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New Methods and Applications in Multiple Attribute Decision Making (MADM)

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To:
Our families

Contents

Preface

Decision making is one of the most important and fundamental tasks of management as an organizational goal achievement that depends on its quality. It includes the correct expression of objectives, determining different and possible solutions, evaluating their feasibility, assessing the consequences, and the results of implementing each solution, and finally, selecting and implementing the solution. Regarding the experts' perspective, decision making is the main essence of management. Decision making has different stages such as identifying and determining the problem, identifying possible solutions, choosing a criterion, determining the results of each solution, evaluating solutions, and choosing the best solution, respectively. Multiple Criteria Decision Making (MCDM) is sum of the decision-making techniques. Multiple Criteria Decision Making (MCDM) has been taught for several years at different educational levels in the fields of industrial engineering, management, and applied mathematics. Nowadays, a lot of articles, books, and dissertations have been published in this important and applied field of operations research, and scientific and research studies should continue in this regard.

MCDM is divided into the Multiple Objective Decision Making (MODM) for designing the best solution and Multiple Attribute Decision Making (MADM) for selecting the best alternative. Given that the applications of MADM are mostly more than MODM, wide various techniques have been developed for MADM by researchers over the last 60 years. The problems related to Multiple Attribute Decision Making (MADM) include components such as objective or a set of objectives, decision maker or a group of decision makers, a set of evaluation attribute, a set of decision alternatives, a set of unknown variables, or decision variables and a set of results obtained from any pair of alternatives. The criterion for the central element of this structure is a decision matrix such as a set of rows and columns by expressing the decision outcomes for a set of alternatives and evaluation attribute.

In the current book, 27 methods from the MADM are discussed, which are not presented in the existing books or are not studied in details, involving more applications. In each chapter of that, a technique is considered and introduced, along with explaining the steps of the method and providing an example for the technique.

We gratefully acknowledge those who have contributed to the compilation of this book, and it is hoped that this book would be useful for readers, researchers, and managers.

Alireza Alinezhad
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Introduction

Nowadays, experts use many empirical and scientific methods to decide on choosing a superior alternative among several alternatives. The use of experiences is only effective in a relevant specific field and cannot be equally applicable in decision making in all areas. However, the use of a scientific method is effective and efficient in choosing the best alternative in any context, regardless of the studied context. The Multiple Criteria Decision Making (MCDM) is regarded as one of the most important scientific methods used by many experts. When a decision maker considers more than one attribute, the discussion of MCDM is proposed which allocates a large part of everyday decisions in organizations and human societies to itself. MCDM is divided into Multiple Objective Decision Making (MODM) and Multiple Attribute Decision Making (MADM). The mathematical model for a single-objective decision is as in Eq. (1).

$$max\,(min)\,f(\underline{x}) : IR^n \rightarrow IR^1 \tag{1}$$

$$\text{S.t:}\,\underline{x} \in X : IR^n \rightarrow IR^m$$

where n represents the number of decision variables, m denotes the number of functional constraints of problem, and $\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1}$ and the possible space for a set of constraints are $X = \left\{ \underline{x} \in IR^n | g_i(\underline{x}) \begin{bmatrix} \leq \\ \geq \\ = \end{bmatrix} b_i; i = 1, \ldots, m \right\}$. Now if $\underline{x} \in X$, the solution is possible and if the solution is impossible if $\underline{x} \notin X$. In this section, the discussions of multi-objective models mainly focus on linear multi-objective functions, and its mathematical model is as shown in Eq. (2).

$$\underline{F}^{t}(\underline{x}) = (f_1(\underline{x}), f_2(\underline{x}), \cdots, f_k(\underline{x})) \quad (2)$$

$$max(min)\underline{F}(\underline{x}) : IR^n \rightarrow IR^k$$

$$\text{S.t:}\ \underline{x} \in X : IR^n \rightarrow IR^m$$

The optimal solution $(\underline{x}^{*1}, \underline{x}^{*2}, \cdots, \underline{x}^{*k})$ is the solution by which all the objective functions are optimized. However, this case rarely occurs due to the contradiction objectives. If the obtained solution is feasible and non-optimal, the efficient and inefficient solutions are created. Obviously, this model can be a Multiple Objective Linear Programming (MOLP) or Multiple Objective NonLinear Programming (MONLP). The purpose of solving Eq. (2) is to achieve the best solution by improving all target functions. In fact, MODM models are used for designing. MADM models are selector models, applied to select the best and the most appropriate alternative among existing m alternatives based on attributes. MADM is usually formulated by the decision matrix (Eq. 3). In MADM, decision maker chooses the best alternative among the multiple independent alternatives $A_i, i = 1, \ldots, m$, according to the attributes $C_j, j = 1, \ldots, n$. The following definitions of MADM literature are necessary to be considered.

Alternative: In MADM, there exists a number of predetermined, limited, and independent alternatives, and each of them satisfy a level of the desired attributes of the decision maker.

Criterion: The criterion is the basis for evaluation, which means measuring the effectiveness rate and is divided into the objective and attribute.

Objective: It is something pursued until its final achievement.

Attribute: It is the property which should be in an alternative. Depending on the idea of decision maker, each alternative is associated with a number of relevant attributes.

Decision matrix: A matrix with the rank of $m \times n$ which is generally demonstrated as in Eq. (3) where $A_i, i = 1, \ldots, m$, denotes alternatives and $C_j, j = 1, \ldots, n$, indicates attributes and $r_{ij}, i = 1, \ldots, m$ and $j = 1, \ldots, n$, represents the value of alternatives for each attribute.

$$\mathrm{D}_{m\times n} = \begin{array}{c} \\ \mathrm{A}_1 \\ \mathrm{A}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{A}_m \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{C}_1 & \mathrm{C}_2 & & & & \mathrm{C}_n \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & \cdot & \cdot & \cdot & r_{1n} \\ r_{21} & r_{22} & \cdot & \cdot & \cdot & r_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ r_{m1} & r_{m2} & \cdot & \cdot & \cdot & r_{mn} \end{pmatrix}_{m\times n} \end{array} \quad (3)$$

Positive attributes: They refer to the attributes of $C_j, j = 1, \ldots, n$, with positive desirability from the perspective of decision maker; namely, their greater amount is more favorable for the decision maker. Positive attributes are usually as the profit, income or productivity.

Negative attributes: Attributes of $C_j, j = 1, \ldots, n$, with negative desirability from the perspective of decision maker, which means that their lower amount is more desirable to the decision maker. Negative attributes are usually as loss or cost.

Non-compensatory attributes: The attributes in which the disadvantage of an undesirable value in an attribute cannot be covered by the advantage of a desirable value in another attribute.

Compensatory attributes: These attributes can interact with each other; in other words, the disadvantage of an undesirable value in an attribute can be covered by the advantage of a desirable value in another attribute.

Independent attributes: Attributes which are absolutely uncorrelated to the other attributes.

Dependent attributes: Attributes which are correlated to at least one of the other attributes.

Quantitative attributes: They are attributes with a unit of measurement, which are expressed numerically and are measurable.

Qualitative attributes: They are attributes usually without a unit of measurement which cannot be expressed numerically and are immeasurable.

The present book considers 27 new methods in MADM, and most of them are among the compensatory techniques. In the current book, the methods such as SMART, REGIME, ORESTE, VIKOR, PROMETHEE I-II-III, QUALIFLEX, SIR, EVAMIX, ARAS, Taxonomy, MOORA, COPRAS, WASPAS, SWARA, DEMATEL, MACBETH, ANP, MAUT, IDOCRIW, TODIM, EDAS, PAMSSEM I & II, ELECTRE I-II-III, EXPROM I & II, MABAC, CRITIC and KEMIRA are examined, respectively. A brief explanation of each technique is presented for familiarization in the following.

The SMART method, introduced in 1986, is a suitable technique for decision making based on qualitative and quantitative attributes. Based on this method, the qualitative attribute is converted to the quantitative attribute and the effective weight of alternative is calculated in each attribute. Then, the alternatives are evaluated by calculating the final weight and the best alternative is selected by providing the rating of other alternatives for the decision maker.

The REGIME method was introduced in 1983. In this method, there is no need to convert qualitative attribute to the quantitative attribute and attributes are independent. First, it calculates the superiority identifier and impacts matrix using superiority attributes, by representing a set of attribute in which an alternative is at least as good as the other alternative. Ultimately, this technique introduces the superior alternative using the REGIME matrix and provides the ranking of other alternatives.

The ORESTE method was introduced in 1980. The decision maker provides an analyst with an initial ranking of attribute and alternatives for decision making, and there is no need to convert the qualitative attribute into quantitative. First, the position matrix, where the ranking of alternatives is based on the attribute, is formed and values of block distances are calculated. Then, the superior alternative is introduced, and the ranking of other alternatives is presented by the block distance matrix.

The VIKOR method, which was proposed in 1998, is one of the compromising methods by finding the closest alternative to the optimal solution using the LP-metric method. In this method, the attribute should be independent and the qualitative attribute should be converted to the quantitative attribute.

The PROMETHEE I method was first introduced in 1986. The providers of this technique have sought to find an essential solution to improve decision-making evaluation. Therefore, it is known as an efficient method. Also, only the partial ranking of alternatives is done. However, in the PROMETHEE II method, a full ranking of the alternatives is done according to the net flow. In the PROMETHEE III method, the final ranking is done based on the intervals. In these methods, the independence of the attributes is not obligatory.

The QUALIFLEX method was introduced in 1975 and its root dated back to the permutation method. In this technique, every possible ranking of the existing alternatives is examined; namely, ranking the alternatives is evaluated based on the number of permutations, and ultimately, the most appropriate alternatives are chosen for the final ranking. On the other hand, the attribute should be independent and there is no need to convert the qualitative attribute to the quantitative attribute.

The SIR method was introduced in 2001. The attribute should be independent and qualitative attribute should be converted to the quantitative attribute. In fact, the basis of this technique is the use of superiority and inferiority values by determining the type of preference function, similar to the PROMETHEE method. Then, the flow is calculated using the weighted matrix, similar to the Simple Additive Weighting (SAW) and TOPSIS methods. Finally, the best alternative is selected among the alternatives obtained from the superiority and inferiority matrix.

The EVAMIX method, introduced in 1982, is one of the compensatory methods, with two completely different approaches to the quantitative and qualitative attributes, which calculates the total dominance, as well as the rating score of each alternative by performing separate operations on quantitative and qualitative attributes. Then, it introduces the best alternative and ranks the alternatives.

The ARAS method, which was suggested in 2010, aims to select the best alternative based on a number of attributes. In this technique, as one of the multiple attribute decision-making method, the qualitative attributes should be converted to the quantitative attributes and attributes should be independent to choose the best alternative.

The Taxonomy method was introduced in 1763 and was proposed as a tool for classifying and determining the degree of development in 1968. This is a compensatory method in which the attributes are independent from each other. The final ranking of the alternatives is done according to the degree of development of different alternatives from the attributes.

The MOORA method, introduced in 2004, is a compensatory method and is also considered as an objective (non-subjective) technique, in which desirable and undesirable attributes are simultaneously used for ranking. Also, attributes are independent.

The COPRAS method was proposed in 1994 and as a compensatory method was used to evaluate the value of both the maximizing and minimizing indexes. The effects of the maximizing and minimizing indexes of attributes on the outcome evaluation are considered separately.

The WASPAS method was proposed in 2012. This technique is a combination of Weighted Sum Model (WSM) and Weighted Product Model (WPM). Also, the attributes are independent and the qualitative attributes are converted to the quantitative attributes.

The SWARA method was suggested in 2010. This method was done by weighting method, and the relative significance and initial priority of the independent attributes are determined according to the opinion of the decision maker, and then, the relative weight of each attribute is determined. Finally, the priority and ranking of the attributes are done.

The DEMATEL method, which was introduced in 1971 as a compensatory method, was used to construct a network design to examine the internal relation among the attributes. Further, this method has been successfully applied in many situations such as the development of strategies, managerial systems, and knowledge management.

The MACBETH method was introduced in 1990. This interactive technique examines alternatives with multi-attribute and opposite objectives. Given that there is no need to convert the qualitative attributes to the quantitative attributes, a wide range of qualitative and quantitative attributes are examined.

The ANP method was introduced in 1996 as a compensatory method, the independence of the attributes is not obligatory and a decision-making problem is decomposed into several different levels, the sum of these decision-making levels forms a hierarchy, and solves the problems of interdependence and the feedback among attributes and alternatives in the real world by considering all types of dependency.

The MAUT method was introduced in 1976. The simplicity of this technique and abundant freedom of action of decision makers make the results of this technique more accurate and realistic. In addition, this is a compensatory method and attributes are independent of each other.

The IDOCRIW method was introduced in 2016. This compensatory method utilizes of entropy and Criterion Impact LOSs (CILOS) methods to determine the relative impact loss of attributes and determines the weight of attributes in a combination of these techniques.

The TODIM method was introduced in 1992. The main idea of this compensatory method is to measure the dominance degree of each alternative over other alternatives using the overall value and alternatives are evaluated and ranked with respect to the independence of the attributes.

The EDAS method was introduced in 2015. This compensatory method is largely applied in conditions with contradictory attributes, and the best alternative is chosen by calculating the distance of each alternative from the optimal amount. Additionally, qualitative attributes are converted to the quantitative attributes and attributes are independent of each other.

The PAMSSEM methods were introduced in 1996. In this compensatory method, the independence of the attributes is not necessary. This technique models the preferences of decision makers using an outranking approach and according to the ordinal or cardinal type of the values of each attribute to choose the best alternative. Also, only the partial ranking of alternatives is done. However, in the PAMSSEM II method, the net flow is determined as the final values and alternatives are ranked completely.

