

Edith Hemaspaandra and Lane A. Hemaspaandra

Abstract We believe that economic design and computational complexity—while already important to each other—should become even more important to each other with each passing year. But for that to happen, experts in on the one hand such areas as social choice, economics, and political science and on the other hand computational complexity will have to better understand each other's worldviews. This article, written by two complexity theorists who also work in computational social choice theory, focuses on one direction of that process by presenting a brief overview of how most computational complexity theorists view the world. Although our immediate motivation is to make the lens through which complexity theorists see the world be better understood by those in the social sciences, we also feel that even within computer science it is very important for nontheoreticians to understand how theoreticians to understand how nontheoreticians think, just as it is equally important within computer science for theoreticians to understand how nontheoreticians think.

1 Introduction

Predictions are cheap. Our cheap prediction is:

Economic design and computational complexity should and will in the future be even more deeply intertwined than they currently are.

E. Hemaspaandra

Department of Computer Science, Rochester Institute of Technology,

Rochester, NY 14623, USA

e-mail: eh@cs.rit.edu

L. A. Hemaspaandra (⋈)

Department of Computer Science, University of Rochester,

Rochester, NY 14627, USA

URL: http://www.cs.rochester.edu/u/lane

What is a bit less cheap is working to make predictions come true. If the prediction is broad or ambitious enough, doing that is often a task beyond one paper, one lifetime, or one generation.

Nonetheless, in this article we will seek to make a small contribution toward our predication's eventual realization. In particular, as complexity theorists who have for more than a decade also been working in computational social choice theory, we have seen first-hand how deeply important computational social choice theory and computational complexity have been to each other. And the "to each other" there is not written casually. As argued in a separate paper (Hemaspaandra 2018), the benefit of the interaction of those areas has been very much a two-way street.

However, to increase the strength and quality of the interaction, and to thus reap even more benefits and insights than are currently being gained, a needed foundation will be *mutual understanding*. After all, even the different subareas of computer science have quite different views of what computer science is about, and sometimes it seems that computer scientists don't understand even each other's worldviews.

As complexity theorists, we can expertly address only one direction in which explanation is needed: trying to explain the—perhaps strange to those who are not complexity theorists—way that complexity theorists tend to view the world. We sincerely hope that the reciprocal directions will be addressed by appropriate experts from the many other disciplines whose practitioners are part of the study of economic design.

Beyond that, we also embrace and somewhat generalize a hope that for the case of (computational) social choice is expressed in the above article (Hemaspaandra 2018): We hope that in time there will be a generation of researchers who are trained through graduate programs that make students simultaneously expert in computational complexity and one of the other disciplines underpinning economic design—researchers who in a single person achieve a shared understanding of two areas. But for now, most researchers have as their core training one area, even if they do reach out to work in—or work with experts in—another area. And thus we write this article to try to make as transparent as we can within a few pages the crazy, yet (to our taste) just-right, way that complexity theorists view the world.

The remainder of this article is organized as follows. Section 2 further discusses the need for, and the importance of improving, mutual understanding. That section argues that the way that complexity theorists view the world is rarely understood even by the other areas of computer science, and that the areas of computer science themselves are separated by huge cultural gaps. Section 3 presents what we feel is the heart of how computational complexity theorists view the world, which is:

We as complexity theorists believe that there is a landscape of beautiful mathematical richness, coherence, and elegance—waiting for researchers to perceive it better and better with the passing of time—in which problems are grouped by their computational properties.

2 The Need for Creed: Why Understanding Each Other Is Hard yet Needed and Important

In this section, we briefly mention some cultural chasms between complexity and social choice—and even between complexity and other areas of computer science—and suggest that shrinking or removing those chasms is important: Understanding between collaborators is of great value to the collaboration.

2.1 Spanning to Computational Social Choice, Economics, and Beyond

In this section, we will focus on computational social choice, as that is the particular facet of economic design that the authors are most familiar with.

This paper will not itself present the many ways that computational complexity and computational social choice have interacted positively and in ways that benefit *both* areas. As mentioned above, a separate paper (Hemaspaandra 2018) already makes that case, for example pointing out: how computational social choice has populated with problems such classes as the Θ_2^p level of the polynomial hierarchy and NP^{PP}; how computational social choice has provided the first natural domain in which complexity theory's search-versus-decision separation machinery could be applied; how that application itself gives insight into how to best frame the manipulative-attack definitions in computational social choice; how complexity's "join" operation has been valuable in proving the impossibility of obtaining certain impossibility results in computational social choice theory; how the study of online control yielded a completely new quantifier-alternation characterization of coNP; and much more, such as how important a role approximation, dichotomy results, and parameterized complexity have played in computational social choice.

