Chapter 15
The Professional Development of Mathematics Teachers

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Abstract The chapter offers an overview of different approaches to the professional development of mathematics teachers. Starting point is the expert-novice-approach establishing both a distinction between experts and novices as well as an attempt to characterise expertise. The question of how to conceptualise professional competence of mathematics teachers is subsequently deepened by a detailed description of related prominent theoretical and empirical approaches followed by a discussion of central empirical results. The chapter closes with a short summary of recent research emphases especially taking into account the various discussions on teachers' professional development at ICME-13.

Keywords Professional development · Expert-novice-approach · Mathematics teachers' professional competence · Mathematics teachers' knowledge

15.1 Introduction

Professional development of teachers is currently seen as a central influential factor for the efficiency of school education, which has been shown by empirical results identifying relations between teachers' professional knowledge and students' achievement (e.g., Blömeke and Delaney 2012). However, reflections on the professional development of mathematics teachers put two different perspectives in the foreground, namely, the development of a teacher from a novice to an expert, and conceptualizations and assessments of teachers' professional competence and its development. Naturally, both perspectives overlap in manifold ways and cannot be separated in a strict way.

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The focus of this paper is set at a general level, as differences between teacher education systems at least across countries, often also within countries, are significant (cf. Blömeke and Kaiser 2012 with an approach to cross-nationally identify profiles of teacher education). Overall, the paper aims to offer an overview on important trends within the professional development of mathematics teachers and related discussions. However, the aim is not an entire and detailed description of the aspects mentioned. To use a metaphor, the paper unfolds a map of some central areas concerning the landscape of teachers' professional development. This might help the reader to identify the area in which she or he is interested, and to find the beginning of paths to an in-depth-analysis in this area.

In the following we use as departure point for the description, the currently most influential approach for the professional development of teachers, namely the expert-novice-approach. Its importance also follows from its inherent combination of two central aspects of expertise: an also intuitionally plausible distinction between experts and novices for describing various degrees of experience in a field of profession and a theoretical sound foundation of characteristics of expertise. Especially the latter allows several opportunities for operationalising expertise and led to many empirical studies in the last two decades based on this theoretical approach.

15.2 The Expert-Novice Approach

Discussions about teachers' expertise can be characterised to a large extent by the fundamental distinction between experts and novices, an approach strongly influenced by Berliner (2004). Even though this distinction seems to be self-evident as especially older discussions often focused only on experts as humans who are exceptional in their domain. Against this background the cognitive processes or structures, conditions under which these humans exceptionally perform, or the way in which they practice are analysed (Chi 2011). In contrast to this absolute approach, subsequently the relative approach was introduced building up on the distinction between experts and novices (Chi 2011).

Independently from the chosen approach, a central problem is the identification of expert teachers. As there is no consensus on this question across various studies, this leads to different criteria for what makes a teacher an expert teacher. Examples of characteristics used in such identification are their years of teaching experience, their educational background, their academic performance during their education, or estimation or recommendation by peers or administrators (Li and Kaiser 2011). Furthermore also the particular researcher's beliefs about what characterizes an expert or what constitutes a 'good' performance in the classroom, influences the particular identification of expert teachers (Schoenfeld 2011).

Coming back to the distinction between the absolute and the relative approach reveals several advantages of the relative approach. Thus, in a relative approach an expert is not regarded as an exceptional individual but as someone who is just more advanced in terms of the various measurements of expertise (e.g., degrees, professional assessment, years of experience). Conceptualizing an expert this way allows the assumption that a novice can become an expert, i.e., expertise in the relative approach can be defined by the teachers' knowledge in contrast to innate capability in the absolute approach (Chi 2011).

Concerning the distinctions between experts and novices, it is obvious that a simple distinction only between experts and novices might be too imprecise for several reasons. Instead, a more precise distinction with intermediate levels seems to be an appropriate way to describe the professional development of a mathematics teacher. Following different theoretical backgrounds leads to different distinctions, both with regard to the number of stages, as well as with regard to the description of each particular stage. Before introducing his own model, Berliner (2004) for example distinguished the following theoretical approaches and their related stage-models:

- Studies of Psychomotor Learning: A novice stage, an intermediate stage and a stage with high levels of performance are distinguished. The focus strongly lies on psychomotor skills, whereas, for example, mistakes are characteristic of the novice stage, and the development of automaticity characterizes the intermediate stage. Accordingly, this approach is not an adequate attempt to describe the professional development in cognitive skills.
- Cognitive Psychology: Also from this perspective, several stages can be distinguished. The stages differ with regard to changing agency, that is, a progression in the stages comes along with a decreasing proportion of support and an increase of self-controlled learning processes. The model seems to be adequate for describing the development of learning in areas of individual performances (e.g., chess) but is less helpful for areas with a stronger social influence on behaviour. Hence, it is less adequate for describing the learning processes of teachers.
- Model of Domain Learning: The focus of this model is the development when attempting to learn a discipline such as mathematics. It again differentiates between three stages, a stage of acclimation to the appropriate discipline, a stage of competence, and a stage of proficiency or expertise (Alexander 1997). The stages, though, follow the process of learning a subject beginning with fragmentary knowledge and ending with integrated knowledge. Because teachers, as part of their professional development, also have to learn topics of several subjects, the model is helpful for describing the development of teachers' expertise with regard to subject matter knowledge. In contrast, it is less helpful for describing the development of pedagogical skills.

All that these models have in common is that they are at most partly suitable for describing the professional development of teachers. Therefore, Berliner (2004) introduced his fundamental five-stage theory, which was developed with reference to Dreyfus and Dreyfus (1986) as a central heuristic theory. The model is based on studies comparing teachers in different phases of their professional development.

Thus, in contrast to the models described above, it was explicitly developed for analysing teachers' professional development. In addition, it offers a rough orientation suggesting how much time a teacher normally takes to reach each particular stage. The five stages can be summarised briefly as follows (Berliner 2004, pp. 205–208):

- Novice stage: This first stage is the stage of student teachers and first-year teachers. "At this stage, the commonplaces of an environment must be discriminated, the elements of the tasks to be performed need to be labeled and learned, and the novice must be given a set of context free rules" (p. 205). Normally the novice is quite inflexible and follows given rules.
- Advanced beginner: This stage is normally reached by second- and third-year teachers. "This is when experience can become melded with verbal knowledge, where episodic and case knowledge is built up. Without meaningful past episodes and cases to relate the experience of the present to, individuals are unsure of themselves; they do not know what to do or what not to do. Through case knowledge, similarities across contexts can be recognized." (p. 206). In particular, this stage has been reached when the acquisition of practical knowledge starts, which will continue during the following stages. This aspect is especially important as "it is practical knowledge, not theories or textbooks, that is the proximal guide for a good deal of a teacher's classroom behaviour" (p. 206).
- Stage of competence: Although not every advanced beginner reaches this stage, the regular case is that teachers come up to this stage in their third to fifth year or later. Teachers in this stage "make conscious choices about what they are going to do. They set priorities and decide on plans. They have rational goals and choose sensible means for reaching the ends that they have in mind. [...] While enacting their skills, they can determine what is and what is not important. From their experience, they know what to attend to and what to ignore" (p. 207).
- Proficient stage: This stage is the first stage that is not regularly reached by many teachers, but instead is reached by only a small number after about five years. "This is the stage at which intuition or know-how becomes prominent" (p. 207). Due to their experience "at some higher level of pattern recognition, the similarities between disparate events are understood" (p. 207). Proficient teachers can use this understanding of similarities to predict possible problems and counteract the problems in advance.
- Expert stage: This stage is the highest level, reached by only a few teachers. It is harder to discriminate this stage from the proficient stage than to discriminate the other stages from each other. "Experts have both an intuitive grasp of the situation and seem to sense in nonanalytic and nondeliberative ways the appropriate response to be made. They show fluid performance" (p. 207). The behaviour of expert teachers corresponds with Schön's (1983) discussion of the practitioner's knowledge-in-action and Polya's (1954) considerations of the role of tacit knowledge in the process of problem solving.

However, independent of the concrete research attempt and the concrete distinction between experts and novices, there are some core ideas of expertise, which can be identified across these research approaches. Central aspects of these core ideas are a broad and substantial subject-related knowledge together with deep representations of the taught mathematics topics and better strategies in problem-solving processes. Concerning the teaching process, experts show a higher flexibility and the use of automatisms for recurrent teaching activities. A very important aspect of teachers' expertise furthermore is a fast, holistic and accurate perception of classroom situations together with a categorial interpretation of these situations using categories for pattern identification, based on their knowledge and previous experiences. This category-led interpretation thereby allows teachers to make fast and meaningful decisions for the further process of instruction, to recognize and anticipate problems, and to react sensibly (see, e.g., Chi 2011; Kaiser and Li 2011; Berliner 2001, 2004).

Finally, this combination of both a widely accepted core of expertise and different attempts to characterize expertise in particular, leads to the question of how to conceptualize professional competence, which is dealt with in the next section.

15.3 Conceptualisation and Assessment of Mathematics Teachers' Professional Competence

Along with various approaches to expertise as described in the last section there are also different conceptualisations of professional competence of mathematics teachers, both from a theoretical as well as from an empirical perspective. In the following, prominent concepts are described mainly in the order in which they were developed.