The ELECTRE methods were first suggested in 1990, and all alternatives are evaluated using outranking comparisons, and ineffective and low-attractive alternatives are eliminated. Therefore, the final ranking of alternatives may be increasingly problematic, and ELECTRE II and ELECTRE III methods are presented to solve this problem. Further, the qualitative attributes should be converted to the quantitative attributes.

The EXPROM methods, introduced in 1991, seek to find a solution for evaluating and rank the alternatives by various and widely available information more accurately. In this compensatory method, the qualitative attributes should be converted to the quantitative attributes and the independence of the attributes is not obligatory. In the EXPROM I method, alternatives are partially ranked using the obtained entering and leaving flows. In the EXPROM II method, the net flow is determined as the final values and alternatives are fully ranked.

The MABAC method was introduced in 2015. The basic assumption in this compensatory method is the definition of the distance of the alternatives from the border approximate area. In fact, each alternative can be evaluated and ranked by specifying the difference between the distances. In addition, experts convert the qualitative attributes to the quantitative attributes.

The CRITIC method was introduced in 1995 and mainly used to determine the weight of the attributes. In this compensatory method, the qualitative attributes should be converted into the quantitative ones in the decision matrix and the independence of the attributes is not obligatory.

The KEMIRA method was introduced in 2014, after determining the priority and weight of the attributes in two different groups, and in the form of decision matrix determined by the experts, the final ranking of alternatives is performed. This technique is one of the compensatory methods and requires the conversion of qualitative attributes into quantitative ones.

Chapter 1
SMART Method

1.1 Introduction

The Simple Multi-Attribute Rating Technique (SMART) was introduced by Winterfeldt and Edwards in 1986 [1, 2], in which a limited number of alternatives are examined based on a limited number of attributes. The present method aimed to rank the alternatives by a combination of quantitative and qualitative attributes. This is a convenient technique because of its ease of use, which is used in many cases such as evaluation of nuclear waste disposal sites [3] and ERP system selection [4]. The SMART method has the following features:

- It is regarded as one of the compensatory methods;
- It is possible to use independent and dependent attributes;
- The qualitative attributes should be converted into the quantitative attributes.

Initially, the matrix of alternatives and attributes is formed based on the information received from the decision maker, which is as shown in Eq. (1.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} \quad ; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{1.1}$$

According to the matrix of Eq. (1.1), r_{ij} is the element of decision matrix for ith alternative in jth attribute. In this technique, the qualitative attributes are ranked as shown in Table 1.1.

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Table 1.1 Seven ranking of qualitative attributes [5]

Poor	Fairly weak	Medium	Fairly good	Good	Very good	Excellent
4	5	6	7	8	9	10

1.2 Description of SMART Method

1.2.1 Rating the Attributes

In the first step, the minimum (P_{min}) and maximum value (P_{max}) are defined for all attributes by decision maker.

Therefore, the decision maker obviously chooses in the interval of (P_{min}, P_{max}). The entire decision-making interval is divided into sub-intervals with equal lengths from Eq. (1.2).

$$P_{min}, P_{min} + e_0, P_{min} + e_1, \ldots \tag{1.2}$$

Eq. (1.3) is used to calculate e.

$$e_{\mathrm{v}} - e_{\mathrm{v}-1} = \varepsilon e_{\mathrm{v}-1} \tag{1.3}$$

The geometric progression is created, and Eq. (1.4) is obtained.

$$e_{\mathrm{v}} = (1 + \varepsilon)e_{\mathrm{v}-1} = (1 + \varepsilon)^2 e_{\mathrm{v}-2} = (1 + \varepsilon)^{\mathrm{v}} e_0 \tag{1.4}$$

Finally, Eq. (1.5) can be deduced [5].

$$P_{max} = e_v + P_{min} \tag{1.5}$$

1.2.2 The Effective Weights of Alternatives

g_{ij} is the effective weight of alternatives and it is obtained from judgment of the decision maker about the alternative A_i against the attribute C_j.

Initially, the qualitative attributes are ranked based on the attribute situation expressed by the decision maker according to Table 1.1. Also, the Eq. (1.6) is used for the quantitative attributes (P_v indicates the value of alternative in the studied attribute) [5].

$$v = log_2^{\frac{P_v - P_{min}}{P_{max} - P_{min}} \times 64} \tag{1.6}$$

According to Eq. (1.6), g_{ij} is obtained for positive attributes (the higher amount of attribute is better like speed), when the value of v is summed with the number 4 to match the quantitative and qualitative attributes in Table 1.1.

On the other hand, g_{ij} is obtained for negative attributes (the lower amount of attribute is better like price), when the value of v is subtracted from 10 to match the quantitative and qualitative attributes in Table 1.1.

1.2.3 The Normalized Weights

Initially, the decision maker is asked to rank the attributes according to his priority and Table 1.1 from 4 to 10. The following definitions are considered to formulate the model:

$\boldsymbol{A_i}$ Alternatives, $i = 1, \ldots, m$ (m represents the number of alternatives);
$\boldsymbol{C_j}$ Attributes, $j = 1, \ldots, n$ (n denotes the number of attributes);
$\boldsymbol{h_j}$ The rank allocated to the attribute C_j by the decision maker ($j = 1, \ldots, n$);
$\boldsymbol{w_j}$ The denormalized weight obtained from Eq. (1.7) [5]

$$w_j = \left(\sqrt{2}\right)^{h_j}; \quad j = 1, \ldots, n \tag{1.7}$$

Now, the value of each attribute is calculated after normalization as shown in Eq. (1.8) [5].

$$w_j = \frac{\left(\sqrt{2}\right)^{h_j}}{\sum_{j=1}^{n} \sqrt{2}^{h_j}} \tag{1.8}$$

1.2.4 The Final Ranking of Alternatives

According to Eq. (1.6), f_i is the final weight, which is introduced as shown in Eq. (1.9).

$$f_i = \sum_{j=1}^{n} w_j . g_{ij}; \quad i = 1, \ldots, m \tag{1.9}$$

The alternative with the highest f_i amount is the best alternative, and others are also ranked.

1.3 Case Study

A manufacturer, aiming to produce a car from three different car models (A_1, A_2 and A_3), expresses the selection interval for the attributes of price (C_1), maximum speed (C_2), acceleration between 0 and 100 (C_3), and the trunk volume of car (C_4) as follows:

Consumer price: 20,000\$–40,000\$
Maximum speed: 140–220 km/h
Acceleration 0–100: 8–20 s
Trunk volume of car: 200–2000 dm^3

Further, the attributes are ranked as shown in Table 1.2. The attributes are represented by experts, and the decision matrix is as the matrix of Fig. 1.1.

It is desirable to select the best alternative according to the ranking of attributes by the manufacturer by the SMART method.

❖ **Solution**

(A) **Rating the attributes**

The rating of attributes is as shown in Table 1.3.

Table 1.2 Ranking the attributes

Attribute	C_1	C_2	C_3	C_4
Rank	9	5	7	6

Fig. 1.1 Decision matrix of car production

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{cccc} - & + & - & + \\ C_1 & C_2 & C_3 & C_4 \\ \end{array}$$
$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \begin{bmatrix} 25000 & 153 & 15.300 & 250 \\ 33000 & 177 & 12.300 & 380 \\ 40000 & 199 & 11.100 & 480 \end{bmatrix}$$

Table 1.3 Rating the attributes

Rank	Performance	C_1	C_2	C_3	C_4
10	Excellent	20,312.500	220	8.188	2000
9		20,625	180	8.375	1100
8	Good	21,250	160	8.750	650
7		22,500	150	9.500	425
6	Medium	25,000	145	11	312.500
5		30,000	142.500	14	256.250
4	Poor	40,000	141.250	20	228.125

(B) **The effective weights of alternatives**

The effective weight of alternatives is calculated according to the proposed tips and Eq. (1.4). For example, the effective weight of alternative 1, under the negative price attribute, is computed as follows:

$$g_{11} = 10 - log_2^{\frac{25000-20312.500}{40000-20312.500}\times 64} = 6.070$$

The effective weight of alternative 1 under the positive attribute of the maximum speed is determined as follows:

$$g_{12} = 4 + log_2^{\frac{153-141.250}{220-141.250}\times 64} = 7.255$$

The effective weight of alternative 1 under the negative attribute of acceleration (the lower the time of reaching a speed of 100 km/h, the machine is better) is calculated as follows:

$$g_{13} = 10 - log_2^{\frac{15.300-8.188}{20-8.188}\times 64} = 4.732$$

The effective weight of alternative 1 under the positive attribute of the trunk volume is obtained as follows:

$$g_{14} = 4 + log_2^{\frac{250-228.125}{2000-228.125}\times 64} = 3.660$$

Also, the effective weight of other alternatives is according to Table 1.4.

Table 1.4 Effective weight of attributes

	Car 1	Car 2	Car 3
C_1	6.070	4.634	4
C_2	7.255	8.861	9.552
C_3	4.732	5.522	6.020
C_4	3.660	6.456	7.186

Table 1.5 Normalized weight of attributes

	C_1	C_2	C_3	C_4
Value	0.475	0.119	0.238	0.168

(C) **The normalized weights**

At first, the normalized weight of four attributes is obtained:

Weight of the price attribute: $\left(\sqrt{2}\right)^9 = 22.627$

Weight of the maximum speed attribute: $\left(\sqrt{2}\right)^5 = 5.657$

Weight of the acceleration attribute: $\left(\sqrt{2}\right)^7 = 11.314$

Weight of the trunk volume attribute: $\left(\sqrt{2}\right)^6 = 8$

The normalized weight of attributes is as shown in Table 1.5.

(D) **The final ranking of alternatives**

The final weight of the alternatives is determined as follows:

$$f_1 = (0.475 \times 6.070) + (0.119 \times 7.255) + (0.238 \times 4.732) + (0.168 \times 3.660) = 5.488$$

$$f_2 = (0.475 \times 4.634) + (0.119 \times 8.861) + (0.238 \times 5.522) + (0.168 \times 6.456) = 5.654$$

$$f_3 = (0.475 \times 4) + (0.119 \times 9.552) + (0.238 \times 6.020) + (0.168 \times 7.186) = 5.677$$

Therefore, the final ranking is:

$$A_3 > A_2 > A_1$$

Accordingly, the car 3 is selected.

1.4 Conclusion

The SMART method, presented by Winterfeldt and Edwards to select the best alternative among the different alternatives, has been emphasized due to the combined utilization of qualitative and quantitative attributes, as well as the lack of need for the dependence or independence of attributes. These features led to the technique development after 1986. The SMART method with the feature of compensatory attributes can be considered as one of the appropriate techniques for choosing the best alternative according to the different attributes. In addition, the low number of steps (Fig. 1.2) accelerates the problem-solving process.

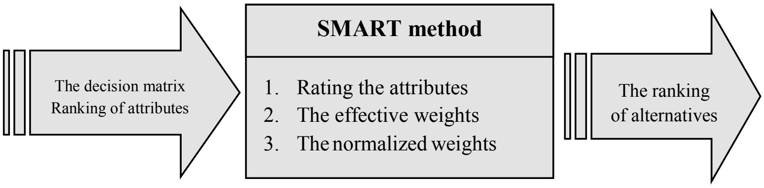

Fig. 1.2 A summary of the SMART method

Chapter 2
REGIME Method

2.1 Introduction

The REGIME method, initially introduced by Hinloopen, Nijkamp, and Rietveld in 1983 [6, 7] is a multiple attribute qualitative method which solves the problem using the REGIME matrix, and a final ranking of the alternatives is done. In the final ranking, the weight of attributes, introduced by the decision maker, is important and can influence the results. This technique is used for ranking the sawability of ornamental and building stones [8] and evaluation and ranking of coastal areas [9] due to its features. The REGIME method, used in various fields, has the following features:

- It is one of the compensatory methods;
- The attributes are independent of each other;
- There is no need to convert the qualitative attributes into the quantitative attributes.

In this method, the matrix of alternatives and attributes is firstly formed based on the information received from the decision maker as in Eq. (2.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} \quad ; \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{2.1}$$

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2.2 Description of REGIME Method

2.2.1 Superiority Index

In decision matrix of Eq. (2.1), r_{ij} is the element of the decision matrix for ith alternative in jth attribute. Then, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$.

The set of attributes in which alternative A_f is at least as good as alternative A_l, displayed by $\tilde{E}_{fl}$.

2.2.2 Superiority Identifier

The superiority identifier is calculated by Eq. (2.2).

$$\hat{E}_{fl} = \sum_{j \in \tilde{E}_{fl}} w_j; \quad j = 1, \ldots, n \tag{2.2}$$

where w_j represents the weight of attributes provided by the decision maker.

2.2.3 Impacts Matrix

This matrix is derived from ranking the alternatives based on the attributes which rank the alternatives from decision-matrix information.

2.2.4 REGIME Matrix

The REGIME matrix is derived from pairwise comparison of alternatives. For example, if two alternatives of $A_1, A_2 \in A$ are considered, the status of A_1, A_2 alternatives should be compared to each other in all attributes.

