



# Balancing of a Wire Rope Hoist Using a Cam Mechanism

Jacek Buśkiewicz<sup>(✉)</sup>

Poznan University of Technology, Poznań, Poland  
jacek.buskiewicz@put.poznan.pl

**Abstract.** To design machines that perform their mechanical functions with minimal power consumption is an important and challenging issue. It may be obtained by applying a gravity balanced mechanical systems. The objective of the paper is to design a cam-roller follower mechanism to gravity balance a wire rope hoist for lifting loads. It is analysed how various geometric features affect the applicability of the cam mechanism (jerks, pressure angle, roller radius). The paper provides the method to synthesize cams with different geometries realizing the same performance, among which a designer can choose the one that is optimal with respect to strength properties.

**Keywords:** Cam · Wire rope hoist · Gravity balancing

## 1 Introduction

Lifts, hoists, elevators and all the machines that raise or lower a load have an enormous number of applications in industry. In numerous cases they require high power engines to operate. The minimization of power consumption then is an important issue. It may be obtained, for example, by applying an additional, i.e. gravity balanced, mechanical system. The literature on gravity balancing is very extensive. The attention is focused on spring-linkage systems. Either zero-free-length springs or non-zero-free-length springs are utilised to statically balance various spatial and planar mechanical systems [1–12]. Compared to counterweights, springs do not increase significantly the mass of the mechanism. Cams are also used to counterbalance mechanisms. An interior cam mechanism is applied as gravity-balancing mechanism for robot arms [13]. Cam is a convenient device for transforming one type of motion into another. The large number of cam-follower combinations has been designed and applied in various applications [14–19]. Their major advantage over the mechanisms with lower kinematic pairs is simpler synthesis [14]. Nonetheless, a lower number of scientific papers deals with the applications of cams in balancing of various systems. It is not a trivial problem to effectively optimise the cam geometry for the prescribed performance. The influence of various parameters on under-cutting and pressure angle problems is underlined in [14].

The objective of the paper is to design a cam mechanism to gravity balance a wire winch hoist for raising a load. A cam-roller follower along with spring is proposed to either minimise the power of the engine or shorten the time of lifting. The cams are synthesized for a prescribed set of initial parameters: the load mass, the height to which

a load is elevated and the spring stiffness. It is analysed how the geometric features affect the properties of cams which are responsible for a proper machine functioning (jerks, pressure angle, roller radius). Such an analysis of cams applied in the gravity balanced cable drum hoists is not widely dealt with in the literature.

## 2 The Concept of the Mechanical System

Figure 1 presents the system which consists of the rope drum (winch), spur gears, cam-roller follower and spring. The sizes of all the elements may not correspond to their sizes in an actual hoist as the scheme is to distinctly illustrate all the components. The drum wraps around a rope with the platform attached at the free end. The weight of the load determines the power of the hoist engine. To increase the area of the possible applications, spur gears are applied. Without gears the hoist elevates the loads to relatively low heights (i.e. devices for elevating patients with disabled neuro-muscular systems) – the working cycle lasts one full rotation of the cam and drum. One spur gear and rope drum are assembled along with an engine on the first shaft, whereas the other gear and cam are mounted on the other shaft. When a load is being elevated, the cam through the roller follower compresses the spring in such a way that the potential energy of the system is conserved. Let the following data be given:

