Effect of Co-flow Stream on a Plane Turbulent Heated Jet: Concept of Entropy Generation

Amel Elkaroui[1(✉)], Amani Amamou[1], Mohamed Hichem Gazzah[2], Nejla Mahjoub Saïd[3], and Georges Le Palec[4]

[1] LGM, National Engineering School of Monastir, University of Monastir, Monastir, Tunisia
amel.karoui@hotmail.fr, amani.amamou@yahoo.fr
[2] URPQ, Faculty of Science of Monastir, University of Monastir, Monastir, Tunisia
hichem.gazzah@fsm.rnu.tn
[3] LGM, Preparatory Institute for Engineering Studies, University of Monastir, Monastir, Tunisia
nejla.mahjoub@fsm.rnu.tn
[4] Aix-Marseille University, CNRS, IUSTI, Marseille, France
georges.lepalec@univ-amu.fr

Abstract. The present paper numerically investigates the effect of a co-flowing stream on the mean and turbulent flow properties, air entrainment and entropy generation rate of a heated turbulent plane jet emerging in a co-flowing stream. The first order k-ε turbulence model is used and compared to the existing experimental data. The Finite Volume Method (FVM) is used to discretize the governing equations. The predicted results were consistent with the experimental data. It is found that a jet in a co-flowing stream is known to be a quicker mixer than a jet in a quiescent ambient, and it is proved that the presence of a co-flow enhances mixing. Therefore, the mixing of a jet in co-flow is more efficient.

Keywords: Numerical modeling · k-ε model · Co-flow · Entropy generation concept

1 Introduction

Free turbulent jets in a stagnant ambient stream as well as in a moving external stream have been the interest of numerous investigations. This interest is due to their practical applications as turbulent diffusion flames in combustion chambers when a fuel jet flow is commonly injected into a co-flowing stream. The important parameters that influence the mixing characteristics of a jet are the presence of density difference and a co-flowing between the jet and its surroundings. The entropy generation concept, which is based on the second law of thermodynamics, has recently appeared in many applications in engineering, such as combustion engines and convective heat transfer system flows. Much more attention is given to round jets emerging in a co-flowing stream. Indeed, some experimental investigations were conducted on co-flowing jets. Nickels and Perry [1], Antonia and Bilger [2], Smith and Hughes [3] made measurements in the

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strong region of round jets as well as in the strong-to-weak jet-transition region. However, the measurements of Davidson and Wang [4] extend into the weak region. Furthermore, Gazzah et al. [5, 6] have numerically investigated the co-flow effect on heated turbulent round jets. Less works, however, deal with co-flowing plane jets. Bradbury [7] and Bradbury and Riley [8] have experimentally studied a turbulent plane jet in a slow moving air stream. They found that the spread of jets could be merged, with varying co-flow velocity ratio, when accounting for the effective origins. Elkaroui et al. [9] have investigated the dynamic and thermal behavior of a free turbulent plane jet, including the influence of temperature on entropy generation. They found that high rates of entropy generation correspond to higher inlet jet temperatures. The present contribution extends this study by investigating the effect of a moving external stream on the jet development and entropy generation. The presence of co-flow stream is known to improve the mixing process for a non-reactive jet and to generate flame stability. Free and co-flowing round jets, which can be used in chambers and the mixer, have received much more attention than plane jets. That is why this paper aims to investigate the configuration of turbulent plane jets issuing in a co-flowing stream and applicable in air curtains. The effect of co-flow on entropy generation, which can be used as an effective tool for the optimal design of thermal systems, is studied through a numerical simulation using the first order k-ε model. The effect of the co-flow velocity (U_{co} = 0.0 m/s, U_{co} = 1.20 m/s and U_{co} = 2.0 m/s) on the dynamic and thermal behavior and local entropy generation for a turbulent plane jet are predicted.