For each C_j attribute, the $E_{fl,j}$ identifier is defined for each (A_f, A_l) alternative as in Eq. (2.3) [6, 9].

$$E_{fl,j} = \begin{cases} -1 & if \quad r_{fj} < r_{lj} \\ 0 & if \quad r_{fj} = r_{lj}; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \\ +1 & if \quad r_{fj} > r_{lj} \end{cases} \tag{2.3}$$

where $\left(r_{lj}, r_{fj}\right)$ indicates the rank of $\left(A_l, A_f\right)$ alternative based on the attribute C_j. When two alternatives are examined in all attributes, a vector is defined as in Eq. (2.4) [6, 9].

$$E_{fl} = \left(E_{fl,1}, \ldots, E_{fl,j}, \ldots, E_{fl,n}\right), \quad j = 1, \ldots, n \tag{2.4}$$

The vector of Eq. (2.4) is called the REGIME, and the total matrix is result of the REGIME vectors.

2.2.5 The Guide Index

The first technique: The guide index $\bar{E}_{jl}$ is introduced as in Eq. (2.5) [9].

$$\bar{E}_{fl} = \sum_{j=1}^{n} E_{fl,j}.w_j \tag{2.5}$$

$\bar{E}_{fl}$ obtains a final ranking of alternatives.

The second technique: The value of the best alternative is obtained by the superior identifier $\hat{E}_{fl}$. In fact, the REGIME method is based on the $\hat{E}_{fl} - \hat{E}_{lf}$ subtract. The positive result of subtract indicates that alternative A_f is superior to the alternative A_l, and the negative result demonstrates the superiority of alternative A_l over alternative A_f.

2.2.6 The Final Ranking of Alternatives

According to the two techniques presented in the previous step, the final ranking of alternatives can be determined based on the guide index.

2.3 Case Study

The dam construction project should be implemented by the relevant ministry. The project can be implemented by the relevant ministry (A_1), domestic contractor (A_2), or foreign contractor (A_3). Attributes such as cost (C_1), strength (C_2), national

Fig. 2.1 Decision matrix of dam construction project

	C1 (−)	C2 (+)	C3 (+)	C4 (+)	C5 (−)
A1	3	Moderate	Very high	24000	Very high
A2	1.200	High	Moderate	25000	High
A3	1.500	Very high	Low	32000	Low

Table 2.1 Weight of the attributes

Attribute	C_1	C_2	C_3	C_4	C_5
w_j	0.100	0.175	0.250	0.350	0.125

reputation (C_3), capacity (C_4), and work hardness (C_5) are available for decision making. Fig. 2.1 displays the decision matrix. Further, Table 2.1 indicates the weight of the attributes.

The purpose is to select the best contractor and express the final ranking of alternatives by the REGIME method.

❖ Solution

(A) Superiority index

The superiority attribute is computed as follows:

$$\tilde{E}_{12} = \{C_3\}, \quad \tilde{E}_{21} = \{C_1, C_2, C_4, C_5\}$$

$$\tilde{E}_{13} = \{C_3\}, \quad \tilde{E}_{31} = \{C_1, C_2, C_4, C_5\}$$

$$\tilde{E}_{23} = \{C_1, C_3\}, \quad \tilde{E}_{32} = \{C_2, C_4, C_5\}$$

(B) Superiority identifier

The superiority identifier is as follows:

$$\hat{E}_{12} = 0.250, \quad \hat{E}_{21} = 0.750$$

$$\hat{E}_{13} = 0.250, \quad \hat{E}_{31} = 0.750$$

$$\hat{E}_{23} = 0.350, \quad \hat{E}_{32} = 0.650$$

Fig. 2.2 Impacts matrix

	C_1	C_2	C_3	C_4	C_5
A_1	3	3	1	3	3
A_2	1	2	2	2	2
A_3	2	1	3	1	1

(C) Impacts matrix

Initially, the impacts matrix is formed based on ranking the alternatives. The negative attributes of cost and work hardness should be considered in this matrix. The first rank belongs to the lowest value, and the impacts matrix is as shown in Fig. 2.2.

(D) REGIME matrix

The REGIME matrix, obtained from pairwise comparison in the impact matrix, is formed as shown in Fig. 2.3.

As:

$$E_{12} = (-1, -1, +1, -1, -1)$$
$$E_{13} = (-1, -1, +1, -1, -1)$$
$$E_{21} = (+1, +1, -1, +1, +1)$$
$$E_{23} = (+1, -1, +1, -1, -1)$$
$$E_{31} = (+1, +1, -1, +1, +1)$$
$$E_{32} = (-1, +1, -1, +1, +1)$$

Fig. 2.3 REGIME matrix

	C_1	C_2	C_3	C_4	C_5
(A_1, A_2)	-1	-1	+1	-1	-1
(A_1, A_3)	-1	-1	+1	-1	-1
(A_2, A_1)	+1	+1	-1	+1	+1
(A_2, A_3)	+1	-1	+1	-1	-1
(A_3, A_1)	+1	+1	-1	+1	+1
(A_3, A_2)	-1	+1	-1	+1	+1

(E) The guide index

The guide index is obtained as follows:

$$\bar{E}_{12} = -0.500$$
$$\bar{E}_{13} = -0.500$$
$$\bar{E}_{21} = 0.500$$
$$\bar{E}_{23} = -0.300$$
$$\bar{E}_{31} = 0.500$$
$$\bar{E}_{32} = 0.300$$

(F) The final ranking of alternatives

The positive values of guide index in the first technique revealed that the alternative A_3 is better than alternative A_2 and both alternatives are better than alternative A_1. Therefore, the ranking of alternatives is as follows:

$$A_3 > A_2 > A_1$$

In the second technique, the pairwise comparison of alternatives is considered by comparing the superiority attribute:

$$\hat{E}_{21} - \hat{E}_{12} = 0.750 - 0.250 = 0.500 \rightarrow A_2 > A_1$$

$$\hat{E}_{31} - \hat{E}_{13} = 0.750 - 0.250 = 0.500 \rightarrow A_3 > A_1$$

$$\hat{E}_{32} - \hat{E}_{23} = 0.650 - 0.350 = 0.300 \rightarrow A_3 > A_2$$

Consequently, the foreign contractor (A_3) is the best alternative, and final ranking is as follows:

$$A_3 > A_2 > A_1$$

2.4 Conclusion

The REGIME method is considered as one of the most important methods for experts in multiple attribute decision making to rank alternatives, due to the lack of direct use of qualitative attributes. In fact, the strength point of this method is the

REGIME matrix formation, which is a combination of quantitative and qualitative attributes at penultimate stage (Fig. 2.4). It allows decision makers to use this technique in many cases without any need to convert the qualitative attributes into quantitative attributes.

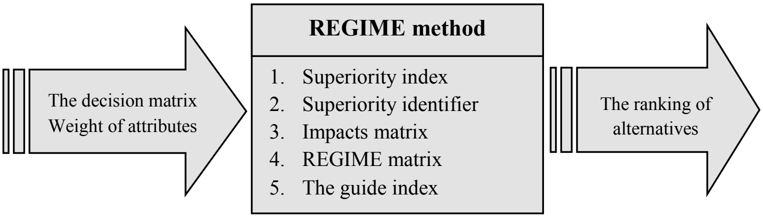

Fig. 2.4 A summary of the REGIME method

Chapter 3
ORESTE Method

3.1 Introduction

The ORESTE method was initially introduced by Roubens at a conference in 1980 [10–12] and then was expanded in an article in 1980. ORESTE is used when the decision maker provides an analyst with an initial ranking of the attributes for decision making. Also, the best alternative is selected among the various alternatives, which is accompanied by different qualitative and quantitative attributes. This technique is used in many cases such as ranking of Web design firms [13], material selection [14], and insurance company selection [15]. The ORESTE has the following features:

- It is one of the compensatory methods;
- Attributes should be independent;
- There is no need to convert the qualitative attributes into the quantitative attributes.

In the ORESTE method, the matrix of alternatives and attributes is initially formed based on the information received from the decision maker as in Eq. (3.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{3.1}$$

With respect to the matrix of Eq. (3.1), r_{ij} illustrates the element of decision matrix for ith alternative in jth attribute. In addition, the attributes are initially ranked by the decision maker.

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3.2 Description of ORESTE Method

3.2.1 The Position Matrix

In this matrix, the alternatives are ranked based on the attributes and according to the decision matrix.

3.2.2 The Block Distance

The block distance of each alternative is obtained from Eq. (3.2) [12].

$$d\left(0, A_{ij}\right) = \alpha r_{ij}(\alpha) + (1 - \alpha) r_j; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{3.2}$$

where α represents the succession rate, and $0 < \alpha < 1$, r_j is the prioritized values by the decision maker, and r_{ij} is the value of the position matrix of *i*th alternative in *j*th attribute.

3.2.3 The Block Distance Matrix

The block distance of each element in the position matrix is computed and placed in the block distance matrix.

3.2.4 The Final Ranking of Alternatives

The ranking technique based on the pairwise comparison of block distances is as; if $d\left(0, A_{ij}\right) \leq d\left(0, A_{i'j}\right)$, consequently, $R\left(A_{ij}\right) \leq R\left(A_{i'j}\right)$ [12]. Usually, $\frac{1}{3} \leq \alpha \leq \frac{1}{2}$ is considered. The total ranking of alternatives is derived from allocating the rank to any value of alternative attribute and aggregating all common attributes as in Eq. (3.3) [10–12].

$$R(A_i) = \sum_{j=1}^{n} R\left(A_{ij}\right); \quad i = 1, \ldots, m \tag{3.3}$$

Fig. 3.1 Decision matrix for hospital construction projects

	+	−	+
	C_1	C_2	C_3
A_1	Very high	3	High
A_2	Moderate	1.200	Moderate
A_3	Low	1.500	Very high

3.3 Case Study

Three projects of A_1, A_2, and A_3 were proposed by experts for constructing a hospital. The attributes such as the suitability of the construction site (C_1), cost (C_2), and strength (C_3) are available for decision making and the decision matrix as shown in Fig. 3.1.

The decision maker expresses the order of importance of the attributes as follows:

$$C_1 > C_2 > C_3$$

The purpose is to choose the best project, and the final ranking of alternatives is expressed by the ORESTE method.

❖ **Solution**

(A) **The position matrix**

Initially, the position matrix is formed as shown in Fig. 3.2, in which alternatives are ranked based on the attributes. The negative cost attribute should be considered in this matrix formation, and the first rank belongs to the lowest value.

According to the order of the attributes expressed by the decision maker:

$$r_1 = 1, \quad r_2 = 2, \quad r_3 = 3$$

(B) **The block distance**

The block distance values are obtained from the following and where the values of $r_{ij}(\alpha)$ are obtained from the values of the position matrix, and r_j represents the values of the attributes prioritized by the decision maker. Thus, the following result is obtained as an example:

Fig. 3.2 Position matrix

	C_1	C_2	C_3
A_1	1	3	2
A_2	2	1	3
A_3	3	2	1

$$d(0, A_{11}) = \alpha r_{11}(\alpha) + (1-\alpha)r_1 = 1\alpha + (1-\alpha) = 1$$

where $r_{11}(a)$ means the amount of position matrix of the first alternative in first attribute. Similarly, the other block distance values are obtained as follows:

$$d(0, A_{12}) = \alpha r_{12}(\alpha) + (1-\alpha)r_2 = 3\alpha + 2(1-\alpha) = 2+\alpha$$
$$d(0, A_{13}) = \alpha r_{13}(\alpha) + (1-\alpha)r_3 = 2\alpha + 3(1-\alpha) = 3-\alpha$$
$$d(0, A_{21}) = \alpha r_{21}(\alpha) + (1-\alpha)r_1 = 2\alpha + (1-\alpha) = 1+\alpha$$
$$d(0, A_{22}) = \alpha r_{22}(\alpha) + (1-\alpha)r_2 = 1\alpha + 2(1-\alpha) = 2-\alpha$$
$$d(0, A_{23}) = \alpha r_{23}(\alpha) + (1-\alpha)r_3 = 3\alpha + 3(1-\alpha) = 3$$
$$d(0, A_{31}) = \alpha r_{31}(\alpha) + (1-\alpha)r_1 = 3\alpha + (1-\alpha) = 1+2\alpha$$
$$d(0, A_{32}) = \alpha r_{32}(\alpha) + (1-\alpha)r_2 = 2\alpha + 2(1-\alpha) = 2$$
$$d(0, A_{33}) = \alpha r_{33}(\alpha) + (1-\alpha)r_3 = \alpha + 3(1-\alpha) = 3-2\alpha$$

(C) **The block distance matrix**

Fig. 3.3 indicates the block distance matrix.

(D) **The final ranking of alternatives**

First, the pairwise comparison of the block distances is as follows:

$$\begin{matrix} 1 & <1+\alpha & <2-\alpha & <1+2\alpha & <2 & <3-2\alpha & <2+\alpha & <3-\alpha & <3 \\ R(A_{11}) & <R(A_{21}) & <R(A_{22}) & <R(A_{31}) & <R(A_{32}) & <R(A_{33}) & <R(A_{12}) & <R(A_{13}) & <R(A_{23}) \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix}$$

The ranking of alternatives and their severity is calculated as follows:

$$R(A_1) = 1+7+8 = 16$$
$$R(A_2) = 2+3+9 = 14$$
$$R(A_3) = 4+5+6 = 15$$

Thus, the fist project (A_1) is the best alternative, and the final ranking is obtained as follows:

$$A_1 > A_3 > A_2$$

Fig. 3.3 Block distance matrix

$$\begin{matrix} & C_1 & C_2 & C_3 \\ A_1 & 1 & 2+\alpha & 3-\alpha \\ A_2 & 1+\alpha & 2-\alpha & 3 \\ A_3 & 1+2\alpha & 2 & 3-2\alpha \end{matrix}$$

3.4 Conclusion

The ORESTE method is another important decision-making method for selecting the best alternative used by managers, experts, and even ordinary people. In this technique, the severity of alternatives is determined using a combination of the quantitative and qualitative attributes and without the need to convert the qualitative attributes into the quantitative attributes and based on the block distance, and then, they are ranked. Further, having the short steps is another advantage of this method as shown in Fig. 3.4.

