to strengthen their reflection and question the effects of learning supports they design.

I first overview the origins and theoretical framework of the model developed with Georget. I then specify the research question and present successively the methodology, the results, and the concluding discussion.

[3] Acknowledgment: The author of this section would like to thank Jean-Philippe Georget, co-designer of the model in a training context.

11.3.1 A Model Based on an Exploratory Study on the Documentation Work of Teacher Trainers

In 2016, I conducted an exploratory study on the teachers' trainers and their design practice with resources (Leroyer in press). The documentational approach to didactics (Gueudet and Trouche 2012b) provided a framework for this exploratory study. In training preparation, a teacher trainer utilizes and transforms existing resources and designs new resources (recombined resources). Four questions structured this study in which trainers' interactions with material resources (textbook, curriculum, scientific paper, video recording of a lesson, etc.) were considered:

- What are the resources used by the trainers when they design training?
- How do the trainers access these resources?
- What are the training supports designed by the trainers from these resources?
- What are the intended uses of the training supports designed?

- In line with the instrumental approach (Rabardel 2002), I considered a training support to be a material artifact made up of a medium (e.g., paper, digital, video) and a content. For example, a texts corpus, a worksheet, and a slideshow are training supports. I distinguished training supports from supports intended for the trainer only, such as preparation worksheet, or reading notes. A learning support becomes an instrument when the trainer uses it. Rabardel (2002) wrote "the instrument is a composite entity made up of an artifact component … and a scheme component … An instrument therefore consists of two types of entities: a material or symbolic artifact produced by the subject or others; one or more associated utilization schemes" (p. 86). Referring to the DAD, a learning support contains "recombined resources," which, associated with "schemes of utilization," correspond to a "document." The DAD considers the document as the result of the subject's activity, which captures subject's interactions with resources. Therefore, the subject develops his/her own resources and uses.

In my exploratory study, I documented that the trainers relied on different training supports and differed in their intended uses. It appeared that the trainers' perspectives on knowledge, trainees, and training seemed to influence their documentation work. For example, in one of his training sessions, one trainer planned to present specific theoretical knowledge by means of a slideshow. The trainees were then expected to use this knowledge to analyze textbooks. For this trainer, theoretical knowledge was very important for teaching. Another trainer planned to engage teachers in an activity designed for pupils' learning. The trainees were given the pupils' materials and a worksheet to note the possible pupils' difficulties. For this trainer, it was important to propose concrete situations that could emerge in teachers' classrooms and in this way allow for her trainees' learning by doing.

In their paper entitled *Training engineering: formalization of teacher training experiences*[4], Bailleul and Thémines (2013) distinguish four trainers' postures[5]—epistemologist, guide, engineer, and didactician—referring, respectively, to the relationship to science, to the others, to time and organization, and to tools. For them, the trainer is led to ask himself several questions in training preparation. These questions are organized along two axes. The first axis focuses on the knowledge to be studied and transmitted and the logic of its presentation. The second axis concerns the place given to individual in training and the activities offered to them. The authors identify tensions between these axes and thus highlight four trainers' postures. They formalize the axes, tensions, and postures in a model.

With Georget, we have adopted this model and sought to identify how these postures are "reflected" in the training supports. We then developed and tested a model in which trainers' postures and training supports are linked.

11.3.2 The Model to Transfer: A Model in Which the Training Support Crystallizes and Materializes the Trainer Postures

For the trainers' postures, we relied on Bailleul and Thémines (2013) model with several modifications presented in Leroyer and Georget (2017). The modified model includes two axes of questions that the trainer is led to ask himself during his/her planning, and the four training dimensions related to these axes. These training dimensions are professional knowledge, organization/operationalization (which includes spatial, material, temporal organization), trainees, and tasks. Between these dimensions, four pairs of tensions are identified: continuity/rupture, involvement/application, transmission/construction, and theorization/pragmatism (see Fig. 11.1).

- From the point of view of trainees' place regarding knowledge: if trainees are taken into account (what they know and what they need), continuity is privileged; if they are not, rupture is privileged.
- From the point of view of the role of knowledge regarding trainee's task: if knowledge is an end in itself, theorization is privileged; if knowledge is an answer to a professional problem, pragmatism is privileged.
- From the point of view of trainees' place regarding the organization: if the organization allows interaction with trainees, the trainees are involved (involvement); if it is not the case, application is privileged.
- From the point of view of the logic on which the organization and tasks are based: if the aim of the organization and task is to get trainees to build the knowledge themselves, the construction of knowledge is privileged and put into the

[4] Translated from the French by the author of this contribution.

[5] A closer definition of this term is provided in the following section.

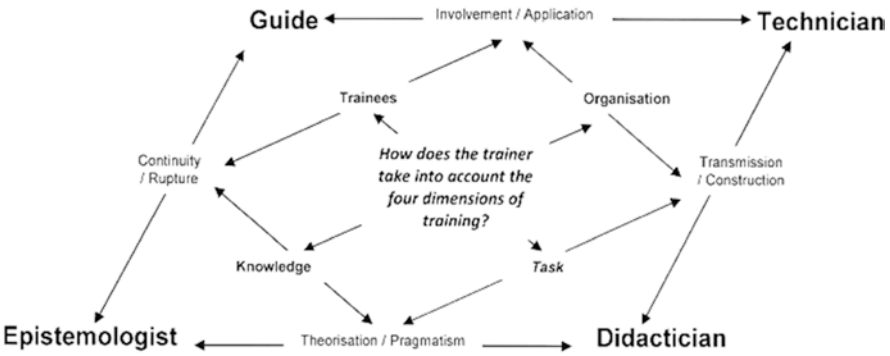

Fig. 11.1 The four trainers' postures

hands of trainees; if organization and task aim at the transmission of knowledge to trainees, transmission is privileged.

Among these tensions, we can identify four trainers' postures: s/he can be regarded as a guide, as an epistemologist, as a didactician and as a technician (see Fig. 11.1). To clarify the term posture, we refer to Bucheton and Soulé (2009) who define a posture as a pre-constructed scheme of "think-say-do" that the individual summons in response to a given school situation or task. This posture is constructed in the social, personal, and scholarly history of the individual. And, these individuals may change posture in the course of the task according to the new meaning they attribute to it. The posture is both on the side of the individual in a given context and on the side of the object and the situation. These postures are:

- The *epistemologist* refers to the knowledge presented/constructed without taking learners into account—knowledge is an end in itself.
- The *didactician* refers to knowledge-construction tasks given to the trainees in a pragmatic concern, where knowledge is an answer to a professional problem.
- The *technician* refers to the knowledge used by trainees that can only be transmitted and not reconstructed—this posture also refers to the control of the training process.
- The *guide* refers to the consideration of trainees' needs in a search for continuity and involvement.

From this work and from the result of the exploratory study, we thought that the learning supports that have been designed by the trainers crystallize and materialize trainers' postures. We thus included a dimension related to each identified posture in our model of training supports that the teacher trainers design:

- The epistemological dimension related to the epistemologist posture
- The didactic dimension related to the didactician posture
- The technical dimension related to the technician posture
- The relational dimension related to the guide posture

We defined each training support dimension based on what the support tool aims to develop:

- Related to the epistemological dimension, it aims at developing knowledge. It brings explicit professional knowledge.
- Related to the didactic dimension, it aims at developing actions/elaborations. It generates actions that sustain the elaboration of knowledge by the trainees.
- Related to the technical dimension, it aims at developing specific way of working. It leads trainees to comply with the trainer's plan.
- Related to the relational dimension, the training support aims at developing interactions between participants in teacher training. It generates relationships between trainees.

In adapting this model to a classroom teaching context, I suggest several changes. I substitute "trainees" with "pupils" and "professional knowledge" with "academic knowledge." The tensions remain the same as well as the postures. I present below the model adapted to a classroom teaching context (see Fig. 11.2). The research question can be clarified: do the trainers' postures and the training supports dimensions apply to the teachers' postures and the learning supports?

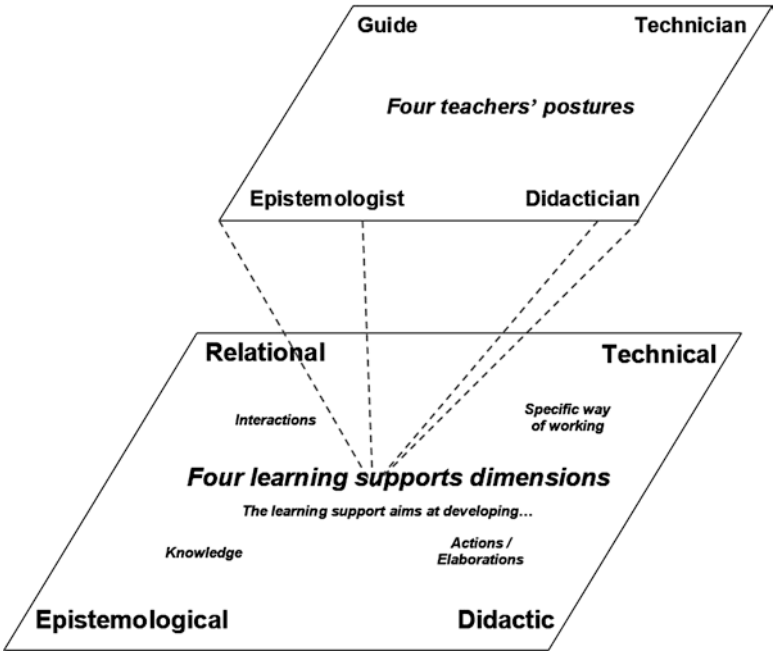

Fig. 11.2 How learning supports crystallize the teacher's postures

11.3.3 Methodology

To test the model in a classroom teaching context, I used case study methodology. Results reported here come from the first case study conducted with a male primary school teacher. Further case studies will be conducted to test and refine the model.

11.3.3.1 Data Collection

The case study is based on a semi-structured interview that addressed the following themes:

- The teacher characteristics and contextual factors (teacher's course of study, professional background, mathematical knowledge for teaching, and teaching contexts). The aim was to contextualize the teacher's point of view.
- The presentation and explanation of one of the teacher's lesson plans in mathematics. To do this, the teacher relied on his preparation and his learning supports. After the interview, the preparation and learning supports were kept by the interviewer and constitute data for purposes of the analysis.
- The teacher's understanding of knowledge, teaching, learners, and learning were investigated. At the end of the interview, I asked this question: In your opinion, how did what you are and what you think about mathematics knowledge, mathematics teaching, and pupils, influence your designed session? The aim was to access:
- The teacher's intentions both in the learning support design and in its intended use, which may remain implicit in his teaching preparation worksheet
- The teacher's relationships with each teaching dimension

The case study was conducted with a French teacher, Matthew, who was in his 13th year of primary school teaching. For the last 2 years, he taught special-needs children with cognitive function disorders. Some of his pupils had difficulties processing information, memorizing the tasks to be performed, planning their work, verbalizing, etc.

Matthew's university education background is in English studies. During the semi-structured interview, he chose to present a geometry session designed for a group of three pupils, even though he never received training on geometry as part of his in-service teacher training. The objective of Matthew's geometry lesson was "reproducing geometric figures on a grid." During the interview, he presented his third classroom session. To contextualize this session, he provided an overview of the two previous sessions.

During the first session, the pupils were asked to reproduce a complex geometric figure that corresponded to tangram pieces on a grid. In view of the difficulties experienced by his pupils, the teacher simplified the task for the second session, where the pupils were asked to reproduce a simple geometric figure on the grid: a square. During the third session, Matthew wanted the pupils to compare their

produced square to the model square. The goal for the pupils was to identify how well they could use instruments to trace a figure and compare/discuss the techniques to correctly reproduce the figure on the grid.

11.3.3.2 Data Analysis

I adopted a three-stage analysis. Step 1: I identified each dimension of the learning supports and of the lesson plan. I listed, in a table, the learning supports and their function within the tasks given to the pupils introduced in the lesson plan. In Matthew's lesson plan, I identified eight learning supports (see column 1, Table 11.1). For each learning support, I indicated the pupils' task. This made it possible to deduce a function for each of them. For example, the enlargement of pupils' worksheets handed out by the teacher allowed me to understand that the learning support aimed to both highlight the mathematical content in the performed task and remind the learners what they needed to compare (see line 3, Table 11.1).

I took note of whether the learning supports relied on interactions, knowledge, specific way of working, or actions/elaborations. In the previous example, I identified two predominant dimensions in the learning support, a relational dimension because it supported exchanges about the meaning of the elements contained and an organizational dimension because the display of this learning support allowed pupils to remember what they needed to compare (see columns 4–7, line 3, Table 11.1).

To quantify whether some of the dimensions prevailed, I noted the timeframe needed for each of these dimensions. This timeframe refers to the time of the phase in which the learning support is used. This analysis relies on the lesson plan containing chronological indications. When a learning support contained several dimensions, I indicated the same timeframe for each dimension.

Step 2: I identified the teacher's postures based on the lesson plan and on the interview. I list the learning supports and the tasks given to the pupils. I added the teacher's comments on each task and learning support, and his general comments, based on the interviews (see excerpts in Table 11.2). I used this to infer the teacher's relationship to knowledge, pupils, tasks, and organization.

Step 3: I compared the dimensions of the learning supports with that of the teacher's postures to check their adequacy.

The model will be considered transferable to a teaching context if, for subsequent cases studied, I find an adequate relationship between the teachers' postures and the dimensions of the learning supports.