The central starting point for the more recent discussions concerning teachers' professional competence is the famous paper by Shulman (1986) in which he distinguished several areas of teacher knowledge. In a first step, he differentiated between general pedagogical knowledge and content knowledge. With regard to the teaching of mathematics, especially the subsequent distinction between several categories of content knowledge is of special interest. In this regard Shulman (1986) distinguished the following categories:

- Subject matter content knowledge: This area covers the body of knowledge of the domain, in this context mathematics, but also covers aspects "going beyond knowledge of the facts or concepts of a domain" (p. 9). The latter means that also knowledge about the structure of the particular subject is necessary for a teacher. "The teacher needs not only understand that something is so; the teacher must further understand *why* it is so […]. Moreover, we expect the teacher to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral" (p. 9).

- Pedagogical content knowledge: Shulman described this area as "subject matter knowledge *for teaching*" (p. 9). It covers knowledge about the typical representations of topics to be taught as well as knowledge about typical students' preconceptions when learning a topic. As these preconceptions also can be misconceptions, this area further includes knowledge about how to deal with those misconceptions.
- Curricular knowledge: This category focuses the area of knowledge about the whole field of curriculum in a wider sense. "The curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10). The teacher needs to know about this curriculum to choose the parts of it that are relevant for teaching. In addition, she or he should know when teaching a class in a certain grade about the curriculum of the preceding and following grades and about the curriculum in other subjects at the same grade.

Although regarded as a milestone and still often referred to in recent studies, Shulman's position was also criticized from different perspectives. Amongst others, it was emphasised that Shulman's distinction implies a certain image of the taught subject (Meredith 1995). With regard to teaching mathematics, Meredith specifies "that the concept of pedagogical content knowledge […] is perfectly adequate if subject knowledge is seen as absolute, incontestable, unidimensional and static. On the other hand, teachers who conceive of subject knowledge as multidimensional, dynamic and generated through problem solving may require and develop very different knowledge for teaching" (p. 184).

Fennema and Franke (1992) formulated a critique from another perspective, demanding a more precise consideration of interaction processes between students and teachers. They claimed that "teachers' use of their knowledge must change as the context in which they work changes" (p. 162), as for example the students change during the process of teaching and learning. Therefore, they further developed the model by Shulman by integrating the "interactive and dynamic nature" (p. 162) of teacher knowledge and set "each component in context" (p. 162). The resulting model is illustrated in Fig. 15.1.

Fennema and Franke (1992) explained their model as follows: "The center triangle of our model indicates the teachers' knowledge and beliefs in context or as situated. The context is the structure that defines the components of knowledge and beliefs that come into play. Within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that drives classroom behaviour" (p. 162).

Another prominent critical position taken against Shulman's approach considered the description of the various areas of content knowledge as not sufficiently precise, for example with regard to an operationalisation. One approach to fill this

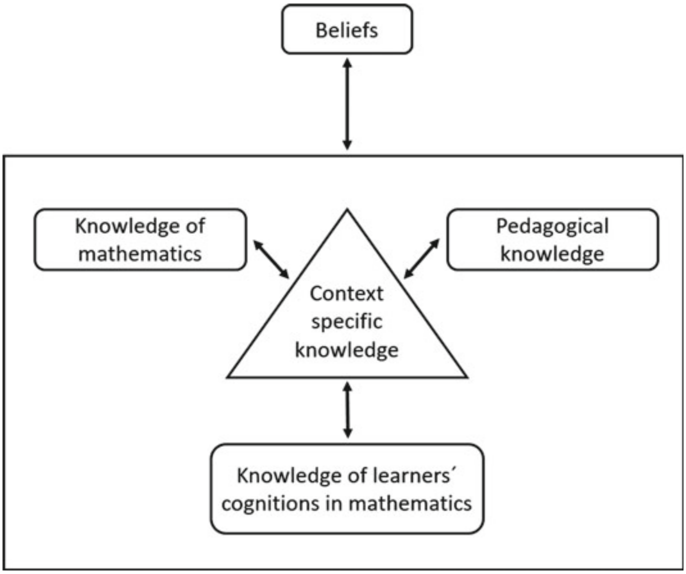

Fig. 15.1 Teachers' knowledge developing in context (Fennema and Franke 1992, p. 162)

gap was the attempt by researchers of the University of Michigan within the "Mathematics Teaching and Learning to Teach Project" and the "Learning Mathematics for Teaching Project" (Ball et al. 2008). The theoretical focal point of the projects was the concept of "mathematical knowledge for teaching" understood by the group as "mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). The construct consists of several domains, which can be assigned to subject matter knowledge and pedagogical content knowledge according to Fig. 15.2.