It in fact is quite remarkable how strongly complexity has helped the study of computational social choice, and is even more remarkable—since this is the direction that might not have been apparent beforehand—how strongly computational social choice has helped the study of computational complexity theory. And most remarkable of all is that these results have usually been obtained by researchers from one side, although ones who were very interested in the other side. This clearly is a pair of areas that already is showing very strong content interactions and mutual benefits. Think of how much more waits to be seen and achieved when computational social choice theorists/social choice theorists and complexity theorists deepen their understanding of each other.

Some might worry about the "computational" in "computational social choice" above—namely, worrying that computational social choice is such a young area that no one is "native" to it. We disagree. It is true that much of the key, early work on this area—as it was emerging *as* an area with a distinct identity—was done by researchers whose training was in operations research, logic, artificial intelligence (AI), theoret-

ical computer science, economics, social choice, political science, or mathematics. But already a generation of students, now in their 20s and 30s, has been trained whose thesis work was on computational social choice theory: researchers whose "native" area and identity is—despite the fact that their thesis advisors view themselves as at their core part of one of the older areas just listed—that they are computational social choice theory researchers. This is a very good development, and yet we are asking even more: Now that the area has its own identity, one can hope to grow researchers whose core training and identity embraces both that young area and the area of computational complexity.

2.2 Spanning to Other Computer-Science Areas

It is often discussed within computer science departments whether computer science is even a coherent discipline. After all, if one thinks about which other department the areas of computer science feel kinship to, for theoreticians that generally would be mathematics, for systems people that generally would be electrical and computer engineering, for symbolic AI people that often would be one of brain and cognitive sciences, linguistics, philosophy, or psychology, and for vision/robotics AI people that might be mechanical engineering, electrical and computer engineering, or visual science.

The cultural differences are also stark. For example, taking as our examples the two subareas of computer science that are most strongly represented in computational social choice research—AI and theory—we have the following contrasts in culture. Anonymous submissions at the main conferences versus submissions with the authors' identities open. Intermediate feedback and rebuttals at the main conferences versus no such round. Authors' names generally being ordered by contribution versus authors names always being listed alphabetically. Large hierarchical program committees versus small almost-flat program committees. And that listing is not even mentioning the issue of the contrasting content of the areas, or their differing views on conference versus journal publication.

Almost any theoretical computer scientist will have stories of how sharply his or her perspective has differed from those of his or her nontheory colleagues, e.g., a nontheory colleague who firmly felt that 8x8 chess—not *NxN* chess (Storer 1983) but actual 8x8 chess—under the standard rules (which implicitly limit the length

 $^{^1}$ One of us once asked a colleague, who at one point was the president of AAAI, whether he, upon seeing a paper with a very large number of authors with them all in alphabetical order, would really assume that Dr. Aardvark had made the largest contribution. The colleague looked back as if he'd been asked whether he really believed that 1+1=2 and said that he of course would. In fact, in the different area within computer science known as systems, there is a running semi-joke—that excellently corresponds with reality—that one can tell how theoretical a given systems conference is by looking at what portion of its papers list the authors in alphabetical order; in fact, there is a very funny joke-paper (Appel 1992) that quantifies this—more rigorously than the earlier part of this sentence does—to prove that the POPL conference is quite theoretical.

of any game) is a great example of *asymptotic* complexity, and who advised the theoretician to go use Google to learn more about this. We suspect that nontheory computer science faculty members could write quite similar sentences—with different examples—from their own points of view, regarding the things theory faculty members say.

So even the subareas of computer science have some gaps between them as to understanding, or at least have rather large agree-to-disagree differences. Our hope is that, regarding the former, this short article may be helpful.

We mention, however, that we do not agree that anything said above shows that computer science is not a coherent discipline. To us, and in this we are merely relating an important, much loved insight that has been around in one form or another for many decades (Knuth 1997; Harel 1987), there is a unifying core to the field of computer science: algorithmic thought (and the study of algorithms). That core underpins AI, systems, and theory, and makes computer science an at least decently coherent discipline.