2 Problem Formulation

The considered flow is a heated turbulent plane jet issuing in a co-flowing ambient stream with various velocity ratios, temperature ratios. The actual dimension of the slot width H is equal to 0.04 m. The flow is only weakly compressible in the sense that the

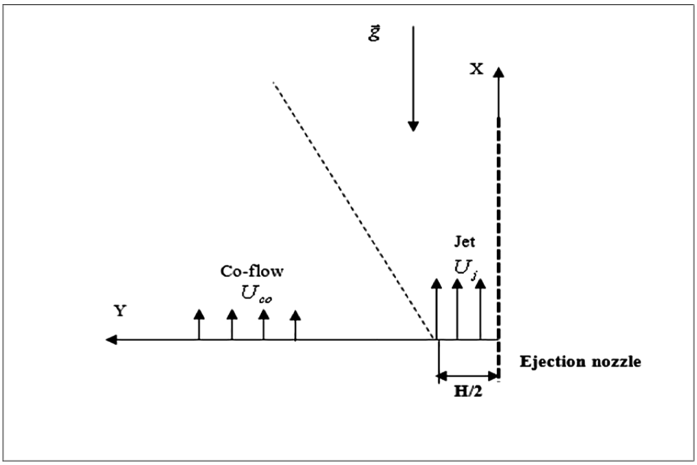

Fig. 1. Geometric configuration with co-flow

Mach number is low. Hence, the flow development is characterised by the Reynolds number $\mathrm{Re} = \frac{U_j H}{\upsilon}$, the densimetric Froude number $Fr = \frac{\rho_j U_j^2}{\left(gH\left(\rho_{co} - \rho_j\right)\right)}$ as defined by Chen and Rodi [10], where U_j = 18 m/s is the jet velocity (Fig. 1).

2.1 Governing Equations

Under these assumptions, the governing equations including continuity (1), momentum (2) and temperature (3) conservation equations are considered in the cartesian coordinates system as follows:

$$\frac{\partial \rho U_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_i U_j\right) = -\frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\overline{\rho u_i'' u_j''}\right) + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial U_k}{\partial x_k}\delta_{ij}\right] \tag{2}$$

$$\frac{\partial}{\partial x_j}\left(\rho T U_j\right) = \frac{\partial}{\partial x_j}\left(\frac{\lambda}{C_P}\frac{\partial T}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\left(\overline{\rho t'' u_j''}\right) \tag{3}$$

With δ_{ij}: is the Kronecker symbol; δ_{ij} = 1 if i = j, 0 if not

$\overline{\rho u_i'' u_j''} = -\mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{2}{3}\rho k \delta_{ij}$: is Reynolds stress tensor

$\overline{\rho t'' u_j''} = -\frac{\mu_t}{\sigma_t}\frac{\partial T}{\partial x_j}$: is the Reynolds heat flux vector

The $k - \varepsilon$ model is used for the closure of this system: the equations of the turbulent kinetic energy (k) and its dissipation rate (ε) can be written as [11]:

$$\frac{\partial}{\partial x_j}\left(\rho U_j k\right) = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) - \overline{\rho u_i'' u_j''}\frac{\partial U_i}{\partial x_j} - \rho\varepsilon \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j \varepsilon\right) = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right) + C_{\varepsilon 1} P_k \frac{\varepsilon}{k} - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} \tag{5}$$

With $\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$ is the turbulent viscosity

P_k is the production term of turbulent energy (k), which is described as follows:

$$P_k = \left(-\rho\overline{u_i'' u_j''}\frac{\partial U_i}{\partial x_j}\right) \approx \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} \tag{6}$$

This model requires the use of different empirical constants [12]:

$$\sigma_k = 1, \sigma_\varepsilon = 1.3, C_{\varepsilon 1} = 1.44, C_{\varepsilon 2} = 1.92, C_\mu = 0.09, \sigma_t = 0.7$$

2.2 Local Entropy Generation Rate

In the non-reacting jet flow, when both temperature and velocity fields are known and based on the second law of thermodynamics, the volumetric entropy generation rate ($\overset{\bullet}{S}_{gen}$) as given by Bejan [13] at each point in the fluid, is calculated as follows:

$$\left(\overset{\bullet}{S}_{gen}\right) = \left(\overset{\bullet}{S}_{gen}\right)_{heat} + \left(\overset{\bullet}{S}_{gen}\right)_{fric} \tag{7}$$

