



Optimization of Bridges Reinforcements with Tied-Arch Using Moth Search Algorithm

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Abstract. The deterioration in the bridges that cross the watercourses is a situation that must be resolved in a timely manner to avoid the collapse of its structure. Its repair can mean a high cost, road and environmental alteration. An effective solution, which minimizes this impact, is the installation of a superstructure in the form of an arch that covers the entire length of the bridge and which, by means of a hook anchored to the deck of the bridge, allows the arch to support the weight. This structure must try to maintain the original properties of the bridge, so the calculation of the magnitude of tension of the hangers and the order in which it is applied should not cause damage to the structure. In this document, we propose to optimize the process of calculating the hanger magnitudes and the order in which they must be applied using the moth search algorithm, in order to obtain one or several satisfactory solutions. Finally, we present the results obtained for an arch bridge and three hangers and, thus, evaluate the efficiency and effectiveness in the process of obtaining results in comparison with the Black Hole Algorithm.

Keywords: Reinforcement of bridges · Metaheuristics ·
Moth search algorithm · Combinatorial optimization

1 Introduction

Computer science is a transversal discipline for many areas of study and human activities. One of them is the construction area. Many constructive designs are based on models that predict how the behavior of the design will be, in order to avoid risks or unexpected results. The more data representative of reality is able to capture the model, it will be possible to generate information to make better decisions. However, the quality of the information depends on the quality of the data and the accuracy in them is a fundamental requirement. This is where

computing comes into play, by processing data more accurately and quickly, delivering timely, useful and error-free information.

A bridge is a structure built with the purpose of saving a geographical accident, road, water course or other obstacle that comes before it. Its design depends on its function and the nature of the terrain on which it rests. They are usually built on the basis of metallic structures, reinforced concrete, wood or a combination of these. Basically, the forms that the bridges adopt are three, being directly related to the efforts that support their constructive elements. These are: beam bridges, arches and pendants. The present paper will address the reinforcement of a beam bridge through an arch with tension hangers.

Every structure created by the human being has a useful life. In the case of bridges vary depending on various causes. In the case of beam bridges, the undermining of the piers is one of the most frequent causes and damage to these structures can be very serious, such as collapse. Repairing them has a high impact on cost, time and use, so a suitable solution alternative is one that has the least impact. The design of a cable-stayed bridge with a lower deck is a viable solution. This consists of an arc that covers the longitudinal extension of the bridge from which hangings hang anchored to the board, supporting the weight of the structure, in this way it is possible to do without the piers where it rested.

This paper is organized in the following way, in Sect. 2 the problem to be solved is presented, in Sect. 3 Moth Search algorithm is explained, in Sect. 4 the integration of metaheuristics to the bridges problem, and in Sect. 5 the results obtained with their respective statistical tests to evaluate performance, ending with the conclusion and future work.

2 Problem

The problem to solve was proposed by Valenzuela [10–12] using Algorithm Genetic [4] and then by Black Hole [3] in order to determine which of the algorithms achieved better results. The best of both turned out to be Black Hole. Now we will use Moth Search (MS) [13] to determine if it achieves better results than Black Hole [7].

The structure of the bridge is modeled in a CAD application called SAP2000 [1], which provides an API that allows it to be operated externally by our algorithm. Thanks to this, the application will perform many complex structural calculations, so that our algorithm will provide the necessary data to perform these calculations and, thus, obtain a solution.

2.1 Objective Function

The objective function is defined as the sum of the tense difference between the original and the modified bridge for each of K cuts in each of the beams, minimizing this difference.

$$\min \sum_{i=1}^2 \sum_{k=1}^K |\sigma_{o_{i,k}} - \sigma m_{i,k}|; \quad i \in \{1, 2\}, \quad k \in \{1, 2, \dots, k\} \quad (1)$$

Minimize the difference between the stresses of the original bridge and the modified arch bridge of the hangers. As long as the optimum result tends to zero, it means that the stress calculations will tend to equal the stresses of the models of both bridges (original and modified), preserving the original properties of the bridge. The tensions are obtained through the following formulas.