Table 11.1 Determination and quantification of the dimensions in Matthew's learning supports

Learning support (*) displayed or (**) handed out by the teacher	**Function of the learning support** It allows to …	**Task given to the pupil**	**Learning support aims at developing…**			
			Interactions	Knowledge	Specific way of working	Actions elaborations
Geometric figures drawn by the pupils in previous sessions (*)	- make it easier to recall and verbalize what has been done in the last sessions.	Listen to the teacher (recall the project activity and previous tasks) Acknowledge one's own production and make personal remarks			X 2 min	
Enlargement (A3) of pupils' worksheet (*) (one square correctly drawn—identified by a tick—and three squares (A, B, C) not correctly drawn—identified by a cross).	- understand the content of the support to perform the task. - keep in mind what has to be compared.	Observe and discuss the meaning of the elements of the worksheet	X 3 min		X 3 min	
Poster (with the title: "To succeed in drawing a figure, I must …: ") (*)	- anticipate what is expected.	Listen to the aims of the activity (look for mistakes and list the elements needed to correctly draw a square on the grid).			X 1 min	
Pupil's worksheet (A4), same as enlarged pupil worksheet (A3). (**)	- compare the figures.	In pairs, look for what is wrong on figures A B C / Find the differences between figures A B C and the square that has been correctly drawn.	X 7 min		X 7 min	X 7 min
Tracing paper showing a square correctly drawn (**) Magnifying glass (**)	Easily make comparisons.					
Enlargement of pupil's worksheet (*)	- establish landmarks during the collective exchange. - make interactions easier and thus progress in the subject. - highlight all mistakes so that all pupils can see and explain them.	Watching the squares (A, B, C) in succession, point out the mistakes identified in each figure and express them in a sentence. Reword sentences using appropriate vocabulary.	X 15 min	X 15 min	X 15 min	X 15 min
Poster (*)	- keep a written record of what has been said.	Tell how to correctly draw a square on a grid (pupils' sentences are written down on the poster)				
Enlargement of pupil's worksheet with the mistakes being circled. Poster completed (*)	- make it easier to recall and verbalize	Answer the question: "What have we done today?"		X 2 min		
			25 min	17 min	30 min	22 min

Table 11.2 Extracts from Matthew's interview

1	"I noticed that my pupils had … needs in terms of drawing and dealing with a grid … They like to manipulate this kind of things (tangrams) and to reproduce the figures … They can gather ideas and see if they have any ideas in common with the other group."
2	"I want them to start from an observation that comes in their minds … that their reproductions are not precise enough … and that they can find out what their own needs are. That is for me the key element of the sequence."
3	[Regarding the order in which the figures are presented] "The mistakes are sorted in an ascending order … Here in figure (A), there are only few mistakes, here in figure (B), we also have new mistakes and here in figure (C) once again, there are many more new mistakes. I didn't want to start directly with figure C that shows several things."
4	"It suits me fine because this is a learning project that we can build together; it's important that everyone gets awareness that they need to progress because their current level does not allow them to do what they are told; they also need to know that I'm going to help them, and finally, that we will all help each other, that together we will find ways to progress and succeed in reaching our goal."
5	"The goal was to deal with the needs that pupils would have to correctly draw a geometric figure on a grid; those needs can be identified through a comparison process between the wrong reproductions and a model."

11.3.4 Initial Findings

The analysis as presented in Table 11.1 and, more specifically, its last four columns related to the dimensions of Matthew's learning supports, highlights the predominance of three dimensions: the technical, relational, and didactical dimensions. The epistemological dimension remains present but to a lower extent.

In three of Matthew's learning supports, several dimensions dominate. I will take the example of the pupils' worksheet that Matthew designed to allow pupils to compare their own square with the expected one.

The worksheet showed four squares drawn on a grid. The first row of two squares showed a square accompanied by a tick and a second square, with the letter A inside, accompanied by a cross. The second row showed two squares, marked as B and C, each accompanied by a cross. The squares A, B, and C reproduced the drawings made by some of the pupils during the previous session. The tick meant that the trace was correct and the cross meant that the trace was incorrect. This pupils' worksheet included a technical dimension. The symbols and the letters were there to guide the pupils in their allocated task. The letters involved a comparison between the incorrect squares and the correct one in a specific order. This worksheet also included group work. As such, this learning support also had a relational dimension. Moreover, it made it possible for the pupils to identify their mistakes, which can be regarded as a didactic dimension.

Other learning supports presented a single dimension, such as a tracing paper showing a square correctly drawn, which could be overlapped with the produced squares to help identify mistakes. At the end of the session, the poster indicating the "rules" to avoid mistakes represented the epistemological dimension.

Matthew's comments highlighted several postures. Taking into account the needs of pupils and what they were used to rely on, Matthew demonstrated a guide posture. For him, it was important that pupils interacted with each other (see Table 11.2, extract 1). The didactician posture appeared in Matthew's intentions for the pupils to develop their own sense of knowledge by getting fully involved in the activity (see Table 11.2, extract 2). The order in which the figures were presented was meant to guide the content of the progression emerging from collective interactions (see Table 11.2, extract 3), suggesting the technician posture. Matthew's ideas of learners, learning, and teaching were reflected in the postures identified (Table 11.2, extract 4). Matthew did not explicitly address pupils' mathematical knowledge during the interview. He instead focused on his teaching approach (Table 11.2, extract 5).

The analysis of Matthew's interview makes it possible to identify three dominant postures (a guide, a didactician, and a technician postures). In his learning supports, three dimensions dominate (technical, relational, and didactician dimensions). These dimensions correspond to the predominant postures. Therefore, there is an adequate relationship between teacher postures and the learning support dimensions: the learning supports crystallize the teacher's postures. In this case, the use of model in a teaching context is conclusive.

11.3.5 Discussion of the Theoretical and Methodological Aspects of the Contribution

In the DAD, when a teacher interacts with resources, for a given class of situations, instrumentalization and instrumentation processes take place. During this documentational genesis, teacher and resource characteristics both influence the processes that contribute to the development of a document. The reflective investigation of the teachers' documentation work—a methodology developed at the origin of the DAD—takes into account the history of the teacher and the context in which he/she evolves. Thus, in the data collection system, a questionnaire provides information about "the teacher's career" and "current working environment" but also of his/her "professional and personal history" (Gueudet and Trouche 2012b).

The model presented above intends to contribute to this very point—to elaborate how the DAD approach can usefully clarify the teacher's characteristics. The four teacher's postures refer to the teacher's professional and personal history and to his/her relationship to knowledge, pupils, tools, and organization of learning. Theorizing that the teacher's postures influence how this teacher designs the learning supports for his/her pupils, these supports can then be seen as crystallizing the elements of these postures.

From a methodological point of view, we can analyze what teachers tell us about their documentation work when they design their teaching and learning supports and consider these teachers' claims as traces of the documentational genesis process.

The additional collection and analysis of the teacher-designed learning supports, as described in the presented model, provides access to elements that do not necessarily appear in interview analysis alone. In this way, the collection and analysis of learning supports, and the links postulated by the presented model between learning supports and the underlying teacher postures, present an investigative tool that enriches substantially the analysis of the comments collected during teacher interviews.

11.3.6 Perspectives

In the presented model, the dimensions of the learning supports are based on their effects on pupils. Introducing a third level, "the pupils' level," could further enrich the model and allow analyzing both the design and implementation of teaching. The interactions between learning supports and pupils could be clarified by focusing on pupils' postures. It could also be interesting to integrate into the model the contextual elements such as the pupils' characteristics, the aims of teaching expected by society, or the contingencies of the profession.

The use of this model in training could be used to develop teachers' reflection on their documentation work based on a critical analysis of their learning supports and postures, thus contributing to teacher professional development. Finally, it would be interesting to understand the effects of the use of this model in teacher professional development on the teachers' documentation work and evolution, requiring longitudinal study designs and data collection.

11.4 Forms of Relation between Teaching and Research at University

Hussein Sabra and Suzane El Hage

The professional activity of an academic often involves teaching and research. We aim to study the relation between teaching and research activities and uncover the disciplinary specificities in this relation. In the present study, we consider particularly the disciplines of physics and mathematics. In France, academics categorized as "teachers–researchers" are responsible for both research and teaching at different program levels (i.e., tertiary level and master's degree programs). They typically do not have the luxury to choose which courses they teach. Some university teachers consider the introductory courses that address the basics of classical physics or mathematics as important, general and necessary for students, but too basic regarding their expertise in their field of research. In contrast, teaching at master's level

enables researchers to teach scientific ideas of their research interest to a classroom of their potential research assistants.

We present an exploratory study and aim to contribute to the understanding of teaching practices at university and the factors underpinning it. We study the research activity of university teachers through the lens of the interactions with resources. Following Adler (2000), we give to the "resource" here, a meaning related to the verb "re-source," to source again or differently. Our study is closely related to those considering resources for teaching at university level (Gueudet et al. 2014; Gueudet 2017; Gonzàlez-Martin et al. 2018). We particularly focus on the place of *research resources* and their impact on the design and use of *resources in and for teaching*.

Some researches in science education attempted to find evidence of positive or negative correlations between academics' research and teaching without taking into account a specific discipline (Elton 1986; Neumann 1992). For instance, some tried to characterize the relation that may occur between teaching activity and research activity (symbiosis, conflict, tension, etc.). Neumann (1992) presented three aspects of what he called "nexus" that can exist between teaching and research: (1) the tangible aspects, generally link to an articulation between content transfer of knowledge from research in teaching; (2) intangible aspects, which relate to the actions of the researcher in the teaching activity and vice versa; and (3) the global aspect, which relates to nexus between teaching institution and research institution. In a more recent study, Elton (2001) examined the reasons behind the presence or absence of the relation between teaching and research in the practice of university teachers. In a perspective of transformation of practice, he suggested ways that could reinforce "positive" articulations between the two kinds of activities.

The question of the correlations between the two activities of a university teacher has been studied recently in relation to the discipline involved. As an example, Madsen and Winsløw (2009) emphasized that the relation between research and teaching in the case of mathematics significantly differs from the physical geography discipline. In their comparative study between teachers in geography and mathematics, they emphasized the fact that the forms of relation between teaching and research strongly depended on the disciplinary specificities (institutional and epistemological characteristics of the discipline). They also stressed that the relation that can take place between both teaching and research activities depended on the perceptions the university teachers had of the specificities of their disciplines.

We aim to understand the relation between teaching and research within the mathematics and physics disciplines through the lens of interaction with resources. They can take place at different moments of teaching practices: in the design of the classroom sessions, the choice of the contents, the implementation of resources in the classroom, and in the evaluation of learning. In addition, university teachers could use the same resources in their teaching practices and in their research activities (Broley 2016). Indeed, our general research question is: when and how do the resources coming from research activity enrich the teacher's capacity to re-design them for his/her teaching work?

To address this question, we use the documentational approach to didactics (DAD; Gueudet et al. 2012). We will discuss the scope of DAD to consider the university teachers' interactions with resources in mathematics and physics across teaching and research institutions.

11.4.1 Documentational Work in Research and Teaching Institutions

The DAD proposes a holistic point of view on teachers' work, considering the activity of the teacher as a continuous process. In the DAD, there is a distinction between resources and documents. We define here resources as all the things that could resource a university teacher activity (research and teaching). The interaction with the resources generates a document, which is the association of resources and a scheme of use of these resources. We can assume that in the case of university teachers the research resources re-source particularly the research activity. However, this dimension is not investigated here. We are interested in how research resources influence the design of resources for teaching.

A scheme is used here as defined by Vergnaud (1998), as the invariant organization of conduct for a set of situations having the same aim. According to Vergnaud (1998), a scheme is a dynamic structure that has four interacting components: aim, rules of actions, operational invariants, and possibilities of inferences. A class of situations includes all the situations having the same aim.

A university teacher develops a professional experience by interacting with the teaching institution and the research institution simultaneously (Madsen and Winsløw 2009). The interactions with resources in each of the institutions are related on the one hand to the specific classes of situations (research classes of situations, teaching classes of situations) and, on the other hand, to the specificities of the discipline. The relation between research and teaching could take place as a migration and adaptation of the resources between institutions, or also like a dissemination by a university teacher of the professional knowledge and mode of teaching (the "operational invariants"—component of scheme of resource use, Gueudet and Trouche 2009).

We distinguish between (1) the teaching document (aims related to the class of situations of teaching, resources for teaching, rules of action, and operational invariants) in the meaning of Gueudet (2017) and (2) the research document (aims related to the class of situations of research, resources for research, rules of action, and operational invariants). Each kind of document is considered in its institution with corresponding conditions and constraints. Gueudet (2017) notes that university teachers develop a resource system for research in the research institution and a resource system for teaching in the teaching institution. The study of both resource systems and their interaction is not our aim here. We are interested in the process of

interaction between both systems from the point of view of "pivotal" resources in research activities of the university teacher.

The concept of *pivotal resources* is characterized in previous studies using DAD as resources that intervene in several classes of situations (Gueudet 2017, see also Gueudet, Chap. 2). In these studies, the pivotal resource is considered in documentation work related to teaching. In our contribution, we define a pivotal resource as a resource that contributes for a given teacher to the development of many research documents. We consider that a pivotal resource is used in several classes of research situations. Frequent use of a pivotal resource could influence a part of the research activity. We assume that if there are relations between research and teaching activities, these will take place in terms of the classes of situations where pivotal resources from research are mobilized. We hypothesized that there is at least one pivotal resource in the research work of a given mathematician or a given physicist. It could be a software for numerical computation, a foundational book in his/her field of research, or others. Consequently, our research questions were:

Q1: How do pivotal research resources inform us about the teaching practices at university?

Q2: How do the pivotal resources coming from the research institution enrich the teacher's capacity to re-design and use them for his/her teaching work?

11.4.2 Context and Methodology of the Study

11.4.2.1 Data Collection

This study is based on six interviews with French university teachers: three mathematicians and three physicists (see Table 11.3). To keep the anonymity of the university teachers interviewed, we will call them M1, M2, and M3 for the mathematicians and P1, P2, and P3 for the physicists. We note that every university teacher conducts research in a specific area of their discipline; however, this is not the case for their teaching. A university teacher teaches a variety of what is called "teaching units" in the French system in each semester. He/she must teach at different levels, a variety of subjects and topics ranging from the basic level in a discipline to very specialized courses in his/her field of research.