Where $\left(\overset{\bullet}{S}_{gen}\right)_{heat}$ and $\left(\overset{\bullet}{S}_{gen}\right)_{fric}$ represent the volumetric entropy generation rates due to heat transfer and fluid friction, respectively with the following expressions [17]:

$$\left(\overset{\bullet}{S}_{gen}\right)_{heat} = \frac{K_{eff}}{T^2}\left(\frac{\partial T}{\partial x_j}\frac{\partial T}{\partial x_j}\right) \tag{8}$$

$$\left(\overset{\bullet}{S}_{gen}\right)_{fric} = 2\frac{\mu_{eff}}{T}\left(S_{ij}S_{ij}\right) \tag{9}$$

Where K_{eff} and μ_{eff} are the effective thermal conductivity and the effective viscosity respectively.

S_{ij} is the mean strain rate and: $S_{ij} = \frac{1}{2}\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right)$.

3 Boundary Conditions

At the jet exit, a uniform velocity U_j, and a temperature T_j are imposed. The jet discharged into a co-flowing stream U_{co} is symmetric to the plane (y = 0) and the calculation is performed over half of the flow. The computational domain is rectangular with a domain size, in terms of the jet inlet height H, of $L_x/H = 60$ and $L_y/H = 25$ in the longitudinal and transversal directions, respectively. The foregoing system of equations is completed with the following boundary conditions:

– In the free boundary parallel to the axis, the considered conditions are as follows:

$$\left(\frac{\partial V}{\partial y}\right)_{y=wall} = 0; U = 0; T = T_{co}; k = 0; \varepsilon = 0$$

– On the symmetry axis, the lateral velocity and gradients of all variables are set to zero.

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = 0;\ \Phi = U,\ T,\ k,\ \varepsilon;\ V = 0$$

– At the outflow boundary, the gradient of dependent variables in the axial direction and the lateral velocity are set to zero.

$$\left(\frac{\partial \Phi}{\partial x}\right)_{outlet} = 0;\ \Phi = U,\ T,\ k,\ \varepsilon;\ V = 0$$

– At the inlet, and in order to overcome the jet emission influence as much as possible, the axial velocity profile was calculated from the following relation (10):

$$x = 0: \begin{cases} 0 < y < H/2: & U = U_j; & V = 0; & T = T_j; & k = 10^{-3} U_j^2; & \varepsilon = \frac{C_\mu k^{3/2}}{0.03H} \\ y \geq H/2: & U = U_{co}; & V = 0; & T = T_{co}; & k = 0; & \varepsilon = 0 \end{cases}$$

4 Numerical Solution Method

The equations are solved by the Finite Volume Method (FVM) with a staggered grid as described by Patankar [15]. The transport equations of momentum, energy, turbulent kinetic energy and its dissipation rate are discretized through this equation:

$$A_P \Phi_P = \sum_{nb} A_{nb} \Phi_{nb} + \Phi_P. \tag{10}$$

Where, the subscript ‘nb’ designates neighbours which mean (i + 1, j), (i − 1, j), (i, j + 1) or (i, j − 1). In the present study, the diffusion and the convection coefficients are discretized using a hybrid scheme [16], which is first order or second order, depending on the local cell Reynolds number.

Thus, to find the numerical solution of these equations, a computer code was developed. The velocity-pressure coupling is solved with the SIMPLE algorithm (Semi-Implicit-Method for Pressure-Linked-Equations). The system of algebraic equations is solved line by line using the TDMA (Tri-Diagonal Matrix Algorithm). The TDMA is based on the Gaussian elimination procedure associated with the over-relaxation technique described by Patankar [15].

The used mesh is not uniform and gradually extends according to the longitudinal and transversal directions.

5 Results and Discussion

5.1 Mean Centerline Velocity Variation

Figure 2 shows the axial evolution of the centerline longitudinal velocity $\left(\left(U_j - U_{co}\right)/\left(U_c - U_{co}\right)\right)^2$ as a function of the normalized distance x/H, where U_c is the jet centerline mean axial velocity. Indeed, the normalized centerline longitudinal velocity obeys the self-similar decay law as: $\left(\frac{U_j - U_{co}}{U_c - U_{co}}\right)^2 = K_1\left(\frac{x}{H} - C_1\right)$.

