Chapter 8
Numerical Synthesis of Stephenson Six-Bar Mechanism Using a CAD Geometric Approach

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Abstract Geometric construction and analytical calculation are recognised as two basic approaches for planar mechanism design. However the geometric construction is only available for relatively simple cases while the analytical calculation is quite complicated and far from being visualised too. Both of them are complicated to be used for the Stephenson six-bar mechanism design due to complexity. Therefore the Computer-aided Design (CAD) geometric approach is developed to fulfil research needs. In this study, an approximate position and posture synthesis for the Stephenson six-bar mechanism is investigated via using the CAD geometric approach. Firstly using the geometric constraint and dimension driving techniques, a primary simulated mechanism is generated. Then based on different tasks of path and motion generations for the dimensional synthesis, the simulated mechanisms of the Stephenson six-bar mechanism are developed from primary simulation. The computer simulation results on approximation dimensional synthesis of the mechanism prove that the CAD geometric approach not only visualises the mechanism accurately and reliably but also increases the number of prescribed positions of synthesis for mechanism.

Keywords Stephenson six-bar mechanism · CAD geometric approach · Path sythesis · Motion Sythesis

8.1 Introduction

The purpose of synthesis of mechanisms is to achieve motion transformation from input to output of such mechanisms, which can generate specifically desired motions at certain positions with specified postures [1]. The main outputs of the mechanisms include function, path, and motion and so on.

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The approximate mechanism synthesis refers to that if the mechanism can produce accurate positions and postures at some positions, but which can produce approximate positions and postures at other positions, and the differences between actual and provided positions and postures must be within an allowable limit. The geometric method and analytic method are two basic methods commonly used [2]. The geometric method is simple and visualised, but inaccurate, less repeatable, and provides only a few numbers of synthesis positions. The analytic method is accurate and repeatable, but complex and not visualised [3–5]. Thus the CAD geometric approach was developed for mechanism synthesis. This method is very useful and powerful to research all kinds of mechanisms for researchers.

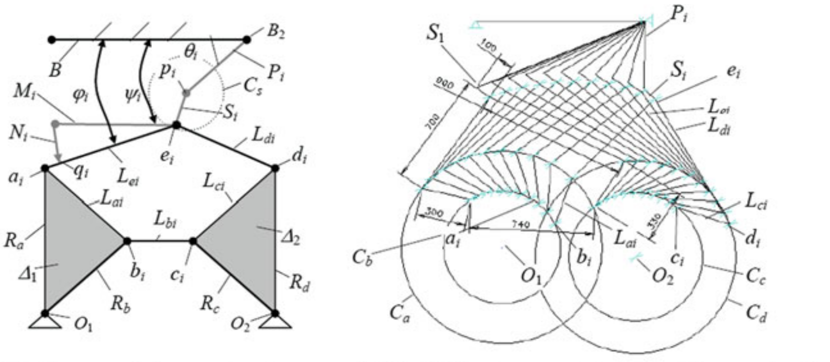
In this study, the CAD geometric approach is developed and utilised for conducting the approximate position and motion syntheses for Stephenson six-bar mechanism via using the CAD geometric approach as a showcase. Firstly using the geometric constraint and dimension driving techniques, a primary simulated mechanism is constructed. Then based on different tasks of path and motion generations for the dimensional synthesis, the simulated mechanisms of the Stephenson six-bar mechanism are developed from the primary simulated model. The computer model and simulations are further used for the path synthesis and motion synthesis to extract approximated solutions, which are further validated by comparing with those accurate solutions availiable.