We constructed the interview guidelines in two distinct parts: the research activity part and the teaching activity part. We did not ask direct questions about resources so that the interviewee could express him/herself freely about research and teaching activities. This choice allowed us to identify the resources quoted in their answers to be considered as a pivotal resource. The semi-structured interviews lasted between 60 and 90 minutes and took place in the office of the university teacher. All the interviews were audio recorded and conducted in French.

Table 11.3 Profiles of the six university teachers

	Gender	Research experience	Research domain	Teaching experience	Teaching level
M1	Male	16 years	Mathematical modeling of physical phenomena	16 years	Undergraduate degree (mathematics and computer sciences) and master's degree (applied mathematics)
M2	Male	6 years	Mathematical modeling of scientific phenomena	6 years	Undergraduate degree (mathematics)
M3	Male	17 years	Mathematician (number theory)	17 years	Undergraduate (mathematics) and master's degree (pure mathematics)
P1	Female	19 years	Nuclear physics	19 years	Undergraduate degree (chemistry)
P2	Male	11 years	Nuclear physics	11 years	Undergraduate degree (chemistry) and master's degrees (nuclear material and aging of nuclear materials)
P3	Male	25 years	Electronic microscopy	25 years	Undergraduate degree (physics) and master's degree (scientific instrumentation and bioimaging)

Table 11.4 The research document table and the teaching document table

Research document table			
Research aims	Resources	Rules of action	Operational invariants
Teaching document table			
Teaching aims	Resources	Rules of action	Operational invariants

11.4.2.2 Analyzing the Data

The transcripts of the interviews were coded according to the theoretical framework and our adaptation in order to build two tables for each interview. The teaching documents table corresponds to the teaching activity, and the research documents table corresponds to the research activity (see Table 11.4). The tables allowed us to consider the list of documents in the two institutions: research institution and teaching institution.

To build the teaching documents tables, we proceeded in the same way as Gueudet (2017). Specifically, in the transcript of the teaching part of each interview, we tracked the given answer for the aim of the teaching activity (e.g., "preparing tutorial project"). For each aim, we added the resources explicitly mentioned in the transcribed declaration. Then, we identified stable elements in the way these resources were used (rules of actions). Concerning stability, we relied on the teacher's declarations (e.g., "for preparing tutorial project, we always start by elaborating

Table 11.5 Identifying pivotal resource (Resource 1) in this research documents table. Resource 1 appears also in the teaching documents table

Research documents table			
Aims (Ai)	Resources	Rules of actions (RA)	Operational invariants (OI)
A1	Resource 1	RA1	OI1
A2	Resource 2	RA2	OI2
A3	Resource 3, Resource 1	RA3	OI3
…	…	…	…
An	Resource 4, Resource 1	RAn	OIn
Teaching documents table			
Aims (Ai)	Resources	Rules of actions (RA)	Operational invariants (OI)
A1	Resource 1	RA1	OI1
A2	Resource 5	RA2	OI2
A3	Resource 6, Resource 1	RA3	OI3
…	…	…	…
An	Resource 7, Resource 8	RAn	OIn

many projects simultaneously"). Finally, we noted the operational invariants. This corresponded to statements in the interview such as "I do it this way … because I think that …"

We proceeded in the same way for the research part of the interview in order to build the research documents table. First, we defined a research aim (e.g., "improve the absorption of sunlight by cells"). Then we added resources (e.g., "Coating material," "bibliographical references") and identified rules of actions in the declaration (e.g., "doing a literature review," "Have a hypothesis"). Finally, we noted the operational invariants (e.g., "hypothesis based on the bibliography").

Once both tables were built, we first identified the pivotal resources in the research documents table (see Table 11.5).

We proceeded to check whether the pivotal resource in the research documents table (Resource 1 in Table 11.5) was mentioned or not in the teaching documents table. When this was the case, we considered the teaching document where this resource appears (the table line corresponding to the document). If not, we tried to understand the reason behind the lack of this resource regarding the operational invariant in research institution and/or the consideration and constraints in the teaching institution.

This methodology enabled us to question the resource mobilization process from research institutions to teaching institutions, by considering a horizontal analysis of each document in each institution.

11.4.3 Forms of Relation Between Research and Teaching in Terms of Resources

We now present our analysis of the cases and the corresponding results. We identified two forms of relation between research and teaching in terms of resources in both disciplines (physics and mathematics). We characterize those forms, and we present forms of relation that appear in the case of one discipline.

11.4.3.1 First Form: Research Resource in Instantiation Processes

In the case of P2, we identified five research aims in the research institution. The resource *nuclear material* was the pivotal resource in his research activity (it appeared in four out of the five identified aims). P2's research activity of using nuclear material entailed carrying out experiments. The results of those experiments had different features depending on the research aims (develop a coherent protocol, develop a simulation, compare empirical results with theoretical mathematical results). In the teaching institution, we identified two teaching documents where P2 mentioned explicitly the resource "nuclear material." Let us develop the case of one of the two teaching documents. While P2 planed the courses with his colleagues, he taught the content of the unit entitled *Diffusion and crystallography applied to nuclear material* alone. The unit aimed to sensitize students to some parameters (specifically two or three characteristics) related to the nuclear material (see Table 11.6).

In the case of M1, we identified seven research aims in the research institution. The resource *software* (Matlab, Maple, etc.) was the pivotal resource in numerical modeling research (it appeared in six out of the seven aims). M1 used the software to generate conjectures and to validate a conjecture or a modeling method. The software occupied the central place in his research approach. In the teaching institution, we identified two teaching documents where the software was used. M1 used the software with the master's degree students in order to sensitize students to the characteristics of the software in the activity of mathematical modeling (see Table 11.6).

In Table 11.6, we present two teaching documents corresponding to P2 and M1, respectively. The two teaching documents can be subsumed under the more general aim "sensitizing students to the characteristics of a resource from research."

In both cases (Table 11.6), we qualify the use of pivotal research resources in the teaching institution as an action of instantiation of it. The instantiation of this resource consists of the mobilization of the research resource from the research institution in the teaching institution in, as far as possible, the similar situations and in the similar role but in a more restricted domain of validity.

Table 11.6 Two teaching documents related to the same general aim of "sensitizing students to the characteristics of a resource from research"

	P2—teaching document	M1—teaching document
Aims	Sensitizing students to two to three specific parameters (characteristics) related to the *nuclear material*	Sensitizing students to the characteristics of *software* in the activity of mathematical modeling
Resources	Resources from previous teaching years of the same module (diffusion and crystallography applied to *nuclear material*). Colleagues	*Software* for numerical computing Resources from previous teaching years that contain problems to solve
Rules of action (way to use the resources)	Discussions with colleagues Collaboratively choosing the two or three parameters to teach	Choosing *software* used in the research Choosing and adapting a problem so that the selected software would provide an interface for manipulation, observation, or experimentation
Operational invariants (reasons for using them this way)	The *nuclear material* is such a wide domain. There are many parameters to take into account in an experiment. We have to raise awareness on two or three specific parameters.	The modeling activity in mathematics is exploratory and experimental

11.4.3.2 Second Form: Research Resource as a Scaffold for the Learning of Disciplinary Content

In the case of P3, we identified three research aims in the research institution. The resource *electronic microscope* was the pivotal resource. It was explicitly mentioned in two out of the three identified research documents. In his research activities, P3 used the electronic microscope to observe and study objects that ranged in millimeters. In the teaching institution, the resource *electronic microscope* was mentioned explicitly in three teaching aims out of the four we identified. We present and develop only one teaching document here (see Table 11.7). The aim of P3's teaching was the students' understanding of theoretical ideas related to the design and the use of the electronic microscope. P3 explained that his priority was to teach "strong scientific bases," because students needed to consider many parameters which were interrelated and depended on each other. P3 taught what he called "basic theoretical knowledge" that he saw as useful in understanding how the electronic microscope works. In his teaching, he did not include examples of how he uses the microscope in his research.

In the case of M2, we identified six aims related to his research activities. In three of these, *software* (Matlab, Maple, Scilab, etc.) was the pivotal resource for numerical computation and graphical simulations. M2's research activity using a

Table 11.7 Two teaching documents related to the same general aim of "supporting the use of specific resources by students"

	P3—teaching document	M2—teaching document
Aims	Co-development and co-implementation of M2 level courses (nano-characterization module)	Designing sessions to experiment and discover mathematical properties with software
Resources	Colleagues *Electronic microscope*	Software for numerical computation Resources corresponding to the course in question
Rules of action (way to use the resources)	Elaborate the content of the module with my colleague	Select a phenomenon of stability of differential equation Show the stability by using a graphical representation Offer the possibility to vary values and parameters in order to lead a discussion about hidden properties.
Operational invariants (reasons for using them this way)	We must teach the theoretical bases of how the two devices work: scanning electron microscopy and transmission electron microscopy We do not have time to handle all electronic microscope devices	Software is a tool that gives the results in a visual way and hides the properties We have to stimulate the spirit of imagination to make links between representations and mathematical properties that underpin these.

software particularly consisted of analyzing, modeling biological phenomena, validating the experimental results, and communicating results to the biologists he worked with. In the teaching institution, the software for numerical simulations appeared in two teaching documents. One of them corresponded to the aim "designing sessions to experiment and discover mathematical properties with software" (see Table 11.7).

In Table 11.7, we present two teaching documents corresponding to P3 and M2, respectively. These can be subsumed under the more general aim "supporting the use of specific resources by students."

M2 assigns the same role to the software in the construction of knowledge in both institutions (research and teaching), while the operational invariants show that M2 uses software in the teaching institution to scaffold disciplinary content related to the design of the resource as well as to its use.

In both cases (Table 11.7), the university teachers use the pivotal research resource to scaffold the disciplinary content for teaching. The scaffolding takes place during the designing process and the implementation of the disciplinary content.

11.4.3.3 Other Forms That Appeared in One Case

In the case of P1, we identified five research aims in the research institution. According to the research documents table, the *bibliographic references* (which include searching for references and reading them) were the pivotal resources. They appeared in two research aims. P1's research activities that used bibliographic references consisted of (a) knowing not only what has already been done in the field about the topic but also what has not been done yet and (b) being able to have a valid idea/hypothesis based on references. In the teaching documents table, we identified two teaching documents where the resource *bibliographic references* was mentioned explicitly in the column "resources." We noticed that in the two teaching documents, *bibliographic references* were used to put students in a research situation (one of the two teaching documents is presented in Table 11.8). We highlight that when P1 talked about "students actually doing research" she meant that students were involved in a process based on scientific methods. It was not research as such because the open problem that students worked on has already been solved (P1 knew the answer). Therefore, bibliographic references mobilized in the teaching institution were not the same as those that P1 mobilizes in her research. However, we can describe the relation in terms of development of the research process attitude: students were learning how to build a relevant bibliography on a subject and how to read it.

We qualify the interactions between research and teaching institutions as an action of spreading scientific attitude (research process) in the teaching task. P1 seems to give an importance to the functions of the bibliographical references. She encourages the students to do a systematic literature review and read articles and is spreading her scientific attitude in learning situations.

In the case of M3, there is a pivotal resource in the research documents table; however, it is not mentioned in the teaching documents table. This result is strengthened by the words of M3 during the interview acknowledging that there is a gap between mathematics research activity and mathematics teaching activity. From his

Table 11.8 One teaching document where *bibliographic references* is a key resource

	P1—teaching document
Aims	Follow tutored project
Resources	Ceramic and Pigment subjects *Bibliographic references*
Rules of action (way to use the resources)	Elaborate the contents of many tutored projects Ask students to work in groups of 6 after choosing a project Ask students to do a bibliography research and to carry out experiments Support students when they ask for help (answer the questions; change the orientation of students during research processes) Ask students to give an oral presentation related to the tutored projects
Operational invariants (reasons for using them this way)	Students have to be active, have to work by themselves, and have to feel responsible from A to Z

point of view, if there is a link, it will be in the way of teaching (operational invariant). M3 teaches the proof following the same process as in his research: he makes hypotheses, and then he determines the properties to be mobilized. There are no resources in common between teaching institution and research institution. He has a perception of a "separation" between the two institutions. He does not place his students in research situations. According to him, to be able to learn, the whole community of the class does not have to know how to solve tasks. This case shows that the relations that can exist might not always be tangible (Neumann 1992, see Sect. 11.4.1). The relations between the way of teaching and research could be captured as "when you teach, follow the same approach as in your research" in the treatment of a proof.

In both cases reported in this section, there is a relation between teaching and research which could be seen through the process of using the resources in the classroom and not only as a process of migration of resources from the research institution to the teaching institution.

11.4.4 Findings, Discussion, and Perspectives

It appears that the relation maintained between research and teaching depends closely on the university teachers' perceptions of his/her research resources. We remind the reader that our methodological choice requires identification of the pivotal research resources of university teachers and then study of their use in teaching. The results support our hypothesis that the pivotal resources from research tend to be mobilized in classes of teaching situations. We identified two forms of relation between research and teaching that appear in both mathematics and physics cases: (1) adapting a research resource to teaching through instantiation processes, and (2) using a research resource to scaffold disciplinary content.

As a result of our study, we can conclude that the DAD helps to determine some aspects of relation between research activity and teaching practices at university. The DAD offers a possibility to characterize a tangible nexus (Neumann 1992) between research and teaching (via the kind of interaction with resources) but also an intangible nexus (Neumann 1992) related to the interaction links to the specific professional knowledge of the university teachers; the operational invariants resulting from the research activity partly determine teaching practices.

We have extended the use of the DAD to consider the interactions with the resources in the research institution. In terms of their use, there are many differences concerning the teaching interactions with them. In this proposed extension, an important notion could construct a direction for a new perspective, which is "research aim" or "research interests." In fact, researchers do not know precisely what they are aiming at.

