Chapter 3
The Strasbourg Period: Radio-engineering

3.1 Radio-engineering at Strasbourg University

In Strasbourg, Mandelstam conducted research in radio-engineering and optics. By working in radio-engineering, he followed F. Braun and he was involved in the mainstream of research which was conducted at Strasbourg Institute of Physics, the institute which was headed by F. Braun. As was noted above with reference to the biography of F. Braun written by F. Haas, since 1901 all the dissertations supervised by Braun had been dedicated to wireless telegraphy.

Let me cite from Strasbourg University newspaper (Straßburger Akademische Mitteilungen für die Studierenden Der Keiser-Wilhelms-Universtät in Straßburg. Winter term 1909/1910. No. 4. Wednesday, 15 December 1909):

> The first experiments in the field of wireless telegraphy were conducted by F. Braun in Strasbourg in 1898. That year researches on telegraphy through water were completed. These experiments were conducted in the old defensive ditches which have been scheduled for backfilling. The first Braun patent was received in 1898. Next year in summer in Cuxhaven the experiments were conducted by Prof. Dr. Kantor and Prof. Dr. Zenneck. On 1900 November Braun reported about his research for the first time... In 1901 in summer in Strasbourg fortifications he conducted experiments with the help of Captain von Siegfeld, who was known by his scientific aeronautics and got into trouble later. Next year with the help of Dr. Mandelstam and Dr. Brandes Braun conducted the successful radio-transmission from the fort. In the same place he conducted the initial experiment of a directional receiving. In 1905 Dr. Papalexy took part in research in the directional telegraphy.

Till 1905 in Strasbourg in radio research, not only Ferdinand Braun, but also his First Assistant J. Zenneck played a decisive role (in 1905, Zenneck became Professor in Danzig and then in Braunschweig). In his monograph and textbooks (they were mentioned in the previous chapter, Sect. 2.2), Zenneck had systematized his Strasbourg colleagues' achievements and compared them with the world level.

As already mentioned, in Strasbourg, Mandelstam, working in radio-engineering, began to cooperate with N.D. Papalexy, and their cooperation has been continuing throughout his life. Reminiscing about his first collaboration with L.I. Mandelstam, N.D. Papalexy with a share of nostalgia writes about the epoch of "stridulous spark",

A. Pechenkin, *L.I. Mandelstam and His School in Physics*,
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about the time “when the discharge of a capacitor was a generator of oscillations” [267, p. 376]. The theme of the oscillatory discharge of a capacitor was in the center of L.I. Mandelstam’s first research and his degree work, or, as it was called in German universities, dissertation.

Is it worth to describe Mandelstam’s early research? Since that time, radio-engineering passed through a revolution or rather through a number of revolutions. Is not mere applied science, but it formulates its own volitions [239]. Let us take these Mandelstam’s early papers as elegant etudes. Besides, starting with these early papers one can trace back Mandelstam’s mature research (Central Radio Laboratory, Moscow State University, the Academy of Sciences) and the examples in his lectures delivered in Moscow State University.

3.2 Mandelstam’s Degree Work

This research was dedicated to the development of an indirect method of frequency measurements (to put it more precisely, of measurements of the period of oscillatory capacitor discharge). According to conventional terminology, indirect measurement is a measurement in which the value of a magnitude is found on the base of the law which connects this magnitude with another magnitude that could be directly measured. A direct measurement is a measurement in which the value of magnitude is directly found from sense data (see [32, p. 4, 88, p. 6]).

The direct measurement of the period of the oscillatory discharge of the capacitor is possible with the use of the Braun tube. A bright spot is forced to oscillate with the help of the coil, with a streamlined alternating current which is under investigation. By means of a rotating mirror, these oscillations are transformed into a curve.

However, such a measurement is suitable for relatively small frequencies.

F. Braun put before L.I. Mandelstam a problem to elaborate an indirect method which would differ from those which were already in application. As Mandelstam wrote,

> Professor Braun proposed me to follow the principle: an oscillating current divides into two branches, one of which consists of a self-inductance and one wire of the differential thermometer, and another consists of a non-inductive resistance and the second wire of the thermometer.
>
> It is necessary to find an ohmic resistance, in which the two wires give the same heat. This would mean that the ohmic resistance is equal to the apparent inductive resistance, from here under a given self-inductance frequency is determined. [1, Vol. 1, p. 69]

This problem can be elucidated as follows. Let us take a parallel connection of two conductors. One of them is the inductance which equals L, and the other is the ohmic resistance which equals R. Let the signal having the frequency ω be going along this connection. Then, the impedance of the first branch equals $i\omega L$, and the impedance of the second branch equals R. We need to choose such a value of R

as would provide in both branches the currents with the same amplitude. The heat effect is used here: By changing R, we obtain equal heat on both branches. Then, the formula $\frac{R}{L}$ gives the frequency.

Mandelstam, however, formulated the problem in another way: He elaborated the method of frequency measurement for "such discharges of a condenser which do not give simple sinusoidal current". He used such a shunt chain which joining with the main chain does not change the period of oscillations in the latter. A shunt circuit has two branches: The first contains an ohmic resistance, and the other has a self-inductance. As a matter of fact, measurement is provided by means of this shunt circuit.

Mandelstam used the differential equation technique. The differential thermometer shows a temperature difference and allows us to compare the heat effects in the branches of the shunt circuit. The following formula expresses an equality of heat effects: $\int (i_1^2 - i_2^2)\mathrm{d}t = 0$, where i_1 and i_2 are the currents in the two branches.

The solution of this equation (with the help of approximations and simplifications) leads to formulas which provide the calculation of the period of oscillations in the main chain by using its parameters.

In his book "Electromagnetic oscillations and radio", J. Zenneck discussed the methods of measurement of the oscillatory discharge of the condenser. Zenneck compares two indirect methods (see Fig. 3.1). Firstly, Zenneck describes a method which is alternative to the above which has been worked out by Mandelstam (Fig. 3.1, on the left): The capacitor circuit is inductively coupled with the circuit containing a thermoelectric wire of the thermometer. This second circuit must be so removed from the first, so as not to cause an impact on it. Then, a sharp rise in temperature, fixed by the thermometer, shows a resonance: the proximity of the frequencies of oscillations in the condenser circuit, with respect to which the measurement is produced, and the measuring circuit. Clearly, the measuring circuit must be prescaled; i.e., its eigenfrequencies (natural frequencies) must be known.

Although the resonance method has for a long time been used in radio measurement, it suffered from a significant drawback: It is still not possible to avoid a disturbance of the capacitor circuit from the side of the measuring circuit. The method developed by L.I. Mandelstam minimized this disturbance if the resistance and inductance of the measuring circuit are quite high.

It is interesting that J. Zenneck mentioned L.I. Mandelstam along with a number of physicists who worked in the same direction (the most famous is E. Rutherford, who published his method in 1897 in "Transactions"). Mandelstam's method is one of a number.[1]

In Zenneck's subsequent books [e.g., in his textbook on wireless telegraphy ("Leitfaden")], L.I. Mandelstam's method was not mentioned. Radio measurements have evolved rapidly.

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[1] Compare with the enthusiastic estimation of Mandelstam's degree work in the Soviet 1951 book [349, p. 196].