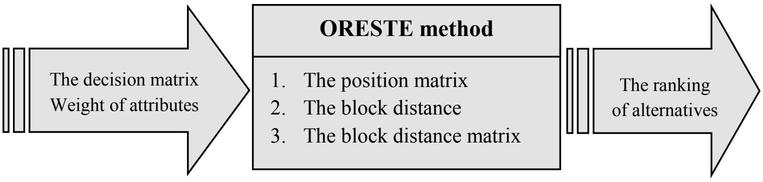

Fig. 3.4 A summary of the ORESTE method

Chapter 4
VIKOR Method

4.1 Introduction

The VIKOR method was introduced by Opricovic in 1998 [16–19]. This technique is one of the compromising methods in compensatory models, as the closest alternative to the ideal solution is preferred in this subgroup. Generally, the technique focuses on the alternatives ranking and selecting an alternative with a set of contradictory attributes, and ultimately, provide a compromise solution, contributing the decision maker to reach the final solution. The VIKOR has been abundantly applied in decision making to select the ideal alternative since its introduction and has been used in analyzing the logistic outsourcing [20], selection of suppliers [21], and airport location selection [22]. This technique has the following features:

- It is one of the compensatory methods;
- The attributes should be independent;
- The qualitative attributes should be converted into the quantitative attributes.

Further, the decision matrix is used in the VIKOR method based on the information received from the decision maker as in Eq. (4.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} \quad ; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{4.1}$$

In Eq. (4.1), r_{ij} denotes the element of the decision matrix for ith alternative in jth attribute. In the VIKOR, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$ by taking into account the normalized property $(\sum_{j=1}^{n} w_j = 1)$.

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4.2 Description of LP-Metric

This is a compromise planning method, which uses the LP-metric method as in Eq. (4.2) to find the closest alternative to the optimal solution [23, 24].

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[\frac{w_j \left(f_j^* - f_{ij} \right)}{\left(f_j^* - f_j^- \right)} \right]^p \right\}^{1/p} ; \quad i = 1, \ldots, m, \ 1 \le P \le \infty \tag{4.2}$$

where w_j indicates the weight of attribute declared by the decision maker, p represents the parameter specifying the LP family, f_{ij} denotes the value of ith alternative in jth attribute, f_j^* is the best f_{ij}, and f_j^- is the worst f_{ij}. L_{1i} is introduced by S_i and is equal to Eq. (4.3).

$$S_i = \sum_{j=1}^{n} w_j \frac{\left(f_j^* - f_{ij} \right)}{\left(f_j^* - f_j^- \right)} ; \quad i = 1, \ldots, m \tag{4.3}$$

$L_{\infty i}$ is introduced by R_i and is equal to Eq. (4.4).

$$R_i = \max_j \left[w_j \frac{\left(f_j^* - f_{ij} \right)}{\left(f_j^* - f_j^- \right)} \right] ; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{4.4}$$

4.3 Description of VIKOR Method

4.3.1 *The f^* and f^- Indexes*

For each attribute $j = 1, \ldots,$ n, the best f_{ij} is specified as f_j^* and the worst f_{ij} as f_j^-. The f_j^* and f_j^- indexes are computed for the positive attributes using Eq. (4.5) [24].

$$\begin{cases} f_j^* = \max\limits_i f_{ij} \\ f_j^- = \min\limits_i f_{ij} \end{cases} ; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{4.5}$$

The f_j^* and f_j^- indexes are determined for the negative attributes from Eq. (4.6) [24].

$$\begin{cases} f_j^* = \min\limits_i f_{ij} \\ f_j^- = \max\limits_i f_{ij} \end{cases} ; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{4.6}$$

4.3.2 The S and R Indexes

The S and R indexes are obtained for each alternative using Eqs. (4.7) and (4.8) [24].

$$S_i = \sum_{j=1}^{n} w_j \frac{\left(f_j^* - f_{ij}\right)}{\left(f_j^* - f_j^-\right)}; \quad i = 1, \ldots, m \tag{4.7}$$

$$R_i = \max_j \left[w_j \frac{\left(f_j^* - f_{ij}\right)}{\left(f_j^* - f_j^-\right)} \right]; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{4.8}$$

4.3.3 The VIKOR Index

The VIKOR index is also calculated for each alternative as in Eq. (4.9) [24].

$$\begin{cases} Q_i = v \times [(S_i - S^*)/(S^- - S^*)] + (1 - \mathrm{v}) \times [(R_i - R^*)/(R^- - R^*)] \\ S^* = \min_i S_i, \quad S^- = \max_i S_i, \quad R^* = \min_i R_i, \quad R^- = \max_i R_i \end{cases} \tag{4.9}$$

where v indicates the strategic weight, which is often considered equal to 0.5 [24].

4.3.4 The Final Ranking of Alternatives

During this step, the alternatives are ranked as descending in values of (S) and (R) and (Q). The alternative with the lowest amount in attributes is the superior alternative.

4.4 Case Study

A workshop wants to buy the best CNC lathe model among the models of A_1, A_2, and A_3. The decision attributes are the amount of coolant consumption in liters (C_1) and number of pieces produced per day (C_2), and attributes have equal weights. Fig. 4.1 demonstrates the decision matrix.

It aims to choose the best CNC lathe using the VIKOR method.

Fig. 4.1 Decision matrix of purchasing CNC lathe

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{cc} - & + \\ C_1 & C_2 \\ \end{array}$$

$$\begin{matrix} & \begin{matrix} - & + \\ C_1 & C_2 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} 1 & 3000 \\ 2 & 3750 \\ 5 & 4500 \end{bmatrix} \end{matrix}$$

❖ **Solution**

(A) The f^* and f^- indexes

The best and worst amounts for each attribute are as follows:

$$\begin{aligned} f_1^* &= 1, & f_1^- &= 5 \\ f_2^* &= 4500, & f_2^- &= 3000 \end{aligned}$$

(B) The S and R indexes

The S index values for alternatives are computed as follows:

$$S_1 = \frac{1}{2} \times \left[\frac{1-1}{1-5}\right] + \frac{1}{2} \times \left[\frac{4500-3000}{4500-3000}\right] = 0 + \frac{1}{2} = \frac{1}{2}$$
$$S_2 = \frac{1}{2} \times \left[\frac{1-2}{1-5}\right] + \frac{1}{2} \times \left[\frac{4500-3750}{4500-3000}\right] = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$
$$S_3 = \frac{1}{2} \times \left[\frac{1-5}{1-5}\right] + \frac{1}{2} \times \left[\frac{4500-4500}{4500-3000}\right] = \frac{1}{2} + 0 = \frac{1}{2}$$

In addition, the R index values for alternatives are as follows:

$$R_1 = max\left\{\frac{1}{2} \times \left[\frac{1-1}{1-5}\right], \frac{1}{2} \times \left[\frac{4500-3000}{4500-3000}\right]\right\} = \frac{1}{2}$$
$$R_2 = max\left\{\frac{1}{2} \times \left[\frac{1-2}{1-5}\right], \frac{1}{2} \times \left[\frac{4500-3750}{4500-3000}\right]\right\} = \frac{1}{4}$$
$$R_3 = max\left\{\frac{1}{2} \times \left[\frac{1-5}{1-5}\right], \frac{1}{2} \times \left[\frac{4500-4500}{4500-3000}\right]\right\} = \frac{1}{2}$$

The R^-, R^*, S^-, and S^* indexes for three alternatives are as follows:

$$\begin{aligned} R^* &= min\, R_i = \tfrac{1}{4}, & R^- &= max\, R_i = \tfrac{1}{2} \\ S^* &= min\, S_i = \tfrac{3}{8}, & S^- &= max\, S_i = \tfrac{1}{2} \end{aligned}$$

(C) The VIKOR index

The VIKOR index (Q) values for three alternatives are as follows:

$$Q_1 = \frac{1}{2} \times \left[\frac{\frac{1}{2} - \frac{3}{8}}{\frac{1}{2} - \frac{3}{8}}\right] + \frac{1}{2} \times \left[\frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2} - \frac{1}{4}}\right] = 1$$

$$Q_2 = 0$$

$$Q_3 = \frac{1}{2} \times \left[\frac{\frac{1}{2} - \frac{3}{8}}{\frac{1}{2} - \frac{3}{8}}\right] + \frac{1}{2} \times \left[\frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2} - \frac{1}{4}}\right] = 1$$

(D) **The final ranking of alternatives**

The alternative with the lowest values of (S) and (R) and (Q) is selected as the superior alternative. Therefore, the second CNC lathe model (A_2) is the best alternative, and the alternatives are ranked as follows:

$$\left.\begin{array}{l} S_2 < S_3 = S_1 \\ R_2 < R_3 = R_1 \\ Q_2 < Q_3 = Q_1 \end{array}\right\} \quad A_2 > A_3 = A_1$$

4.5 Conclusion

The abundant utilization of the VIKOR method demonstrates its importance and popularity, due to its short steps and the factors of (S), (R), and (Q), which increase the accuracy of ranking alternatives. As illustrated in Fig. 4.2, the final ranking of alternatives is done by receiving the basic information and based on the different and compensatory attributes and then provided for the decision makers. Nowadays, the relevant software is provided for convenient utilization of the VIKOR method.

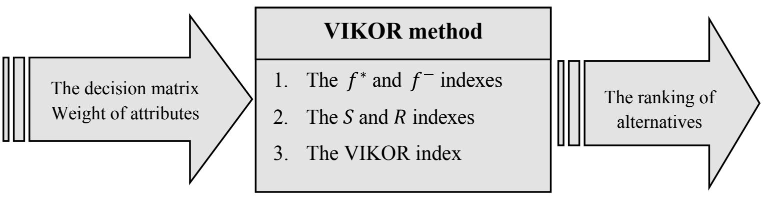


Fig. 4.2 A summary of the VIKOR method

Chapter 5
PROMETHEE I-II-III Methods

5.1 Introduction

The Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) methods was first introduced by Brans, Vincke and Mareschal in 1986 [25–27], which has been widely used so far. As the name indicates, the providers of this technique have sought to find a basic solution to improve decision-making evaluation. Therefore, it is recognized as an efficient method. The PROMETHEE I method only examines the obtained output and input flows and ranks alternatives partially.

However, in the PROMETHEE II method, the net flow is determined as the final values and the full ranking of the alternatives is done. In the PROMETHEE III method, the final ranking is performed based on the intervals and has abundant application, as presented in different studies such as facility location selection [28], ranking of accredited laboratories [29], selection of industrial robot [30], and selecting the ERP system [31]. The PROMETHEE method has the following features:

- Belonging to the compensatory methods;
- Converting qualitative attributes into the quantitative attributes;
- No need for the independence of attributes.

In this method, the matrix of alternatives and attributes is formed based on the information received from the decision maker as in Eq. (5.1).

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_5

$$F = \begin{bmatrix} f_1(A_1) & \cdots & f_j(A_1) & \cdots & f_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_i) & \cdots & f_j(A_i) & \cdots & f_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_m) & \cdots & f_j(A_m) & \cdots & f_n(A_m) \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{5.1}$$

In Eq. (5.1), $B = \{A_1, A_2, \ldots, A_m\}$ is a finite set of alternatives, and $C = \{f_1(0), f_2(0), \ldots, f_n(0)\}$ is a set of evaluation attributes of the alternatives of the set B.

Further, in this method, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$, as well as all the parameters of preference function like l in the quasi-criterion, m in the V-shape criterion, s and r in the linear criterion, q and p in the level criterion, and σ in the Gaussian criterion.

5.2 Description of PROMETHEE Methods

5.2.1 The Preference Function

In order to determine the value of the preference function, the difference between the pair of alternatives is initially obtained as in Eq. (5.2).

$$d_j(A_i, A_{i'}) = f_j(A_i) - f_j(A_{i'}); \quad i, i' \in \{1, \ldots, m\}, \ j = 1, \ldots, n \tag{5.2}$$

Therefore, the value of the preference function is computed from the function (5.3) [32].

$$P_j(A_i, A_{i'}) = f_j[d_j(A_i, A_{i'})]; \quad 0 \le P_j(A_i, A_{i'}) \le 1 \tag{5.3}$$

The type of the function should first be specified to determine the values of the preference function. Therefore, the type of each function is determined according to the type of each attribute evaluated through Tables 5.1 , 5.2, 5.3, 5.4, 5.5, and 5.6.