The study of the relation between the research resource system and the teaching resource system deserves further—possibly a long-term—study that would also comprise observations. We assume that a teacher may show both identified forms of relation between teaching and research depending on the teaching aims (indeed, the associate class of situation). This is a field to explore in order to understand the interactions between the teaching resources system and the research resources system. Lastly, designing resources collectively, with peers, in both teaching and research institutions constitutes an important direction for future research. It can elucidate the complex forms of relation between research and teaching in the practices of university teachers.

11.5 Toward a Conceptual Model of Documentation Expertise

Chongyang Wang

In a time of information technologies, teachers' resource work is getting more convenient but not necessarily more efficient. The resources for users are richer; the possibility of working with/as resource designers in potential communities is rising due to the emergence of new technologies (Pepin et al. 2015). At the same time, richness comes along with the problem of "resource quality" (Pepin et al. 2013) and the importance of users' resource appropriation (Trouche et al. 2013). Important questions are as follows: How to work more productively with resources (Kim, Sect. 11.6)? How to better transmit the knowledge with learning supports (Leroyer, Sect. 11.3)? How to be qualified for multiple working roles when working as both a researcher and a teacher (Sabra and El Hage, Sect. 11.4)?

Facing an immensity of potentially suitable resources, teachers need some relevant expertise allowing their successful resource integration (Ruthven 2014). This section aims at exploring this expertise aspect of teachers' documentation work, which is termed as documentation expertise (DE; Wang 2018) for distinguishing it from the related concepts. To propose a conceptual model of DE, efforts are made in two steps: a first model of DE is proposed based on a literature review and reflections on a pilot study in China (Pepin et al. 2016); a refined model through two contrasting cases studies. This section presents the preliminary results for the first DE model in four sections: firstly, a discussion on the key issues for proposing the notion of DE; secondly, some reflections on the Chinese pilot study; thirdly, the conceptual DE model; and finally, a conclusion.

11.5.1 Three Key Issues Drawn for DE

This section concerns three issues: (1) the necessity to propose the notion of DE from the perspective of terminology choice; (2) the specificity of DE compared with the related concepts; and (3) the links between DE and DAD through the notions of resource system and scheme.

11.5.1.1 Documentation and Expertise: Terminology Choice

The notion of DE is proposed based on two considerations: the origin of the term *documentation* in DAD, and the match of expertise with the nature of teacher's work and resources.

Firstly, the term "documentation" in DAD was drawn from the French word "ingénierie documentaire" (Gueudet et al. 2012, p. ix), referring to the terminology of "document management research" (Gueudet and Trouche 2009, p. 205). The roots of this term reveal a potential aspect of documentation work: there could exist specific knowledge or expertise (as engineering), with systematic and operational principles (as management).

Secondly, the term "expertise" matches the nature of resource and teacher's work. On the one hand, teaching is described as inherently a cultural activity (Stigler and Hiebert 1999) and as culturally shaped (Bishop 2002). Resources also bear some cultural and contextual imprints, such as in Adler's (2000) socio-cultural resources or in Brown's (2002) conception of resources as cultural artifacts (cultural tools). On the other hand, expertise is often considered as "highly contextualized" (Berliner 1988, p. 6), "culture-bound" (Schoenfeld 2011, p. 328), and needs to be understood in terms of socio-cultural contexts and education systems (Li and Kaiser 2011). Thus, to study teachers' expertise in their resource work, the cultural and institutional contexts need to be considered, and empirical research with contrasting case studies is especially relevant.

11.5.1.2 Distinguishing DE from Teacher Design Capacity and Pedagogical Design Capacity

Concepts related to capacity in teachers' resource work have been developed, such as pedagogical design capacity (PDC; Brown 2002) and teacher design capacity (TDC; Pepin et al. 2017). The specificity of DE is claimed through a demarcation with the two capacities.

PDC was proposed as a capacity that individual teachers exhibit to "craft" episodes to achieve their instructional goals (Brown 2002), through perceiving and mobilizing the existing resources. Perceiving refers to the ability to notice and recognize potential resources, while "mobilize" was claimed to be the fundamental term of PDC (Remillard 2005). It was further pointed out by Leshota and Adler

(2018) that PDC "is not what a teacher 'has', like knowledge" (p. 92), and that each teacher's PDC has its own specificity, reflecting his/her preferences, contexts and own understandings of different features of the resources. The work in PDC emphasized design but also included "perceiving affordances, making decisions and following through plans" (Brown 2009, p. 29).

TDC (Pepin et al. 2013) was proposed based on the notion of design by Brown, regarding the practice of designing in both phases of lesson preparation and teaching (design-in-use). It was initially dedicated to (digital) curriculum resources use, with three essential components: goal(s) of the design activity, a set of principles (robustness and flexibility), and reflection-in-action (Pepin et al. 2017).

DE is distinguished from the other two capacities by three aspects. Firstly, as the expertise of documentation work, DE is evidenced in all teacher–resources interactions inside and outside of the classroom. It covers more than the phases of designing resources (perceiving and mobilizing) and design-in-use, and also includes management of the resources. Secondly, DE concerns teachers' views of resources, which makes the resources (both the scale and category) diverse and extends the resources beyond instructional resources (Brown 2002), curriculum materials (Remillard 2005) or digital curriculum resources (Pepin et al. 2017). Thirdly, DE is linked with the individual teachers' multiple work roles at school, with the value of expertise in their cultural/institutional contexts.

Along with PDC and TDC, DE is not a uniform standard or ideal state for teachers to achieve, but a framework for reflecting on teachers' expertise as they make use of resources, for understanding their resources, resource systems, and the diverse schemes they develop for fulfilling their tasks and adapting resources to their working contexts.

11.5.1.3 Two Concepts for Constructing the DE Model: Resource System and Scheme

Resource system and scheme are considered as two key concepts in DAD (Trouche, Chap. 13). To propose and explore the DE model (with its structure and components), this section presents the two concepts and how they support in framing the DE model.

A resource system is "the set of resources accumulated and organized (over time) by a teacher in line with his/her regular teaching activity" (Trouche et al. 2018). It is not merely a collection of resources, but a functional entity and a coherent system (Ruthven 2009). "The word 'system' is purposefully chosen to emphasize that this system is highly structured, the structure being linked, more or less explicitly, to teacher activity" (Gueudet et al. 2013, p. 1004). For Ruthven (Chap. 3), the resource system expands both the notion of textbook (into a systematic curriculum scheme combining diverse resources into a coherent program), and the notion of library (into a resource repository organized systematically to make contents readily searchable and usable). These statements emphasize three aspects of resource system: it is a structured, systematical, and functioning repository of resources; it is

formed and organized with personal preferences of the teachers; and it is dynamically developed along with teacher's documentation work and professional development. The documentation work can be viewed as a process of interaction between the outside world and teachers' resource system: the teacher adapts resources and schemes to the resource system while organizing, maintaining, and managing it.

Scheme, the second key concept, was defined by Vergnaud (2009) as "the invariant organization of activity for a certain class of situations" (p. 88) with four components: (1) goal/sub-goals, which is connected to the target of specific situations; (2) rules of action, for transforming reality, searching information and controlling the results; (3) operational invariants, the conceptual basis for selecting the appropriate information and identifying the most appropriate rules of action; and (4) possibilities of inferences, which allow the subject to think and compute the activities in different situations. For Vergnaud (1998), "competences are composed of schemes aimed at facing situations" (p. 230); and schemes are the operational side of knowledge (knowledge in action). This makes schemes analyzable and enables to see teacher expertise through the lens of schemes.

Schemes can be named and classified by situations: Schemes get developed by being adapted to situations, while situations work as a key to understand and analyze schemes, they are so intricate that we can use an expression concerning situations to refer to a scheme, or an expression concerning schemes to refer to a situation (Vergnaud 2009). Situation was considered as the problem to be dealt with (Vergnaud 1998) and categorized into two classes (Vergnaud 1990): one is familiar for the subject, and the necessary treatments and competences are ready in his/her repertoire, and the other is new to/for the subject and requires the subject to reflect and explore. A scheme can be expressed in a form of scheme/sub-schemes along with goal/sub-goals of the situation.

In this way, DE is characterized through two dimensions: a static dimension evidenced from the structure and elements of the resource system; a dynamic dimension evidenced from the process of integrating resources, including (1) the systematic management of the resource system; and (2) the appropriation and transformation of the resources for specific documentation work, such as selecting, modifying and creating new resources, by individuals or by a group of teachers working together, in-class and out-of-class (Gueudet and Trouche 2012b). The concrete potential components of each dimension will be presented after the reflections on a pilot study in the following section.

11.5.2 A Reflection on Methodology and DE Model with a Chinese Pilot Study

In 2014, a pilot study of teacher expertise through resource system analysis was conducted, which involved two in-depth interviews with three Chinese mathematics teachers (Pepin et al. 2016). The process and results provoke some reflections on the potential components for formulating the DE model and for the methodological tools.

The three teacher participants in the pilot study were colleagues with whom I worked in a high school for more than 6 months in the same office. After a long-term observation of their office work and classroom teaching, two rounds of in-depth interviews were conducted: a first interview was about their resource work including the resources used in their daily work. Each of them was invited to draw a *schematic representation of the resource system* (SRRS; Gueudet et al. 2012) to represent the structure of the resources they mentioned in a specific lesson preparation/implementation activity. The second interview was about their perceptions of teacher expertise and suggestions for novice teachers about how to get it developed.

The results showed the diverse structures and components of their resource systems, even though they worked in the same physical space with a lot of shared resources and frequent exchanges: (1) In order to categorize the resources in their resource systems, some referred to the location (at home/office or in computer/notebooks), or to the source (from colleagues or self-purchased), or to the function (for preparing exams or for homework); (2) in order to organize their resource systems, some centered their resources on printed curriculum materials and kept paper-and-pencil notes, while others focused on digital resources and on linking the resources through cloud drives; and (3) in order to denote the resource elements in their resource systems, some considered only material resources like textbooks, while others also referred to the collective discussion with colleagues, social communications, and cooperative projects.

There were also some strategies in developing expertise revealed from their self-descriptions of specific lesson preparation activities. One teacher valued the openness of the resource system, and the spirit of sharing and exchanging resources/experiences with others. Another teacher stressed that combining teaching practices with the contemporary educational theories was important. Yet another suggested that it was essential to keep up with the requirements and trends of the curriculum program and high school entrance exams.

The pilot study yielded reflections on the DE model. When studying an individual teacher's resource system, three aspects are worth considering: (1) the collective aspect, especially those with cultural and institutional characters, such as the Chinese Teaching Research Group (Wang 2018); (2) the student aspect, which was emphasized by a Chinese teacher in the pilot study as important to get teaching effects feedbacks for better adjusting their following lessons; (3) the design aspect, which reflects to what extent the resources were proceeded, forming teachers' personal resources. DE could differ in terms of these aspects for different teachers.

In addition, the pilot study inspired some methodological considerations. Schoenfeld (2011) pointed out that one needs to be careful about researchers' own orientations on expertise when studying teacher expertise. This echoes the principle of "confronting teachers' view" (Trouche et al. 2018) in DAD. To see the difference between the teacher's view and the researcher's view on the structure and elements of the teacher's resource system, the tool of SRRS is expanded into *inferred mapping of resource system* (IMRS)" (created by the researcher based on the observa-

tions of and interviews with the teachers about their resource work) and *reflective mapping of resource system* (RMRS) (created by the teachers based on their own reflection) (Wang 2018). In this way, the tools of IMRS and RMRS could bring more information for a more precise description of SRRS. These mappings need to be adapted in several times for obtaining different descriptions of resource system from the teachers and for catching the changes in the mappings caused by teachers' better understanding of their resource system or by its development. Considering the flexibility, the order in which IMRS and RMRS are constructed can be different: a RMRS can be developed through a further interview based on the previous IMRS, and vice versa. Besides, for better understanding of teacher's collective aspect of documentation work, the notion of documentation-working mate (Wang 2018) was proposed as someone who has close interactions in documentation work with the targeted teacher therefore forming his/her smallest collective. The documentation-working mate is chosen by the targeted teacher and followed in the same way.

A conceptual DE model will be presented next based on the literature review and the pilot study.

11.5.3 A First Conceptual Model of DE

This section contains three parts: a discussion on the nature of DE, followed by a description of the static dimension of DE comprising six views of the resource system, and the description of the dynamic dimension of DE encompassing five schemes.

11.5.3.1 Some Ideas on the Nature of DE

This section presents some ideas about the nature of DE:

- Unnormalized and off standard. DE is neither a standard nor a universal stereotype for all mathematics teachers, but a framework to be verified and enriched through more contrasting cases.
- Contextually diverse. DE contains a contextual and culture-bound character in different contexts. It does not only inherit the culture-bound nature of expertise, but also echoes the cultural aspect of resources, which makes DE diverse in different cultural and institutional contexts.
- Bi-directionality of adapting and self-adaptiveness. As the expertise aspect of documentation work, DE should be evidenced in both the process of adapting resources (instrumentalization) and self-adapting to resources (instrumentation).
- Multidimensional framework. DE could be analyzed in terms of two dimensions: the static dimension, i.e., the structure and elements of resource system, and the dynamic dimension, i.e., schemes related to teachers' specific documentation

activities, including how they manage their resource systems. The resource system develops dynamically along with teacher professional development, but it can be analyzed as a dynamic process composed of static moments, like making screenshots from a video.

11.5.3.2 A Static Dimension of the Structure and Elements of Resource System

A resource system is dynamically developed along with teachers' documentation work and professional development. It can be studied in specific moments, which is referred to as a *static dimension*, relating to the structure and elements of teachers' resource system. It contains six views (see in Fig. 11.3). A "view" could be understood as a lens used by the researcher to study the resource system and its structure/elements from a particular perspective. Three of them (collective, student, and design), as discussed before, were inspired by the pilot study. The other three were chosen concerning the keywords of the research field: mathematics and didactic (Gueudet and Trouche 2009, p. 214), and curriculum (Pepin et al. 2017). There could be more views included if it is necessary for other research interests.