The domains can be summarised as follows:

- Common content knowledge: This is "the mathematical knowledge and skill used in settings other than teaching" (Ball et al. 2008, p. 399) and is reasoned by the fact that teachers of course also need to have knowledge about mathematics itself.
- Specialized content knowledge: This "is the mathematical knowledge and skill unique to teaching" (p. 400) and therefore furthermore "mathematical knowledge not typically needed for purposes other than teaching" (p. 400). This domain for example covers the identification of patterns in mathematical errors.
- Knowledge of content and students: This "is knowledge that combines knowing about students and knowing about mathematics" (p. 401). This for example contains the knowledge about what students are likely do with an assigned task or what might confuse students. Another example of this domain is knowledge about students' conceptions or misconceptions about certain topics. This domain therefore is related to the "interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (p. 401).

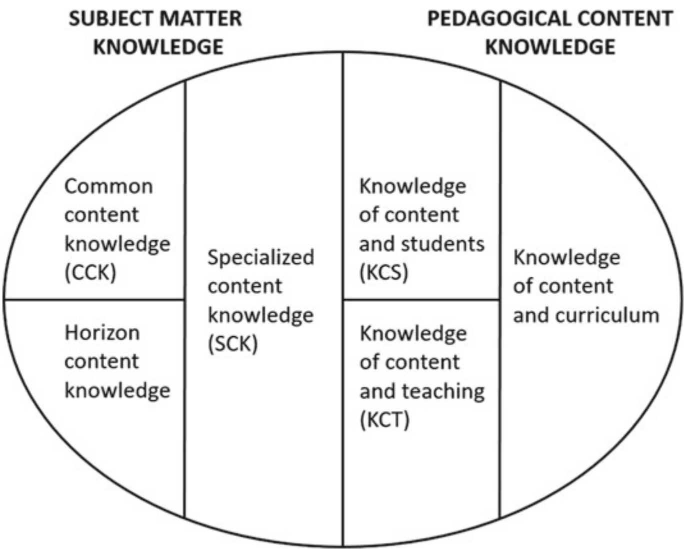

Fig. 15.2 Domains of mathematical knowledge for teaching (Ball et al. 2008, p. 403)

- Knowledge of content and teaching: This domain "combines knowing about teaching and knowing about mathematics" (p. 401). Examples of this domain are the choice of an order according to which examples are taught, or the evaluation of different representations with regard to their instructional advantages or disadvantages. The domain is therefore related to "interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (p. 401).
- The curricular knowledge is taken from Shulman's distinction and "provisionally placed […] within pedagogical content knowledge" (p. 403) according to later publications of researchers of Shulman's group. Ball et al. (2008) left it open whether it is a part of knowledge of content and teaching or a new domain, or whether it runs across the domains.
- Similarly, the domain "horizon knowledge" is provisionally included, meaning "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). Like the preceding domain, this one also could ran across the other domains.

The last two points especially illustrate one difficulty of the approach, namely how to distinguish between domains, which are quite close to each other. Another aspect is that beliefs are not taken into consideration within the model. However, a central achievement of this approach is that it not only developed a theoretical conception of mathematical knowledge for teaching but also developed instruments to measure it with multiple-choice-items (see also, e.g., Hill et al. 2004). Moreover, the ability of the project to identify a relation between mathematical knowledge and student achievement (Hill et al. 2005) is of special importance.

Another project aiming at both conceptual development and empirical research, is the German COACTIV project (Baumert and Kunter 2013). Its theoretical framework also has roots in approaches to teachers' professional knowledge and integrates the concept of professional competence (Weinert 2001). The developed "nonhierarchical model of professional competence is a generic structural model that needs to be specified for the context of teaching" (Baumert and Kunter 2013, p. 28). The result is displayed in Fig. 15.3 and shows that COACTIV "distinguish between four *aspects* of competence (knowledge, beliefs, motivation, and self-regulation), each of which comprises more specific *domains* derived from the available research literature. These domains are further differentiated into *facets*, which are operationalized by concrete indicators" (p. 28).

With regard to the subject-related domains, from a theoretical perspective COACTIV divided mathematical knowledge into four parts, which distinguish from academic mathematical knowledge over advanced and basic perspectives on school mathematics to everyday knowledge. From an empirical perspective, content knowledge was regarded as "teachers' understanding of the mathematical concepts underlying the content taught in middle school" (Baumert and Kunter 2013, p. 34). Content knowledge was in addition described "as a necessary condition for the development of […] PCK" (p. 33). The construct of pedagogical content