Table 5.1 Usual criterion (Type I) [32]

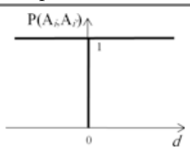
Description	Graph	Function	Condition	Parameter
Absence of difference in interval (d $\le$ 0), the existence of a complete priority of an alternative in interval (d > 0)		$f(d) = \begin{cases} 0 & d \le 0 \\ 1 & d > 0 \end{cases}$	Impacts and the issues related to the ecology	–

Table 5.2 Quasi-criterion (Type II) [32]

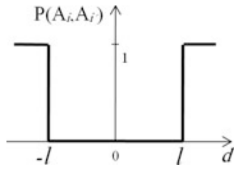
Description	Graph	Function	Condition	Parameter
Lack of differences in the interval (d $\leq$ *l*), the existence of a complete priority of an alternative in interval (d > *l*)		$f(d) = \begin{cases} 0 & d \leq l \\ 1 & d > l \end{cases}$	Attributes related to the discrete sources	*l*

Table 5.3 V-shape criterion (Type III) [32]

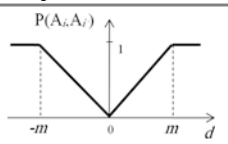
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d $\leq$ *m*), the existence of a complete priority of an alternative in interval (d > m)		$f(d) = \begin{cases} \frac{d}{m} & d \leq m \\ 1 & d > m \end{cases}$	Operational attributes, purchase costs	*m*

Table 5.4 Level criterion (Type IV) [32]

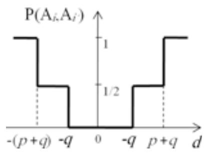
Description	Graph	Function	Condition	Parameter
Lack of difference in interval (d $\leq$ q), change in priority value of alternative linearly in the interval ($q < d \leq q + p$), the existence of a complete priority of an alternative in the interval (d > $q + p$)		$f(d) = \begin{cases} 0 & d \leq \mathrm{q} \\ \frac{1}{2} & \mathrm{q} < d \leq \mathrm{q} + \mathrm{p} \\ 1 & d > q + p \end{cases}$	Long-term benefit, maintenance cost, lifetime cost	*q,p*

Table 5.5 Linear criterion (Type V) [32]

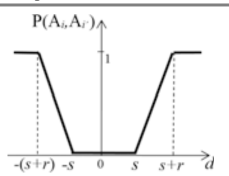
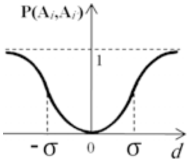
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d $\leq$ s), change in the priority value of alternative linearly in the interval (s < d $\leq$ s + r), the existence of the full priority of an alternative in interval (d > s + r)		$f(d) = \begin{cases} 0 & d \leq s \\ \frac{\mathrm{d-s}}{\mathrm{r}} & s < d \leq s + r \\ 1 & d > s + r \end{cases}$	Exploration cost, short-term profit, constructing cost	*s,r*

Table 5.6 Gaussian criterion (Type VI) [32]

Description	Graph	Function	Condition	Parameter
Lack of difference in the interval (d $\leq$ 0), an increase in the priority rate of alternative in the interval (d > 0)		$f(d) = \begin{cases} 0 & d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}} & d > 0 \end{cases}$	Appearance, quality, and safety	σ

5.2.2 *The Preference Index*

With respect to the weight of attributes, the preference index is calculated as in Eq. (5.4) [32].

$$\pi(A_{\mathrm{i}}, A_{\mathrm{i}'}) = \sum_{j=1}^{n} P_j(A_{\mathrm{i}}, A_{\mathrm{i}'}).w_j; \quad i, i' \in \{1, \ldots, m\} \tag{5.4}$$

5.2.3 *The Leaving and Entering Flows*

The leaving and entering flows are determined through Eqs. (5.5) and (5.6) [32].

$$\varphi^{+}(A_i) = \frac{1}{m-1} \sum_{A_i \epsilon A} \pi(A_i, A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.5}$$

$$\varphi^{-}(A_i) = \frac{1}{m-1} \sum_{A_i \epsilon A} \pi(A_{i'}, A_i); \quad i, i' \in \{1, \ldots, m\} \tag{5.6}$$

5.2.4 *The Net Flow*

In this step, the full ranking $\left(\mathrm{P}^{\mathrm{II}}, \mathrm{I}^{\mathrm{II}}\right)$ is performed. When the decision maker needs to rank alternatives completely, he computed the net flow using Eq. (5.7) and then ranks.

$$\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i); \quad i = 1, \ldots, m \tag{5.7}$$

5.2.5 Final Ranking of Alternatives (PROMETHEE I Method)

At first, Eqs. (5.8) to (5.11) are considered [32].

$$A_i P^+ A_{i'} \quad if \quad \varphi^+(A_i) > \varphi^+(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.8}$$

$$A_i I^+ A_{i'} \quad if \quad \varphi^+(A_i) = \varphi^+(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.9}$$

$$A_i P^- A_{i'} \quad if \quad \varphi^-(A_i) < \varphi^-(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.10}$$

$$A_i I^- A_{i'} \quad if \quad \varphi^-(A_i) = \varphi^-(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.11}$$

In this method, the A_i alternative is better than the alternative $A_{i'}$, if:

$$A_i\, P\, A_{i'} \quad if \quad \begin{cases} A_i\, P^+\, A_{i'} \quad and \quad A_i\, P^-\, A_{i'} \\ A_i\, P^+\, A_{i'} \quad and \quad A_i\, I^-\, A_{i'} \\ A_i\, I^+\, A_{i'} \quad and \quad A_i\, P^+\, A_{i'} \end{cases}; \quad i, i' \in \{1, \ldots, m\} \tag{5.12}$$

In addition, the alternatives A_i and $A_{i'}$ are indifferent to each other, if:

$$A_i\, I\, A_{i'} \quad if \quad A_i\, I^-\, A_{i'} \quad and \quad A_{i'}\, I^-\, A_i; \quad i, i' \in \{1, \ldots, m\} \tag{5.13}$$

Accordingly, all alternatives are ranked.

5.2.6 Final Ranking of Alternatives (PROMETHEE II Method)

In PROMETHEE II method, the A_i alternative is better than the alternative $A_{i'}$, if:

$$A_i\, P^{II}\, A_{i'} \quad if \quad \varphi(A_i) > \varphi(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.14}$$

Further, the alternatives A_i and $A_{i'}$ are indifferent to each other, if:

$$A_i\, I^{II}\, A_{i'} \quad if \quad \varphi(A_i) = \varphi(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{5.15}$$

Accordingly, all alternatives are ranked.

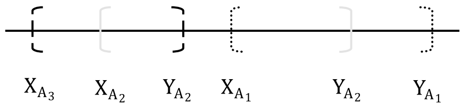


Fig. 5.1 Relation between the three alternatives A_1, A_2, and A_3 [27, 34]

5.2.7 *Final Ranking of Alternatives (PROMETHEE III Method)*

In this technique, an interval with the value of $[X_{A_i}, Y_{A_i}]$ is considered, and alternative A_i is better than the alternative $A_{i'}$, if:

$$A_i\, P^{III}\, A_{i'} \quad if \quad X_{A_i} > X_{A_{i'}}; \quad i, i' \in \{1, \ldots, m\} \tag{5.16}$$

And, the alternatives A_i and $A_{i'}$ are indifferent to each other, if:

$$A_i\, I^{III}\, A_{i'} \quad if \quad X_{A_i} \leq X_{A_{i'}} \quad and \quad X_{A_i} \geq X_{A_{i'}}; \quad i, i' \in \{1, \ldots, m\} \tag{5.17}$$

The interval values are obtained from Eqs. (5.18) and (5.19) [27, 33].

$$\begin{cases} X_{A_i} = \bar{\varphi}(A_i) - \alpha\sigma_{A_i} \\ Y_{A_i} = \bar{\varphi}(A_i) + \alpha\sigma_{A_i} \end{cases}; \quad i = 1, \ldots, m \tag{5.18}$$

$$\begin{cases} \bar{\varphi}(A_i) = \frac{1}{n} \sum\limits_{A_{i'} \in A} [\pi(A_i, A_{i'}) - \pi(A_{i'}, A_i)] = \frac{1}{n}\varphi(A_i) \\ \sigma^2_{A_i} = \frac{1}{n} \sum\limits_{A_{i'} \in A} [\pi(A_i, A_{i'}) - \pi(A_{i'}, A_i) - \bar{\varphi}(A_i)]^2 \end{cases}; \quad i, i' \in \{1, \ldots, m\} \tag{5.19}$$

In addition, the values of $\alpha > 0$ are used as a parameter greater than zero in these equations. Therefore, by assuming three intervals for the three alternatives, A_1, A_2, and A_3 are in accordance with Fig. 5.1.

With respect to Fig. 5.1, the relation between alternatives is as $A_1 I^{III} A_2$ and $A_2 I^{III} A_3$ and $A_1 P^{III} A_3$. The final ranking of alternatives and choosing the best alternative are performed using interval values.

5.3 Case Study

The board of directors of an automotive factory seeks to select the best alternative among the six projects proposed by the research and development unit to choose its production. The decision attributes include the volume of workforce (C_1), reduction in the operations rate required to construct (C_2), construction costs (C_3), vehicle maintenance costs (C_4), environmental degradation effects (C_5), and beauty (C_6), respectively. The research and development unit, after holding successive sessions, have converted the qualitative attributes into the quantitative attributes and

$$
\begin{array}{c}
 \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc}
- & + & - & - & - & + \\
C_1 & C_2 & C_3 & C_4 & C_5 & C_6
\end{array} \\
\begin{bmatrix}
80 & 90 & 600 & 54 & 8 & 5 \\
65 & 58 & 200 & 97 & 1 & 1 \\
83 & 60 & 400 & 72 & 4 & 7 \\
40 & 80 & 1000 & 75 & 7 & 10 \\
52 & 72 & 600 & 20 & 3 & 8 \\
94 & 96 & 700 & 36 & 5 & 6
\end{bmatrix}
\end{array}
$$

Fig. 5.2 Decision matrix for car production

Table 5.7 Type and values of parameters in the preference function

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
Type	II	III	V	IV	I	VI
Value	l = 10	m = 30	s = 50 r = 450	q = 10 p = 50	–	$\sigma = 5$

presented the matrix of Fig. 5.2 to the board for decision making. Furthermore, the weights of attributes are equal, and the value of α equals 0.160, and Table 5.7 illustrates the type and values of parameters of the preference function.

The purpose is to determine and compare the best alternative to produce cars using the PROMETHEE I, PROMETHEE II, and PROMETHEE III methods.

❖ Solution

(A) The preference function

First, the value difference between the pair of alternatives is computed. For example, for the pair of alternative A_2 and A_3 for attribute 6 as follows:

$$\mathrm{d}_6(A_2, A_3) = 1 - 7 = -6$$

$$\mathrm{d}_6(A_3, A_2) = 7 - 1 = 6$$

Thus, the values of the preference function according to the type of preference function and its parameters, for example, are calculated as follows:

$$\mathrm{P}_6(A_2, A_3) = 0$$

$$\mathrm{P}_6(A_3, A_2) = 1 - e^{-\left(\frac{(6)^2}{2(5)^2}\right)} = 0.513$$

Finally, the values of the preference function for the other alternatives are computed as shown in Table 5.8.

Table 5.8 Preference function values

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	1.776	1.500	1.608	0.600	1.110
A_2	2.772	–	2.388	1.998	1.776	3.000
A_3	1.416	1.080	–	1.998	0.336	2.574
A_4	2.394	3.030	1.830	–	1.338	1.272
A_5	2.640	3.090	2.922	2.380	–	2.688
A_6	1.716	2.394	1.500	2.592	0.798	–

Table 5.9 Preference index values

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	0.296	0.250	0.268	0.100	0.185
A_2	0.462	–	0.389	0.333	0.296	0.500
A_3	0.236	0.180	–	0.333	0.056	0.429
A_4	0.399	0.505	0.305	–	0.223	0.212
A_5	0.444	0.515	0.487	0.380	–	0.448
A_6	0.286	0.399	0.250	0.432	0.133	–

(B) **The preference index**

Given the weights of attributes, for example, the preference index value for alternatives A_2 and A_3 is as follows:

$$\pi(A_2, A_3) = \frac{1}{6}(1 + 0 + 0.334 + 0 + 1 + 0) = 0.389$$

$$\pi(A_3, A_2) = \frac{1}{6}(0 + 0.067 + 0 + 0.500 + 0 + 0.513) = 0.180$$

Other values are determined as shown in Table 5.9.

(C) **The leaving and entering flows**

For instance, the leaving and entering flows for A_1 is calculated as follows:

$$\varphi^{+}(A_1) = \frac{1}{6-1}(0.296 + 0.250 + 0.268 + 0.100 + 0.185) = 0.220$$

$$\varphi^{-}(A_1) = \frac{1}{6-1}(0.462 + 0.236 + 0.399 + 0.444 + 0.286) = 0.365$$

Table 5.10 indicates the other leaving and entering flows.