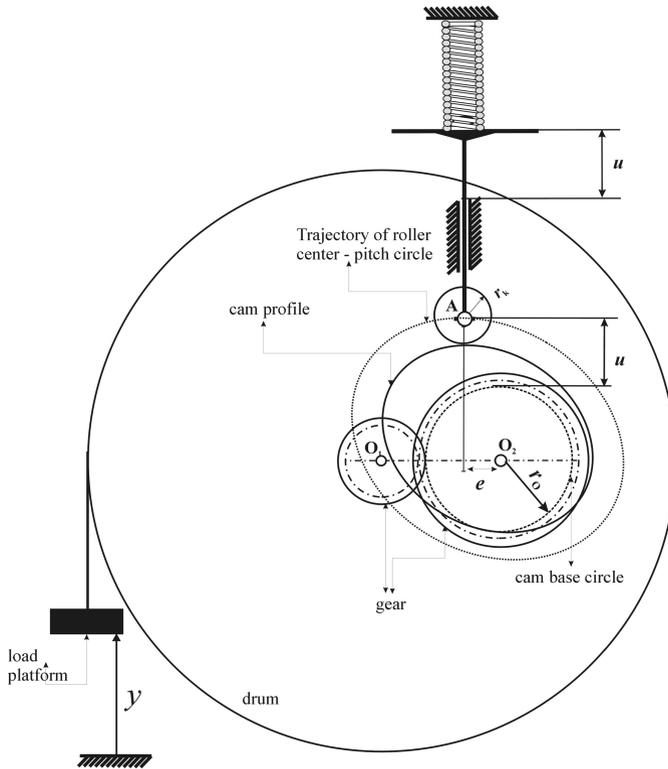
- $m$  - the mass of the hoist platform and load,
- $k$  - the spring stiffness,
- $a$  - the radius of the rope drum,
- $y_{\max}$  - the maximum height of the platform,
- $i$  - gear transmission ratio.

It is also denoted:

- $u$  - the displacement of the follower and spring compression,
- $u_{\max}$ ,  $u_{\min}$  - the maximum and minimum spring compression,
- $y$  - the height of the platform measured from the lowest position,
- $\phi$  - the angular position of the cam ( $\phi$  varies from 0, corresponding to the lowest platform position, and  $\phi_{\max}$  at which the platform is in the highest position),
- $r_0$  - the radius of the cam base circle,
- $e$  - the eccentricity of the follower,
- $r_k$  - radius of the roller.

The displacement of the follower is derived from the principle of the potential energy conservation. If the potential energy is conserved for any platform position, the mechanism is in equilibrium at each position. Assuming that friction forces can be disregarded, the total energy is the sum of the gravitational potential energy of the load and elastic spring potential energy.

$$V = \frac{1}{2}ku^2 + mgy = C. \quad (1)$$



**Fig. 1.** The concept of a hoist.

When the cam rotates about  $\varphi$ , the drum rotates by  $i\varphi$  and, as a consequence, the length of the rope wrapped around the drum equals  $ai\varphi$ . Then by equalling the potential energies at the highest and arbitrary positions.

$$V = \frac{1}{2}ku_{\max}^2 = \frac{1}{2}ku^2 + mgai\varphi, \quad (2)$$

one obtains the follower displacement:

$$u(\varphi) = \sqrt{u_{\max}^2 - \frac{2mgai\varphi}{k}}, \quad (3)$$

where:  $0 \leq \varphi \leq \varphi_{\max}$ . The parameters:  $u_{\max}$ ,  $m$ ,  $a$ ,  $k$  have to be prescribed.

The hoist then is designed for a preset mass of the load. In order not to change the mechanism when mass  $m$  of the load is changed, one has to select new stiffness  $k$  of the spring. The compression function  $u$  fails to change, when  $m/k$  is constant. The radius  $a$  cannot be changed since then  $y_{\max}$  will change.

*Properties of the cam-roller follower*

The displacement of the follower is frequently defined in the intervals to regard various phases of the mechanism operation. The displacement is measured from the base circle to the tracer point i.e. the centre of the roller which generates the pitch circle.

$$f = \begin{cases} s_1, 0 \leq \varphi < \varphi_1 \\ s_2, \varphi_1 \leq \varphi < \varphi_2 \\ \dots \\ s_n, \varphi_{n-1} \leq \varphi < \varphi_n \end{cases} \quad (4)$$

The pressure angle is an important parameter for a proper functioning of the cam. It is the angle between the axis of the follower and the normal to the cam profile. The angle is kept as small as possible. If this angle becomes too large, the cam shaft will be excessively bent.