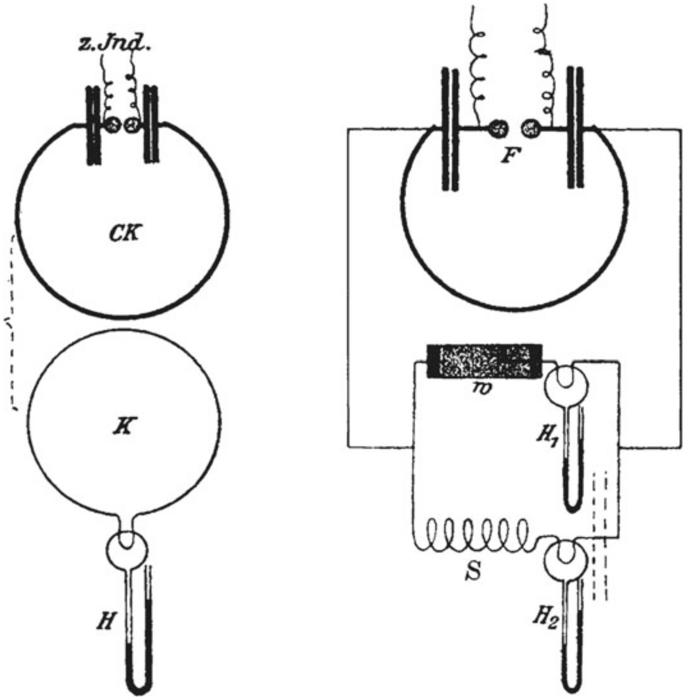

Fig. 3.1 From Zenneck's book "Electromagnetic oscillations". Two ways of how to measure the oscillatory discharge of a condenser. *On the right*: *CK* is a condenser circuit which induces e.m.f. in the closed circuit *K*, equipped by the measuring instrument *H*. *On the left*: the current of the discharge goes along two branches. The upper branch has a considerable electrolytic resistance *w*, and the lower branch has a conductor with the inductance *S*. Both upper and lower branches are equipped by their measuring instruments. The instruments H_1 and H_2 should be identical

3.3 Experiments with Loose Coupling

In Sect. 3.1, Strasbourg University newspaper was cited. The experiments conducted by Mandelstam and Brandes were mentioned there. These were the experiments with loose coupling between primary and secondary circuits in the Braun transmitter and receiver. F. Braun himself wrote about the Mandelstam–Brandes experiments in the printed version of his lecture "Wireless telegraphy and the problems of physical cognition" (1905) [62], and he mentioned them in his Nobel Lecture. About these experiments J. Zenneck wrote in his book "Electromagnetic oscillations…", which was mentioned above [386, p. 986]. In the biography written by N.D. Papalexy, it is said that Mandelstam together with Brandes invented the loose coupling [1, Vol. 1, p. 51]. Zenneck wrote more gingerly: The phenomenon of loose coupling was "theoretically described by Max Wien and almost at the same time experimentally invented by Mandelstam and Brandes" [381, p. 387, a comment in the footnote].[2] Everybody points to the applied importance of Mandelstam–Brandes' experiments that led to the improvement of reception and to the rise of the receiver selectivity.

Let me cite F. Braun's Nobel Lecture [63, p. 234]:

> In the summer of 1902 I was able to erect two experimental stations on two forts at Strasbourg for the purpose of closer study. The task which I had set for us was to determine the most favorable conditions in the receiver. We adopted the resonant circuit, in which known capacitances were combined with calculated self-inductances, so as to bring both parts of the transmitter system into the same natural frequency of oscillation. We fixed likewise the two oscillations arising from the coupling and searched for these with the receiver. The result of the test was, for that time, surprising, as an example will show. If, by means of a coil in the receiver circuit, the oscillations were transferred inductively into a second coil located in a tuned circuit containing the indicator (parallel to a small capacitor), not only was the sharpness of the resonance but also—and here was the surprise—the *intensity* of the excitation was raised as soon as the two coils were *moved away* from one another. The intensity increased with increasing distance between the coils, though naturally beyond a certain limit there was again a decrease. Described in the customary expression, the effectiveness increased with *looser* coupling. This result in the receiver was *not* subject to a similar loose coupling in the transmitter.

What is this "loose coupling"? This is a coupling between an antenna and a closed circuit in the Braun transmitter and in his receiver. By leaving aside the strict definitions which Mandelstam formulates in his 1930–32 lectures, it is possible to provide the following picture: The force of coupling is proportional to the coefficient of mutual inductance of coils coupling the primary and secondary circuits, and it is inversely proportional to the product of the inductances of these coils. In the case of a strong coupling of primary and secondary circuits, their system works as a single whole. The primary system acts on the secondary one, and the secondary one acts on the primary one (as a mechanical model can be taken the oscillations of two pendulums connected by a spring). In this case by altering the parameters of the system, it is possible to reach an increase of the amplitude in the secondary circuit. The strong coupling allows us to set up the situation where the amplitude of voltage oscillations across the capacitor plates would exceed the amplitude of the oscillations of the spark voltage. "By increasing the capacity of the primary circuit, Max Wien writes, and by decreasing its self-inductance, it is possible to considerably increase the ratio of amplitudes V_1/V (V_1 is the voltage across the condenser plates of the secondary circuit, V is the spark voltage)" [382, p. 46] (a Russian translation is cited). However, the damping of oscillations in the system consisting of strongly coupled circuits is more intensive than in each circuit taken separately.

In the case of the loose coupling, the back action of the secondary circuit on the primary circuit is small. It allows us to reach a weak damping of forced oscillations in the secondary circuit. However, the amplitude of these oscillations will be less than in the case of the strong coupling.

Note that, according to Zenneck, L. Mandelstam and Brandes experimentally and Wien theoretically opened namely the loose coupling. The case of an infinitely loose

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[2] For M. Wien's papers, see [381, 382].

coupling was described by V. Bjerknes as far back as in 1895 (see [47]).[3] However, Mandelstam and Brandes applied this idea in radio-engineering. They showed that the quality of reception increased in the case of an infinitely loose coupling when "two oscillations approximate the natural frequency, that is to say, become equal to each other" [61, p. 231]. In this case, the secondary system demonstrates two kinds of oscillations: the forced oscillations corresponding to the oscillations of the primary system and oscillations which are close to its (secondary system's) natural oscillations. If the frequencies of the primary and secondary systems approach each other (the systems are tuned in on each other), one can reach an almost full energy transfer from the primary circuit to the secondary circuit and the maximal amplitude of oscillations in the secondary system. In his 1931 lectures, Mandelstam mentioned that in the case of loose coupling one can reach "strong coherence".

For a certain time, a receiver with an infinitely loose coupling became the "calling card" of Telefunken Company which was founded with the participation of Ferdinand Braun (see, e.g., "The specifics and novelty of the schemes and constructions of the "Telefunken" system").

The Braun transmitter allows one to regulate the coupling force. If one needs the high transmission range and hence a high power of the transmission, one should prefer the strong coupling. If the problem of noise immunity arises, a loose coupling between the primary circuit and the antenna is preferable.

3.4 The F. Braun Energy Scheme

L.I. Mandelstam participated in Braun's two-circuit scheme which was mentioned in Chap. 2 (Sect. 2.3) and in the present chapter (Sect. 3.1). Together with his teacher, he considered the problem of how to increase the transmission range and hence the power of the transmitter. The F. Braun energy scheme (1902) appeared as a result of this research. This scheme became a noticeable event in the history of radio-engineering.

Mandelstam did not publish any paper on this theme. His participation in this research is only captured in Papalexy's recollections and in a patent which was taken by him and Papalexy due to their development of an apparatus theoretically connected with the Braun scheme. There is more evidence. Siemens Company Archives keep F. Braun's letter to W. von Siemens. In this letter, Braun recommends Mandelstam as a talented young engineer who participated in the development of his two-circuit scheme [400, LK 205] (the 1903 letter).