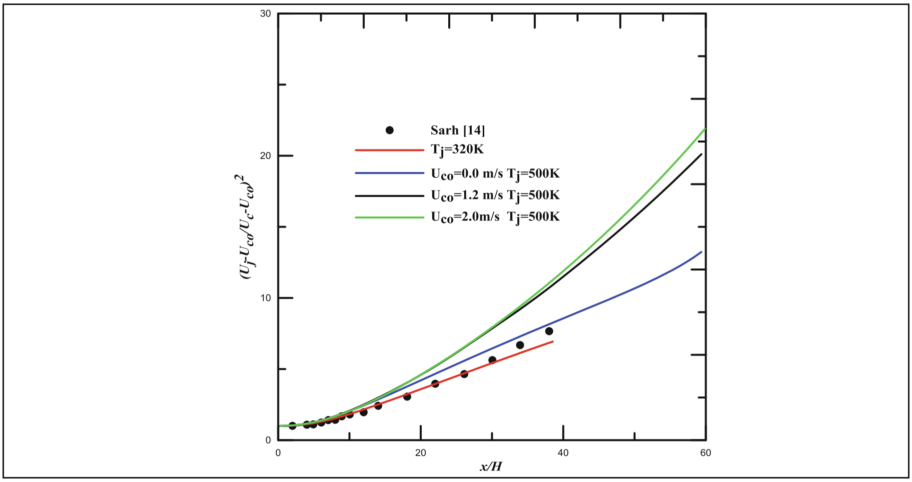

Fig. 2. Effect of co-flow variation on the axial evolution of the centerline longitudinal velocity

It appears clearly that the computed results agree well with the experimental values of Sarh [14]. For three different co-flow velocities (U_{co} = 0.0 m/s, U_{co} = 1.20 m/s and U_{co} = 2.0 m/s), the centerline longitudinal velocities are constant in the potential core area, near the jet exit, and are equal to the centerline exit velocity. In this region, the co-flow velocity has no effects on this parameter and the flow behaves as a jet in a stream at rest. However, in the similarity zone, away from the jet exit, the centerline velocity decreases as ($x^{1/2}$). It is found that the jet in a co-flowing stream is a quicker mixer than a jet in a quiescent ambient, which means that the presence of a co-flow enhances mixing. Therefore, the mixing of a jet in co-flow is more efficient.

5.2 Entrainment Variation

Figure 3 represents the axial evolution of air entrainment with T_j = 500 K and for three different co-flow velocities (U_{co} = 0.0 m/s, U_{co} = 1.20 m/s and U_{co} = 2.0 m/s). The amount of air entrainment by the jet can be determined by the time-average lateral profiles of velocity. This quantity, which relates the mass flow rate of the surrounding fluid entrained into the jet to the characteristic velocity difference between the jet and the co-flow, is given by:

$$E = 2 \int_{0}^{y(U=U_{co})} \rho (U - U_{co}) dy \tag{11}$$

It is shown that the presence of co-flow stream decreases considerably the air entrainment. This is due to the reduction of the jet lateral expansion in the presence of co-flow. Qualitative analyses suggest that a co-flowing stream would restrict the lateral