In the case of the cam-roller follower, the tangent of this angle is expressed as:

$$\operatorname{tg} \alpha = (f_{,\varphi} - e) / f. \quad (5)$$

Sometimes, it is convenient to assemble the follower eccentrically with respect to the cam shaft. When eccentricity  $e$  is non-zero, the trajectory coordinates of roller centre A (pitch circle) for cam angular position  $\varphi$  are as follows.

$$y_A = -e \sin \varphi + f \cos \varphi, \quad (6.1)$$

$$x_A = e \cos \varphi + f \sin \varphi. \quad (6.2)$$

To prevent from under-cutting effect and enable the proper motion of the roller on the cam, the roller radius must be less than the curvature radius of the pitch circle. The curvature formula is expressed as follows:

$$1/\rho = \kappa = \frac{f^2 + 2f_{,\varphi}^2 - ff_{,\varphi\varphi}}{(f^2 + f_{,\varphi}^2)^{3/2}}. \quad (7.1)$$

When the follower is shifted eccentrically, it is convenient to compute the curvature using formula:

$$1/\rho = \kappa = \frac{y_{A,\varphi\varphi}x_{A,\varphi} - y_{A,\varphi}x_{A,\varphi\varphi}}{(x_{A,\varphi}^2 + y_{A,\varphi}^2)^{3/2}}. \quad (7.2)$$

The cam profile is the envelope of circles with radii equal to the roller radius and centred at point A.

$$F(x_{\text{OK}}, y_{\text{OK}}, \varphi) = (x_{\text{OK}} - x_{\text{A}}(\varphi))^2 + (y_{\text{OK}} - y_{\text{A}}(\varphi))^2 - r_k^2 = 0, \quad (8.1)$$

$$\frac{\partial F(x_{\text{OK}}, y_{\text{OK}}, \varphi)}{\partial \varphi} = -2(x_{\text{OK}} - x_{\text{A}}(\varphi)) \frac{dx_{\text{A}}(\varphi)}{d\varphi} - 2(y_{\text{OK}} - y_{\text{A}}(\varphi)) \frac{dy_{\text{A}}(\varphi)}{d\varphi} = 0. \quad (8.2)$$

The cam profile is then computed as the equidistance of the pitch circle using the following equations:

$$x_{\text{OK}} = x_{\text{A}} + r_k y_{\text{A},\varphi} / \sqrt{x_{\text{A},\varphi}^2 + y_{\text{A},\varphi}^2}, \quad (9.1)$$

$$y_{\text{OK}} = y_{\text{A}} - r_k x_{\text{A},\varphi} / \sqrt{y_{\text{A},\varphi}^2 + x_{\text{A},\varphi}^2}, \quad (9.2)$$

which satisfy Eqs. (8.1 and 8.2). The greater roller radius, the smaller stresses in the contact surface of the cam and roller.

### 3 Numerical Analysis and Discussion

It is assumed that the following parameters are given:

- spring stiffness  $k = 40000$  N/m,
- load and platform mass  $m = 50$  kg,
- maximum load rise  $y_{\text{max}} = 2$  m,
- eccentricity  $e = 0, 0.1, 0.15$  m,
- gear transmission ratio  $i = 1, 2$ ;
- minimum spring compression  $u_{\text{min}} = 0.1, 0.2$  m.

The solutions for various: minimum spring compression, eccentricity  $e$ , and transmission ratios are presented. The displacement of the follower is determined from the formula resulting from the principle of the potential energy conservation. This formula is not harmonic, therefore the cam profile obtained for the full rotation would be open. It is assumed that the cam turns by  $\varphi_{\text{max}} = 3/2\pi$  when raising a load to the highest level. The cam profile for the angle from  $\varphi_{\text{max}}$  to  $2\pi$  is determined for the following, prescribed motion conditions:

- $s_2 = \sum_{i=0}^5 a_i t^i$ ,
- the contour must be continuous,
- the velocity and acceleration of the follower at the boundary points  $\varphi_{\text{max}}, 2\pi$  are continuous.

Velocities and accelerations are not essential in this analysis, but the demand for the continuity up to the second derivatives ensures the continuous curvature along the whole profile. Then the follower displacement measured from the base circle is as follows:

$$f = \begin{cases} s_1 = \sqrt{u_{\max}^2 - \frac{2mga_i\phi}{k}}, 0 \leq \phi < \phi_{\max} \\ s_2 = \sum_{i=0}^5 a_i t^i, \phi_{\max} \leq \phi < 2\pi \end{cases}, \quad (10)$$

As the minimum string compression is non-zero, the displacement function  $f$  is also non-zero, and, as a consequence, the base circle radius can be 0.