N.D. Papalexy provides the following description of the problem which F. Braun formulated [267]. The theory of electricity (the energy stored in a condenser equals $\frac{1}{2}CU^2$, where C is the capacity and U the voltage) points to two ways of how to

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[3] Vilhelm Bjerknes (1862–1951) was Assistant to H. Hertz in 1890–91. The concept of the extremely loose coupling he mainly developed in 1895–1898. He became a famous earth physicist and meteorologist later.

increase the transmitter power: (1) to increase the capacity of the condenser and (2) to increase the charging voltage. As F. Braun explained, both methods are vulnerable. If we increase the capacity of the condensers, we decrease the self-inductance. However, the coupling of the whole system decreases with the reduction in the self-induction of the oscillating circuit (containing a spark gap) and the secondary circuit (containing an antenna). Besides, practical difficulties arise: An increase of capacity of condensers leads to an increase of their self-inductance of the condensers originating in the coating of the jars themselves but especially in the connections.

The increase of the charging voltage (even if this is possible from an engineering point of view) leads to an increase of the length of the spark and hence to the considerable energy loss. Here, difficulties with insulation arise, too.

> It was tried out many different schemes, partly suggested by L.I. Mandelstam, writes Papalexy. About these unsuccessful attempts, Leonid Isaakovich shared with me and our mutual friend Brandes at meetings over lunch. Have been struggling with this problem for a week and getting no result, Prof. Braun and Mandelstam left the laboratory in the reduced mood. The next day (Sunday) for lunch, Leonid Isaakovich told us that he was from yesterday all the time thinking about this problem and apparently solved it. In his characteristic caution and modesty, Leonid Isaakovich put it this way: "I seem to have found a way to solve, but I fear that Professor Braun has also come to this solution". Indeed, when the next morning, Mandelstam entered the laboratory of Professor Braun, then he met this exclamation: "Herr Dr., Wissen Sie, ich habe gekriegt!" Thus arose the famous "Braun energy scheme", the essence of which lies in the ingenious use of a parallel charging of the capacitors appearing in n oscillatory circuits, and sequential discharging them through the self-inductances of these circuits, which along with the capacitors constitute a serial (main) circuit of the transmitter. [267, pp. 376–377]

By following F. Braun's article in the journal "Electrician" [61], there were two stages in the development of the original scheme. The first result was reached in the same 1898, when Braun created his inductive coupling system, alternative to Marconi's scheme. Braun's reflections had led him to the arrangement of Fig. 3.2. Three condenser circuits of exactly the same frequency are connected in series. "The transmitter passes through them all. As shown by the polarity signs (±), the condensers were charged in series. The energy of n capacities C is $\frac{1}{2}(C/n)(nV)^2 = nCV^2/2$. Each spark, however, has only a damping corresponding to the part V of the *P.D*". [Ibidem, p. 20].

Figure 3.2 represents the Braun scheme consisting of three capacitor circuits. Figure 3.3 shows how the Braun transmitter works (three capacitor circuits, taken together, are inductively connected with the antenna).

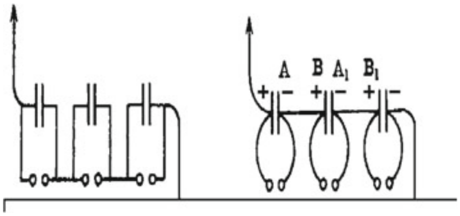

Fig. 3.2 From Braun's article in the journal "Electrician". Two series connections of the three condenser circuits

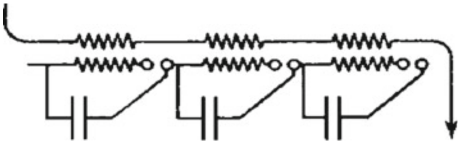

Fig. 3.3 Three capacitor circuits, taken together, are inductively connected with the antenna

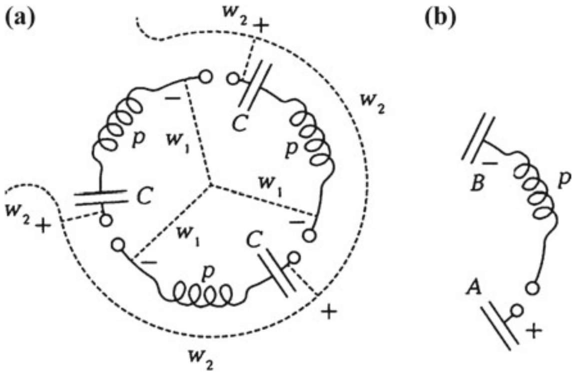

Fig. 3.4 Braun energy scheme. *On the right*: the partial system. From Braun's article in "Electrician"

The problem of synchronism was left. "The difficulty was to so couple circuits of this kind together that they would all start to discharge at the same moment, for example within exactly 1/10 of a millionth of a second", Braun said in his Nobel Lecture. "This task occupied me on repeated occasions". In 1902 autumn, Braun started to solve this problem by recruiting the newly made Doctor Leonid Mandelstam.

As a result of this work, the Braun energy scheme arose, the scheme about which Papalexy said: three oscillatory circuits, arranged in a circle (Fig. 3.4). "I reverted to these arrangements in the autumn of 1902,—F. Braun writes,—after having developed methods which allowed the phase-difference between rapid oscillations to be detected and measured without in influencing in a disturbing manner the system to be tested. The outcome of these experiments was that the oscillatory circuit were coupled more closely; such a coupling, which assists the simultaneous jumping of the spark, exists already in the isochronous oscillations of the transmitter, which passes through the entire system" [61, p. 20].

Braun's energy scheme maintained the advantages which were reached by him in 1898. However, the energy scheme showed synchronism. This system had symmetry: If the parameters of the circuits were equal to each other, all three circuits acted identically.

> The condensers are charged in parallel, and are connected for this purpose with non-inductive or inductive resistances ω_1 and ω_2. As soon as one spark occurs the whole circuit closes within itself ....
>
> The considerations from which this arrangement has been developed are very simple and clear. Assuming, for reasons of simplification, that all capacities C and all self-inductance are equal, n capacities then contain a total energy $nCV^2/2$, in which V equals a charging voltage. The period of an oscillation, however, depends upon the product Cp, because the

> capacities when discharging are in series, all have capacity of *C/n*. They have capacity of *C/n* only, when self-inductance is then represented by *np*. Therefore a period of oscillations is the same when one condenser is closed by one self-inductance *p*. [61, p. 20]

For the F. Braun *energy scheme*, a description was possible that was not valid for the 1898 system of circuits arranged in a line. This scheme could be treated as a system consisting of three circuits each of which contains a capacity, an inductance, and a spark gap. It could also be treated as a set of the "partial systems" (Fig. 3.4, on the right). By a "partial system", Braun meant the circuit consisting of the following elements: a capacitor plate, an inductance, a spark gap, and a next capacitor plate. The "partial systems" had the same electrical characteristics as the circuits composing the energy scheme.

As was noted, the Mandelstam–Papalexy 1913 patent application arose as an extension of the Braun–Mandelstam research. Mandelstam and Papalexy proposed an improvement of Braun's energy scheme. They struggled against the energy loss which resulted from the spark. Mandelstam and Papalexy used Max Wien's conception of the transmitter (it was mentioned in Chap. 2, Sect. 2.3). The Wien transmitter is close to the Braun scheme. However, a very short spark (a very small discharge gap) is used in it. By having excited oscillations in the secondary circuit (antenna), the spark dies away. Thus, the primary circuit turns out to be disconnected and there is no energy loss in it.[4]

In their 1913 scheme, Mandelstam and Papalexy combined two ideas: Winn's idea about the disconnection of the primary circuit and the F. Braun energy scheme. However, this disconnection is provided by a special construction of the transmitter and its parameters.