Table 5.10 Leaving and entering flows

	φ^+	φ^-
A_1	0.220	0.365
A_2	0.396	0.379
A_3	0.247	0.336
A_4	0.329	0.349
A_5	0.455	0.162
A_6	0.300	0.355

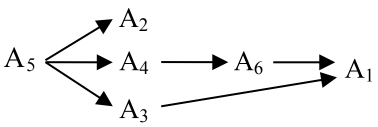
(D) **The net flow**

By calculating the leaving and entering flows, the net flows of all alternatives are as follows:

$$\varphi_1 = -0.145,\ \varphi_2 = 0.017,\ \varphi_3 = -0.089,\ \varphi_4 = -0.020,\ \varphi_5 = 0.293,\ \varphi_6 = -0.055$$

(E) **Final ranking of alternatives (PROMETHEE I method)**

The priority of alternatives to each other is determined based on the leaving and entering flows as shown in Table 5.11. The ranking of alternatives is as follows:



(F) **Final ranking of alternatives (PROMETHEE II method)**

Given the net flow of the alternatives, the fifth project (A_5) is the best alternative for car production. The ranking of other alternatives is as follows:

$$A_5 > A_2 > A_4 > A_6 > A_3 > A_1$$

Table 5.11 Priority of alternatives

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	I	–	–	–	–	–
A_2	–	I	–	–	–	–
A_3	P	–	I	–	–	–
A_4	P	–	–	I	–	P
A_5	P	P	P	P	I	P
A_6	P	–	–	–	–	I

Table 5.12 Interval values of alternatives

	X_A	Y_A
A_1	−0.137	−0.105
A_2	0.012	0.016
A_3	−0.085	−0.065
A_4	−0.019	−0.015
A_5	0.211	0.277
A_6	−0.052	−0.040

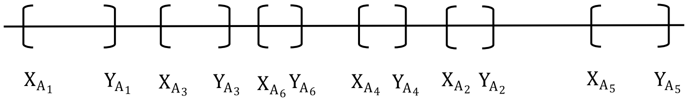


Fig. 5.3 Interval relation between the alternatives

(G) Final ranking of alternatives (PROMETHEE III method)

For example, considering the value of α, the interval of alternative A_1 is determined as follows:

$$\begin{cases} X_{A_1} = -0.121 - 0.160(0.101) = -0.137 \\ Y_{A_1} = -0.121 + 0.160(0.101) = -0.105 \end{cases}$$

$$\begin{cases} \bar{\varphi}(A_1) = \frac{1.099-1.827}{6} = -0.121 \\ \sigma_{A_1} = 0.101 \end{cases}$$

The other values of alternatives are obtained as shown in Table 5.12. Further, Fig. 5.3 demonstrates the relation between alternatives.

According to the obtained values and Fig. 5.3, the fifth project (A_5) is selected as the best alternative for the car production, and the final ranking is determined as follows:

$$A_5 > A_2 > A_4 > A_6 > A_3 > A_1$$

5.4 Conclusion

With the advent of PROMETHEE method, many researchers and experts in the field of MADM focused on this technique. In the following years, this method was developed, and even PROMETHEE-based methods were presented, and a lot of papers were published. The use of six-type preference function is regarded as one of the most prominent features of PROMETHEE method, which is used to determine the ideal alternative more precisely, due to the dependence of attributes. On the other hand, the development of this technique, aiming to increase its accuracy and develop its utilization in many areas, increases the use of PROMETHEE. Fig. 5.4 illustrates a summary of this method.

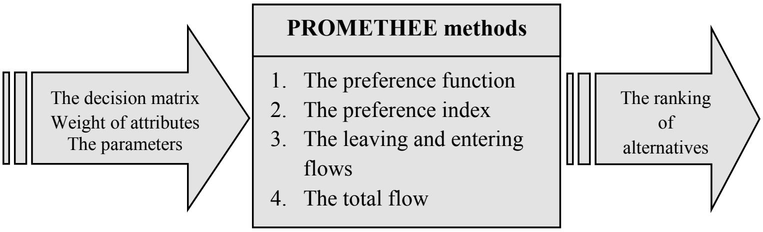


Fig. 5.4 A summary of PROMETHEE I-II-III methods

Chapter 6
QUALIFLEX Method

6.1 Introduction

The QUALIFLEX method was introduced by Paelinck in 1975 [35–37], which is rooted in the permutation method, introduced by Jacquet Lagreze [16, 27]. In QUALIFLEX, each possible ranking of existing m alternative is evaluated. In other words, the ranking of alternatives is evaluated to the number of $m!$ permutation, and finally, the most appropriate ones are selected for the final ranking.

Also, it is assumed that the decision matrix $D = \left\| f_{ij} \right\|$ is clear and the weights w_j are calculated for existing attributes by one of the proposed algorithms such as entropy [38, 39]. Similar to the other alternatives ranking methods, this technique is also used for airport location selection [39], the optimal site selection for the nuclear power plant [40], and supplier evaluation and selection [41]. The QUALIFLEX method has the following features:

- This technique, similar to the permutation method, is in the boundary of compensatory and non-compensatory methods;
- Attributes should be independent;
- There is no need to convert the qualitative attributes into the quantitative attributes.

The entering information of the QUALIFLEX method is as decision matrix and based on the information received from the decision maker as shown in Eq. (6.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{6.1}$$

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_6

In matrix of Eq. (6.1), r_{ij} displays the element of decision matrix for ith alternative in jth attribute. In addition, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$ by considering the normalized property $\left(\sum_{j=1}^{n} w_j = 1\right)$.

6.2 Description of QUALIFLEX Method

6.2.1 The Initial Permutation of Alternatives

The possible permutations are made up of the existing m alternative, for example, if $m = 3$, consequently, $m! = 3! = 6$.

Therefore, with the assumption of three alternatives of permutations, the alternatives are as set of Eq. (6.2).

$$\begin{aligned} \text{per}_1 &= \{A_1, A_2, A_3\} \\ \text{per}_2 &= \{A_1, A_3, A_2\} \\ \text{per}_3 &= \{A_2, A_1, A_3\} \\ \text{per}_4 &= \{A_2, A_3, A_1\} \\ \text{per}_5 &= \{A_3, A_1, A_2\} \\ \text{per}_6 &= \{A_3, A_2, A_1\} \end{aligned} \tag{6.2}$$

6.2.2 The Initial Ranking of Alternatives

At this stage, the decision matrix, provided by the decision maker, is ranked based on the strengths. The number 1 is given to an alternative which is better than the rest in an attribute, and the other alternatives are ranked similarly.

6.2.3 The Dominant and Dominated Values

If the permutation matches the amounts of ranking, the value is 1, and if it does not match, the value is -1. When two alternatives are identical in one attribute, the amount of zero is allocated. For example, it is assumed that the values of Eq. (6.3) to be in the hypothetical permutation and ranking of alternatives are $A_1 > A_2 = A_3$.

$$\text{per} = \{A_2, A_1, A_3\}: \tag{6.3}$$

$$A_2 < A_1 \rightarrow -1$$
$$A_2 = A_3 \rightarrow 0$$
$$A_3 < A_1 \rightarrow 1$$

In Eq. (6.3), the permutation equals to $A_2 > A_1$, but the ranking of alternatives are $A_2 < A_1$, and as this value mismatches the ranking of attributes, the value becomes -1. Further, the permutation equals to $A_2 = A_3$, and the value becomes zero. Eventually, the permutation equals to $A_3 < A_1$, but the ranking of alternatives are $A_3 < A_1$ which match each other and amount becomes 1.

6.2.4 *The Permutation Values of Attributes*

The values computed in the previous step are aggregated together and are calculated separately for all permutations and attributes.

6.2.5 *The Permutation Values of Alternatives*

The permutation value of each attribute is multiplied by its weight and is aggregated together and is introduced as the permutation value.

6.2.6 *The Final Ranking of Alternatives*

After determining the permutation values of alternatives, the alternative with the highest permutation value represents the best alternative.

6.3 Case Study

A refinery intends to buy a liquefied petroleum gas (LPG) bunker. A LPG bunker should be purchased among the three models (A_1, $A_{2,}$ and A_3). The attributes such as price (C_1), working pressure (C_2), and capacity (C_3) are considered for decision making, and the decision matrix is as shown in Fig. 6.1.

The weight of the attributes is considered equal. It is desirable to select the best model of LPG bunker and rank the alternatives by the QUALIFLEX method.

Fig. 6.1 Decision matrix of buying LPG bunker

$$\begin{array}{cc} & \begin{array}{ccc} - & + & + \\ C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1.200 & 18 & 32000 \\ 2 & 23 & 24000 \\ 2 & 15 & 25000 \end{bmatrix} \end{array}$$

❖ Solution

(A) **The initial permutation of alternatives**

There are six permutations for three alternatives as follows:

$$\begin{aligned} \text{per}_1 &= A_1 > A_2 > A_3 \\ \text{per}_2 &= A_2 > A_1 > A_3 \\ \text{per}_3 &= A_2 > A_3 > A_1 \\ \text{per}_4 &= A_3 > A_2 > A_1 \\ \text{per}_5 &= A_3 > A_1 > A_2 \\ \text{per}_6 &= A_1 > A_3 > A_2 \end{aligned}$$

(B) **The initial ranking of alternatives**

Here, the first attribute is negative. In other words, lower number leads to a better attribute. In addition, the other two attributes are positive, that is, the higher the better. Therefore, ranking alternatives is based on the attributes as shown in Fig. 6.2.

(C) **The dominant and dominated values**

For instance, the dominant and dominated values are obtained for firth permutation:

$$\begin{aligned} &\text{Per}_1 = A_1 > A_2 > A_3 \text{ for } C_1 : \\ &A_2 < A_1 \rightarrow 1 \\ &A_2 = A_3 \rightarrow 0 \\ &A_3 < A_1 \rightarrow 1 \end{aligned}$$

Fig. 6.2 Initial ranking of alternatives

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \end{array}$$

Table 6.1 Permutation values

	C_1	C_2	C_3
per_1	2	1	1
per_2	0	3	−1
per_3	−2	1	−3
per_4	−2	−1	−1
per_5	0	−3	1
per_6	2	−1	3

$$\begin{aligned}&\mathrm{Per}_1 = A_1 > A_2 > A_3 \text{ for } C_2 :\\&A_2 > A_1 \rightarrow -1\\&A_2 > A_3 \rightarrow 1\\&A_3 < A_1 \rightarrow 1\end{aligned}$$

$$\begin{aligned}&\mathrm{Per}_1 = A_1 > A_2 > A_3 \text{ for } C_3 :\\&A_2 < A_1 \rightarrow 1\\&A_2 < A_3 \rightarrow -1\\&A_3 < A_1 \rightarrow 1\end{aligned}$$

Accordingly, the values are obtained for other attributes.

(D) **The permutation values of attributes**

Table 6.1 indicates the permutation values of attributes.

(E) **The permutation values of alternatives**

The permutation values of alternatives are as follows:

$$\begin{aligned}\mathrm{per}_1 &= 1.333\\\mathrm{per}_2 &= 0.667\\\mathrm{per}_3 &= -1.333\\\mathrm{per}_4 &= -1.333\\\mathrm{per}_5 &= -0.667\\\mathrm{per}_6 &= 1.333\end{aligned}$$

(F) **The final ranking of alternatives**

According to the permutations of alternatives, the permutations of 1 and 6 are selected:

$$\begin{aligned}\mathrm{per}_1 &= A_1 > A_2 > A_3\\\mathrm{per}_6 &= A_1 > A_3 > A_2\end{aligned}$$

As a result, the first model of LPG bunker (A_1) is chosen as the best alternative.

6.4 Conclusion

Paelinck could present the QUALIFLEX method as one of the precise techniques for ranking alternatives using the permutation method. Considering the independent and compensatory features of this technique, the permutation of attributes is determined, due to the use of the dominant and dominated property of alternatives based on the attributes. Accordingly, it is possible to find the most ideal alternative, and actually, this process has differentiated the QUALIFLEX method in comparison to the other methods. Fig. 6.3 indicates the process of determining the best alternative, now used in various papers with different applications.

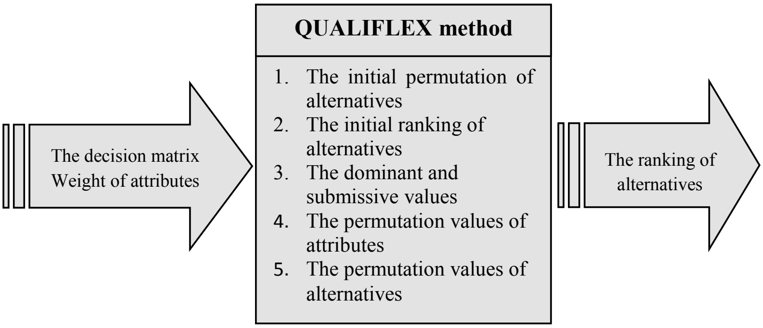


Fig. 6.3 A summary of QUALIFLEX method

Chapter 7
SIR Method

7.1 Introduction

Superiority and Inferiority Ranking (SIR) method was introduced by Xu in 2001 [42–44]. The basis of this technique is the utilization of superiority and inferiority values, by determining the type of the preference function, similar to the PROMETHEE method. Then, the net flow is calculated using the weight matrix, similar to the simple additive weighting (SAW) method and technique of order preference by similarity to the ideal solution (TOPSIS). Finally, the optimal solution is chosen among the solutions obtained from the superiority and inferiority matrix. The SIR method has various applications such as selection of solar energy for green building [45], choosing the concrete pump [46] and contractor selection [47], and has the following features:

- This is one of the compensatory methods;
- Attributes should be independent;
- The qualitative attributes are converted into quantitative attributes.

The decision matrix is formed according to the information received from the decision maker as Eq. (7.1).

$$\mathrm{F} = \begin{bmatrix} f_1(A_1) & \cdots & f_j(A_1) & \cdots & f_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_i) & \cdots & f_j(A_i) & \cdots & f_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_m) & \cdots & f_j(A_m) & \cdots & f_n(A_m) \end{bmatrix}_{m\times n} \quad ; \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{7.1}$$

In Eq. (7.1), $\mathrm{B} = \{A_1, A_2, \ldots, A_m\}$ is a finite set of alternatives, and $\mathrm{C} = \{f_1(0), f_2(0), \ldots, f_n(0)\}$ is the set of evaluation attributes for the alternatives of set B.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_7

On the other hand, the decision maker determines the weight of attributes $[w_1, w_2, \ldots, w_n]$ and provides all parameters of preference such as l in the quasi-criterion, m in the V-shape criterion, s and r in the linear criterion, q and p in the level criterion, and σ in the Gaussian criterion.