Table 1 presents the parameters corresponding to the cases that are analysed. Parameters:  $u_{\min}$ ,  $i$  and  $e$  are the input data, whereas the remaining  $u_{\max}$ ,  $r_k$ ,  $\alpha$  and  $a$  are computed as the functions of the input parameters. The spring is maximally compressed in the initial (lowest) position, whereas the spring compression is minimal when the load is in its upper position.

**Table 1.** The set of geometric parameters of the system

Case	$u_{\min}$ [m]	$i$	$e$ [m]	$u_{\max}$ [m]	$r_k$ [m]	$\alpha$ [°]	$a$ [m]
I	0.1	1	0	0.243	<0.09	<25	0.424413
II	0.1	1	0.1	0.243	<0.12	<26	0.424413
III	0.1	1	0.15	0.243	<0.15	<45	0.424413
IV	0.1	2	0	0.243	<0.09	<25	0.212207
V	0.1	2	0.1	0.243	<0.12	<26	0.212207
VI	0.2	1	0	0.298412	<0.22	<10	0.424413
VII	0.2	1	0.1	0.298412	<0.23	<22	0.424413

The positive  $e$  is measured horizontally to the left direction, the displacement of the follower to the right worsens the pressure angle significantly. It can be seen from Table 1 that:

- The higher eccentricity  $e$ , the greater pressure angle. Nevertheless, the angle increases significantly to magnitude generally considered as unacceptable, when  $e$  is greater than 0.1.
- Higher eccentricity  $e$  is accompanied by a less curvature. Higher curvature radius allows to apply a larger roller, which decreases the contact forces between the roller and cam.
- The higher minimum spring compression, the larger size of the cam and the larger curvature radiuses.

It is obvious that the higher transmission ratio, the smaller radius of the drum.

In cases I–V the minimum spring compression is the same and equal to 0.1. Similarly, the maximum spring compressions are equal to each other as they depend on the minimum spring compression and the product  $ia$ . Displacement function  $u$  is the function of the maximum spring compression and product  $ia$ . The curvature and pressure angle depend on the displacement function that is function of  $e$ .

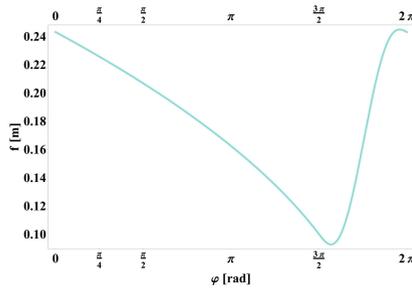
The displacement function for cases I–V is expressed by:

$$s_1(\varphi) = \sqrt{0.05905 - 0.0104087\varphi}, \text{ for } 0 < \varphi < \varphi_{\max},$$

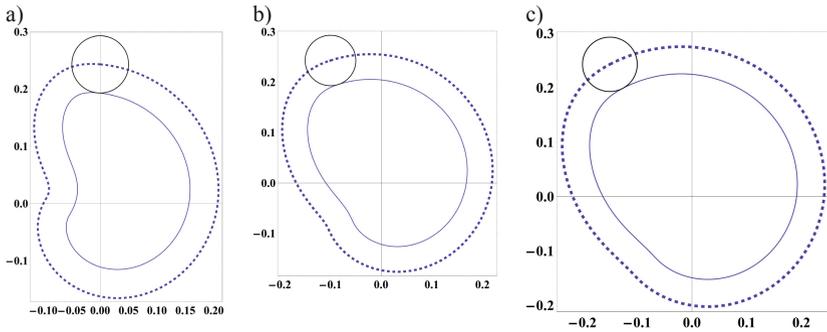
$$s_2(\varphi) = -611.797 + 570.786\varphi - 211.543\varphi^2 + 38.9312\varphi^3 - 3.55765\varphi^4 + 0.129171\varphi^5,$$

for:  $\varphi_{\max} < \varphi < 2\pi$ ,

and shown in Fig. 2. Cam profiles in the initial positions are shown in Fig. 3.