$$\sigma_{o_{i,k}} = \frac{M_{o_{i,k}} \cdot v_i}{I_{o_i}}; \quad i \in \{1, 2\}, \quad k \in \{1, 2, \dots, k\} \quad (2)$$

$$\sigma m_{i,k} = \frac{P}{A} + \frac{P \cdot e \cdot v_i}{I m_{TOTAL}} + \frac{M m_{i,k} \cdot v_i}{I m_i}; \quad i \in \{1, 2\}, \quad k \in \{1, 2, \dots, k\} \quad (3)$$

2.2 Decision Variable

For a bridge of three hanging will be 6 decision variables, three to indicate the order of tension and the other three to determine the magnitude of tension of each hanger.

$$ord_1, ord_2, \dots, ord_n \in \{1, 2, \dots, n\} \quad (Orders) \quad (4)$$

$$mag_1, mag_2, \dots, mag_n \in [T_{min}, T_{max}] \quad (Magnitudes) \quad (5)$$

2.3 Constraints

The problem have two constraints that must be met to satisfy the objective function.

- The hangers cannot be jacking simultaneously.

$$ord_w \neq ord_j; \quad \forall w, j \quad con \quad w \neq j \quad w, j \in \{1, 2, \dots, n\} \quad (6)$$

- The effort of the modified bridge deck should not pass the limits of the BAM:

$$\sigma m_{i,k} \geq \sigma o \quad (7)$$

$$\sigma m_{i,k} \geq f_{ct} \quad (8)$$

$$\sigma m_{i,k} \leq f_{cmax2} \quad (in \quad internmediate \quad stages) \quad (9)$$

$$\sigma m_{i,k} \leq f_{cmax} \quad (in \quad final \quad stages) \quad (10)$$

$\sigma m_{i,k}$: Tension (top or bottom) in the beams of the cable-stayed bridge.

f_{ct} : Maximum tension of admissible traction by the concrete.

f_{cmax} : Maximum compressive stress admissible by the concrete.

f_{cmax2} : Maximum compressive stress admissible for expanded concrete.

3 Moth Search

Moth Search is a bio-inspired metaheuristic algorithm to respond to global optimization problems. It was created by Wang [13].

Moths are a type of insects that belong to the order Lepidoptera. Among the various characteristics of moths, phototaxis and Levy flights are the most representative characteristics described below.

Phototaxis is the orientation reaction of free cellular organisms in response to a luminous stimulus. In general, moths tend to fly around the light source, in the form of Lévy flights.

3.1 Lévy Flights

Lévy flights are one of the most important flight patterns in nature. Many species move following a flight pattern of Lévy and describe a type of random walks, whose length steps are taken from the distribution of Lévy. The distribution of Lévy can be expressed mathematically in the form of a power law formula.

$$L(s) \sim |s|^{-\beta} \quad (11)$$

where $1 < \beta \leq 3$

Lévy flights can maximize the efficiency of finding resources in uncertain environments. Therefore, $\beta = 1.5$ is used to optimize benchmarks and engineering cases.

The moths, which have a smaller distance than the best, will fly around it in the form of Lévy flights. In other words, their positions are updated by Lévy flights.

For moth i in each variable, it can be updated as:

$$x_i^{t+1} = x_i^t + \alpha L(S) \quad (12)$$

$$x_i^{t+1} = x_i^t + \alpha \left(\frac{(\beta - 1)\Gamma(\beta - 1)\sin\left(\frac{\pi(\beta - 1)}{2}\right)}{\pi S^\beta} \right) \quad (13)$$

Where x_i^{t+1} and x_i^t are respectively the updated original position in generation t , and t is the current generation. $L(s)$ is the passage of Lévy flights. The parameter is the scale factor related to the problem of interest. In our current work, it can be given as:

$$\alpha = \frac{S_{max}}{t^2} \quad (14)$$

Where S_{max} is the maximum walking step and its value is established according to the given problem. Lévy distribution $L(s)$ in equation can be formulated as follows:

$$L(s) = \frac{(\beta - 1)\Gamma(\beta - 1)\sin\left(\frac{\pi(\beta - 1)}{2}\right)}{\pi S^\beta} \quad (15)$$

Where s is greater than 0, $\Gamma(s)$ it is the gamma function.

3.2 Straight Flight

Certain moths that are far from the source of light will fly towards that source of light in line. This process can be described below.