The horizontal axis denotes that DE is developing continuously along with time, but it does not mean that an advanced or expert teacher must be strong within each view. Besides, the evaluation of DE is not discussed in this study. On the vertical axis, there is no hierarchical order among these six views. One resource can be seen from multiple views. For instance, inside a teacher's resource system, a curriculum program could be considered through both the views of didactic and curriculum.

(1) The mathematics view allows the teachers to gather mathematical information and make logical considerations from the perspective of mathematics. For example, when teaching the notion of algorithm, the teacher might reflect on its different definitions in mathematics and in informatics, where this concept orig-

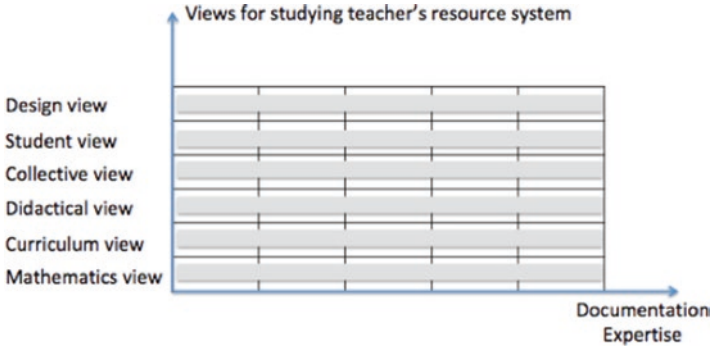

Fig. 11.3 The six views for studying teachers' resource system

inated and how it was developed, and the links with other mathematics knowledge.
(2) The curriculum view assists the teacher to catch the ideas and requirements from the curriculum program or the textbooks. The teacher might consider the curriculum expectations, the suggestions for teachers, and the available interpretative resources.
(3) The didactical view distinguishes teaching as a profession (Berliner 1988), providing the principles to guide teachers' practice and choice of resources related to their teaching and school settings.
(4) The student view allows teachers to arrange their resource design in terms of the students' needs/interests, and take their feedbacks as important references to adjust their subsequent teaching.
(5) The collective view refers mainly to professional collectives, allowing teachers to benefit from collective interactions, enriching their resource system with new resources, or learning new schemes of working with resources.
(6) The design view is closely linked to teachers' personal documentation work habits and preferences. For example, to what extent and how are the resources advanced and designed?

For studying the specific elements of the resource system, three indicators are considered: (1) Content: What is the resource and its function? (2) Structure: What is the position of it in the resource system? Which view does it belong to? What are its links with other resources, inside its view, and across other views? (3) Activeness: Are the resources often used? How are they managed and where are they stored?

The six views and three indicators are proposed for exploring the structure and elements of the resource system through the tools of IMRS and RMRS, which is considered as a static dimension. The management of the resource system will be considered as part of the dynamic dimension in the next section.

11.5.3.3 A Dynamic Dimension of the Schemes in Documentation Work

In DAD, the use of resources and corresponding schemes of usage constitute teacher's documentation work. As a kind of "knowledge in action" (Vergnaud 1998), a scheme is considered here as the basic unit in characterizing the *dynamic dimension* of DE, including how to manage the resource system and how to integrate the available resources to confront situations.

As discussed earlier (in Sect. 11.5.1), schemes (and their four components) are inseparably linked to situations. The situations are either familiar or unfamiliar to teachers. Thus, I assume that no matter if the necessary competences or resources are ready or not for the situations, the schemes can be decomposed into sub-schemes and named based on the goal/sub-goals of the situations.

Similar to what Shulman (1987) proposed in his model of pedagogical reasoning and action, the six activities (comprehension, transformation, instruction, evaluation, reflection, and a new comprehension) form a cycle of teacher's pedagogical

reasoning. Inspired by this, five phases were selected based on the definition of documentation work (Gueudet et al. 2012): searching and selecting (from teacher's resource system, or resources outside), modifying and adapting in the situation, accumulating resources back to the resource system, and reflecting through the whole documentation work (see in Fig. 11.4). They do not necessarily occur in a sequential order. Since documentation work is a continuous process, DE could be evidenced in more than five phases if further studies subdivide the process in depth.

Figure 11.4 shows a process of teacher's documentation work and how a resource system is developed: In front of a given situation either familiar or not, a teacher could search for resources either in (the gray circle filled with stars) or out (the white square filled with black dots) of their resource system. The four-point and five-point gray stars mixed in the resource system refer to different types and functions of resources. For example, a teacher selects resources from his/her resource system (four-point stars in the blue square), adapts and modifies them according to the needs of situation (from four-point stars in the white square to five-point stars in the gray square), and, in the end, accumulates it back to his/her resource system. He/she could also look for resources that are not familiar for him/her (black dots in the white square), make modifications in order to adapt these resources to the situation (from gray dot in the white square to four-point stars in the gray square), and then accumulate them to the resource system. The changes of the colors and shapes refer to the transformation on the resources. Reflecting accompanies the whole documentation work.

The five schemes involved in the current conceptual model are not presented in detail with all of the four components of scheme but based on the third component of operational invariants, namely, the conceptual basis for choosing the most appro-

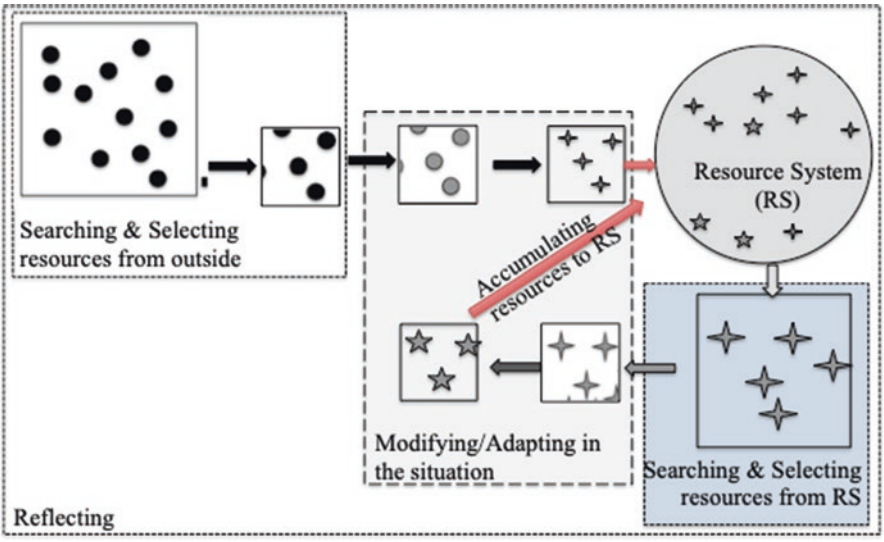

Fig. 11.4 The dynamic process with five phases for evidencing DE

priate rules of action. The specific contents of the five schemes, as well as the remaining three components (goals, rules of action, and inferences), will be illustrated in specific situations of the contrasting cases.

– Scheme related to searching for resources

Searching for resources includes the integration of available resources and experiences. Generally, the expert teachers bring richer and more personal resources to the problem that they are trying to solve (Berliner 2001), draw on their previous teaching experiences as well as the reflections thereon (Borko and Livingston 1989), or use planning materials from previous years as cues (Livingston and Borko 1989).

– Scheme related to selecting resources

Selecting resources is a process of identifying the useful resources by referring to factors like teaching objects, students' learning conditions, requirements from the curriculum program, and teachers' own understanding about what should be taught.

– Scheme related to adapting resources

Adapting resources comprises a process of transforming the resources into a form ready to be used, or familiar for the teacher. Experienced teachers can balance content-centered and student-centered instruction (Borko and Livingston 1989) and adjust syllabus guidelines and institutional expectations with their own educational beliefs and ideologies (Calderhead 1984).

– Scheme related to accumulating resources

Accumulating resources belongs to the management of the resource system. Experienced teachers have the consciousness to include, share off, and store the resources in a structured way. Since accumulating resources depends on the personal working habits, it could differ among different teachers.

– Scheme related to reflecting on the documentation work.

"Doing and thinking are complementary" (Schön 1983, p. 280). Reflection appears in the whole documentation work and makes the development of the resource system and schemes possible.

11.5.4 Section Summary

This study presents the first model of DE mainly based on a literature review and reflections on a Chinese pilot study. Asa kind of expertise shown and evidenced in documentation work, DE inherits the nature of both teacher expertise and documentation work: culturally influenced and evaluated, continuously and dynamically developed, shaping and shaped by resources, able to be evidenced from a static dimension (structure and elements of the resource system) and dynamic dimension (schemes of managing a resource system and of the documentation work in specific

situations). To study the static dimension of a resource system, the new tools of RMRS and IMRS were expanded to differentiate the views from teachers and researchers. The new concept of documentation-working mate was proposed to study the collective aspect of documentation work. The model of DE is thus presented from a static dimension in terms of the structure and elements of a resource system (with six views and three indicators), and a dynamic dimension with five basic schemes as part of documentation work. The aim of the DE model is not to formulate an ideal stereotype with a list of standards, but a framework to see the diverse preferences of teachers from different contexts. This model of DE is only a preliminary result. To propose a richer and more elaborated model of DE, a second step of verifying it by specific case studies (in China and in France) will be conducted in my following work.

11.6 Teacher Capacity for Productive Use of Existing Resources[6]

Ok-Kyeong Kim

Mathematics teachers use a variety of resources to design instruction. How they use, adapt, and transform the resources to teach mathematics influences the quality of instruction, and teachers are required to have the capacity for using them productively. Focusing on curriculum resources that teachers use for daily instruction, this section describes teacher capacity needed for using existing curriculum resources productively. For that purpose, a set of analyses are drawn from the Curriculum Use for Better Teaching (ICUBiT) project, whose goals were (1) to identify components of the capacity that Brown (2009) calls *pedagogical design capacity* (PDC, i.e., a teacher's ability to perceive and mobilize existing curricular resources in order to design instruction) and (2) to develop tools to measure PDC. As such, to investigate the capacity for using existing resources productively, I drew on Brown's (2009) notion of PDC. However, exploring the capacity for productive use of existing resources through the analyses in this section can be one way to study PDC, and I do not intend to equate the capacity elaborated here with PDC. I instead attempt to answer to the following question: What are the components of teacher capacity needed for productive use of existing resources?

The ICUBiT project gathered data from elementary teachers in the United States who were using five different curriculum programs (each program included resources for students and teachers for daily lessons), ranging from commercially developed to reform-oriented. The five curriculum programs were analyzed to

[6]Acknowledgment: This section is based on work supported by the National Science Foundation under grants No. 0918141 and No. 0918126. Any opinions, findings, conclusions, or recommendations expressed in this section are those of the author and do not necessarily reflect the views of the National Science Foundation.

account for the kinds of content and pedagogical support for teachers and ways in which such support was provided. Also, classroom data were analyzed from various perspectives to examine ways in which teachers used their curriculum program to teach everyday lessons. These analyses of curriculum programs and teachers' use of curriculum resources shed light on specific aspects of teacher capacity needed for effective use of existing resources. I describe some significant aspects of the teacher capacity along with examples from the ICUBiT project and related literature.

11.6.1 Theoretical Background

Although Brown's notion of PDC is drawn on initially, investigating teacher capacity for using existing resources productively is situated in a broad research context. First, this capacity seems critical in teachers' documentation work (Gueudet and Trouche 2009) in that one important aspect of the documentation work relates to how teachers use existing resources and how this affects their documentation work. According to Gueudet and Trouche (2009), teachers are engaged in documentation work, such as looking for resources and selecting tasks. Gueudet et al. distinguish between resources and documents. Resources are a range of artifacts for teaching, such as textbooks, software, and discussions with a peer teacher, whereas documents are evolving products of teachers' documentation work, which include resources, usage (action rules), and operational invariants (cognitive structure guiding resource use). How teachers use the resources is observable; in contrast, operational invariants are often invisible but can be interpreted from ways in which teachers use the resources. In the analyses to explore teacher capacity for productive resource use, I mainly focused on artifacts for teaching, especially existing curriculum resources for everyday teaching, i.e., student texts and teacher manuals. However, I attended to teachers' usage, i.e., how teachers read, adapt, and use existing resources to teach mathematics lessons. Also, I inferred teachers' operational invariants to make sense of the ways in which they used the resources. Examining teachers' use of resources along with their operational invariants supports the inquiry into teacher capacity needed for resource use.

I consider teacher decision making around using existing resources as pedagogical reasoning and action (Shulman 1987) and using knowledge in teaching practice as elaborated in knowledge quartet by Rowland and his colleagues (e.g., Rowland et al. 2005). When making decisions in planning and teaching mathematics lessons, teachers use their own personal resources, such as their experiences with and knowledge of mathematics content, curriculum (resources), and students. They also transform the resources they use in a way that fit their instructional goal and their students' need. The notion of contingency in the knowledge quartet by Rowland et al. (2005) highlights teachers' design of instruction beyond the plans they have made and the resources they use. This is similar to what Remillard (1999) calls improvisation, or "on-the-spot curriculum construction" (p. 331), which indicates teacher moves that are not specified in the written lessons (i.e., individual lessons

outlined for teaching in the existing resources). Examining teachers' decisions on how to use resources to design instruction and their improvisations is eventually digging deeper into teachers' reasoning and knowledge in use, which helps explore teacher capacity for productive resource use.

The productivity of using existing resources depends on the opportunity for students to learn during instruction. When the resources are used productively, enacted lessons must create opportunities for students to learn the mathematical points of the lessons with sufficient cognitive demand on the students (Kim 2018). Students need to explore, reason about, and understand the target mathematics of the lessons. Therefore, teacher capacity for productive resource use should be examined in terms of whether the resource use supports students' learning of the mathematical points of the lessons, and what aspects of resource use support or do not support student learning and how. Mathematical points have dual aspects: conceptual foundation and procedural competence. Each lesson contains both aspects even when it places more emphasis on one aspect than the other (Kim 2018, 2019). Generating opportunities for students to engage in the dual aspects of the mathematical points is fundamental for student learning.