During their Strasbourg years, Mandelstam and Papalexy received 15 patents in total [399, 600-2-3]. The above scheme shows the idea of one of them.

3.5 On the Theory of F. Braun's Transmitter. Coupling and Coherency

The first sentence in the title of the present section reproduces the title of L.I. Mandelstam's article published in 1904.

In this article, Mandelstam proposes the unified theory of different modifications of the Braun transmitter. In Chap. 2, a version of the Braun transmitter was described (Fig. 3.5, on the left) within the framework of the comparison of this system with

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[4] L.I. Mandelstam mentioned the principle of the Wien transmitter in his lectures on the theory of oscillations [1, vol. 4, p. 279; 2, p. 256]. "Wien invented a very small spark gap, Mandelstam said. When the breakdown happens in it, it is becoming a conductor. However, under certain conditions the spark gap is losing its conductivity if the current is small. Then, the first circuit is automatically disconnected, when the amplitude of its oscillations is becoming very small, and back energy transfer does not take place".

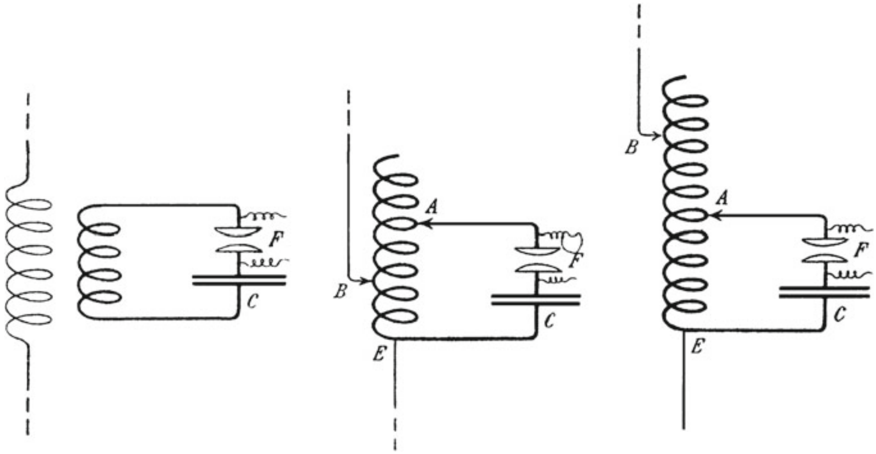

Fig. 3.5 Three modifications of the Braun transmitter. From Zenneck's book "Wireless telegraphy". *On the right* and *in the centrum*: the primary and secondary circuits are connected by autotransformer coupling. *On the left*: the primary and secondary circuits are connected by the inductive coupling

the Marconi scheme. The primary and secondary circuits are inductively connected with each other through a couple of coils (and each coil belongs to its own circuit).

However, besides the inductive coupling F. Braun considered the direct one: The secondary circuit is directly attached to the coil belonging to the primary circuit (Fig. 3.5, two schemes on the right). As a result, the coil turned out to be divided into two parts: the upper which is connected with an antenna (overhead wire, as people called it at that time) and the lower part which is grounded. As the secondary circuit, one should regard the combination consisting of antenna, grounding wire, and that part of the coil which is located lower than the point where the antenna is attached.

The direct coupling is also called the autotransformer. An autotransformer is a transformer in which the primary winding is a continuation of the secondary one.

Max Wien proposed the first mathematical theory of the Braun transmitter, by describing the case of inductive coupling (in 1902—M. Wien's article has already been mentioned in connection with the loose coupling). J. Zenneck, however, pointed out that between the inductive and direct coupling there is no fundamental difference. Mandelstam confirmed Zenneck's conclusion by presenting the mathematical structure of the system (he considered strong coupling only). Mandelstam presented an antenna (an open circuit) as a sequence of m closed circuits, including a circuit, which is directly inductively coupled with the primary closed circuit, and proposed an appropriate system of differential equations. Unlike M. Wien, who considered the Braun transmitter as a system with two degrees of freedom, Mandelstam in the spirit of mathematical physics presented the antenna as a system with many degrees of freedom. If $m = \infty$, the inductance and capacity are continuously distributed along the series of closed circuits. He further showed that the effect of continuity is reached at great but finite m. He also showed that the system with autotransformer coupling is described by the same system of differential equations as the system with inductive coupling. His conclusion on the theory of the Braun transmitter was the following: "the system with autotransformer coupling is reduced to the system with inductive coupling" [1, Vol. 1, p. 98].

J. Zenneck, quoting Mandelstam's article about the Braun transmitter, emphasizes one of the conclusions drawn in it [386, p. 880]. The point here is yet another modification of the Braun transmitter. In Fig. 3.5 is shown the structure in which the antenna is trivially grounded. F. Braun, however, examined a replacement of the so-called ground wire by a "symmetrization", a "counterbalance" (making it a mobile transmitter—it can be placed on a trolley). Instead of grounding, the antenna was attached to the bottom of the iron lattice, oriented parallel to the ground. Mandelstam showed that "grounding is not equivalent to the inclusion of "symmetrizating wire", but rather corresponds to a strong coupling between the primary and secondary circuits" (ibid.).

In his book, J. Zenneck proceeded from the usual definition of the coupling strength. He calculated the coefficient of coupling strength as follows: The numerator equals the square of the mutual inductance of coils, and the denominator equals the product of the inductances of the primary and secondary circuits. When lengthening the antenna, the numerator is not changing, and the denominator increases, since the inductance of the antenna increases. Introduction of the symmetrizating wire means that the antenna is becoming longer. This results in an increase of its inductance and therefore in a reduction in the coupling strength.

L.I. Mandelstam, however, conducted a more sophisticated analysis. As noted above, he started out from the system of differential equations, one of which represented the primary circuit, the rest represented the elements. In analyzing the system of equations and resorting to a number of assumptions, L.I. Mandelstam derived a relation that allows one to calculate the frequency:

$$\frac{1}{L_1 n} - C_1 n = \frac{L_2}{L_1}\sqrt{\frac{E}{L}} \cdot \frac{\sin n\sqrt{LEl} \cdot \sin n\sqrt{LEl_1}}{\sin n\sqrt{LE(l_1 + l)}},$$

where C_1 and L_1 are the capacitance and inductance, respectively, the primary circuit, but also L_2 is the inductance of the secondary circuit, E and L are capacitance and inductance per unit length of the antenna, and l and l_1 are lengths of right and left sides, respectively, of the antenna (secondary circuit).

The derived equation has an infinite number of roots representing the fundamental oscillations (harmonics) of a system of two circuits. The practical significance, however, has a range of

$$\frac{\pi}{\sqrt{LEl}} > n > 0,$$

and one of the fundamental oscillations of the system will be lower, and the other will be higher than the lowest and highest, respectively, eigenoscillations of both circuits taken separately. Strengthening the coupling between the primary circuit and the antenna corresponds to the convergence of the frequencies of the "normal" oscillations. If the antenna included “symmetrizating wire”, the maximum coupling strength occurred in the “symmetrical excitation”. This means that the circuit inductively coupled to the primary circuit is located right in the middle antenna length (as a result, the lengths of the air and symmetrization parts are equal). However, Mandelstam conducted a calculation which showed that the grounding of the antenna leads to an even stronger connection between it and the primary circuit.