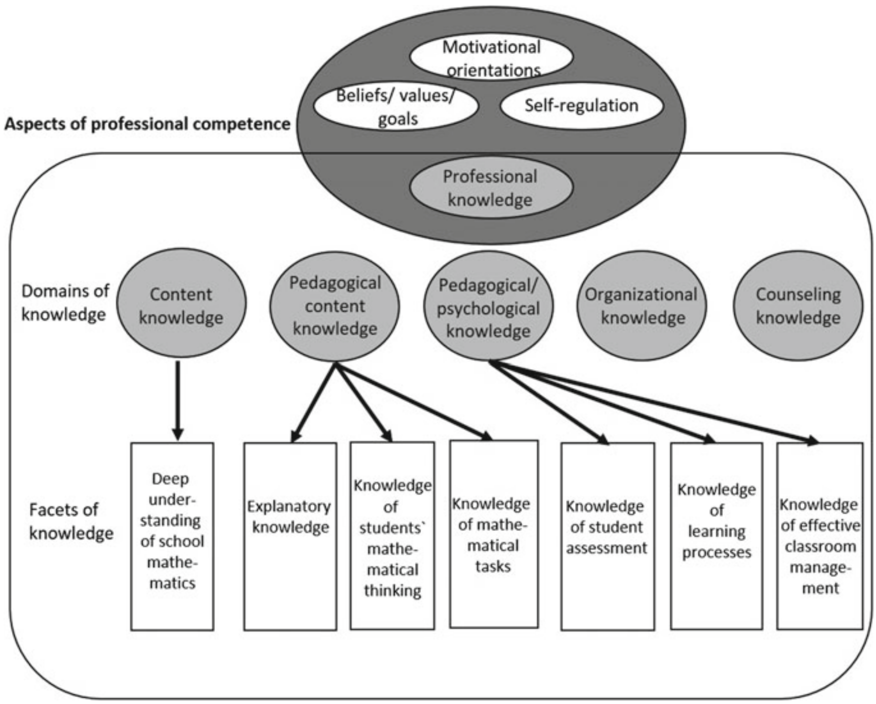

Fig. 15.3 The COACTIV model of professional competence, with the aspect of professional knowledge specified for the context of teaching (Baumert and Kunter 2013, p. 29)

knowledge is therefore theoretically and empirically distinguished from content knowledge and contains three dimensions, as follows:

- "Knowledge of the didactic and diagnostic potential of tasks, their cognitive demands and the prior knowledge they implicitly require, their effective orchestration in the classroom, and the long-term sequencing of learning content in the curriculum
- Knowledge of student cognitions (misconceptions, typical errors, strategies) and ways of assessing student knowledge and comprehension processes
- Knowledge of explanations and multiple representations" (Baumert and Kunter 2013, p. 33).

Similarly to the Michigan-group, COACTIV also could identify as a central result empirical relations especially between teachers' pedagogical content knowledge and students' achievement (Baumert et al. 2010).

Another study aiming at both theoretical development and empirical research, is the TEDS-M study (Blömeke et al. 2014; Döhrmann et al. 2012, 2018), which consisted of two sub-studies, one for primary teachers and one for secondary teachers. In contrast to the other studies described above, TEDS-M particularly was designed as an international comparative study, including 23,000 participants from 17 countries. "Its aim was to understand how national policies and institutional practices influence the outcomes of mathematics teacher education" (Döhrmann et al. 2018, p. 65). With regard to the distinction between different parts of teachers' professional knowledge, TEDS-M therefore understood and measured subject matter knowledge and pedagogical content knowledge as outcomes of the various national teacher education systems. Similarly to the approach of COACTIV, TEDS-M was based on the concept of competence by Weinert (2001). The detailed conceptual model of TEDS-M is illustrated in Fig. 15.4.

With reference to Shulman, TEDS-M differentiated professional knowledge as a cognitive part of teachers' competence, amongst others, in content knowledge and pedagogical knowledge. The development of items for the domain of content knowledge thereby was guided by a two-dimensional approach functioning as heuristic tool. Thus, the first dimension covered different areas of mathematics (algebra, geometry, number and data, with the latter only scarcely represented), which were derived from the theoretical conceptualizations of TIMSS 2007. The other dimension covered, again according to TIMSS, cognitive domains (knowing, applying, reasoning). The items furthermore were categorised into three levels of difficulty, ranging from topics taught at the various school levels to topics "typically taught three or more years beyond the highest grade the future teacher will teach" (Tatto et al. 2008, p. 37). The development of items for the domain of pedagogical content knowledge likewise was guided by the distinction between the different areas of mathematics and the three levels of difficulty. In addition, two sub-domains were distinguished according to a distinction of pre-instructional demands and demands during teaching, in detail "(a) *curricular knowledge and knowledge of planning for mathematics teaching and learning* and (b) *knowledge of enacting mathematics for teaching and learning*." (Döhrmann et al. 2018, p. 73).