Finally, I insist that exploring teacher capacity of productive resource use is based on the participatory relationship between teachers and resources (Remillard 2005). Using notions of instrumentation and instrumentalization, Gueudet and Trouche (2009) also illustrate the mutual interaction between a teacher and resources in documentation work. Teacher capacity needed for using resources productively is grounded in such bilateral influences that shape both parties. This relationship generates the research context that examines not only the components of the teacher capacity needed for using resources productively but also the role of the resources in supporting teachers to develop such a capacity.

11.6.2 Data Sources

In order to explore the capacity needed for productive use of existing resources, I drew on data gathered from 25 teachers in grades 3–5 in the ICUBiT project in the United States. These teachers were using five different curriculum programs, each of which contained resources for teachers and students, such as the teacher's guide for everyday teaching, student materials, and the implementation guide (five teachers per curriculum program), ranging from reform-oriented to commercially developed. The teachers were (1) asked to keep a Curriculum Reading Log (i.e., on a copy of written lessons they indicated parts they read, parts they planned for instruction, and parts influenced their planning), (2) observed in three consecutive lessons in each of two rounds, and (3) interviewed after each round of observations. All observations were videotaped, and all interviews were audiotaped. Then, both video- and audio-taped data were transcribed for analysis. Scrutinizing teacher capacity for productive resource use, this section draws on a range of analyses on various aspects of resource use by the teachers, such as sequencing lessons, using

intervention resources, and deciding whether to follow the guidance in the written lessons (e.g., Kim 2015, 2018, 2019). I documented patterns of the teachers' resource use within each analysis (e.g., using, omitting, or changing a significant component in the written lesson), their effectiveness in terms of the mathematical points of the lessons, and teachers' rationale for their decisions. The mathematical points of individual lessons were determined based on a careful reading of objectives, directions for teachers and students, tasks and problems, and other descriptions about the lessons. Then, for every significant teacher move, it was determined whether it supported or hindered the mathematical points of each lesson. Searching for patterns of use and their productivity in these analyses revealed critical components of the capacity for productive resource use. I also drew on literature related to teacher capacity and resource use to compare the patterns that surfaced in the analyses. For more details about how the data were analyzed in each investigation, see the papers cited.

11.6.3 Teacher Capacity

The teachers in the ICUBiT project made various decisions regarding how to use their curriculum program. Some decisions impacted enacted lessons positively toward students' learning of the mathematics of the lessons; others did not. Although a range of support features were provided in the written lessons, it was evident that teacher improvisations occurred quite often regardless of programs used (Kim 2019). Various teacher decisions on resource use, kinds of improvisations, and teachers' reasoning behind their decisions revealed different aspects of resource use and teacher capacity needed. Below, five specific aspects of teacher capacity for productive resource use are described along with brief examples from the data in the ICUBiT project. Although described individually, I view these as interrelated components of teacher capacity, which are not mutually exclusive.

11.6.3.1 Articulating Mathematical Points of the Lessons

Using existing resources to teach mathematics, teachers first read and make sense of the written lessons. In doing so, they need to identify the mathematical points of the lessons and evaluate how well the lesson activities, tasks, and problems support students' learning of the mathematical points (Remillard and Kim 2017; Sleep 2012). Then, they need to organize lesson activities toward the mathematical points in instruction (Brown et al. 2009; Sleep 2012). Failing to identify the mathematical points of the lessons, teachers orchestrate lesson activities away from the mathematical points (Kim 2015, 2018, 2019).

One third-grade teacher in the ICUBiT project considered identifying and using keywords as the goal of the lessons on creating and solving multiplication and division story problems, and emphasized keywords instead of the meaning of the two

operations in instruction (Kim 2015, 2019). The mathematical points of the written lessons she used were (1) understanding the meaning of multiplication and division, and their relationship, and (2) using them to solve and create multiplication and division story problems. Placing greater emphasis on keywords in place of the meaning of the two operations, the teacher said, "The keywords for recognizing what multiplication and division are, and how to pull those out of a story problem and use them to advantage for the kids. … it was definitely valuable." The teacher thought keywords were important to highlight although using them was not suggested in the teacher's guide.

With keywords as the goal of the lessons in her mind, the teacher altered or omitted important lesson components that had a great potential to support students' understanding of multiplication and division. For example, there was a lesson component that asked students in pairs to come up with story problems for two related expressions (i.e., 6×3 and $18 \div 3$) so that students could see the differences between multiplication and division contexts. Instead of this task, the teacher asked students to generate a list of keywords for each of the two operations. The teacher made comments, as students offered some expressions as keywords, whether each suggested word would be acceptable for each operation. In doing so, she lost an opportunity to highlight characteristics of multiplication and division in relation to each other. The loss of meaning continued in the following lesson when students were creating multiplication and division story problems. The teacher often said, "If it says 'in each', it's gonna be a division problem." Or, "Now remind me, what are our multiplication keywords? If it's a multiplication story problem it's gonna have what key words in it?" While focusing on keywords, such as "in all," and "share equally," the teacher did not use the important terms, such as number of groups, number in each group, and equal groups, to explain the characteristics of and differences between multiplication and division. As a result, after spending 2 days on generating multiplication and division story problems, still more than half of her students were not able to complete the task. On the third day of classroom observations, there was a range of student-generated story problems. Some students had stories but no questions; some students did not have multiplication or division contexts (addition or subtraction instead); and some students had only one type of story problems (all multiplication or all division)

11.6.3.2 Steering Lessons Toward the Mathematical Points

Sometimes teachers identified the mathematical points of the lessons properly and yet had hard time steering instruction toward these points. This was observable when they were challenged by students' difficulty in understanding the mathematical idea, when they did not use proper resources available in the written lesson, or when the written lesson did not provide sufficient resources for the mathematical points (Kim 2018, 2019). An example of the last case is one teacher using a commercially developed program who taught a lesson on mean. The written lesson indicated the meaning of a mean in different places for teachers as follows (Charles et al. 2008);

> Like the median and the mode, the mean tells what is typical of the numbers in a set of data. The mean is sometimes called the average (p. 404). Explain that an average levels off or evens out the numbers in the data set so that all the numbers are the same (p. 404). Averaging involves distributing numerical data evenly across a set of numbers and provides a single number to describe what is typical in that set of numbers (p. 404A).

The lesson also included a picture of cube towers in the case of 7, 4, and 4 (see Fig. 11.5) to illustrate what the mean of a set of numbers means. The mathematical points of the lesson were (1) understanding what the mean of a set of numbers means and the procedure to find the mean and (2) finding the mean of a set of numbers. The explanations above and the picture of cube towers made the teacher think hard about the concept of the lesson and set that as an important goal for the lesson. She said, "I liked the idea of how it levels out. So that's what conceptually I was thinking in my head." She also related this meaning with the procedure to find the mean of a set of numbers. She said, "They've got a whole bunch of chips. This person only has three chips. But when you combine everything and then divide it evenly, how it levels out. … division is dividing up as evenly as you can, the leveling out part." She also recognized the importance of the term, "typical" as the meaning of the mean of a data set.

However, steering the lesson toward the mathematical points identified was very challenging for this teacher, especially without sufficient resources in the written lesson. Other than the explanations provided for teachers and the picture of the cube towers, the lesson was mostly about the procedure to find the mean of a set of numbers, namely, "adding all and dividing the sum by the number of data." Different sets of numbers were given to students, who were asked to calculate mean. The teacher attempted to incorporate the idea of "leveling out" by using cubes. She put students in groups, distributed cubes to each student, and asked them to count how many each had, and then "combine all in the center and share them evenly." Unfortunately, this activity of using cubes, and the teachers' directions and additional comments did not create an opportunity for students to make sense of what the mean of a data set means, let alone how the procedure to find the mean of a data set works. During the follow-up interview, she confessed, "I probably didn't articulate it well to my students," although it made sense to her.

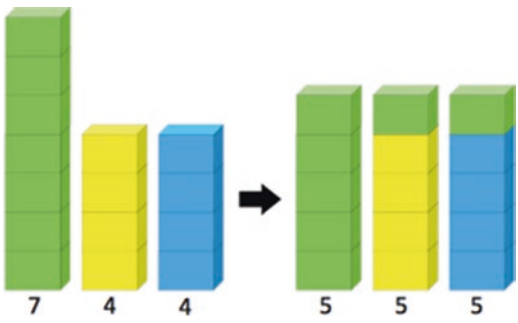

Fig. 11.5 Representing the meaning of the mean of 7, 4, and 4

Articulating mathematical points and steering lessons toward the mathematical points are aspects of teacher capacity that are not limited to individual lessons. Teachers need to articulate mathematical points of a series of lessons (an entire unit or a set of consecutive lessons) and teach students through a proper mathematical pathway so that the students can understand the connections and relationships in the mathematical points and develop a coherent mathematical storyline, or "a deliberate progression of mathematical ideas" (Sleep 2012, p. 954) across lessons. Teachers need to envision how mathematical ideas are intended to develop over a series of lessons, and sequence tasks and lessons according to this progression. Otherwise, students may have difficulty develop a proper understanding of the complete ideas across lessons. For example, sequencing tasks and lessons in a way that eased up on the first 2 days and then enacted a series of important explorations all on one single day, a fifth-grade teacher rushed students to make sense of common fractions (specifically 1/4, 3/4, 1/8, 3/8, 1/3, 2/3, 1/6, etc.) and their percent equivalents (Kim 2018). Although the teacher was aware of the mathematical goals of the lessons, this way of steering a series of lessons significantly hindered the students' engagement in the mathematical points of the lessons: (1) understanding relationships between percent and fractions, and (2) using these relationships, known equivalents, and representations to determine fraction equivalents of common fractions.

11.6.3.3 Recognizing Affordances and Constraints of the Resource in Use

As teachers read and make sense of the resources and identify the mathematical points of the lessons/activities/tasks, they also recognize what aspects/components/features of the lessons/activities/tasks support or do not support students' learning of the mathematical points. In order to use existing resources productively, teachers need to recognize such affordances and constraints of the resources they use, with respect to their students' learning of the mathematical points (Atanga 2014; Brown 2009; Choppin 2011; Kim 2015, 2018, 2019; Kim and Son 2017). Teachers who were not able to recognize the affordances may not use them in instruction. Also, teachers who do not recognize the constraints hardly try to make up the limitations. Depending on their evaluation of the affordances and constraints along with their students' need, teachers can decide whether they use, change, or omit components of lessons/activities/tasks, or add new elements to enact lessons (Kim 2019). Therefore, recognizing affordances and constraints is critical for using the existing resources productively.

For example, even when one third-grade teacher identified the mathematical points of the lessons, she did not recognize that visual representations (fraction circles or pictures, bars, and number line) provided in the resources were useful to promote students' understanding. Not seeing the usefulness of those representations for the procedure of subtracting a fraction from a whole number, the teacher dismissed the need for using the representations in supporting students' conceptual understanding of the procedure for subtracting a fraction from a whole number (Kim 2018). Even when students suggested to use a representation, the teacher

refused to do so. Mentioning that the representations were too simplified and tended to confuse students, the teacher did not recognize the affordances of the representations in supporting students' conceptual understanding of the procedure. On the contrary, she saw those representations as constraints and avoided them in all the lessons observed. After three days of listening to the teachers' explanations and using the procedure, the students in this class still had hard time making sense of what they did.

The third-grade teacher using keywords also did not see the affordances of several activities and representations. Whereas the teacher emphasizing keywords did not identify the mathematical points of the lessons accurately, the teacher above was able to clearly identify the mathematical points of the lessons she taught. In fact, she was trying to steer instruction in order to support students to (1) understand the relationship between improper fractions and mixed numbers and (2) use the relationship to add fractions to get a mixed number or subtract a fraction from a whole number. Yet, not seeing the usefulness of the representations provided in the written lessons, the teacher dismissed them entirely while orchestrating classroom activities.

11.6.3.4 Using Affordances

Recognizing the affordances of existing resources is important; so is using those affordances in instruction. Brown's (2009) definition of PDC includes both "perceive and mobilize" the existing resources. In particular, using those resources together as a coherent set seems critical in using the existing resources well (Atanga 2014). Various components of the resources are designed to support students' learning of the mathematical points. Resources as a set rather than separate elements indicate the synergy that they can generate in supporting teachers to steer instruction toward the mathematical points. In the ICUBiT project, when using resources productively to teach lessons, teachers were using a range of elements provided in the resources toward the mathematical points of the lessons. Otherwise, as seen in the earlier example of the teacher focusing on keywords, teachers altered or omitted useful, important resources (e.g., representations and tasks). Sometimes they added new elements which were not productive in place of critical resources suggested. In other cases, teachers used the affordances unproductively.

The fifth-grade teacher mentioned earlier recognized the usefulness of 10×10 grids to relate fractions and their percent equivalences (e.g., 3/4 = 75%). But, the teacher used the grids not very effectively in the second observed lesson, by asking students to shade their own grids and write the fraction and the percent that each of their grids represented. Students shaded their grids randomly and wrote a fraction and percent pair mainly by counting the number of squares shaded (e.g., 79 squares shaded, so the grid represents 79/100 and 79%) without much attention given to the relationship between fractions and percent, especially percent equivalents of target common fractions, such as halves, tenths, fourths, eighths, thirds, and sixths. This was problematic because the mathematical point of the lessons was not about deter-

mining fraction-percent pairs of 10 × 10 grids shaded randomly. The written lessons were deliberately focusing on using the grids to relate common fractions and their percent equivalents, moving from easy fractions (e.g., 1/2 = 50%) to harder fractions (e.g., 1/4 = 25%) and finally to more complex fractions (e.g., 1/3 = 33%). As explained earlier, the teacher identified the mathematical points of the lessons, but her instructional moves led away from the learning pathway carefully laid out across the lessons. The biggest step away from the pathway was misusing the 10 × 10 grid in the second lesson.