It is interesting that L.I. Mandelstam came back to the problem of coupling in his 1930–1932 “Lectures on oscillations” delivered at Moscow State University [2].

L.I. Mandelstam distinguished between coupling and coherency of two connected systems (he called them “partial systems” by referring to a couple of connected systems each of which can be “fastened”). The adequate treatment of the phenomenon of coherency requires delving into the mathematics of the theory of oscillations. However, it can be explained by referring to the example of two pendulums connected by a spring (Mandelstam himself constructed such an example in his lectures). If one of the pendulums is fastened, the other can be treated as a partial system which has its own frequency. The coherence of the oscillations of two pendulums depends on the difference between the “partial frequencies”. The coherence is higher if the difference is smaller. In turn, the coupling depends on the strength of the spring. As a matter of fact, strong coherence can be achieved under loose coupling.

In essence, in his early writings on the loose coupling L.I. Mandelstam implicitly introduced the concept of coherency. He described the phenomenon of strong coherence under loose coupling.

Let us turn to the L.I. Mandelstam and N.D. Papalexy achievement which is noted in many publications on radio-engineering (not only in Zenneck’s books and not only in German publications in general).

3.6 A Definite Phase Difference

N.D. Papalexy writes in his biography of Mandelstam: “1904 is the year of our first collaboratory work in the field of oscillations and radio, which continued in Strasbourg, and in Russia until recently. This work was dedicated to a method of obtaining the phase lagging, but identical in shape oscillations, which formed the basis of experiments on the directional wireless telegraphy and radio interference” [1, Vol. 1, p. 9].

A brief comment. N.D. Papalexy writes about the collaborative work carried out in 1904. Mandelstam–Papalexy’s joint paper on the method of obtaining lagging but identical in shape oscillations was not published until 1906. However, already in 1904 F. Braun in formulating his method of the directional radiotelegraphy referred to the result of Mandelstam and Papalexy. He also mentioned this result in his 1905 lecture “On wireless telegraphy and new research in physics”, which was cited above.

We present the problem by following J. Zenneck (we shall not follow his book quoted above, “Electromagnetic oscillations…”, published in 1905, but shall follow his textbook on wireless telegraphy, published in 1908) [387] (for Russian translation [389, p. 390], see also [162, p. 61]).

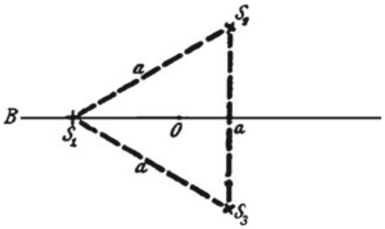

Fig. 3.6 From Zenneck's book "Wireless telegraphy". The F. Braun antenna. View from above

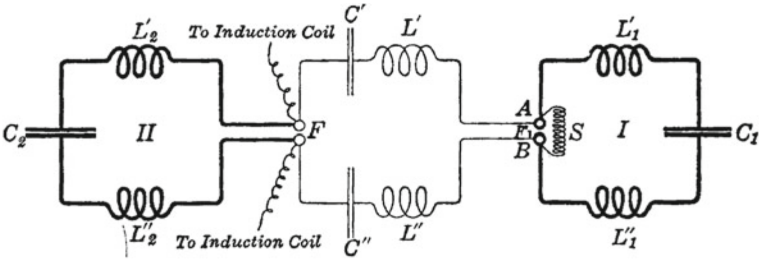

Fig. 3.7 Mandelstam–Papalexy scheme providing the phase lagging, but identical in shape oscillations. From Zenneck's book "Leitfaden der drahtlosen Telegraphie"

Figure 3.6 shows the 1904 Braun arrangement. In the antennas S_2 and S_3 oscillations are in phase, in contrast, in the antenna S_1 oscillations are phase-shifted by 270. The amplitudes in S_1, S_2, S_3 are in the ratio of 1:0.5:0.5; distance $a = \lambda/4$. In this case, the calculation shows that maximum radiation is in the direction of the SO, and in the opposite direction there is no radiation at all. In agreement with this, the experiments provide a particularly strong effect in the direction of the SO, and in the opposite direction the effect does not disappear, but it turns out to be negligible. The main difficulty with such a setup consists in excitation in different antennas of the oscillations with the described phase difference. This difficulty was solved by Mandelstam and Papalexy.

L.I. Mandelstam and N.D. Papalexy proceeded from the known fact that a high self-induction coil locks high-frequency oscillations, but ignores the low frequency. They had constructed the arrangement consisting of three circuits which contain capacities, self-inductances, and spark gaps (Fig. 3.7). The second circuit is connected to the transformer of high frequency. The first circuit provides the result. The third circuit $\underline{FC'L'AL'_1C_1L''BL''_1C''_1F}$ connects the first and second and partially includes the first circuit. The first and second circuits must be identical. The oscillations in them should come with a specified phase difference.

Let the circuits *II* and *I* ($\underline{FL'_2C_2L''_2F}$ and $\underline{F_1L'_1C_1L''_1F_1}$) be tuned up to the same frequency. The wavelength in the circuit $\underline{FC'L'AL'_1C_1L''_1BL''C''F}$ is larger than the wavelength in the circuit $\underline{F_1L'_1C_1L''_1F_1}$.

The coil S has a considerable reactance for high frequency, and it provides a short circuit (Kurzschluss) across the capacity C_1, while C_2 is being charged by supplying voltage (of low frequency).

The coil S does not influence the processes of high frequency. Spark gap AB is adjusted in such a way as to have a spark occurring at a time when the potential difference across the contacts AB is at the high (more precisely—when the amplitude of the voltage oscillation is on the high side). When a spark arises in the spark gap F, high frequency oscillations appear in the circuit II and in the interconnecting circuit $\underline{FC'L'AL'_1C_1L''BL''_1C''_1E}$. Thus as the amplitude of the high voltage reaches its maximum, a spark breaks the spark gap F_1 and the circuit I becomes active. This happens after half of an oscillation in the circuit $\underline{FC'L'AL'_1C_1L''BL''_1C_1F}$ (or, equivalently, in a time equal to a half of the period of the circuit).

Why namely "after half of an oscillation"? In their article, L.I. Mandelstam and N.D. Papalexy explained the phase relations in the circuits I and II by formulating the equations. By retelling their article, J. Zenneck, however, used graphic diagrams. Anyway, an explanation is based on two well-known facts: In the capacity, the phase of the current lags that of the voltage by $\pi/2$, and in the inductance the phase of the voltage lags that of the current by $\pi/2$.

The Mandelstam–Papalexy method was quoted not only by F. Braun and J. Zenneck but also by a person of a competitive community, namely J.A. Fleming who was the closest collaborator of Marconi's and who seldom mentioned the German radio-engineers in his publications [106, 109–111]. As was noted, Mandelstam criticized Fleming by considering his approach to the directional radiotelegraphy. The time has come to explain the world of radio-engineers as it was in the beginning of the twentieth century. Hence, it is worth to have an excursion to the history of radio business.

3.7 Radio Business

Radio in those days was not only a subject of research, but also an area of businesses, and research and business were not independent of each other. On the one hand, the radio immediately became an attractive area of capital investment. Especially, the militaries and shipowners had needs in the development of radio. On the other hand, the scientific development of the radio since 1898 required significant financial resources.