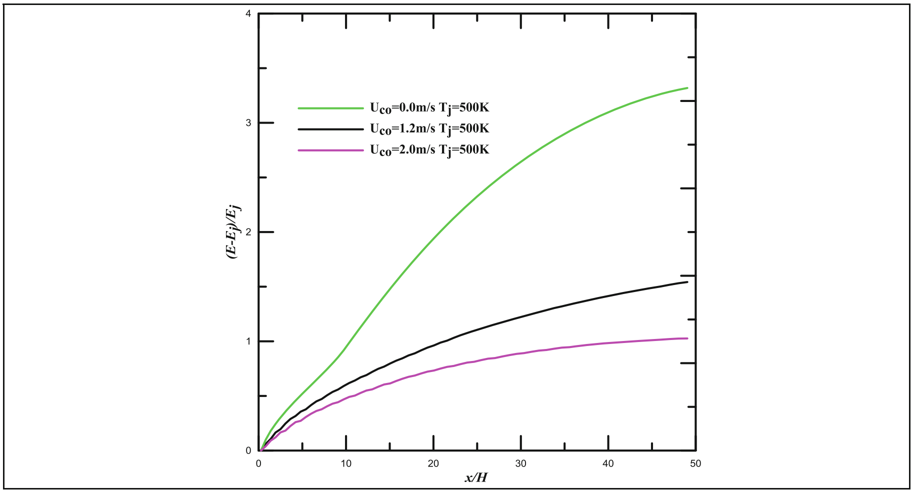

Fig. 3. Effect of co-flow on the axial evolution of the air entrainment

in flow of air into the jet. The free jet entrains from 30% to 75% more air than the co-flowing jet at any given axial location.

5.3 Entropy Generation Rate Variation

Figure 4 represents the axial evolution of the total entropy generation rate in the jet with a fixed inlet jet temperature $T_j = 500$ K and for three co-flow velocities ($U_{co} = 0.0$ m/s, $U_{co} = 1.20$ m/s and $U_{co} = 2.0$ m/s).

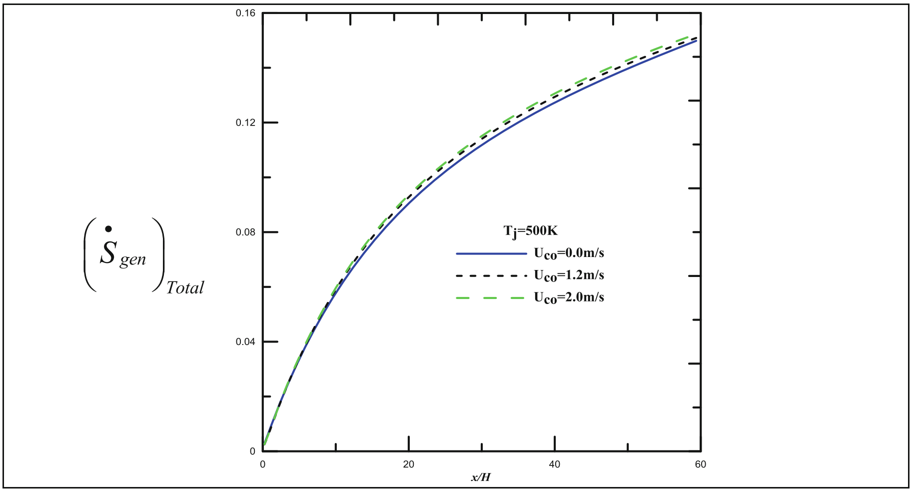

Fig. 4. Effect of co-flow on the axial evolution of the total entropy generation rate

In the potential cone region, it is seen that the entropy generation rate grows as x/H increases. Furthermore, there is a sudden increase in entropy generation in the flow region where x/H is less than 20. In this region, the jet is more unsteady and also has the most distorted profiles, right before, it reaches the self-similarity region. Moreover, in the self-similarity region, (x/H > 20) there is a smooth increase of entropy generation with respect to the observed longitudinal coordinate. The obvious conclusion from this Figure is that the co-flow velocity has very little effect.

6 Conclusion

A comprehensive analysis of the evolution of a heated turbulent plane jet emerging in a co-flowing stream is provided. The entropy generation rate in a turbulent plane jet is investigated, taking into account the effect of co-flow. Numerical simulations are carried out using the first order k-ε turbulence model. In particular, the numerical results for mean and turbulent quantities, air entrainment, entropy generation rate are studied. The main conclusions from the present study can be summarized as follows:

First of all, the predicted results agree reasonably well with the experimental data available in the literature for plane jets.

The increase of the co-flow velocity reduces the amount of air entrainment and the mixing efficiency of the jet. Moreover, the total entropy generation rate decreases with the increase of the co-flowing velocity.

The calculation results confirm that the entropy generation rate grows and attains an asymptotic value along the flow direction and depends directly on the entrainment with the still ambient fluid.

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