For moth i in each variable, its flights can be formulated as:

$$x_i^{t+1} = \lambda(x_i^t + \phi(x_{best}^t - x_i^t)) \quad (16)$$

where x_{best}^t is the best moth in generation t , and ϕ is an acceleration factor established in golden relation in our current work. λ is a scale factor. On the other hand, the moth can fly towards the final position that is beyond the light source. For this case, the final position for moth i can be formulated as:

$$x_i^{t+1} = \lambda(x_i^t + \frac{1}{\phi}(x_{best}^t - x_i^t)) \quad (17)$$

For simplicity, for moth i , its position will be updated by Eqs. 16 or 17 with the possibility of 50%. In addition, these two update processes mentioned above can be represented in (Fig. 1a, b) respectively. In Fig. 1, x_{best} , x_i and $x_{i,new}$ are respectively the best, original and updated position for moth i , and are considered as a light source, start point and end point. λ is a scale factor that can control the speed of convergence of the algorithm and improve the diversity of the population. In our current work, the scale factor is set to a random number drawn by the standard uniform distribution.

4 Integration

In the MS method, for simplicity, the entire population of moths is divided into two equal subpopulations (Subpopulation 1 and Subpopulation 2) according to their suitability, and they are updated according to the Lévy flights or in a straight line, respectively. This is equivalent to saying that the moths in Subpopulation 1 are much closer to the better than Subpopulation 2. In addition, like many other metaheuristic algorithms, an elitism strategy is incorporated in order to accelerate the convergence of the MS method. *MaxIterations* is the initial maximum generation that can be considered as the term condition. Algorithm 1 describes this process.

In synthesis, two models will be used: one of the original bridge and another with the modifications to compare it with the first one. Therefore, the metaheuristic will provide the voltage magnitudes for SAP2000 to perform and thus obtain a solution. The communication between metaheuristics and SAP2000 will be made through the API provided by the latter.

Algorithm 1. Integration between SAP2000 and Moth Search algorithmic.

```

1: Establish bridge models to evaluate (Instance).
2: for all Instance do
3:   SAP2000.Open
4:   SAP2000.Load(Original Bridge Model)
5:   Randomly initialize the population  $P$  of  $NP$  moths randomly
6:   Set MaxIterations, MaxWalkStep  $S_{max}$ ,  $\beta$ ,  $\varphi$ 
7:   for Iteration  $\leq$  MaxIterations do
8:     SAP2000.Load(Modified Bridge Model)
9:     SAP2000.ApplyTension()
10:    SAP2000.GetFitness()
11:    Sort all the moth individuals as per their fitness and Save BestMoth.
12:    Save Best Moth
13:    for  $i = 1$  to  $NP/2$  (for all moth individuals in Subpopulation 1) do
14:      for  $j = 1$  to  $D$  do
15:        Generate  $x_{ij}^{t+1}$  by (15) performing Lévy flights.
16:      end for
17:    end for
18:    for  $i = NP/2 + 1$  to  $NP$  do
19:      for  $j = 1$  to  $D$  do
20:        if  $rand > 0.5$  then
21:          Generate  $x_{ij}^{t+1}$  by (16)
22:        else
23:          Generate  $x_{ij}^{t+1}$  by (17)
24:        end if
25:      end for
26:    end for
27:  end for
28:  SAP2000.Close()
29: end for

```

The configuration was parameters used for the execution, it was obtained through parametric sweep are shown in the Table 1 [5].

Table 1. Parameters for execution.

Population	Executions	Iterations	Alpha
48	15	200	0.000001

5 Experimental Results

The Tables 2, 3 and 4 summarize the best solutions achieved in each of the 15 executions for instance.