8.2 Simulation of Basic Stephenson Six-Bar Mechanism

The Stephenson six-bar mechanism is shown in Fig. 8.1a, which is used to construct a mechanism synthesis at the k prescribed position via using the CAD geometric approach. When the mechanism is at the ith position, two triangles Δ_1 and Δ_2, and four lines O_1O_2, L_{bi}, L_{di} and L_{ei} constitute the mechanism. Line O_1O_2 is fixed, and line L_{ei} is a floating bar. In order to produce an approximate orientation of the bar L_{ei}, the two lines M_i and N_i are constructed using the suitable geometric constraint and dimension driving. In order to produce an approximate trajectory positions of the bar L_{ei}, the line S_i is constructed using the suitable geometric constraint and dimension driving. The multiple approximate positions and postures of bar L_{ei} are then produced.

First, the basic mechanism at the kth position is constructed using the geometric constraint and dimension driving, before constructing the Stephenson six-bar simulation mechanism, as shown in Fig. 8.1b. The construction process is described as follows:

1. First, construct one line L_{a1} and a dimension D_{a1}, then the same k lines are moved and copied using ‘Move’ and ‘Copy’ commands, where k is the prescribed number of positions. The dimensions D_{ai} are used as driving dimensions, setting $D_{ai} = D_{a1}$ ($i = 2, 3\ldots k$), and the dimensions D_{ai} of the line L_{ai} ($i = 2, 3\ldots k$) are changed as driven dimensions using the constraint function of the dimension



(a) Stephenson six-bar mechanism (b) Basic Stephenson six-bar simulation mechanism

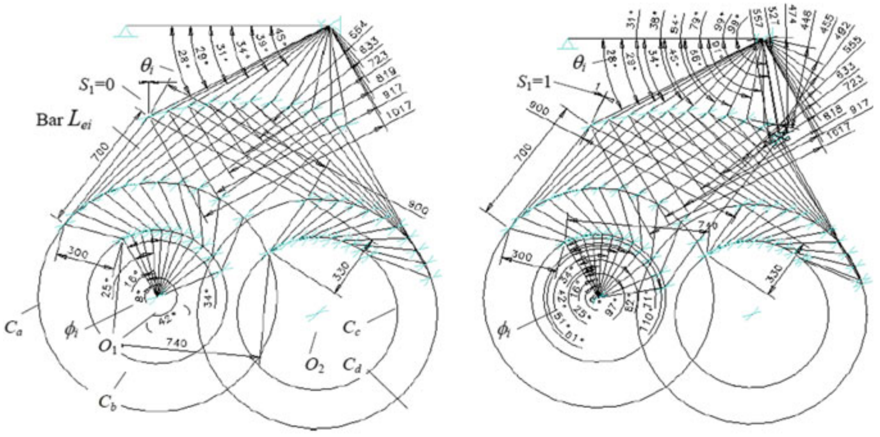
Fig. 8.1 Stephenson six-bar mechanism and its basic simulation

equation. In order to simplify the view, all the driven dimensions D_{ai} ($i = 2, 3 \ldots k$) are hided using the 'Hide' command. All the driven dimensions D_{ai} ($i = 2, 3 \ldots k$) change accordingly when modifying the driving dimension D_{a1}.

2. Second, construct two circles C_a and C_b with the radii R_a and R_b, respectively. The two centres of circles are connected to the O_1 using a geometric constraint command. The two ends of the set of lines are respectively connected to the point a_i on the C_a and the point b_i on C_b ($i = 2, 3 \ldots k$) using coincide command. Thus three sets of lines R_a, R_b and L_{ai} can made up k identical equivalent triangle Δ_{1i} ($i = 1, 2, \ldots k$), and the triangle Δ_{1i} can rotate with O_1.
3. Repeat Steps 1 and 2, construct a line L_{ci}, two circles C_c and C_d, and respectively dimensioned D_{ci}, R_c and R_d, O_2 as the center of the circle. Thus three sets of lines R_c, R_d and L_{ci} can made up k identical equivalent triangle Δ_{2i} ($i = 1, 2, \ldots k$), and the triangle Δ_{2i} can rotate with respect to O_2.
4. Repeat Step 1, construct three sets of lines L_{bi}, L_{di} and L_{ei} ($i = 1, 2, 3 \ldots k$).
5. Lastly, the two lines L_{ei} and L_{di} are connected to the point e_{i} at the ith position using the 'Coincide' command. The free ends of L_{ei} and L_{di} are connected to a_i on the C_a and the point d_i on C_d using the 'Coincide' command, respectively. The two ends of the L_{bi} are connected to b_i on the C_b and the point c_i on C_c.