In 1900, a limited company—Professor Braun Telegraph or abbreviated Telebraun—was formed. The principal contribution to the capital of this partnership came from the patent (1898) belonging to Braun. Thus, Braun became a competitor of the company which had been formed around Marconi in England (1897), as well as he was a competitor of German engineers Adolf Slaby and George Count von Arco. Adolf Slaby and George Count von Arco together with electrotechnical company AEG formed the partnership AEG-Slaby-Arco radio in 1898.[5] In 1901, the cooperation of F. Braun with the well-known entrepreneur Siemens resulted in a new company which arose on the base of Telebraun and Siemens and Halske (Siemens & Halske). Its name was "The Prof. Braun and Siemens-Halske partnership of the system of wireless telegraphy" (or for short the "Braun-Siemens partnership"). Telebraun Company, however, continued its activities until 1913, when it was eliminated because of its unprofitability. In 1903 on Kaiser Wilhelm's orders, the two largest companies Braun-Siemens and AEG-Slaby-Arco merged into a powerful company, which became famous under its telegraphic address—Telefunken.

In the book on the history of radio, the situation on the radio market has been summed up as follows: "By 1901 Marconi's Wireless Telegraph Company had already won a leading position in ship to shore communications in Britain, was reaching out for a similar position in the United States, and, with much publicity, was entering the fight for transatlantic traffic in competition with the cables. In Germany there was the Telefunken Company, using the Slaby-Arco-Braun patents and aggressively seeking to establish itself in maritime and European traffic with the strong support of the German Government. In the United States the United Wireless Company, using apparatus based on De Forest patents, operated a network of stations handling ship traffic on the Atlantic and Pacific coasts.[6] And there are other, less formidable rivals" [6, pp. 143–144].

This does not mean that radio-engineering was compartmentalized along "business apartments". There was a unified stream of results in the area of radio. There were international journals, which published both English and German radio-engineers and radiophysics. The major was the English "Electrician" and the German "Jahrbuch der drahtlosen Telegraphie und Telephonie". As noted above, Braun, Zenneck, and Mandelstam entered on the editorial board of "Jahrbuch der drahtlosen Telegraphie und Telephonie". In this editorial board was also included G. Marconi. But the descriptions of the technical devices and inventions have always shown preferences.

Radio industry resulted in the strong competition between the companies. Partially, this competition was represented in the discussions of priority. F. Braun specially proved his priority concerning the two-circuit scheme which he proposed in 1898. In 1900, the scheme which was close to Braun's was put forward by Marconi [60]. F. Braun repeatedly emphasized that his English application for a patent had been ignored (at least up to the date when he secured his first German and equivalent British patent, No. 1862, of January 26, 1899).

The English engineer John Ambrose Fleming (1849–1945) accentuated the secondary role of German inventions. In 1899, Fleming had taken the position of scientific advisor at Marconi's company which was then named *Wireless Telegraph and Signalization Company* [330]. Let us cite a long passage from the first edition of

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[5] AEG—Allgemeine Electricität Gesellschaft.

[6] Lee de Forest was the American radio-engineer and entrepreneur. Having improved the diode invented by Fleming, Forest took out a patent for triode, a three element device. Forest contributed a lot to the development of radio in USA (see, for example, [6]).

Fleming's book "The principles of electric wave telegraphy". Fleming himself called this book his "principal book" [114, p. 175]:

> Braun's suggested direct coupling of an aerial wire a nearly closed oscillation circuit, consisting of a Leyden jar and associated inductance and spark balls, compared with the simple insulated conductor or aerial of Marconi, separated from the earth by a spark gap, does not produce a radiator having any special advantages, unless there is a syntonism between two coupled circuits. Neither is the inductive coupling of any special advantage unless the oscillation transformer is constructed in a particular manner. There is some indication in the opening remarks of Braun's specification, that he considered the real novelty in his invention to be the employment of the oscillations or discharges of a Leyden jar to create electric waves for telegraphic purposes, in place of the oscillations established directly of a simple linear or open circuit radiator containing a spark gap. This conception, however, is seen to have no foundation as soon as we make a metrical study of the phenomena, and the conditions which must be fulfilled for any useful result to take place. There are, in fact, only two modes of coupling an open and closed oscillatory circuit which have any technical value. First, we may couple together the circuits in such a manner that a single pure oscillation or one single period of vibration is forced upon the aerial or radiator, not its own natural period, but that of the actuating closed circuit. Secondly, we may couple together circuits which have the same free natural time period when separate, and thus establish a syntonism between the circuits which, under the condition of somewhat "loose coupling", results in the radiation of waves of two different wave lengths.
>
> The first mode or operation was described by J. S. Stone...,[7] and the second was discovered and worked out practically by Marconi. It has sometimes been suggested that Marconi availed himself of Braun's prior invention, but in truth his (Marconi's) investigations were carried on quite independently, and conducted to a more practical issue than those of Braun. [106, pp. 490–491]

Having described the principle of operation of the Telefunken (Braun-Siemens) transmitter, Fleming stated:

> The comparison... which the German writers and inventors at that date invariably insisted upon making was to take as typical of Marconi's methods of the original single wireaerial transmitter of Marconi, direct connected to one spark ball of the induction coil, the other being earthed. Marconi had advanced far beyond this stage at the end of 1899 and beginning of 1900. [106, p. 500]

Although in the subsequent editions (in total, there were four editions [106, 109–111]) Fleming had excluded this critique, he presented Braun's achievements superficially. Basically, he mentioned his results concerning the directional telegraphy. He even kept this line in third and fourth editions which appeared after the Nobel Prize was given to Ferdinand Braun for his contribution to radio-engineering and radiophysics!

The Papalexy words were above quoted that "the atmosphere of electromagnetic oscillations, in which Leonid Isaakovich found himself by entering into scientific life, had played a very important role in shaping the main directions of his scientific career". But early in his career, L.I. Mandelstam was in the environment of not only radio researches but also radio business. Already the early experiments provided by

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[7] J.A. Stone was the English radio-engineer who contributed to the construction of the transmitter.

Mandelstam and Brandes with the power of communication have been adopted by radio business. These experiments were initially funded by Telebraun and then were supported by Telefunken. Their results were included in the promotional booklet, "The specificity and originality of design and architecture of Telefunken systems". As Papalexy notes, from Telefunken Company L. Mandelstam has received a significant premium for those times for these achievements [1, Vol. 1, p. 12].

As was noted in Sect. 3.4, in 1903 in his letter to Wilhelm von Siemens, F. Braun characterized Mandelstam as a talented engineer who enthusiastically accepted Braun's energy scheme [400, Lb. 703]. In 1904, Mandelstam and Papalexy worked for Telefunken Company (see [5, p. 312]). Judging on F. Braun's 6.11.1912 letter to Zenneck, they continued to work for this company till this time [397, DM, NL 53/032]. Braun had disagreement with the administration of this company, and he recommended Mandelstam and Papalexy to stop their cooperation with this company. Braun disapprovingly wrote that Mandelstam and Papalexy left for Berlin for a half of year to work for Telefunken.

It should be noted that the Mandelstam and Papalexy participation in business was restricted. In contrast to Braun and Zenneck, they did not participate in the administration. They were engineers engaged by Telefunken Company. However, they felt themselves as members of a group involved in radio business.

In Strasbourg, L.I. Mandelstam became a member of the team headed by F. Braun. Having returned to Russia, Mandelstam continued to feel himself as a member of the community which arose around F. Braun. As was mentioned, in "Die Naturwissenschaften" Mandelstam and Papalexy published a paper in commemoration of F. Braun. On October 30,1918, Mandelstam wrote to Richard von Mises: "I was shocked by the news about Braun's death. I was so attached to him". As follows from Mandelstam's wife letter to Richard von Mises [395], he maintained contact with his Strasbourg younger friend Rohman. We have no clear information about the level of Mandelstam's relations with J. Zenneck, who became an important administrative figure after World War I. However, Zenneck's recollections and Papalexy's writings about Mandelstam show that Zenneck and Mandelstam sympathized with each other. In Papalexy's notebook, there is a note that he had a conversation with Zenneck during one of his trips abroad (Autumn 1930) [399, 600-2-33].