7.2 Description of SIR Method

7.2.1 Comparing the Alternatives

Considering the decision matrix in Eq. (7.2).

$$D = \begin{bmatrix} C_1(A_1) & \cdots & C_j(A_1) & \cdots & C_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C_1(A_i) & \cdots & C_j(A_i) & \cdots & C_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C_1(A_m) & \cdots & C_j(A_m) & \cdots & C_n(A_m) \end{bmatrix}_{m\times n} \quad ; \quad i = 1, \ldots, m,\ j = 1, \ldots, n \tag{7.2}$$

First, the alternatives are compared in the decision matrix based on attributes and a function such as $d = C(A_1) - C(A_2)$ is introduced, which implies that the value of d equals to difference of the alternative A_1 and A_2 in attribute C.

d is similarly calculated for all alternatives. Further, the function $P(A_1, A_2)$ is defined as $P(A_1, A_2) = f(C(A_1) - C(A_2)) = f(d)$. It is noteworthy that this amount is for positive attributes (the more the better) and the symmetry of this value should be calculated for negative attributes.

7.2.2 The Preference Function

The type of function should first be specified to determine the values of the preference function. Therefore, the type of each function is determined according to the type of each evaluated attribute through Tables 7.1, 7.2, 7.3, 7.4, 7.5, 7.6.

Table 7.1 Usual criterion (type I) [48]

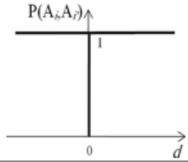
Description	Graph	Function	Condition	Parameter
Absence of difference in interval (d $\leq$ 0), the existence of a complete priority of an alternative in interval (d > 0)		$f(d) = \begin{cases} 0 & d \leq 0 \\ 1 & d > 0 \end{cases}$	Impacts and the issues related to the ecology	–

Table 7.2 Quasi-criterion (type II) [48]

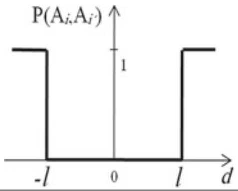
Description	Graph	Function	Condition	Parameter
Lack of differences in the interval (d $\leq$ l) the existence of a complete priority of an alternative in interval (d > l)		$f(d) = \begin{cases} 0 & d \leq l \\ 1 & d > l \end{cases}$	Attributes related to the discrete sources	l

Table 7.3 V-shape criterion (type III) [48]

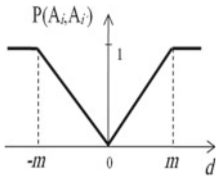
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d $\leq$ m), the existence of a complete priority of an alternative in interval (d > m)		$f(d) = \begin{cases} \frac{d}{m} & d \leq m \\ 1 & d > m \end{cases}$	Operational attributes, purchase costs	m

Table 7.4 Level criterion (type IV) [48]

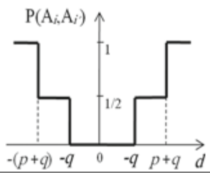
Description	Graph	Function	Condition	Parameter
Lack of difference in interval (d $\leq$ q), change in priority value of alternative linearly in the interval ($q < d \leq q + p$), the existence of a complete priority of an alternative in the interval (d > $q + p$)		$f(d) = \begin{cases} 0 & d \leq \mathrm{q} \\ \frac{1}{2} & \mathrm{q} < d \leq \mathrm{q} + p \\ 1 & d > q + p \end{cases}$	Long-term benefit, maintenance cost, lifetime cost	q, p

Table 7.5 Linear criterion (type V) [48]

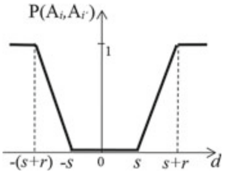
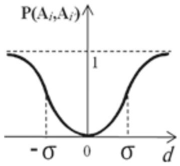
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d ≤ s), change in the priority value of alternative linearly in the interval (s < d ≤ s + r), the existence of the full priority of an alternative in interval (d > s + r)		$f(d)=\begin{cases}0 & d\le s\\ \frac{d-s}{r} & s<d\le s+r\\ 1 & d>s+r\end{cases}$	Exploration cost, short-term profit, constructing cost	$s,\ r$

Table 7.6 Gaussian criterion (type VI) [48]

Description	Graph	Function	Condition	Parameter
Lack of difference in the interval (d ≤ 0), an increase in the priority rate of alternative in the interval (d > 0)		$f(d)=\begin{cases}0 & d\le 0\\ 1-e^{-\frac{d^2}{2\sigma^2}} & d>0\end{cases}$	Appearance, quality, and safety	σ

7.2.3 The (S) and (I) Indexes and (S) and (I) Matrices

The (S) index is determined as Eq. (7.3).

$$S_j(A_i)=\sum_{i=1}^{m}P_j(A_i,A_{i'})=\sum_{i=1}^{m}f_j\big(C_j(A_{i'})-C_j(A_i)\big);\quad j=1,\ldots,n \tag{7.3}$$

Additionally, (I) index is calculated as Eq. (7.4).

$$I_j(A_i)=\sum_{i=1}^{m}P_j(A_i,A_{i'})=\sum_{i=1}^{m}f_j\big(C_j(A_i)-C_j(A_{i'})\big);\quad j=1,\ldots,n \tag{7.4}$$

If these indexes are calculated for all elements of decision matrix (D), the (S) and (I) matrices are obtained as Eqs. (7.5) and (7.6) [48].

$$\mathrm{S}=\begin{bmatrix} S_1(A_1) & \cdots & S_j(A_1) & \cdots & S_n(A_1)\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ S_1(A_i) & \cdots & S_j(A_i) & \cdots & S_n(A_i)\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ S_1(A_m) & \cdots & S_j(A_m) & \cdots & S_n(A_m)\end{bmatrix}_{m\times n};\quad i=1,\ldots,m,\ j=1,\ldots,n \tag{7.5}$$

$$\mathrm{I} = \begin{bmatrix} I_1(A_1) & \cdots & I_j(A_1) & \cdots & I_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ I_1(A_i) & \cdots & I_j(A_i) & \cdots & I_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ I_1(A_m) & \cdots & I_j(A_m) & \cdots & I_n(A_m) \end{bmatrix}_{m\times n} ; \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{7.6}$$

7.2.4 The Flow Matrix

In this step, the (S) and (I) matrices are used and the flow matrix is formed similar to the SAW and TOPSIS methods as the weights introduced for the importance of attribute and are linearly multiplied by values of (S) matrix, and result is placed in the flow matrix. The same is done for (I) matrix. Therefore, the solution technique is as Eqs. (7.7) and (7.8), and $\varphi > (A_i)$ represents the dominant flow and $\varphi < (A_i)$ indicates the dominated flow computed as Eqs. (7.7) and (7.8) [48].

$$\varphi^{>}(A_i) = V\left[S_1(A_i),\ldots,S_\mathrm{j}(A_i),\ldots,S_\mathrm{n}(A_i)\right] = \sum_{j=1}^{n} S_\mathrm{j}(A_i)\times w_j; \quad i = 1,\ldots,m \tag{7.7}$$

$$\varphi^{<}(A_i) = V\left[\mathrm{I}_1(A_i),\ldots,\mathrm{I}_\mathrm{j}(A_i),\ldots,\mathrm{I}_\mathrm{n}(A_i)\right] = \sum_{j=1}^{n} \mathrm{I}_\mathrm{j}(A_i)\times w_j; \quad i = 1,\ldots,m \tag{7.8}$$

7.2.5 The (n) and (r) Flows

The n-flow is calculated as Eq. (7.9) [48].

$$n\text{-flow} = \varphi^{>}(A_i) - \varphi^{<}(A_i); \quad i = 1,\ldots,m \tag{7.9}$$

The r-flow is determined as Eq. (7.10) [48].

$$r\text{-flow} = \frac{\varphi^{>}(A_i)}{\varphi^{>}(A_i) + \varphi^{<}(A_i)}; \quad i = 1,\ldots,m \tag{7.10}$$

7.2.6 Final Ranking of Alternatives (SIR-SAW Method)

The SIR-SAW method has two steps. Initially, the s-flow values, which are the same values of $\varphi^{>}(A_i)$, are ranked in descending order. That is, alternative with the lowest amount of $\varphi^{>}(A_i)$ is the best alternative.

In the next step, the I-flow values, which are the values of $\varphi^{<}(A_i)$, are ranked in an ascending order. The best alternative is selected by considering the share of two categories of ranking, which is called relative ranking method.

7.2.7 Final Ranking of Alternatives (SIR-PROMETHEE I Method)

In SIR-PROMETHEE I method, the n-flow values are computed and then, are ranked in a descending order. Namely, alternative with the lowest n-flow value is the best alternative and is regarded as the full ranking technique.

7.2.8 Final Ranking of Alternatives (SIR-PROMETHEE II Method)

In SIR-PROMETHEE II method, the r-flow values are calculated and ranked in a descending order. That is, the alternative with the lowest r-flow value is the best alternative. Also, it is a full ranking technique.

7.3 Case Study

Experts have proposed six projects for the construction of the hydroelectric power plant. The decision attributes include the volume of workforce (C_1), power generation capacity (C_2), construction cost (C_3), maintenance cost (C_4), environmental destructive impacts (C_5), and security level (C_6). The research and development unit, after holding consecutive meetings, converts the qualitative attributes into the quantitative attributes and presents the matrix of Fig. 7.1 to the board for decision making.

Further, the weights of attributes are equal, and Table 7.7 indicates the type and values of the parameters of the preference function.

$$
\begin{array}{c|cccccc|}
 & - & + & - & - & - & + \\
 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
A_1 & 80 & 90 & 6 & 5.400 & 8 & 5 \\
A_2 & 65 & 58 & 2 & 9.700 & 1 & 1 \\
A_3 & 83 & 60 & 4 & 7.200 & 4 & 7 \\
A_4 & 40 & 80 & 10 & 7.500 & 7 & 10 \\
A_5 & 52 & 72 & 6 & 2 & 3 & 8 \\
A_6 & 94 & 96 & 7 & 3.600 & 5 & 6
\end{array}
$$

Fig. 7.1 Decision matrix for project selection

Table 7.7 Types and values of parameters in the preference function

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
Type	II	III	V	IV	I	VI
Value	$l = 10$	$m = 30$	$s = 0.500$ $r = 5$	$q = 1$ $p = 6$	–	$\sigma = 5$

It is desirable to select the most appropriate project for constructing the hydroelectric power plant using the SIR method.

❖ **Solution**

(A) **Comparing the alternatives**

As the attribute C_1 is negative, it is necessary to subtract in symmetric form for computing d:

$$
\begin{aligned}
&P_1(A_1, A_1) = f(0) \\
&P_1(A_1, A_2) = f(C_1(\mathrm{A}_2) - C_1(\mathrm{A}_1)) = f(65 - 80) = f(-15) \\
&P_1(A_1, A_3) = f(C_1(\mathrm{A}_3) - C_1(\mathrm{A}_1)) = f(83 - 80) = f(3) \\
&P_1(A_1, A_4) = f(C_1(\mathrm{A}_4) - C_1(\mathrm{A}_1)) = f(40 - 80) = f(-40) \\
&P_1(A_1, A_5) = f(C_1(\mathrm{A}_5) - C_1(\mathrm{A}_1)) = f(52 - 80) = f(-28) \\
&P_1(A_1, A_6) = f(C_1(\mathrm{A}_6) - C_1(\mathrm{A}_1)) = f(94 - 80) = f(14)
\end{aligned}
$$

In addition, for attribute C_2, which is positive:

$$
\begin{aligned}
P_2(A_1,A_1) &= f(0)\\
P_2(A_1,A_2) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_2)) = f(90-58) = f(32)\\
P_2(A_1,A_3) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_3)) = f(90-60) = f(30)\\
P_2(A_1,A_4) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_4)) = f(90-80) = f(10)\\
P_2(A_1,A_5) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_5)) = f(90-72) = f(18)\\
P_2(A_1,A_6) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_6)) = f(90-96) = f(-6)
\end{aligned}
$$

(B) **The preference function**

The attribute C_1 belongs to the third type and l = 10 are considered that:

$$
\begin{aligned}
P_1(A_1,A_1) &= f(0), d<10 \to f(\mathrm{d}) = 0\\
P_1(A_1,A_2) &= f(C_1(\mathrm{A}_2) - C_1(\mathrm{A}_1)) = f(65-80) = f(-15), d<10 \to f(\mathrm{d}) = 0\\
P_1(A_1,A_3) &= f(C_1(\mathrm{A}_3) - C_1(\mathrm{A}_1)) = f(83-80) = f(3), d<10 \to f(\mathrm{d}) = 0\\
P_1(A_1,A_4) &= f(C_1(\mathrm{A}_4) - C_1(\mathrm{A}_1)) = f(40-80) = f(-40), d<10 \to f(\mathrm{d}) = 0\\
P_1(A_1,A_5) &= f(C_1(\mathrm{A}_5) - C_1(\mathrm{A}_1)) = f(52-80) = f(-28), d<10 \to f(\mathrm{d}) = 0\\
P_1(A_1,A_6) &= f(C_1(\mathrm{A}_6) - C_1(\mathrm{A}_1)) = f(94-80) = f(14), d > 10 \to f(\mathrm{d}) = 1
\end{aligned}
$$