8.3 Path Synthesis of the Simulated Stephenson Six-Bar Mechanism

The one point at the floating bar should move along the prescribed path relative to the base following the rule of the path synthesis. According to the requirements of the path synthesis, the six-bar mechanism of the approximate path synthesis is constructed on the basis of the basic simulation mechanism, as shown in Fig. 8.2. The input function is determined by the accurate angle Φ_{i} of the crank (the triangle



(a) Accurate path synthesis (b) Approximate path synthesis

Fig. 8.2 Path synthesis of the Stephenson six-bar simulated mechanism

$\varDelta_1$). The position of the output path is determined by the approximate position of the point e_i in the floating bar L_{ei}. At the ith position in Fig. 8.1, the end point p_i of the line S_i is as the accurate position point of the output path, the other point e_i of the line S_i is as the approximate position point of the output path. Obviously, the point e_i is restricted the arc C_s of the radius S_i, so any point on the C_s may be the approximate position of the bar L_{ei}. Thus not only the number of the mechanism constraint is reduced, but also the number of the prescribed synthesis position is added. The path synthesis of the Stephenson six-bar mechanism can be achieved at more positions. When the length of the radius S_i is gradually reduced to the allowable limit, the ideal simulation mechanism of approximate path synthesis can be obtained, as shown in Fig. 8.2b.

The construction process is described as follows:

1. Construct one datum line B, the two ends B_1 and B_2 of the line B are fixed, as shown in Fig. 8.1a.
2. Construct k lines P_i ($i = 1, 2, 3 \ldots k$), the end of the line P_i is connected to point B_2, as shown in Fig. 8.1b.
3. Construct k lines R_{ai} ($i = 1, 2, 3 \ldots k$), the two ends of line R_{ai} are connected to the points O_1 and a_i. The angle of the lines R_{a1} and R_{ai+1} ($i = 1, 2, 3 \ldots k$) is as input function, and label prescribed value.
4. Construct k lines S_i and dimension D_{si}, the two ends of the line S_i are connected to the other end of the line P_i and point e_i of the bar L_{ei}.
5. Gradually reduce the dimension D_{s1} until $D_{s1} = 1$ mm, the point pi and the point ei are almost coincident.
6. Label the prescribed value for angle θi and length of the line Pi. The maximum difference between the accurate k position and the actual position of the floating bar is defined by the dimension D_{s1}.

The results can be obtained from the approximate path synthesis of the six-bar simulation mechanism as follows:

- When the dimension of line S_1 reduced to zero ($D_{s1} = 0$), the approximate path synthesis and the accurate path synthesis of the simulated mechanism are the same, as shown in Fig. 8.2a. When the angle Φ_i is given with a prescribed value of the input function, and the angle θ_i and the length of the line P_i are given with the prescribed value for the output path, the maximum number of positions is found as 6 in the path synthesis of the simulated mechanism.
- When the length of the line S_1 is reduced to 1 mm ($D_{s1} = 1$ mm), the actual path position of the floating bar is close to accurate prescribed position, their difference is within the limit. Thus the simulated mechanism can achieve the approximate synthesis of the path position and orientation. Now the number k of the floating bar can be increased to 11, all the angle Φ_i, the angle θ_i and the length of the lines P_i can be given with initial values of driving dimensions. This shows that the maximum number of prescribed positions can be increased to 11, which are significantly more than 6 of the prescribed positions of the accurate path synthesis of the simulated mechanism. These results show that the six-bar mechanism can complete a path synthesis with more prescribed positions.