3.8 L.I. Mandelstam Criticizes J.A. Fleming

The Mandelstam–Fleming polemics, which is mentioned in Introduction, started in Strasbourg and was induced by Mandelstam and Papalexy's research in directional radiotelegraphy (see above). Mandelstam and Papalexy followed Braun who proposed an antenna consisting of three poles. In turn, Marconi proposed an alternative setup, namely a Γ-shaped antenna (bent antenna) which radiates stronger in the direction opposite its free end. Fleming and Marconi conducted experiments which showed the advantages of the bent antenna. Besides, Fleming developed a theory of this antenna (see, e.g., [107]). Mandelstam published two articles (1907 and 1908) in which he criticized Marconi's experiments and critically analyzed Fleming's theory of the bent antenna.

L.I. Mandelstam described the situation as follows.

> Prof. Braun proposed a setup providing with antisymmetrical radiation. Many years before he described a method of the directional reception.
>
> Marconi published the results of the experiments of the directional reception which is based on the same principle that Braun proposed. Besides, Marconi has described the experiments with the directional transmitter. J.A. Fleming has theoretically described those experiments.
>
> Fleming's calculations are not correct, therefore their confirmation by experiment is only seeming.
>
> Marconi-Fleming's measurements don't form the base for the solution of practical problems since they conducted at short rang [1, vol. 1, pp. 143, 144].

Marconi's Γ-shaped antenna was successfully used for the transatlantic transmission by its inventor. As Zenneck writes, this is the best proof of the advantages of the Γ-shaped antenna [388, p. 427]. Therefore, it is worth to be in touch with Mandelstam's criticism of Fleming.

Like the other radio-engineers, Fleming proceeded from the model of an oscillator: An antenna together with its grounding wire (which is treated as a symmetrical extension down under ground) was taken as a radiating oscillator which is upright standing on the ground. It is obvious that this rod must radiate equally in all directions which have the line of the rods for an axis.

If the rod has two bends made in it so as to make an oscillator resembling in shape three sides of a rectangle, then the symmetry of radiation is destroyed (Fig. 3.8, the left part).

We may consider the oscillator as constructed in the following manner. Imagine a rectangular circuit placed perpendicularly to the earth surface, and let it be traversed by a high-frequency oscillation. Then, if a horizontal line is drawn through the center of rectangle and two points and chosen at equal distances of the right and left sides, the magnetic field at these points will be equal and normal to the plane of rectangle. If at any instant the current is flowing counterclockwise around the circuit, the magnetic forces H at the points will be directed away from the observer (Fig. 3.8, the right part). Next, suppose a wire to be placed in contiguity to the right side of the rectangle and to be traversed by an equal current in opposite phase to that in the side of the rectangle adjacent to it. The field due to this open-circuit current at two points a

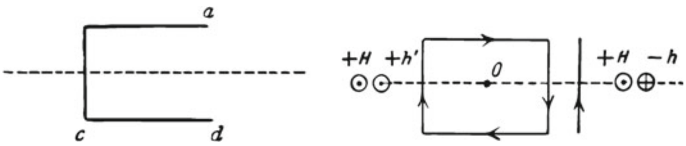

Fig. 3.8 From Mandelstam's article criticizing the Fleming theory of the bent antenna. *On the left*: the ground is taken as an ideal mirror. *On the right*: the bent antenna was supplemented to get a rectangle

and b will be toward the observer at the right-hand point and away from him at the left-hand point. Hence, if h and h' are the magnetic forces due to the open-circuit current at the points in question, the resultant fields are $H - h$ at the right-hand point and $H + h'$ at the left.

To understand Mandelstam's criticism, one needs to turn to Fleming's figure, which precedes the left part of Fig. 3.8. This is a picture of a vertically disposed rod: An antenna together with its grounding wire (which is treated as a symmetrical extension down under ground) was taken as a radiating oscillator which is standing upright on the ground. The coordinates of the middle point of the rod are designated as x', y', z'. To construct the left part of Fig. 3.8, Fleming needs to move the rod to the left by $\delta y/2$. As a result, the coordinates of the middle point have values x', $y' - \delta y/2$, z', where δy is the length of the horizontal oscillator *ba*.

Fleming calculated the electromagnetic field of the rod (dipole) in the x, y, z point. Mandelstam calls it the "point of observation". To calculate the dipole scalar and vector potentials, one needs to differentiate the magnitude containing r (the distance from the point of observation to the middle point of the rod).

Both Fleming and Mandelstam differentiate the magnitude, which is a complex function of the kind $F(f(t))$. Fleming differentiates $F(f(y))$ with respect to y; that is, he really differentiates with respect to the point of observation rather than to the middle point of the rod. Mandelstam differentiates $F(f(y'))$. However, the shift of the rod to the left means the shift of the point of observation to the right. This results in the formulas which Mandelstam emphasizes: $\frac{\partial}{\partial y} = -\frac{\partial}{\partial y'}$. Fleming does not take this into consideration. He differentiates with respect to y as if $y = y'$ [1, Vol. 1, p. 157].

The coordinate of origin of the coordinate system is located in the middle point of the rod. To construct the left part of Fig. 3.8, Fleming needs to move the origin of the coordinate system to the center of the rectangle. He needs to provide $\delta y/2$ shift of the rod, where δy is the length of the horizontal oscillator *ba*. As a result, the middle point of the rod has received the value of $-\delta y/2$.

The German engineer Karl Uller pointed to the same mistake in Fleming's writings [348].[8]

Mandelstam points to another mistake. He emphasized that Fleming had reached the expression of the magnetic field amplitude by neglecting the terms of the second-order infinitesimal. However, by proving the asymmetry of the radiation of the bent antenna Fleming solved the extremum problem where he took the terms of the second-order infinitesimal in the expression of the magnetic field amplitude.

Mandelstam gave an example where such an operation with the second-order infinitesimal resulted in an essential mistake.

Mandelstam resumed his criticism of Fleming as follows: "In his article published in "Proceedings of Royal Society" Mr. Fleming made an algebraic sign mistake and, besides, on the reason of incorrect interpretation of his formulae he came to the mistaken conclusion concerning asymmetry" [1, Vol. 1, p. 181].

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[8] Uller published a number of articles on the propagation of radio waves.

As early as 1905 F.Braun was critical with respect to Fleming but for another reason. In 1905, Braun called Fleming's statement that Marconi's apparatus withstands disturbances from foreign wireless stations a "fantastic story" (see [62, p. 25]). However, L.I. Mandelstam became the main critic of J.A. Fleming.

Fleming has not acknowledged his errors. In [108], Fleming reacted to Mandelstam's criticism. Fleming wrote that he "has not been unable to agree with this criticism" [108, p. 329]. In the same issue, Mandelstam published an article where developed his criticism [1, Vol. 1, pp. 194–161]. Fleming, however, wrote in the second edition of his book "The principles of electric wave telegraphy" the following [109, pp. 650–651]:

> Although the mathematical method of treating the problem of the bent antenna given in this section is based on ideas which are in accordance with experience, and is confirmed by the experimental work of Bellini and Tosi described below, it has been criticized by K. Uller and L. Mandelstam, who have taken exception to it on the ground the above formula… the algebraic sign prefixed to he second term inside each bracket should be changed, and −M written for +M. The author has, however, been unable to agree with this criticism. But for the detailed discussion of this difference of opinion the reader must turn to the critical articles in the Jahrbuch der Drahtlosen Telegraphie und Telephonie, Vol. 1, p. 291 and 333, 1908, and the author's rely on p. 329 of the same journal.