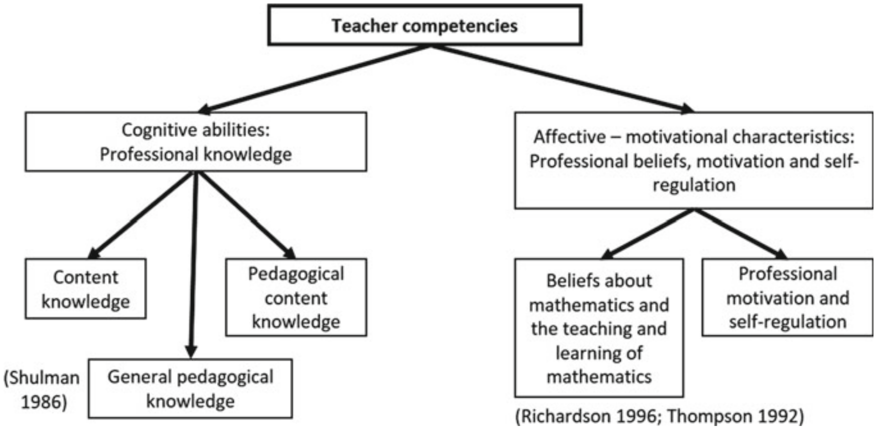

Fig. 15.4 Conceptual model of teachers' professional competencies (Döhrmann et al. 2012, p. 327)

Along with the ongoing development of empirical research on teachers' professional competence, again new theoretical concepts were introduced. A recent approach of conceptualising mathematic teachers' knowledge aimed at bridging the gap between school mathematics and academic mathematics by introducing the concept of "school-related content knowledge" which could be empirically separated from academic content knowledge and pedagogical content knowledge (Dreher et al. 2018). Generally, this knowledge was understood "as a special kind of mathematical CK [content knowledge] for teaching secondary mathematics." (Dreher et al. 2018, p. 329). Conceptually, three facets of school-related content knowledge were derived from corresponding theoretical perspectives on the relationship between school and academic mathematics: "(1) knowledge about the curricular structure and its legitimation in the sense of (meta-)mathematical reasons as well as knowledge about the interrelations between school mathematics and academics mathematics in (2) top-down and in (3) bottom-up directions" (p. 330).

Another recent important milestone with regard to both theoretical and empirical perspectives on teachers' competence was the conceptualisation of teachers' competence as a continuum (Blömeke et al. 2015a). The conceptual starting point was the question of how to overcome dichotomous ways of understanding competence. Amongst others, from a conceptual position these dichotomies were formed by the distinction between an analytic and a holistic position, with each position also implying consequences on the methodological level. Following the analytic position, "competence is analytically divided into several cognitive and affective-motivational traits" (p. 3). In contrast, the holistic position "focuses on the "real-life" part […] and thus on observed behavior in context. Competence itself, then, is assumed to involve a multitude of cognitive abilities and affect-motivation *states* that are ever changing throughout the duration of the performance" (p. 4). Against this background, the idea of Blömeke et al. (2015a)—following the title of their paper—was to go "beyond dichotomies" and model "competence as a continuum" (p. 7). Agreeing on the assumption "that competence ultimately refers to real-world-performance" (p. 6), they aimed to bridge the dichotomy by asking "which *processes* connect cognition and volition-affect-motivation on the one hand and performance on the other hand" (p. 7). In doing so, they identified perception, interpretation, and decision making as "situation-specific skills" and understood them as mediating factors "between disposition and performance" (p. 7). The resulting theoretical model is illustrated in Fig. 15.5.

The authors concluded, "instead of insisting on an unproductive dichotomy view of competence, in particular knowledge *or* performance, competence should be regarded as a process, a continuum with many steps in between" (p. 7). With regard to empirical research on teachers' professional knowledge and teacher education, the model can serve as a heuristic tool from which new conceptualisations and operationalisations for studies can be derived. Concerning evaluation methods Blömeke et al. (2015a) stated: "Besides multiple-choice and constructed-response items or performance assessments in real life or laboratories, they suggested video-based assessments using representative job situations so that the perception of real-life, that is unstructured situations, can be included" (p. 9).