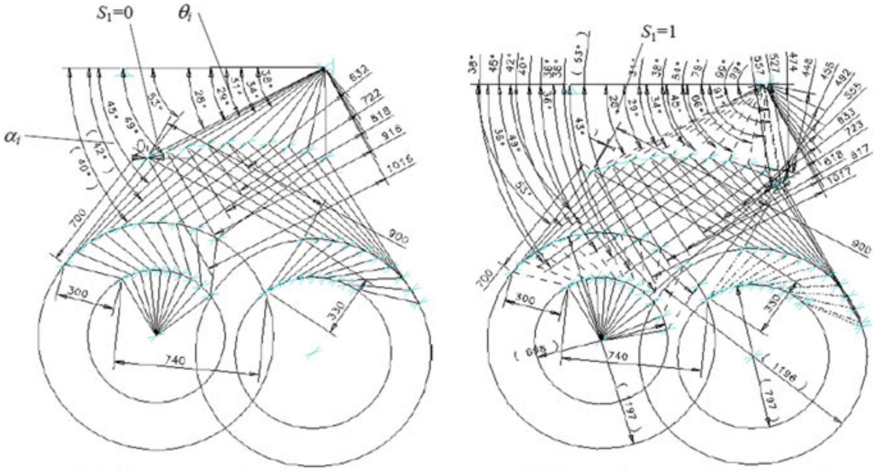
8.4 Motion Synthesis of the Simulated Stephenson Six-Bar Mechanism

According to the requirements of the motion synthesis, two kinds of motion syntheses of the six-bar mechanism are constructed for the basic simulated mechanism, as shown in Figs. 8.3 and 8.4, respectively.

In the first motion synthesis of the simulated mechanism, the azimuth angle of the floating bar L_{ei} is given with the prescribed accurate value at the k prescribed position, and the path position of the floating bar is given with an approximate value of the allowable limit. The constructed method of the first motion synthesis of the simulated mechanism is similar to the approximate path synthesis for the simulated mechanism. The prescribed angle αi ($i = 1, 2, 3 \ldots k{-}1$) between the line B and the bar L_{ei} will be replaced by using accurate input angles Φ_i, as shown in Fig. 8.3b.

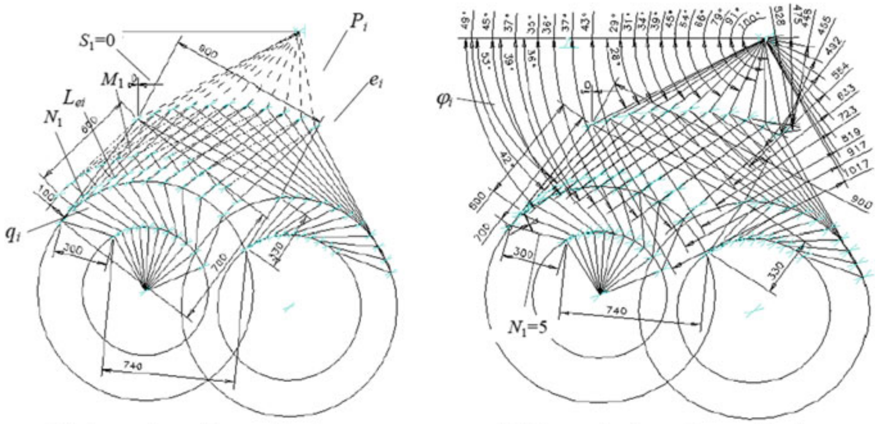
The result can be obtained from the approximate motion synthesis of the simulated six-bar mechanism as follows:

1. When the dimension of the line S_1 is reduced to zero ($D_{s1} = 0$), the actual path position of the floating bar L_{ei} is the same as the accurate path position at the k prescribed positions, as shown in Fig. 8.3a. The maximum number of positions is 4 in the motion synthesis of the simulated mechanism.
2. When the dimension of the line S_1 is reduced to 1 mm ($D_{s1} = 1$ mm), the actual path position of the floating bar L_{ei} is very close to the accurate path position at k prescribed position, as shown in Fig. 8.3b. Using the first six-bar simulation mechanism of approximate motion synthesis, which shows that the maximum