Table 2. Fitness comparison (a)

	PV-TCV		HW-TCV		PT-TCV		AB-TCV	
	BH	MS	BH	MS	BH	MS	BH	MS
1	525788,701	524139,528	522992,527	520707,516	520560,471	522709,168	520152,444	529824,623
2	524362,167	527337,192	518566,592	521467,410	520610,728	523656,669	520848,212	529462,056
3	523983,245	526024,026	520271,778	521921,189	523880,059	523871,984	519373,561	529358,722
4	523941,247	522720,317	517752,373	522429,087	518340,350	520855,130	520872,273	534472,795
5	523427,989	522383,486	518554,722	522249,702	519411,917	523411,190	524023,770	529321,548
6	523381,036	523860,029	521443,876	521619,530	525110,874	521199,953	524821,681	529332,343
7	522809,147	522924,479	523204,076	522837,239	520863,041	525332,737	524246,509	529154,644
8	522648,276	522267,417	520515,557	522237,042	522029,399	523449,317	523785,395	538118,512
9	522138,965	524027,438	520214,641	525463,179	521966,778	522385,429	521241,888	528748,384
10	521950,827	523160,081	518227,336	522954,810	526676,105	523310,390	517068,779	528751,098
11	521351,709	521923,453	519526,112	522385,494	522617,218	521627,024	520203,824	528794,300
12	521114,467	522200,113	519225,675	525625,865	525204,201	521776,053	523623,834	528710,845
13	520927,571	522624,045	516990,684	522918,025	524252,033	526735,660	520622,784	528653,496
14	520202,016	522023,396	521974,260	520395,044	518891,310	524878,383	522447,219	528455,727
15	518788,669	522492,710	522006,562	521272,424	523880,059	525725,692	517564,071	527881,264

Table 3. Fitness comparison (b)

	WR-TCV		VC-TCV		CC-TCV		TC-TCV	
	BH	MS	BH	MS	BH	MS	BH	MS
1	523896,152	529477,706	521676,411	519846,831	518616,048	527953,566	522834,907	520217,120
2	519204,810	531663,354	523586,259	521680,027	524706,864	529698,713	521301,708	524967,128
3	522236,031	531239,237	520710,801	522635,853	522562,147	530439,770	522591,311	524796,056
4	525384,947	531139,511	521939,460	521775,646	517694,832	536831,146	525268,020	522572,121
5	519448,379	529715,769	516764,747	523607,474	520212,174	531236,044	519571,134	523143,451
6	519880,227	528671,041	526420,203	523382,604	520538,000	532152,376	524103,151	523030,723
7	518685,865	529270,895	520958,228	522720,927	515930,442	530039,302	521634,820	521806,946
8	521471,347	529918,358	522523,255	522645,384	520447,927	530745,301	523052,417	523451,586
9	521745,606	530487,258	520900,604	527274,068	520404,149	531022,389	521994,378	524235,633
10	521210,450	533216,584	517454,836	534720,702	520682,197	529892,495	519824,404	522523,995
11	523417,258	530490,245	521104,325	536159,250	523472,335	537619,557	524988,999	523543,682
12	522611,368	528807,423	524091,123	529433,687	524143,332	529270,743	519926,614	523253,590
13	526646,976	528620,138	517878,923	530675,662	522918,409	531145,877	522386,876	523829,050
14	522051,315	529381,392	520548,389	527051,075	515608,056	531853,565	519598,167	523178,525
15	519162,739	530263,611	515963,267	530497,598	519413,560	530729,770	522191,193	523907,308

5.1 Comparing Results

To determine which algorithm achieves the best results, the best solutions of the 15 executions made to each instance have been collected.

This allows obtaining 2 sets of data to be compared for each instance: those obtained with BH and those obtained with MS. These two samples are subjected to two statistical tests. The first allows to determine if both samples are independent, which would clear the doubt if the samples are reliable to compare, and the second to verify the veracity of the hypothesis formulated with respect to the best result obtained.

Table 4. Fitness comparison (c)

	RD-AA10		RC-AA10		CR-AA10	
	BH	MS	BH	MS	BH	MS
1	517200,649	524066,303	514508,111	520920,138	511024,809	521705,173
2	512678,644	521738,302	517790,493	521047,167	517088,897	522055,256
3	511107,489	521460,585	515619,911	522175,344	515740,371	522952,922
4	508556,024	521818,123	514564,776	520969,151	514210,370	521523,765
5	519226,401	521090,646	514605,633	521769,704	514470,308	521105,522
6	513572,503	521829,232	517856,008	522029,575	517102,869	520597,009
7	514930,924	522007,048	513847,138	521218,138	512584,970	521202,212
8	509176,298	521473,608	514603,894	523063,544	517126,961	522001,367
9	514299,205	521996,533	514367,665	522363,607	514913,049	522064,221
10	517038,968	520935,668	514019,852	521063,580	517761,806	521254,353
11	512632,111	520250,193	513776,522	520626,463	516430,949	526959,437
12	516494,402	523463,306	514272,046	520083,744	512943,624	520758,016
13	511665,241	520327,645	514457,070	524280,148	514502,449	520874,602
14	513408,640	521210,740	513593,241	520569,576	512808,821	520048,010
15	515799,785	520597,995	510072,744	520109,998	516154,932	520660,141

The Kolmogorov-Smirnov test, with Lilliefors correction [6], is used to test if a data set fits a normal distribution or not, in our the test concluded that the samples are independent.