An example of such a study using video-vignettes to ensure a more situated item format, is the TEDS-FU-study, a follow up study to the TEDS-M-study. The sample of TEDS-FU consisted of German mathematics teachers in the fourth year of their professional practice. As all participants formerly also participated in TEDS-M, TEDS-FU is a longitudinal study examining mathematics teachers' development from the end of their teacher education into the first years of teaching profession. Its theoretical framework referred to the idea of competence as a continuum, as sketched above, with a special emphasis on the PID-model. Additionally, TEDS-FU also used the concept of expertise together with the distinction between experts and novices as a theoretical starting point. Therefore, the

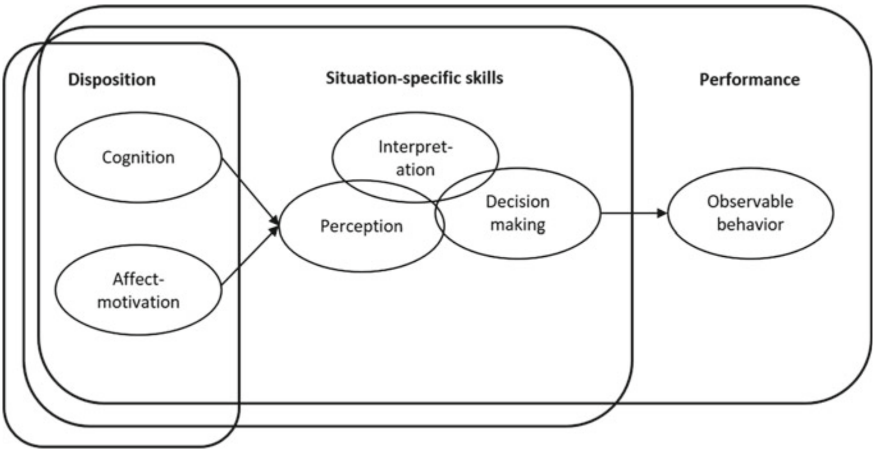

Fig. 15.5 Modelling competence as a continuum (Blömeke et al. 2015a, p. 7)

idea of a more advanced perception of classroom events and the idea of a more integrated knowledge as characteristics of experts' knowledge also functioned as reference for the item development (Kaiser et al. 2015). Selected central results of TEDS-FU are summarised in the following section. A detailed comparison of the cognitive approach in TEDS-M and the situated approach of TEDS-FU together with summarised results of both studies can be found in Kaiser et al. (2017).

Video-based assessment instruments were also used in the COACTIV-video study, a follow-up study to COACTIV. The study aimed at measuring "situated reaction competency". Therefore, teachers were shown videos of classroom situations which stopped at an educationally decisive moment. Using open answer items the teachers should then describe how they would continue within the respective situation (Bruckmaier et al. 2016). Another example for a study using video-based assessments is the v-ACT study, which combined video-based items with other formats such as items based on photos. The theoretical framework of the study thereby distinguished between reflective competence (i.e., abilities concerning pre- and post-instructional phases), action-related competence (i.e., abilities for the phases of instruction itself) and basic knowledge (Knievel et al. 2015).

15.4 Empirical Results Concerning Teachers' Professional Development

Teachers' professional development focuses empirically on two phases, namely, the development during the phase of teacher education and the development within teaching practice. Empirical results from both phases are discussed in the following.

The first kind of studies focused on the growth of knowledge during teacher education. Here, once again it has to be taken into account that there are several very different ways of becoming a teacher, sometimes already within a country and certainly across different countries. Despite this variation, there is empirical evidence of the efficiency of teacher education. Especially, the international comparative study on teacher education Mathematics Teaching in the 21st Century (MT21) (Schmidt et al. 2011) revealed in a quasi longitudinal design that future teachers' achievements in tests on pedagogical content knowledge, content knowledge and general pedagogical knowledge are related to the number of opportunities to learn to which the future teachers could attend. The international comparative study on the efficiency of teacher education, the TEDS-M study, showed remarkable differences referring to the professional knowledge of future teachers at the end of their study, among the participating 16 countries, with the East Asian future teachers outperforming the other groups by far (Blömeke et al., 2014). Furthermore König et al. (2018), in a comparison of German future teachers in the bachelor and the master phase of their university studies, showed that master students performed better in both pedagogical content knowledge as well as general pedagogical knowledge. Moreover, this result was confirmed for future teachers with different subject specializations, namely for mathematics, German and English.