(a) Accurate motion synthesis (b) Approximate motion synthesis

Fig. 8.3 Motion synthesis of the first Stephenson six-bar simulated mechanism



(a) Accurate motion synthesis (b) Approximate motion synthesis

Fig. 8.4 Motion synthesis of the second Stephenson six-bar simulated mechanism

number of prescribed positions can be increased to 11, which is significantly more than 4 of the prescribed positions of the accurate motion synthesis of the simulated mechanism.

In the second motion synthesis of the simulated mechanism, the azimuth angle of the floating bar L_{ei} is given prescribed approximate value at k prescribed positions, and the path position of the floating bar is given with an accurate value of the allowable limit. When the dimension of line S_1 reduced to zero ($D_{s1} = 0$), the floating bar L_{ei} can obtain accurate position. In order to obtain approximate azimuth angle, the two sets of lines M_i and N_i ($i = 1, 2, 3 \ldots k{-}1$) are increased at the basic simulation mechanism in Fig. 8.1, as shown in Fig. 8.4.

The construction process is described as follows:

The first end of the line M_i is connected to the point e_i of the floating bar L_{ei}, and the second end of the line M_i is connected to the first end of line N_i. The second end of the line N_i is connected to the point q_i of the floating bar L_{ei} ($i = 1, 2, 3 \ldots k{-}1$), which is equivalent to a sliding pair of planar mechanism. Because the geometric constraints can make point q_i sliding in line L_{ei}, the line M_i can rotate with point e_i. When the azimuth angle ψi between line B and line Mi is given prescribed accurate value, the actual approximate azimuth angle Φ_{i} ($i = 1, 2, 3 \ldots k{-}1$) of the bar L_{ei} can be obtained between line B and line L_{ei}, as shown in Fig. 8.4. The results show that it not only reduces the constraint number of the second motion synthesis simulation mechanism, but also increases prescribed position number.

When the dimension of line N_i ($i = 1, 2, 3 \ldots k{-}1$) are gradually reduced to small enough, which can construct the motion synthesis of the second Stephenson simulated mechanism as shown in Fig. 8.4. The maximum difference $\Delta\psi_i$ ($i = 1, 2, 3 \ldots k$) between the angle ψ_i and angle φ_i can be obtained using the equation $\Delta\psi = |\varphi_i - \psi_i| = N_i/M_i$ at k prescribed positions. The motion synthesis of the second Stephenson simulated mechanism is under ideal conditions, so long as the $\Delta\psi$ is less than the allowable value.

The result can be obtained from the approximate motion synthesis of the second six-bar simulated mechanism as follows:

When the dimension of line S_1 is reduced to zero ($D_{s1} = 0$), and the dimension of line N_1 is reduced to small enough, and the dimension of the line M_1 is given with an equal value (700 mm) with the floating bar, the actual path position of the floating bar is the same as its exact position, as shown in Fig. 8.4. Based on the approximate motion synthesis of the second six-bar simulated mechanism, the maximum number of the prescribed positions can be increased to 11, which is significantly more than 4 of the prescribed positions of the accurate motion synthesis of the simulated mechanism, as shown in Fig. 8.4.

8.5 Conclusion

The path and motion simulation mechanism have been constructed by using the CAD geometric approach. The six-bar approximate simulation mechanism obtained by using this approach can be used for path and motion syntheses at 11 prescribed positions, which are more than the number of the accurate simulation mechanisms. The results show that the CAD geometric approach is equivalent to the geometric method and the analytic method but the CAD geometric approach is more cost-effective, visualised, accurate and repeatable. Therefore this method can be not only used in simple mechanism, but also can be used to analyse complex mechanisms.

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