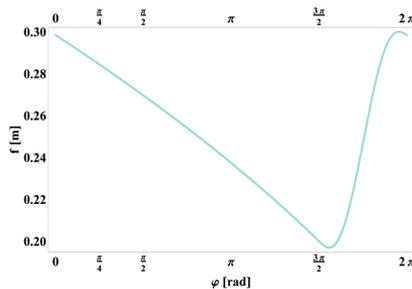


**Fig. 2.** The displacement function of the follower in cases I–V.

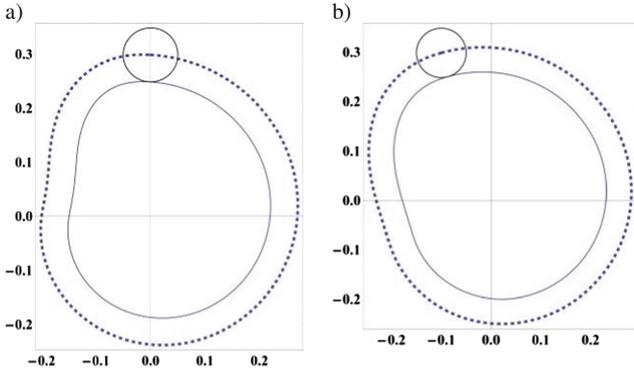


**Fig. 3.** Cam profiles (continuous line), pitch circles (dotted line) and roller (disc): cases I, IV (a), cases II, V (b), case III (c)

The displacement function for cases VI–VII is shown in Fig. 4. The cam profiles in the initial positions are shown in Fig. 5.