11.6.3.5 Filling in Gaps and Holes

Recognizing constraints of the existing resources does not necessarily lead to productive ways of overcoming them, which is another important aspect of the capacity needed for effective use of existing resources. In the ICUBiT project, teachers tended to add new elements to the written lessons to enact them (Kim 2019). Some elements were intentionally added as planned; others were improvised in response to students. Whether these new elements are planned in advance or improvised during instruction, they have to support students' learning of the mathematical points of the lesson. Especially, those intended to overcome the constraints of the written lessons or improve the written lessons must be prepared carefully to increase the opportunity for students to learn the mathematical points of the lessons.

One teacher used a curriculum program whose individual lessons were designed for multiple class periods so that students could explore related mathematical ideas in depth over 2–3 days (Kim and Atanga 2013, Kim 2019). In a lesson written for 3 estimated days, students were asked to use base-ten pieces (i.e., pieces for ones, longs for tens, and flats for hundreds) to measure the area of a coat, and compare and order large numbers. This lesson was designed for geometrical and numerical explorations combined. The conceptual aspect of the mathematical point of the lesson was understanding how to measure an irregular shape and place value; the procedural aspect was using base-ten pieces to measure the area of an irregular shape, and ordering, comparing, and adding four-digit numbers. The students were using the concept of symmetry to efficiently measure the area of a coat (i.e., only measuring a half of the area and doubling the number found) and making sense of the large numbers as the resulting areas would be thousands of single pieces. As the lesson was complex in nature, detailed guidance was provided for instruction in the teacher's guide. However, there was still room for additional elements and improvisations as the teacher enacted the lesson. Noticing that her students needed a review on symmetry before starting a task of finding the area of a coat, the teacher asked students questions about area and symmetry, which effectively supported students' understanding of the nature of measuring the area of a shape like a coat and their work on the task. In fact, the teacher identified the mathematical points accurately, and noticed the affordances of the task and how the task could fall apart because of its nature. From these recognitions, she not only used the task and resources as suggested in the written lesson but also included additional steps to support students to

use the base-ten pieces appropriately to measure the area of a coat. All of these aspects enabled the teacher to orchestrate her instruction toward the mathematical points effectively.

The fourth-grade teacher described earlier, who used blocks in a lesson on mean, recognized that the task for students in the written lesson mainly focused on the procedure to find the mean of a set of numbers and also recognized that the cube towers in Fig. 11.5 could be used to highlight the conceptual foundation for students' understanding of mean. In order to support students to make sense what the mean of a set of numbers means and why the procedure to find the mean works, the teacher asked students to use cubes to determine the mean of four different numbers before moving to the main student task. As described earlier, however, this was not productive because her use of cubes was not supporting students to understand what the mean really means or how the procedure works. Basically, showing the procedure of "add/combine them all and divide by four" with the cubes, the teacher intended, but was not able to highlight the conceptual nature of the mean—what the mean of the four numbers really represents, i.e., levelling out or evening out across the numbers.

There are no perfect curriculum resources that fit in any classroom situation; proper change, omission, or addition is needed as teachers are engaged in documentation work. Yet, the way teachers fill in the gaps and holes in the existing resources should be determined toward students' engagement in the mathematical points of the lessons.

11.6.4 Discussion

It was evident in the ICUBiT project data that different components of the capacity are interrelated. For example, identifying the mathematical points of a lesson was critical in making further decisions and using existing resources. Without accurate mathematical points identified, teachers can hardly steer instruction toward these mathematical points. Moreover, they can seldom recognize the affordances and constraints of the resources in use. The teacher emphasizing keywords had a number of missed opportunities to support students to think about the meaning of multiplication and division to solve and create story problems. Also, not seeing the importance of comparing multiplication and division in contexts and related problems, the teacher eliminated those components from her instruction, which, in fact, would have been good for students' understanding of the meaning of the two operations and their differences. In this sense, helping teachers articulate mathematical points of lessons seems to be a reasonable starting point to support them to develop the capacity for productive resource use.

The data used for this section also revealed that the teachers in the ICUBiT project had certain operational invariants, the notion that Gueudet and Trouche (2009) use to indicate cognitive structure guiding resource use. Unproductive use of existing resources is often rooted in operational invariants or conceptions that are not

appropriate (Kim 2019). For example, the teacher emphasizing keywords in multiplication and division story problems believed keywords helped students' learning of operations and solving story problems. She said, "I know without those key pieces of information these kids can't be successful at finding the answers to story problems." Also, the teacher, not using representations in the lessons on operations with fractions, believed that representations were not helpful but instead were confusing students' thinking. Therefore, in order to support teachers to develop the capacity needed for productive use of existing resources, teacher education (i.e., teacher preparation and professional development) needs to support teachers to examine their own conceptions and generate such opportunities in their resource use.

Teacher knowledge is a critical element in teacher capacity for productive resource use, and developing the teacher capacity draws on different kinds of knowledge and skills. In particular, the knowledge of content and curriculum in mathematical knowledge for teaching [MKT] (Ball et al. 2008) and knowledge of curriculum embedded mathematics [KCEM] (Remillard and Kim 2017) seem very important for all five aspects of the teacher capacity for productive resource use. One approach to building the capacity is increasing teacher knowledge; another is working on the capacity (i.e., learning to make proper decisions in using resources) and cultivating the knowledge needed at the same time. The latter seems more promising. In this way, inappropriate conceptions can also be assessed against specific decisions and their productivity, and revised toward a higher capacity. In fact, increasing knowledge in building the capacity can help develop proper conceptions for the capacity. For example, the teacher using keywords extensively can become aware of her improper use of keywords and make better decisions toward the meaning of multiplication and division next time, by increasing knowledge needed through, for example, (1) unpacking the meaning of multiplication and division carefully, (2) looking into how different problem contexts embed this meaning and how these contexts can support students' thinking and learning of the operations, and (3) examining the actual impact of her use of keywords on students' understanding and thinking. All of these opportunities can help not only develop knowledge needed for the capacity for productive resource use, but also amend any inappropriate conceptions, such as the one on keywords.

11.7 Discussion and Perspectives

Sebastian Rezat, Carole Le Henaff, and Jana Visnovska

All four frameworks that are presented in Sect. 11.2 and are applied in Sects. 11.3, 11.4, 11.5 and 11.6 conceptualize teachers' work with resources. Since they are focusing on the same object, they share some commonalities, but they also put different emphasis on particular aspects of teacher documentation work. This is partly visible in the empirical studies and the theoretical contribution in Sects. 11.3, 11.4,

11.5, and 11.6. While Leroyer (Sect. 11.3) and Sabra and El Hage (Sect. 11.4) only draw on DAD, Wang (Sect. 11.5) explicitly demarcates her notion of DE from PDC and teacher design capacity (Pepin et al. 2017), and Kim (Sect. 11.6) contextualizes her study in both, PDC and DAD.

In light of the four study examples, we now discuss commonalities and differences of the four frameworks presented in Sect. 11.2, highlight some of their affordances, and point out some constraints. In particular, we discuss how the frameworks approach the focused activity and mediational means, the role of students, the role of institutional aspects, issues related to intentions, collective work, the quality of resource use, and the potential of the frameworks to inform change.

11.7.1 Focused Activity and Mediational Means

The four presented frameworks conceptualizing teachers' work with resources are grounded in activity theory. Therefore, they share the focus on mediational means and their capacity to afford and constrain human activity in a participatory relationship with mutual impacts. However, the mediational means are called and conceptualized differently in the four frameworks: artifacts, curriculum materials, or resources.

While artifacts can be both material and nonmaterial objects such as signs, the scope of the artifacts that are considered in the different theories varies. DAD refers to Adler's (2000) wide notion of resources including nonmaterial resources. By referring to resources instead of artifacts, DAD even goes beyond the notion of artifacts in activity theory. The main difference between artifacts in activity theory and resources in DAD is that artifacts are designed by humans with an intention, while a resource neither needs to be designed by humans nor does it need to be designed with a particular intention. DAD focuses on the interplay of these resources in the instrumentalization by and instrumentation of the teacher in the construction of the document. Both Remillard's (2005) framework of components of the teacher–curriculum relationship and Brown's (2002) design capacity for enactment framework in principle focus on material curriculum resources. However, both of them include a number of resources such as subject matter knowledge and pedagogical content knowledge, skills, beliefs, and perceptions of curriculum materials and students, which influence teachers' use of material curriculum resources. While in Remillard's and Brown's frameworks the role and the interplay of all these resources are open and subject to empirical studies, this interplay is partly structured in DAD by the notions of instrumentalization and instrumentation and the related notion of scheme.

The wide notion of resources in DAD may at times create difficulties in identifying the resources that are relevant for a specific purpose and limiting the scope of resources that are included in studies of teacher's documentation work. This becomes particularly apparent in Kim's contribution (Sect. 11.6), in which she conceptualizes teacher's capacity for productive use of resources. The five components

of teacher's productive use of resources she identifies are (1) articulating the mathematical points of the lessons, (2) steering lessons toward the mathematical points of the lessons, (3) recognizing affordances and constraints of the resource in use, (4) using affordances, and (5) filling in gaps and holes. Looking at these components from the perspective of DAD, it would be interesting to go a step further and identify the resources that a teacher must possess or access in order to demonstrate productive use of teaching resources, and thus the capacity Kim outlines.

The frameworks also differ in the activity that is in focus. Studies based on DAD often operationalize the teachers' documentation work by analyzing teachers' reflections and planning activities that take place outside the classroom. In other words, a notion of document constructed in those ways shares similarities with the planned curriculum. According to the definition of documentational work, the document could be understood as the utilization of resources in order to develop the planned curriculum in particular situations. Prior experiences from enacting the planned curriculum, including insights into students' work, are considered to be a resource in the documentation work.

As opposed to the focus of DAD, SDT focuses on the activity inside the classroom, in which teachers and students interact with shared artifacts. Thus, SDT affords the analysis of interactions of teachers and students through artifacts. Consequently, artifacts that are solely used by teachers (or by students) are not the primary focus of SDT. A focus on the artifacts that are only used by the teacher would mean to focus only on one triangle side of the SDT. In such case, the students would become the (imagined) subject at which the teacher's activity is directed.

In comparison, Remillard's (2005) framework aims to represent design-stages of a curriculum resource that include before, during and after classroom practice. She distinguishes written, planned, and enacted curriculum and thus acknowledges that the enacted curriculum might differ significantly from both the planned curriculum (because it is co-constructed by teacher and students) and the written one (if the designers' and teachers' goals differ). Besides their role as co-constructors of the enacted curriculum, students play a role in the resources that the teacher brings to the participatory relationship with curriculum materials as teachers' perception of students.

Brown's (2002, 2009) metaphor of "teaching as design" seems to relate to both teachers' planning activities and the design of instruction in class. Therefore, it comprises the planned and the enacted curriculum.

The role of the attained curriculum, that is, students' actual performance and learning achievements, remains opaque in all four frameworks. The attained curriculum is not mentioned explicitly in any of the frameworks, and therefore, its role within the frameworks is open to suspicion. It could be conceived of as part of teachers' perception of students in Remillard's (2005) framework or as such become a resource in the DAD. Teachers' perceptions of students' attained curriculum might also lead to an adjustment of the learning goals and thus influence teachers' interaction with resources.

11.7.2 The Role of Students

In DAD, PDC, and the framework of the teacher–curriculum relationship, students play a minor role. In DAD the student is not explicitly mentioned. However, in her model of documentational expertise, which is closely related to DAD, Wang (Sect. 11.5) mentions the "student view" as one possible view to study and understand teachers' resource systems. If a teacher's resource system is studied from this perspective, the selection and adjustments of resources with regard to students' needs and feedback are the matter of interest.

In Remillard's (2005) and Brown's (2002) frameworks, students appear in terms of teachers' perception of students and in terms of teachers' knowledge about students' behavior. Siedel and Stylianides (2018) find that many teachers' selection of resources is "student driven," that is, "driven by consideration of their students' [perceived] needs" (p. 132, our insertion). The authors exemplify that the teachers' consideration of students' needs does not only have implications for the selection of resources, but also regarding their use.

In Kim's conceptualization of teacher's capacity for productive resource use (Sect. 11.6), students appear related to each of the five components of teachers' productive resource use. Students' learning processes appear as the objective at which the teacher's productive resource use is directed. A reason might be that these components are partly derived from observing teachers' activities in the classroom with students. It is possible that this is an implicit assumption within DAD, the design capacity for enactment framework, and the framework of components of the teacher–curriculum relationship. Namely, that the goal of teachers' design activities is always to provide students with resources that will best support their learning progress. However, given the competing agendas to which teachers routinely have to attend, is an implicit assumption like this sufficient when we theorize their work with resources?

Several contributions that use the DAD, design-capacity-for-enactment framework, or the framework of components of teacher–curriculum relationship have underlined the effects of the use of resources on students' learning and the search for innovative resources to support students' learning (Argaud et al. 2018; Barbosa and Vale 2018; Leroyer 2018; Rodrigues et al. 2018). However, in these cases, the intentions attributed to the teacher together with the selection and use of resources based on these intentions, are projected on the potential learning that students will carry out on the basis of the used resources. The underlying hypothesis is that the better the teacher knows his/her students, the more expertise he/she develops in his/her design capacity related to the goal of supporting the students' learning in the best possible way. And, in the same way, the more he/she knows about the subject he/she teaches, the more expertise he/she gains. Nevertheless, these assumptions remain to be empirically tested.

Unlike previous frameworks, the socio-didactical tetrahedron (SDT) accounts explicitly for the student as a user of curriculum materials and resources. This is

related to the different focus of the framework, which is to model the overall didactical situation in the classroom focusing on the artifacts that both teachers and students use.

Consequently, with exception of SDT, the frameworks give a relatively minor consideration to students. While SDT acknowledges the student as a coequal user of artifacts, the other frameworks regard students as influences on the written and enacted curriculum, and include them indirectly in terms of teachers' perception of how students influence the next cycle of the planned curriculum. This difference might arise from the different foci of the frameworks: while SDT models the use of artifacts in the classroom, the other frameworks focus on teachers' interaction with curriculum materials during their planning and teaching activities.