3 A Core Belief, and Its Expressions, Interpretations, and Implications

3.1 A Core Belief

We feel that a core view—in fact, *the* core view—of complexity theorists is the following (phrased here both as a profession of belief and as a statement of what is believed).

[Core Belief We as complexity theorists believe that:] There is a land-scape of beautiful mathematical richness, coherence, and elegance—waiting for researchers to perceive it better and better with the passing of time—in which problems are grouped by their computational properties.

If the subfield can be said to have a creed, this is it.

By saying that complexity theorists feel this, we don't mean to suggest that it is exclusive to them. In a less computational vein, the great mathematician Paul Erdős spoke of "The Book," which holds the most elegant proof of each mathematical theorem. He famously said, "You don't have to believe in God, but you should believe in The Book," and surely viewed as moments of true joy those when a proof so beautiful as to belong in the book was discovered. And the great computer scientist Edsger Dijkstra is traditionally credited² with this lovely, insightful comment:

²The quote is attributed to him in works of others as early as 1993 (Haines, 1993, p. 4), though attributing the quote to Dijkstra is disputed, as Michael Fellows published a very similar comment

Computer Science is no more about computers than astronomy is about telescopes. — E. Dijkstra

Though different people interpret that quotation in different ways, we have always interpreted it to suggest almost precisely what our core belief is expressing. Indeed, the quotation's implicitly drawn parallel between astronomy studying the structure of the universe and computer scientists studying a similarly majestic structure is extremely powerful. And things are made even more pointed in the 1991 version by Michael Fellows, which follows the same sentiment as that of the quotation with, "There is an essential unity of mathematics and computer science."

3.2 The Heretics

Having read Sect. 3.1, theoretical researchers from any field may think, "Well, duh!" That is, they may think that the core belief is obvious, and wonder who could possibly think anything else.

The answer is that quite a large portion of the field computer science thinks something else. This was most famously expressed in a 1999 "Best Practices Memo" (Patterson et al. 1999) that was published in *Computing Research News*, the newsletter of a prestigious group, the Computing Research Association, of over two hundred North American organizations involved in computing research, including many universities. To this day, that memo is on the Computing Research Association's web site as a best practices memo (Patterson et al. 2017, although there certainly has been strong pushback on some of its points, see, e.g., Vardi 2009; Fortnow 2009). The most jump-off-the-page lines in that memo are these:

... experimentalists tend to conduct research that involves creating computational artifacts and assessing them. The ideas are embodied in the artifact, which could be a chip, circuit, computer, network, software, robot, etc. Artifacts can be compared to lab apparatus in other physical sciences or engineering in that they are a medium of experimentation. Unlike lab apparatus, however, computational artifacts embody the idea or concept as well as being a means to measure or observe it. Researchers test and measure the performance of the artifacts, evaluating their effectiveness at solving the target problem. A key research tradition is to share artifacts with other researchers to the greatest extent possible. Allowing one's colleagues to examine and use one's creation is a more intimate way of conveying one's ideas than journal publishing, and is seen to be more effective. For experimentalists conference publication is preferred to journal publication, and the premier conferences are generally more selective than the premier journals... In these and other ways experimental research is at variance with conventional academic publication traditions.

in a 1991 manuscript that appeared in a 1993 conference proceedings and published the identical quotation in 1993 in a *Computing Research News* article joint with Ian Parberry.

Underlying this is a worldview that is very different than that of most theoreticians. The worldview is that software systems and devices are often so complex that trying to theoretically capture their behavior and properties is hopeless, and we instead need to experiment on them to make observations. For example, that view might suggest that operating systems are so enormous and complex that we can't really capture or understand precisely their behavior.

Yet theoreticians think otherwise. Theoreticians dream of a time when essentially all programs—of any size—will have a rigorous, formally specified relationship between their inputs and their actions/outputs, and when we will seek to prove that the programs satisfy those relationships (insofar as can be done without running aground on undecidability issues). Perhaps that time will be decades or centuries away for extremely complex programs, but we believe it will come. And in fact, real progress—for example thanks to advances in automated theorem-proving/automated reasoning—has been made in the past few decades on verifying that even some quite large programming systems meet their specifications.

In brief, we don't think that because software systems are complex one can only experiment on them as if they were great mysteries; rather, we think that, precisely because they are so complex, the field should increase its efforts to formally understand them, including working on building the tools and techniques to underpin such an understanding.