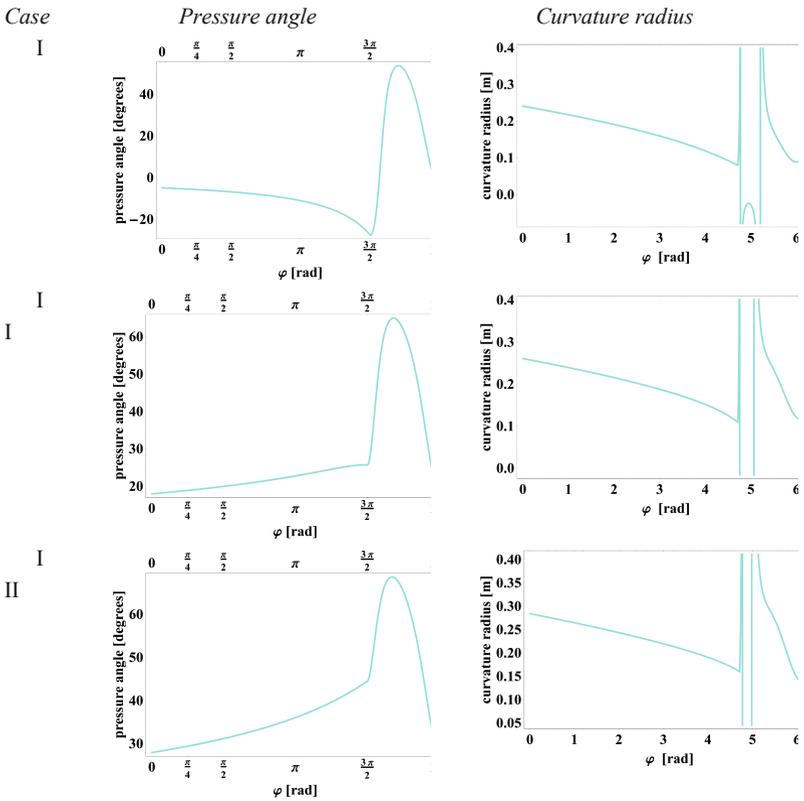


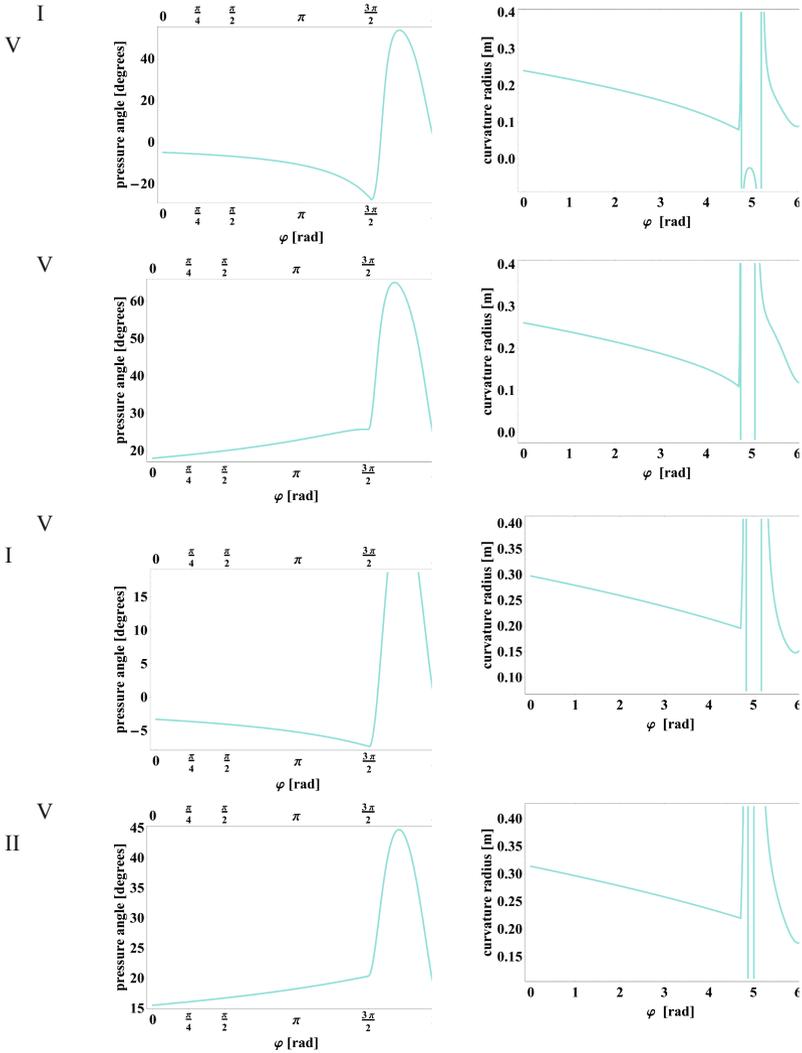
**Fig. 4.** The displacement function of the follower in cases VI and VII.



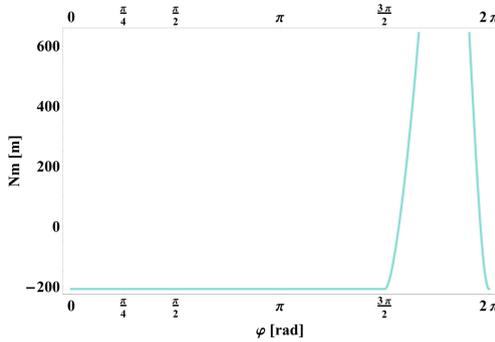
**Fig. 5.** Cam profiles (continuous line), pitch circles (dotted line) and roller (disc): case VI (a), case VII (b)

**Table 2.** The evaluations of pressure angles and curvature radiuses





The magnitudes of pressure angles and roller radii are read from graphs gathered in Table 2. The working phase is up to angle  $\varphi_{\max}$ . A curvature radius at the boundaries is discontinuous as the profile has the inflexion points at the passing between the convex and concave parts. In order to verify the correctness of the profile cam, the moment of the pressure force (exerted by the spring) between the cam and roller about cam axis of rotation is computed and presented in Fig. 6. The geometry from case V is considered as the example. In the working phase the moment is constant and equal to the moment required to counterbalance the load weight. It can be also observed that in the non-working part of the cam profile there are two positions at which the moment equals zero i.e. the equilibrium positions of the unloaded system.



**Fig. 6.** The moment of the pressure force between the cam and roller about the cam centre (case V)

## 4 Conclusions

The applications of cams are very extensive as they can be synthesised to precisely perform various technical functions. It has been confirmed that they can be considered as parts of gravity balanced mechanisms. The paper provides the method to synthesize cam-roller follower mechanisms with different geometries realizing the same performance, among which a designer can choose the one which is optimal with respect to i.e. strength properties. Aspects of strength analysis are not presented here. The other problems not solved here are:

- the return of the platform after discharging a load,
- comparison analysis of the cam with roller follower and cam with flat-faced follower,
- the influence of assembly tolerances and friction forces on the balancing.