11.7.3 The Role of Institutions

It is key to consider what place is occupied by the institution in the frameworks used for studying teachers' work with resources in the instrumentation process. The meaning of institution here is twofold. In the more usual sense of the term (Douglas 1987), the institution is understood as that which organizes, structures, even prescribes, and controls the activity of teachers. But Douglas (1987) also develops a new conception of institutions as "legitimate social groups." For example, Wang (Sect. 11.5), relying on this definition, called groups of teachers who regularly work collectively on a regular basis an institution. The place occupied by the institution, in the second sense of the term, is indeed crucial in the development of teachers' competencies related to the use of resources.

Martinez et al. (2018) problematize the role played by institutional prescriptions in the selection and modification of teachers' resources. The resources provided by the institution are linked to particular intentions and goals. However, teachers have to make use of these resources, or in other words, attribute their individual functions to them (instrumentalization), perceive their affordances and constraints, and incorporate them into the resource system. Thus, they are not released of the problems of selecting "good" materials for teaching mathematics and incorporating these materials in their teaching practice.

Sabra and El Hage use the DAD to investigate the use of resources in a university context. Referring to Madsen and Winsløw (2009), Sabra and El Hage (Sect. 11.4) differentiate between the teaching institution and the research institution of university teachers. Analyzing the use of a pivotal resource, they are able to better understand the relation of the research institution and the teaching institution through the lens of DAD. Thus, they use DAD to understand the use of resources in different institutions and the mutual relationships. Wang (Sect. 11.5) also acknowledges the role of the institution in the notion of DE and stresses the dependency of DE on institutional contexts.

It appears that DAD enables to grasp and better understand institutional aspects in professional work in different contexts through the lens of resource use. Due to their roots in activity theory, it is likely that this is also the case for the other frameworks, since the main assumption there is that the whole activity with its cultural and historical heritage is crystallized in the use of the artifact.

11.7.4 Intentions

The decisive aspect of institutional prescriptions largely forms the basis of the relationship between teachers and their practice, and therefore, the development of teachers' competencies related to their use of resources. This aspect also crosses teaching subjects, as Gruson et al. (2018) have shown, by comparing the design capacity of English and mathematics teachers: "Firstly, they both trust and use consistently 'officially approved' resources. The need to be in line with the official curriculum is an operational invariant (Vergnaud 1998) shared by both teachers."

The design of a resource therefore seems closely linked to the "patterns of intention" (Baxandall 1985) underlying the teachers' use of resources. The initial didactic intention (Margolinas and Wozniak 2010), conceptualized by Sensevy (2011) as "strategic rules," can be readjusted at any time.

When studying use of resources, it seems therefore necessary not to try to study a "reconstituted historical state of mind, (…) but a relation between the object and its circumstances" (Baxandall 1985, p. 42). Sensevy (2011, p. 192) adds that such intentions do not only apply to persons, but also to resources. For instance, developers have specific intentions which they aim to communicate via the designed resources. Similarly, teachers develop and refine their didactic intentions, while they conceive the resources, and while they use them in their class. We can suppose that the intentions resulting from planning and teaching activities are closely related to teachers' documentational expertise (Sect. 11.5) and their usual postures (Sect. 11.3) but could also be shaped by the intentions communicated by the resource itself. For example, students' work with a resource can incite teachers to re-organize their didactic intentions, as well as the use of the resource, in the course of action. In addition, some resources are devoid of initial didactic intentions when teachers select them.

As all discussed frameworks are grounded in activity theory or sociocultural perspectives, they allow for the analysis of this tension of intentions. Resources as mediational means are inherently situated culturally, institutionally, and historically (Wertsch 1998). An analysis of mediated action from a sociocultural or activity theoretical perspective can thus provide insights into the relationship of the intentions inherent in the mediational means and those of the user.

11.7.5 Collective Work and Design Capacity

The place of collective work is a crucial point to question in the study of teachers' use of resources but also of teachers' expertise. Indeed, the work of a teacher is part of both a report, and a path, which are specific to him/her, and which are linked to his/her academic training, as well as to his/her experience in teaching a certain type of knowledge. But design capacity, in a general way, and the development of expertise, is not limited to these aspects, as Wang notably shows it in her section (Sect. 11.5).

Indeed, it is important to take into account, when studying the documentation genesis of a teacher, how he/she fits into a collective, and what this collective brings to him/her in the development of competencies that are related to the use of resources. Wang (2018), Quéré (2018), and Ratnayake and Thomas (2018) also describe that exchanges within a group of teachers, or with other education professionals, have important effects on teachers' didactic reflections. For example, Quéré has shown that teachers who work collectively with the same teaching object, and with the same resource, are led to develop their conception of the knowledge at stake in these resources, but also their conception of the design and use of these resources.

This aspect also echoes the lesson studies (Miyakawa and Winsløw 2009) and their contribution to the development of teachers' competencies that are related to the use of their resources. Indeed, as Scardamalia (2002) put it, "creating a shared intellectual resource and a rallying point for community work helps to provide an alternative to tasks, lessons, projects and other expert-designed motivators of work, replacing them with a system of interactions around ideas that leads to the continual improvement of these ideas" (2002, p. 9). Wang (Sect. 11.5) also highlights the determining aspect that collective work brings to the development of teachers' competencies. She argues that teachers' individual resource systems may be studied and understood in terms of how collectives influence them and how they are used.

We rarely find studies of teachers' collective work with resources based on other frameworks than DAD. This might be due to the very wide notion of resources that is at the heart of DAD as opposed to the other frameworks. In the view of DAD, the collective might become a resource for the individual teacher in his/her documentational work.

The SDT also has the potential to contribute to the understanding of the role of communities and collectives in the use of artifacts, since it includes different communities on a social level. The frameworks by Brown (2002) and Remillard (2005) share their focus on the single teacher and his/her individual resources that he/she brings to the participatory relationship with curriculum materials. Therefore, communities and collectives play a subordinate role so far in the studies using these frameworks.

11.7.6 The Quality of Resource Use

The frameworks presented in Sect. 11.2 provide a language to describe resource use and to understand it from the perspective of activity theory. However, they do not account for the quality of the outcomes of the use of resources. The frameworks are not used to evaluate whether or not a document, the use of a curriculum material, or a "crafted instructional episode" is appropriate for the instructional goal, or whether or not it supports learning in a desired way. Their main intention is to better understand the interaction between the teacher and the (curriculum) resource.

Brown's notion of PDC seems to be a partial exception, because he explains PDC as a relation between teachers' perception of the affordances of the resource and a goal to be achieved by a designer-intended use of the resource. Therefore, this framework appears best suited for adaptations that would aim at evaluating the quality of resource use by different teachers.

Males et al. (2018) and Cooper et al. (2018) suggest two different methodological approaches to teacher's perception of the affordances and constraints of resources. Males et al. (2018) differentiate three interrelated phases of teachers interacting with a resource while reading: (1) curricular attending, (2) curricular interpreting, and (3) curricular responding. In order to grasp precisely what teachers attend to in a curricular resource, they suggest to use an eye-tracking methodology, which records eye movement in between and fixation time on particular locations on the page. While the methodology produces highly detailed data, the interpretation of this data requires further theoretical foundation. In particular, it remains unclear whether or how could long or short fixations of gaze and different patterns of eye-movement across the page contribute to informing us about teacher perceptions of the affordances and constraints of used materials.

Cooper et al. (2018) developed a tagging tool for digital resources. While the main intention is to provide a tool in order to support teachers as co-designers of curriculum, the tool enables the researchers to represent and analyze teachers' perceptions of the curriculum materials, the choices they make, and sequences of the enacted curriculum.

The concern with quality of resource use is also apparent in Kim's notion of teacher's capacity of productive resources use (Sect. 11.6). Based on her analysis of teachers' use of primary curriculum programs, Kim explores the components of teacher's capacity to use resources productively. Her main criterion for productive resource use is that enacted lessons create opportunities to learn the mathematical core of the enacted materials with an appropriate cognitive demand.

11.7.7 Potential of the Frameworks to Inform Change

When researching aspects of (mathematics) teachers' work and professional development, documenting and understanding the change is often the primary aim. This has several reasons. First—like in any profession—an individual teacher's practice and

rationales that underpin his/her decisions are expected to undergo changes with time and teaching experience. Second, institutional expectations for what the job of teaching (mathematics) has to entail, including the tools that teachers are expected to use in their work, also change with time, and teachers are expected to make adaptations to their practices and rationales that would reflect this ever-renewing stream of changing expectations. Third, the most important, it is generally recognized that improvements to what mathematics students get to learn in schools, which students get to learn this mathematics, and how well will they understand it, cannot occur without changes in instructional practices that generate conditions for student learning.

The frameworks discussed in this chapter are currently used—in presented example studies and beyond—to capture and describe teacher's work with resources. We are proposing that in increasing the level of detail in these descriptions, the field might also need to take steps to investigate to what extent the new distinctions could be more broadly useful to inform change, and in particular provide better guidance about how teacher professional development can be supported. This direction is generally of interest to DAD research community, as illustrated by Leroyer (Sect. 11.3) who is anticipating and proposing to study how her framework of teachers' postures can be used for purposes of teacher professional development.

Notably, all discussed frameworks point out the multitude of personal resources that teachers bring to documentation work. Irrespective of the labels used for naming them, these personal resources are considered to be the result of the teacher's history of participation in the profession of teaching. They are assembled through the teacher's responses to opportunities, expectations, and problem situations, and stabilize those responses that prove to be the most reasonable from within this teacher's point of view.

Some of these resources can be explicit and some implicit for the teacher, and researchers often postulate these, based on empirical data, as being assumptions held by teachers about what kind of mathematics is the key for students to learn, what to look for in a curricular/instructional resource, how and when student learning happens, what might be the reasons for students' struggle, or how should teachers organize classroom events to effectively support the learning process. Research strongly suggests that changing curricular or instructional resources rarely fundamentally challenges teachers' assumptions. This was the case even in situations when new resources were produced with the explicit intent to change teachers' focus and rebuild their practices (e.g., NSF-funded curriculum materials in the United States) and where external guidance was provided to teachers (Simon et al. 2000). Indeed, successful cases of supporting productive shifts in some of these assumptions are rare and appear unduly resource-intensive (e.g., Carpenter et al. 1989; Visnovska and Cobb 2019).

However, it appears that to make mathematics learning meaningful for more students, we would benefit from supporting many more mathematics teachers in rethinking and rebuilding their assumptions, and changing their instructional practices. It is our opinion that frameworks that conceptualize teacher–resource interactions and relationships are uniquely positioned to support such work. It would also appear that improvements are possible in this respect.

Let us take DAD as an example. A number of studies conducted within this approach document teachers' resource systems, and explore how these change in response to introduction of a new (often digital) resource (see, for instance, Chap. 12 by Drijvers, Gitirana, et al.). From a design research perspective, insights generated in such studies of change could contribute to design theories of teacher learning. However, it is important to clarify that analyses conducted within DAD do not establish how teachers usually work with resources under unspecified conditions. Their activity theory roots position teacher learning as situated and the findings speak to design theories about teacher learning under particular conditions of support.

Within design research, means of supporting learning are theorized, designed for intentionally, and treated as an inherent aspect of any learning theory. In contrast, DAD subsumes the means of proactively supporting teacher learning within the rather broad notion of resources (which include material, human, and social-cultural). This does not appear problematic when creating point-in-a-time accounts of teachers' resource systems (see, e.g., Sect. 11.6), or retrospective accounts of their development. However, when design of a proactive support for specific changes in teachers' resource systems is the goal (such as changes in teachers' assumptions about nature of students' mathematical learning), the lack of theoretical differentiation between teachers' starting point situation and the required means of support, provision of which would have to be designed for, becomes problematic.

It is equally problematic that when theorizing all elements present within teachers' situation as 'resources', the responsibility to derive support (e.g., gather and select resources) appears to rest primarily—and possibly solely—with the teachers. While variability in teachers' access to resources explains why very different uses of the same classroom instructional resource are the result, this insight (and the theory guiding it) does not seem to distribute the responsibility for the result sufficiently among the players who substantially contributed to it (e.g., designers, school and system instructional leaders). It is our opinion that these kinds of consequences of theoretical tools we produce need to be continuously examined and addressed.

11.8 Conclusions

In this chapter, we provided an overview of four influential frameworks that conceptualize teachers' use of (curriculum) materials in paper or digital format including relevant resources in the interaction with the materials. Additionally, four empirical studies exemplify the application of these frameworks to a wide range of settings and contribute to their further theoretical development and elaboration. Based on the general introduction of the frameworks and the four example studies, we discussed the affordances and constraints of these frameworks.

While all frameworks are grounded in activity theory and thus share their focus on the mediational role of artifacts within activity, they vary in the scope of artifacts and resources and the activity in focus. Due to these differences, they afford or con-

strain the particular focus on different phases or levels of curriculum (e.g., designed, planned, enacted, attained) and the investigation of the role of communities and collectives, institutions, and students.

The scientific interest in understanding teacher's work with curriculum materials and resources grew out of the desire to promote change in teaching and learning mathematics accompanied by the experience that it does not suffice to provide reform-oriented curriculum materials. The frameworks discussed in this chapter contribute to an understanding of the complex relationship and interaction between the curriculum materials and the resources of the teachers. Furthermore, there is a growing interest to account for a quality aspect in teachers' use of resources. This tendency is mirrored in the contributions by Wang (Sect. 11.5) and her notion of DE and Kim (Sect. 11.6) and her aim to investigate teachers' capacity for productive use of resources. However, we argue that the insights in teachers' use of resources need to be used to further exploit the potential of the frameworks in order to proactively support specific changes in teachers' resource systems and their use in order to promote change in mathematics teaching and learning.

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