The second kind of studies on the question of how teachers' knowledge develops sets the focus on the first years of professional experience in school and its influence on teachers' competence development. With regard to this question, Blömeke et al. (2015b) showed for German primary teachers within the framework of TEDS-FU that after about three and a half years in school the teachers' general pedagogical knowledge increased, while mathematical pedagogical content knowledge remained approximately stable and mathematical knowledge decreased slightly. The study reported in addition, that the ranking order of teachers' achievements between the end of their teacher education and their first years of teaching practice remained unchanged concerning mathematical content knowledge, and showed significant changes concerning mathematics pedagogical content knowledge and pedagogical knowledge. Moreover, with regard to their beliefs when teachers started positions in schools, the so-called "practical shock" could not be confirmed, as "none of the facets changed towards more traditional directions" (Blömeke et al. 2015b, p. 300). In contrast "the primary teachers' beliefs about the nature of mathematics even changed significantly towards a more process-oriented direction" (p. 300.). Furthermore, the environment in which the teachers worked had an influence on their professional development. Again based on TEDS-FU, for German teachers in the fourth year of their profession, Blömeke and Klein (2013) revealed similar relations between the teachers' teaching quality and the school environment they experienced. In summary, these researchers amongst others "point out that the extent of teacher support depended on the quality of the school management and was, in turn, an important predictor of the teaching quality" (Blömeke and Klein 2013, p. 1043). Finally, looking on the distinction between declarative knowledge and practical skills, again an influence of professional experience can be identified. König et al. (2015) found for German middle school mathematics teachers that general pedagogical knowledge can be predicted by the grades gained at the end of teacher education, whereas teachers' competence to interpret classroom situations was associated with their amount of time spent on teaching relative to their overall working time. (A broader discussion on the current state of empirical results on teacher's competencies can be found in Kaiser and König, under review).

15.5 Summary and Concluding Remarks

Teacher education and the professional development of mathematics teachers has been an important topic in the last decades and especially at ICME-13. In the Topic Study Group (TSG) on "Knowledge in/ for Teaching Mathematics" (at Primary Level as topic of TSG 45 and at Secondary Level in TSG 46) the increasing broadness of the ongoing debate on theoretical focal points, as well as differing or complementing empirical approaches were discussed. Overall—in line with the development of empirical studies on teachers' professional knowledge, which goes from Shulmans' distinction towards more situated approaches (see above)—also the debate in the two TSGs set a strong focus on aspects of teachers' actual activities in the classroom. Important topics of this discussion, amongst others, covered challenges in empirically analysing context-orientated knowledge and the consequences of an ever-growing number of theoretical approaches towards mathematics teachers' professional knowledge (Maher et al. 2017; Even et al. 2017).

A second important strand of discussion about professional development of mathematics teachers during ICME-13 focused on the actual education of mathematics teachers, which was tackled in four TSGs at ICME-13.

For example, concerning the pre-service education of primary mathematics teachers a clear relation to aspects of professional knowledge, such as content knowledge and knowledge for teaching, became obvious. Besides this, a second focus was on the assessment of future teachers' competences in teacher education and classroom experiences in teacher education. Furthermore, similarly to considerations in TEDS-FU and the understanding of teachers' competence as a continuum (Blömeke et al. 2015a), aspects of noticing were included (Hino et al. 2017; and in more detail, Stylianides and Hino 2018). The particular TSG about pre-service education of secondary mathematics teachers discussed similar topics to a large extent. Furthermore, a focus was placed on technology and tools used within teacher education, and in relation to concepts of competence, questions about professional identities of future mathematics teachers were discussed (Strutchens et al. 2017a; and in more detail Strutchens et al. 2017b).

Summarising the discussions within the TSG on primary teachers’ in-service education, strong emphasis on the situated requirements a teacher has to face and the complexity of mathematics teaching can be identified. Besides considering the requirements of direct in situ teaching activities, aspects of working in school in a broader context was a focus, for example working with new curricula, the use of new technologies for teaching, and aspects of inclusion. With regard to these challenges, the TSG discussed approaches for educating practicing teachers. Again of course, conceptualisations of teachers’ knowledge played a central role, as well as empirical studies on in-service teacher education and an analysis of various policies of in-service teacher education and primary teachers' professional development (Takashashi et al. 2017). This combination of practice, research and policies in discussions also took place in the parallel TSG on secondary teachers. Here, again amongst others, the distinction between knowledge, beliefs and practice was stressed and several programs were discussed. Furthermore the use of several technologies, especially as interaction tools, played a central role (Adler et al. 2017).

In this chapter we aimed to summarise central issues concerning teachers' professional development. It became obvious, that even though the prominent distinction of areas of teachers' professional knowledge by Shulman (1986) is still widely received, the debate went on, accumulating a variety of new conceptions. One motor for this rapid development of theories surely was the remarkably growing number of large-scale empirical studies on teacher education, teacher competence and the development of teachers, such as TEDS-M, COACTIV, the studies of the Michigan-Group or TEDS-FU. All these studies developed their own conceptual frameworks and by this process helped to further elaborate on the theoretical considerations concerning mathematics teachers' competence and its development. However the variety of ongoing discussions, as summarised in Sect. 15.5 with regard to ICME-13, showed that there is still a huge amount of work to do and of discussions to have. This finally leads us back to the intention of the chapter to serve as a map, which may provide individually interesting areas. The ongoing discussions and the related ICME-13 materials may serve as a guidepost to show how to find a path in order to provide greater depth in the discussions. In this spirit: Bon voyage!

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