Because the size of the cam is independent of the gear transmission, the higher gear transmission is, the more sensitive on the manufacturing and assembling tolerances the system. Moreover, the system must have higher stiffness. The compensation of these phenomenon is an interesting problem also not discussed in here. The paper provides a background for further detailed studies.

**Acknowledgments.** The research work reported here was made possible by grant 02/21/DSPB/3513.

## References

1. Nathan, R.H.: A constant force generation mechanism. *Trans. ASME* **107**, 508–512 (1985)
2. Shin, E., Streit, D.A.: Spring equilibrators theory for static balancing of planar pantograph linkages. *Mech. Mach. Theory* **26**(7), 645–657 (1991)

3. Gopalswamy, A., Gupta, P., Vidyasagar, M.: A new parallelogram linkage configuration for gravity compensation using torsional springs. In: Proceedings of the IEEE International Conference on Robotics and Automation (ICRA 1992), Nice, France, pp. 664–669 (1992)
4. Streit, D.A., Shin, E.: Equilibrators for planar linkages. *J. Mech. Des.* **115**, 604–611 (1993)
5. Pons, J.L., Ceres, R., Jimenez, A.R.: Quasi-exact linear spring counter gravity system for robotic manipulators. *Mech. Mach. Theory* **33**, 59–70 (1998)
6. Herder, J.L.: Design of spring force compensation systems. *Mech. Mach. Theory* **33**, 151–161 (1998)
7. Ono, Y., Morita, T.: An underactuated manipulation method using a mechanical gravity canceller. *J. Robot. Mechatron.* **106**(6), 563–569 (2004)
8. Lin, P.Y., Shieh, W.B., Chen, D.Z.: Design of perfectly statically balanced one-DOF planar linkage with revolute joints only. *J. Mech. Des.* **131**(5), 051004 (2009)
9. Lin, P.Y., Shieh, W.B., Chen, D.Z.: A stiffness matrix approach for the design of statically balanced planar articulated manipulators. *Mech. Mach. Theory* **45**, 1877–1891 (2010)
10. Lin, P.Y., Shieh, W.B., Chen, D.Z.: Design of statically balanced planar articulated manipulator with spring suspension. *IEEE Trans. Rob.* **28**(1), 12–21 (2012)
11. Deepak, S.D., Ananthasuresh, G.K.: Perfect static balancing of linkages by addition of springs but not auxiliary bodies. *ASME J. Mech. Rob.* **4**(2), 021014 (2012). <https://doi.org/10.1115/1.4006521>
12. Deepak, S.R., Ananthasuresh, G.K.: Static balancing of a four-bar linkage and its cognates. *Mech. Mach. Theory* **48**, 62–80 (2012)
13. Koser, K.: A cam mechanism for gravity-balancing. *Mech. Res. Commun.* **36**, 523–530 (2009)
14. Erdman, A.G., Sandor, G.N., Kota, S.S.: *Mechanism Design: Analysis & Synthesis*, 4th edn (Web Enhanced), vol. 1. Prentice-Hall, Upper Saddle River (2001)
15. Ananthasuresh, G.K.: Design of fully rotatable, roller-crank-driven, cam mechanisms for arbitrary motion specifications. *Mech. Mach. Theory* **36**, 445–467 (2001)
16. Navarro, O., Wu, C.J., Angeles, J.: The size-minimization of planar cam mechanisms. *Mech. Mach. Theory* **36**, 371–386 (2001)
17. Ye, Z., Smith, R.M.: Synthesis of constant-breadth cam mechanisms. *Mech. Mach. Theory* **37**, 941–953 (2002)
18. Ottaviano, E., Mundo, D., Danieli, G.A., Ceccarelli, M.: Numerical and experimental analysis of non-circular gears and cam-follower systems as function generators. *Mech. Mach. Theory* **43**(8), 996–1008 (2008)
19. Marcinkevicius, A.H.: Synthesis of a cam system for the measurement and follow rest of multi-step shaft necks at machining. *Mech. Mach. Theory* **42**, 1029–1042 (2007)