To be fair to the above-quoted memo, it carefully had a very separate coverage in which it described what theoreticians do. But to many theoreticians, viewing computing systems as too complex to theoretically analyze—and more suitable for experimenting on—is far too pessimistic, at least as a long-term view.

Is our Core Belief utterly optimistic? Not purely so. It is broadly optimistic, in what it believes exists, though to be frank the landscape it is speaking of is typically more about problems and classes than about analyzing operating systems. But embracing the Core Belief does not mean that one must be delusional as to time frames. For example, in Gasarch's P versus NP poll (Gasarch 2012), only 53% percent of those polled felt that P versus NP would be resolved by the year 2100. 3% thought it would never be resolved, and 5% said they simply did not know when/if it will be resolved.

A astounding 92% of the polled theoreticians believe that it will be eventually resolved, even though currently no path for imminently resolving the question is in sight (see also the very grim possibility mentioned in the 1970s by Hartmanis and Hopcroft (1976): that the question might be independent of the axioms of set theory). Theoreticians have generally taken to heart Sir Thomas Bacon's 1605 comment from *The Advancement of Learning*:

They are ill discoverers that think there is no land, when they can see nothing but sea. — Thomas Bacon

3.3 Landscape and Classification

So what is this landscape that the Core Belief speaks of? And how can we bring it into better focus?

3.3.1 Axes and Granularity of Classification

The landscape is one where each problem is located by its classification in terms of various measures. What is its (asymptotic, of course) deterministic time cost? What is its deterministic space cost? What are its nondeterministic costs? Its costs in various probabilistic models? What about in nondeterministic models that forbid ambiguity (i.e., that have at most one accepting path) or that polynomially bound the ambiguity? What about in quantum computing models and biocomputing models? How well can the problem be—in various senses—solved by heuristics or approximations? What types of circuit families can capture the problem? What types of interactive proof classes can capture the problem?

And that is just a quick start to listing aspects of interest. The number of interesting dimensions along which problems can be classified is already large, and continues to grow with time. Our landscape is not a physical one, of course, but is a rich world of mathematical classification.

The granularity with which we group the "locations" in this world itself is interesting. Complexity theorists typically focus on equivalence classes of problems, linked by some type of reduction. For example, the NP-complete problems are all those problems that are many-one, polynomial-time interreducible with the problem of testing the satisfiability of boolean formulas. One can think of the NP-complete sets as an extremely important feature of the landscape. Yet one can also view the landscape with an interest in other degrees of granularity. The class of NP-Turing-complete sets for example contains all the NP-complete sets, and may well contain additional sets (Lutz and Mayordomo 1996), since Turing reductions are a more powerful reduction type than many-one reductions. Going in the other direction, the class of sets that are polynomial-time isomorphic to boolean satisfiability may well be a strict subset of the NP-complete sets, and it is known to be a strict subset with probability one relative to a random oracle (Kurtz et al. 1995).

Briefly put, complexity classes usually are defined by placing a bound on some key resource, e.g., NP is the class of sets that can be accepted by polynomially time-bounded nondeterministic computation. Complexity classes in some sense are upper bounds on some dimension of complexity. Reductions are yardsticks by which sets can be compared. If a set A reduces to a set B by some standard reduction type, we view A as being "easier or not too much harder" than B, with the details depending on what power the reduction itself possesses. There are now a huge number of intensely studied reduction types, capturing such notions as, just as examples, the amount of time or space the reduction is itself allowed to use; whether the reduction is a single query or multiple ones and if the latter how they are used and whether

they are sequential or parallel; and to what extent the reduction itself can act nondeterministically. And completeness for complexity classes combines a class with a reduction type, identifying those sets in the class that are so powerful that every set in the class reduces to them by the given reduction type. In some sense, the completeness equivalence class of a complexity class groups together those problems, if any such problems exist (and some parts of the landscape perhaps lack complete sets Sipser 1982; Gurevich 1983; Hartmanis and Hemachandra 1988; Hemaspaandra et al. 1993), that distill the essence of the potential hardness of the class—they share the same underlying computational challenge. As such, they help complexity theorists focus on what the source of a problem's complexity is.

The joyful obsession and life's work of complexity theorists is to better understand this landscape. This often is done though classifying where important problems—or groups of problems—fall. Far more rarely yet vastly more excitingly, complexity theorists find new relationships between the different dimensions of classification, e.g., by showing that every set in the polynomial hierarchy Turing reduces to probabilistic polynomial time (Toda 1991) or by showing the class of sets having interactive proofs is precisely deterministic polynomial space (PSPACE) (Shamir 1992).