Then, the Mann-Whitney-Wilcoxon test [2] will be applied to the same group of data to determine the veracity of the hypothesis with respect to which algorithm of the comparators presents the best results, based on the following hypotheses:

H_0 : MS is better than BH

H_1 : BH is better tha MS

If the p-value of a hypothesis of one sample with respect to the other is less than 0.05, it can not be assumed to be true.

When applying this test, the results presented in Table 5 were obtained.

5.2 Distribution Comparison

The best solutions obtained in each execution are presented in the following graphs, where you can see a comparison between Black Hole and Moth Search. The boxes of blue color represent the solutions given by BH, the red ones correspond to those of MS.

In all situations, the results obtained by MS do not surpass BH. This is due to a number of insufficient iterations, a small population and parameter settings that allow greater precision (progress at a smaller step).

Table 5. p-value Mann-Whitney-Wilcoxon test

Instance	BH better than MS	MS better than BH
PV-TCV	0,864152	0,13584722
HW-TCV	0,998273	0,00172669
PT-TCV	0,873017	0,12698204
AB-TCV	0,999998	0,00000153
WR-TCV	0,999998	0,00000153
VC-TCV	0,999346	0,00065324
CC-TCV	0,999998	0,00000153
TC-TCV	0,979971	0,02002832
RD-AA10	0,999998	0,00000153
RC-AA10	0,999998	0,00000153
CR-AA10	0,999998	0,00000153

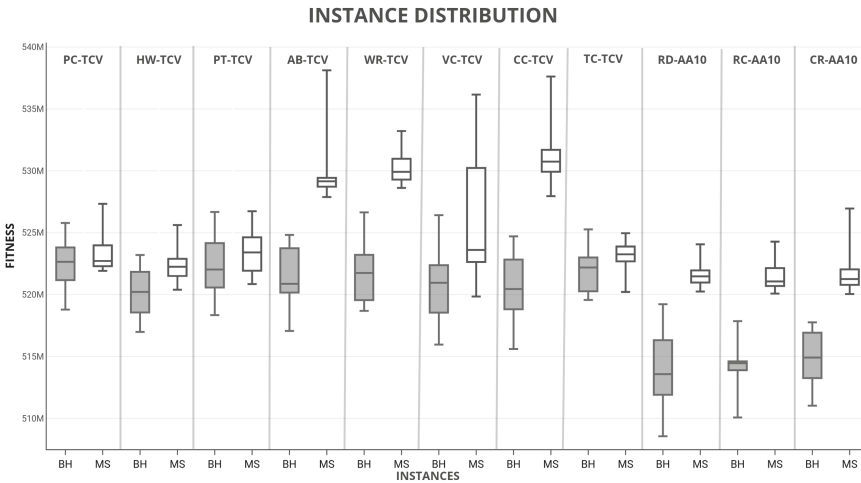


Fig. 1. Best fitness instances

6 Conclusions

The use of optimization techniques, present great advantages when solving problems of great complexity [8,9]. The solutions given by Moth Search in the vast majority of cases do not exceed those achieved by Black Hole. However, in some situations the proximity has been close, so it is possible to infer that by making the necessary adjustments to the algorithm, results such as Black Hole can be achieved.

In two of the eleven instances it is not concluded that Moth Search is unable to achieve results like those of Black Hole, which would lead to promising results if improvements are applied to the algorithm and execution parameters.

In conclusion, it is proposed to make improvements in the parameters such as Population Size, Number of Iterations, Acceleration Factors, Disturbance Operators and in the algorithm as Elitist Strategies, Convergence Acceleration Strategies, Selective Population, Population Grouping, others.

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