Fleming did not mention Mandelstam's reply to his reply in this book. In the third and fourth editions, Fleming did not mention Mandelstam's criticism at all.

3.9 The Mandelstam–Papalexy Induction Dynamometer

Mandelstam and Papalexy have not invented the dynamometer. However, in 1910 they developed the theory of this measuring instrument (this theory is reproduced in Zeneck's textbook) and invented its effective variety—short-circuit frame dynamometer.

The Mandelstam–Papalexy induction dynamometer "made it possible to read directly the values of frequency and damping decrement" [1, Vol. 1, p. 15, a comment in the footnote]. This apparatus had commercial success (see Zenneck [390, pp. 132–136]). The Mandelstam–Papalexy idea was developed in some writings (see the references in Chvolson's textbook on physics [82, the second edition, Vol. 5, pp. 384–385]).

By describing Mandelstam's dissertation (graduation work) in Sect. 3.2, we mentioned the resonance method to measure the frequency. At that time in radio-engineering, there was another problem—the estimation of oscillation damping. To measure it, one should find not simply a resonance, that is, the frequency of the measured system that would correspond to the frequency of the measuring instrument.

To estimate damping, one needs to obtain the resonance curve showing an increase of the amplitude as the difference of frequencies is decreasing.

The resonance curve can be plotted by proceeding from the heating effect of current. Mandelstam and Papalexy used the dynamometer effect. The core of it runs as follows (I follow Zenneck's book cited above). Assume a movable coil, S_2, in a vertical plane, e.g., suspended on a vertical wire, placed within a fixed coil, S_1, also in a vertical plane. If a current I_1 is passed through S_1 and I_2 through S_2, the turning moment to which the movable coil is subjected equals I_1I_2. If I_1 and I_2 vary rapidly with time, e.g., in high-frequency alternating currents, the coil will in general not respond to the rapid variations and its motion will be determined by the average turning moment, i.e., the average value of I_1I_2. This average value is called the dynamometer effect I_1I_2 (from the use of this arrangement from a movable coil in the field of a fixed coil in the well-known dynamometer type of wattmeters).

Assume now that a primary circuit of constant frequency acts inductively upon a secondary circuit of variable wavelength, e.g., an adjustable condenser circuit. Let I_1 and I_2 represent the currents in the primary and secondary circuits, respectively. The dynamometer effect of two currents is measured and a curve plotted in which the abscissa are the wavelengths of the variable secondary circuit, the ordinates being the corresponding dynamometer effect.

The resulting curve passes through the axis of abscissa when the wavelength of the secondary circuit is equal to that of the primary, i.e., when the two circuits are in resonance.

The form of this resonance curve, similar to the current effect curve, depends on:

- the sum of the decrements of the primary and secondary circuits;
- the degree of coupling between two circuits.

If the coupling between primary and secondary circuits is extremely loose, we are able to deduce simple formulas which allow us to calculate the decrements proceeding from the "dissonance values". A dissonance value is the common fraction where the numerator is a difference in constant and variable frequencies and the denominator is a variable frequency. The constant frequency is the frequency at which the dynamometer effect equals zero. The "variable frequencies" are frequencies at which the resonance curve has some specific points. For example, let x_1 and x_2 be the dissonance values at which the dynamometer effect has its maximum positive and negative values, respectively (Fig. 3.9). Then, we have the following formula ($d_1 + d_2$ is the sum of the primary and secondary decrements):

$$d_1 + d_2 = 2\pi x_1 = 2\pi x_2 = 2\pi \frac{x_1 + x_2}{2}$$

It is interesting that in L.I. Mandelstam's 1931–1933 lectures on the theory of oscillations there is a piece which provides the foundations of the above formulas. Mandelstam proceeded from the differential equation for the forced damped harmonic oscillator:

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = E\cos pt,$$

where p is the frequency of the outside force (this is the measured frequency), ω is the frequency of the oscillator (this is the measuring device), δ is damping (here we have no damping of the force acting on the oscillator, for simplicity sake the dynamometer measures its own damping).

The solution of the equation is

$$x = X\cos(pt - \varphi),$$

and

$$tg\,\varphi = \frac{2\delta p}{\omega^2 - p^2}.$$

Mandelstam said in his "Lectures on the theory of oscillations":

Let us take a dynamometer. Between the two coils through which currents i_1 and i_2 pass, there arises a pair of forces giving a turning moment

$$M \approx i_1 i_2$$

Let the current i_1 be directly taken from the source of the variable electromotive force. The current i_2 is passed through the resonance circuit. We have then

$$\begin{aligned} i_1 &= \alpha E \sin\rho t \\ i_2 &= \dot{X}\sin(\rho t - \varphi), \end{aligned}$$

where α is a constant. On the base of these formulas, we have

$$M = \text{const} \sin\varphi\, \overline{\sin pt\ \sin(pt - \varphi)}$$

However, $\overline{\sin pt \sin(pt - \varphi)} = \frac{\cos\varphi}{2}$.

Therefore, we have

$$M = \text{const}\ \sin\varphi\ \cos\varphi = \text{const} \cdot \sin 2\varphi.$$

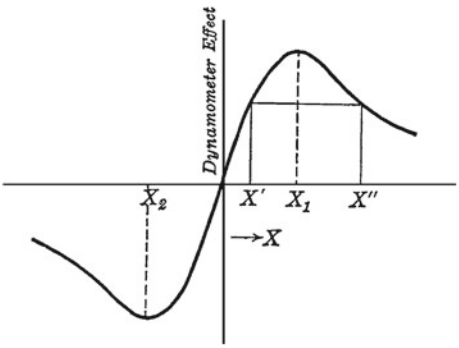

Fig. 3.9 From Zenneck's book "Wireless telegraphy". Abscissa axis—dissonance value, ordinate axis—dynamometer effect

Under the tuning in $\sin\varphi = 1$, $\cos = 0$; that is, in the case of resonance the turning moment equals to zero [2, p. 150].

However, the value of damping is not shown by resonance. This value should be estimated proceeding from the behavior of the resonance curve.

The maximum and minimum of the curve M are reached as $tg\,\phi = 1$, respectively, -1. Since $tg\,\varphi = \frac{2\delta p}{\omega^2 - p^2}$ or approximately $tg\,\varphi = \frac{\delta}{\omega - p}$ [it has been taken into account that at a small detuning $\omega^2 - p^2 = 2p(\omega - p)$]. By introducing the logarithmic decrement d through the formulas $\frac{\delta}{\omega} = \frac{d}{2\pi}$, we have $tg\,\varphi = \frac{\omega d}{2\pi(\omega - p)}$. If $tg\phi$ equals 1 (maximum of the ratio of the difference between variable and constant frequencies to the variable frequency), which is the value which Mandelstam and Papalexy introduced in their article about dynamometer. Let us recall that for the sake of simplicity we consider the measurement of d rather than $d_1 + d_2$ (we follow Mandelstam's "Lectures").

G.S. Gorelik, the disciple of L.I. Mandelstam, writes that in Strasbourg L.I. Mandelstam was occupied by the "countless variety of the phenomenon of resonance" [138, p. 139]. In his Moscow lectures on oscillations, Mandelstam generalized his Strasbourg experience.