3.3.2 Classification Is Done for Insights Into the Landscape

The Core Belief and the previous section should hint at a truth that often is surprising to people who are not complexity theorists. That truth is that complexity theorists want to classify problems as part of the ongoing attempts to better understand the landscape of problem complexity. And in particular, we are interested in doing that even for problems where the classifications we are trying to distinguish between don't in practice differ in what they say about how quickly a problem can be solved.

For example, complexity theorists think that it is a rather big deal whether a problem—if it is an interesting one, such as about logic—is complete for double exponential time versus for example being complete for triple exponential space. This isn't because we think that complete problems for double exponential time are going to be easy to quickly solve. It is because we want to clarify where interesting problems fall in the landscape.

Looking at the other extreme, there is a huge amount of research into complexity classes (such as certain uniform circuit classes and logarithmic-space classes) all of which are contained in deterministic polynomial time. Yet to most people, deterministic polynomial time already is the promised land as to computational cost. Nonetheless, smaller classes are intensely studied, to better understand the rich world of complexities that exist there, and which problems have which complexities, although in fairness we should mention that some of this type of study is also motivated by the issue of whether the problem can or cannot be parallelized (Greenlaw et al. 1995).

But the real kicker here is that even if SAT solvers turn out to be able to do stunningly well on NP-complete problems, complexity theorists still will view the notion of NP-completeness as being of fundamental importance to the landscape. This is not because we don't care about how well heuristics can do—that too is

a dimension of the landscape, and thus something on which rigorous results are important and welcome—but rather we think that the notion of NP-completeness itself is one of the greatest beauties of the landscape, and is natural and compelling in so very many ways.³

To take as an example one of the most beautiful examples of how profound the issue is of whether NP-complete sets belong to P, i.e., whether P=NP, we mention that a not widely known paper by Hartmanis and Yesha (1984) is in effect showing that whether humans can be perfectly replaced by machines in the task of finding and presenting particular-sized mathematical proofs of theorems—loosely put, the issue of whether humans have any chance of having any special creativity and importance in achieving mathematical proofs—can be characterized by the outcome of such basic landscape questions as whether P and NP differ, and whether P and PSPACE differ.

4 Conclusion

To end as we started, we believe that economic design and computational complexity should become even more important to each other with each passing year, but that an improved mutual understanding of the areas' worldviews is important in making that happen. In that spirit, this article sets out the optimistic worldview that we believe is held by most computational complexity theorists. And the most central part of that worldview is that we as complexity theorists believe that there is a landscape of beautiful mathematical richness, coherence, and elegance—waiting for researchers to perceive it better and better with the passing of time—in which problems are grouped by their computational properties.

That is not to say that we believe that the greatest open issues within that landscape will be resolved within our lifetimes. But we believe that—just as that landscape has already been seen to have utter surprises in what it says regarding language theory (Szelepcsényi 1988; Immerman 1988), interactive proofs (Lund et al. 1992; Shamir 1992), branching programs and safe-storage machines (Barrington 1989; Cai and Furst 1991), approximation (Arora et al. 1998), the power and lack of power of probabilistic computation (Nisan and Wigderson 1994; Impagliazzo and Wigderson 1997; Toda 1991), and much more—the landscape contains countless more surprises

³This article is not on the subject of how well heuristics can do on NP-complete problems, or the strengths and limitations of SAT solvers. On one hand, there are theoretical results showing that polynomial-time heuristics cannot have a subexponentially dense set of errors on any NP-hard problem unless the polynomial hierarchy collapses. And if someone says they have a SAT solver that works on any collection of NP problems they ever have encountered, it is interesting to point out to them that factoring numbers that are the product of two large primes can be turned into a SAT problem, and so their amazing SAT solver should be able to break RSA and make them rich... yet no one has yet been able to make that work. On the other hand, SAT solvers undeniably do perform remarkably well on a great range of data sets. For discussion of most of the issues just mentioned, and how they can be at least partially reconciled, see for example the article by Hemaspaandra and Williams (2012).

and advances that will be reached in years, in decades, and in centuries, and we believe that many of them will be in the important, rapidly growing areas at the intersection of economic design and computational complexity.

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