The attribute C_2 belongs to the second type and is as follows for this category of attributes with m = 30:

$$
\begin{aligned}
P_2(A_1,A_1) &= f(0), d<10 \to f(\mathrm{d}) = 0\\
P_2(A_1,A_2) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_2)) = f(90-58) = f(32), d > 30 \to f(\mathrm{d}) = 1\\
P_2(A_1,A_3) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_3)) = f(90-60) = f(30), d = 30 \to f(\mathrm{d}) = 1\\
P_2(A_1,A_4) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_4)) = f(90-80) = f(10), 0<d<30 \to f(\mathrm{d}) = 0.333\\
P_2(A_1,A_5) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_5)) = f(90-72) = f(18), 0<d<30 \to f(\mathrm{d}) = 0.600\\
P_2(A_1,A_6) &= f(C_2(\mathrm{A}_1) - C_2(\mathrm{A}_6)) = f(90-96) = f(-6), d \le 0 \to f(\mathrm{d}) = 0
\end{aligned}
$$

(C) **The (S) and (I) indexes and (S) and (I) matrices**

(S) index values for alternative A_1 and attributes C_1 and C_2, are as follows:

$$
S_1(A_1) = \sum_{i=1}^{m} P_1(A_1, A_\mathrm{i}) = 0+0+0+0+0+1 = 1
$$

$$
S_2(A_1) = \sum_{i=1}^{m} P_2(A_1, A_\mathrm{i}) = 0+1+1+0.333+0.600+0 = 2.933
$$

If the same technique to be used for all elements of decision matrix, the (S) matrix is as follow:

$$S = \begin{pmatrix} 1 & 2.933 & 0.889 & 1.500 & 0 & 0.274 \\ 3 & 0 & 3.889 & 0 & 5 & 0 \\ 1 & 0.067 & 2.222 & 0.500 & 3 & 0.610 \\ 5 & 1.667 & 0 & 0.500 & 1 & 1.711 \\ 4 & 0.867 & 0.889 & 3 & 4 & 0.886 \\ 0 & 3.533 & 0.556 & 2.500 & 2 & 0.413 \end{pmatrix}$$

If the same steps, performed to compute the (S) matrix, are applied for determining the (I) matrix, the (I) matrix is obtained as follows:

$$I = \begin{pmatrix} 3 & 0.200 & 1.111 & 1 & 5 & 0.665 \\ 2 & 3.267 & 0 & 3.500 & 0 & 2.607 \\ 3 & 3.067 & 0.333 & 1.500 & 2 & 0.185 \\ 0 & 0.867 & 4.111 & 1.500 & 4 & 0 \\ 1 & 1.667 & 1.111 & 0 & 1 & 0.077 \\ 5 & 0 & 1.778 & 0.500 & 3 & 0.371 \end{pmatrix}$$

(D) **The flow matrix**

Initially, the values of $\varphi^{>}(A)$ are obtained as follows:

$$\varphi^{>}(A_1) = \sum_{j=1}^{n} S_j(A_1) \times w_j = \frac{1}{6}(1 + 2.933 + 0.889 + 1.500 + 0 + 0.274) = 1.099$$

$$\varphi^{>}(A_2) = \frac{1}{6}(3 + 0 + 3.889 + 0 + 5 + 0) = 1.981$$

$$\varphi^{>}(A_3) = \frac{1}{6}(1 + 0.667 + 2.222 + 0.500 + 3 + 0.610) = 1.332$$

$$\varphi^{>}(A_4) = \frac{1}{6}(5 + 0.867 + 0.889 + 3 + 4 + 0.886) = 2.440$$

$$\varphi^{>}(A_5) = \frac{1}{6}(4 + 0.867 + 0.889 + 3 + 4 + 0) = 2.274$$

$$\varphi^{>}(A_6) = \frac{1}{6}(0 + 3.533 + 0.556 + 2.500 + 2 + 0.413) = 1.500$$

Consequently:

$$\varphi^{>}(A) = \begin{pmatrix} 1.099 \\ 1.981 \\ 1.332 \\ 2.440 \\ 2.274 \\ 1.500 \end{pmatrix}$$

Then, the values of $\varphi^{<}(A)$ are obtained as follows:

$$\varphi^{<}(A_1) = \sum_{j=1}^{n} \mathrm{I_j}(A_1) \times w_\mathrm{j} = \frac{1}{6}(3 + 0.200 + 1.111 + 1 + 5 + 0.665) = 1.829$$
$$\varphi^{<}(A_2) = \frac{1}{6}(2 + 3.267 + 0 + 3.500 + 0 + 2.607) = 1.896$$
$$\varphi^{<}(A_3) = \frac{1}{6}(3 + 3.067 + 0.333 + 1.500 + 2 + 0.185) = 1.681$$
$$\varphi^{<}(A_4) = \frac{1}{6}(0 + 0.867 + 4.111 + 1.500 + 4 + 0) = 1.746$$
$$\varphi^{<}(A_5) = \frac{1}{6}(1 + 1.667 + 1.111 + 0 + 1 + 0.077) = 0.809$$
$$\varphi^{<}(A_6) = \frac{1}{6}(5 + 0 + 1.778 + 0.500 + 3 + 0.371) = 1.775$$

Therefore:

$$\varphi^{<}(A) = \begin{pmatrix} 1.829 \\ 1.896 \\ 1.681 \\ 1.746 \\ 0.809 \\ 1.775 \end{pmatrix}$$

(E) **The (n) and (r) flows**

The n-flow value is determined as follows:

$$n\text{-flow} = \varphi^{>}(A_i) - \varphi^{<}(A_i)$$

Thus, φ_n (A) equals to:

$$\varphi_n(A) = \begin{pmatrix} -0.729 \\ 0.085 \\ -0.349 \\ 0.694 \\ 1.465 \\ -0.275 \end{pmatrix}$$

The r-flow value is computed as follows:

$$r\text{-flow} = \frac{\varphi^{>}(A_i)}{\varphi^{>}(A_i) + \varphi^{<}(A_i)}$$

As a result, the value of φ_{r} (A) equals to:

$$\varphi_r(A) = \begin{pmatrix} 0.375 \\ 0.511 \\ 0.442 \\ 0.583 \\ 0.738 \\ 0.458 \end{pmatrix}$$

(F) **Final ranking of alternatives (SIR-SAW method)**

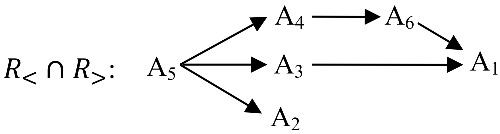
Initially, $\varphi^{>}(A)$ is ranked in descending order:

$$R_{>} : A_4 \rightarrow A_5 \rightarrow A_2 \rightarrow A_6 \rightarrow A_3 \rightarrow A_1$$

Then, $\varphi^{<}(A)$ is ranked in ascending order:

$$R_{<} : A_5 \rightarrow A_3 \rightarrow A_4 \rightarrow A_6 \rightarrow A_1 \rightarrow A_2$$

Therefore, if the share of two ranking methods is calculated, the following result is obtained:



Finally, first project (A_1) is the most appropriate alternative.

(G) **Final ranking of alternatives (SIR-PROMETHEE I method)**

φ_n (A) is ranked in descending order and first project (A_1) is selected:

$$R_n : A_5 \rightarrow A_4 \rightarrow A_2 \rightarrow A_6 \rightarrow A_3 \rightarrow A_1$$

(H) **Final ranking of alternatives (SIR-PROMETHEE II method)**

φ_r (A) is ranked in descending order, and the first project (A_1) is selected as follows:

$$R_r : A_5 \rightarrow A_4 \rightarrow A_2 \rightarrow A_6 \rightarrow A_3 \rightarrow A_1$$

7.4 Conclusion

The SIR method is considered as one of the special techniques for modeling and using PROMETHEE, SAW, and TOPSIS methods for conducting more accurate final evaluation and developing applications range. Additionally, it leads to the elimination of weaknesses of each one using the strengths of other method. Also, SIR method is considered as one of the powerful methods in MADM for alternatives ranking and determining the ideal alternative. Fig. 7.2 represents a summary of the steps of this technique.

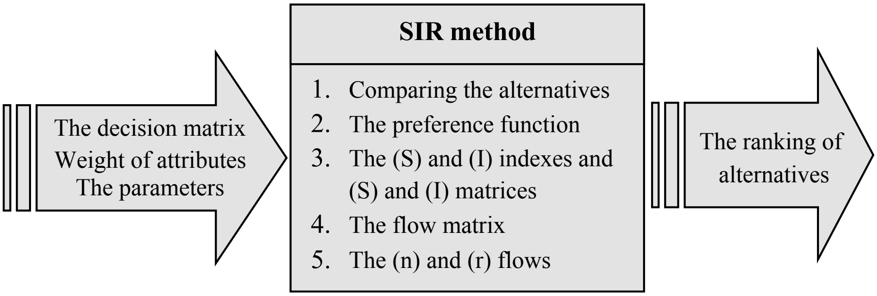


Fig. 7.2 A summary of the SIR method

Chapter 8
EVAMIX Method

8.1 Introduction

The EVAluation of MIXed data (EVAMIX) method, introduced in 1982 by Voogd [49–51], with two completely different approaches to the quantitative and qualitative attributes and attributes should be independent. Therefore, EVAMIX greatly helps experts and managers to reach the solution quickly, due to its insensitivity in converting qualitative attributes into the quantitative attributes. The abundant application of this method includes the choosing of wastewater treatment alternative [52], the agricultural tractor selection [53], ranking approaches to struggle corruption [54] and has the following features:

- It is one of the compensatory methods;
- The attributes are independent of each other;
- It is not necessary to convert the qualitative attributes into the quantitative attributes.

The input information of EVAMIX is expressed using the matrix of alternatives and attributes, based on the information received from the decision maker, as shown in Eq. (8.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{8.1}$$

In addition, r_{ij} is the element of the decision matrix for ith alternative in jth attribute. Furthermore, the decision maker provides the weight of attribute $[w_1, w_2, \ldots, w_\mathrm{n}]$.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_8

8.2 Description of EVAMIX Method

8.2.1 The Superiority Rate of Alternatives

Attributes are divided into two qualitative categories (ordinal), represented by O, and quantitative (cardinal), represented by C. The ordinal attributes are compared by the dominant factor $\alpha_{ii'}$, indicating the superiority of alternative A_i to the alternative A_i', and this value is obtained as shown in Eq. (8.2) [49, 55].

$$\alpha_{ii'} = \left[\sum_{j \in O} \left\{ w_j \times \operatorname{sgn}\left(e_{ij} - e_{i'j}\right) \right\}^c \right]^{1/c} ; \quad i, i' \in \{1, \ldots, m\},\ j = 1, \ldots, n \tag{8.2}$$

where e_{ij} indicates the evaluation of the alternative A_i based on the attribute C_j and $e_{i'j}$ denotes the evaluation of the alternative A_i' based on the attribute C_j as shown in Eq. (8.3) [49, 55].

$$sgn\left(e_{ij} - e_{i'j}\right) = \begin{cases} -1 & if \quad e_{ij} < e_{i'j} \\ 0 & if \quad e_{ij} = e_{i'j} \\ +1 & if \quad e_{ij} > e_{i'j} \end{cases} ; \quad i, i' \in \{1, \ldots, m\},\ j = 1, \ldots, n \tag{8.3}$$

w_j demonstrates the weight allocated to the attribute, calculated by two techniques.

8.2.1.1 The First Technique for Calculating Weights

When quantitative and qualitative attributes are together, the decision maker is asked to allocate the relevant weight and determine the importance of attribute. Additionally, the cardinal attributes are compared through the computation of dominant factor $\alpha_{ii'}$, indicating superiority of the alternative A_i to the alternative A_i', and this value is obtained as shown in Eq. (8.4) [49, 55].

$$\alpha_{ii'} = \left[\sum_{j \in C} \left\{ w_j \times \left(e_{ij} - e_{i'j}\right) \right\}^c \right]^{1/c} ; \quad i, i' \in \{1, \ldots, m\},\ j = 1, \ldots, n \tag{8.4}$$

8.2.1.2 The Second Technique for Calculating Weights

The random weights are used when all attributes are qualitative. Now, the random numbers are frequently repeated to calculate w_j, and then, the number of times which each alternative is located in the first place is examined, and alternative placed in the first place with the highest repetition is chosen as the best alternative. This is repeated for all alternatives to get a final ranking of alternatives.

8.2.2 The Differential Matrix in the Ordinal Attributes

The differential values $(\delta_{ii'})$ are considered as the function $\alpha_{ii'}$, $\delta_{ii'} = h(\alpha_{ii'})$, and its value is obtained as shown in Eq. (8.5) [49, 55].

$$\delta_{ii'} = \frac{(\alpha_{ii'} - \alpha^-)}{(\alpha^+ - \alpha^-)}; \quad i, i' \in \{1, \ldots, m\} \tag{8.5}$$

where α^+ is the maximum amount of dominant factor and α^- is the minimum amount of dominant factor in the ordinal attributes.

The values obtained from the differential values are placed in the differential matrix $(\Delta_{ii'})$ for comparing each alternative.

8.2.3 The Differential Matrix in the Cardinal Attributes

$d_{\mathrm{ii'}}$ is defined as the function $\alpha_{ii'}$, $(d_{ii'} = h(\alpha_{ii'}))$, and its value is obtained as shown in Eq. (8.6) [49, 55].

$$d_{ii'} = \frac{(\alpha_{ii'} - \alpha^-)}{(\alpha^+ - \alpha^-)}; \quad i, i' \in \{1, \ldots, m\} \tag{8.6}$$

where α^+ is the maximum amount of dominant factor and α^- is the minimum amount of dominant factor in the cardinal attributes. The values obtained from the differential values are placed in the differential matrix $(\nabla_{ii'})$ for comparison among each of alternatives.

8.2.4 The Total Dominance

The total dominance is defined by Eq. (8.7) [49, 55].

$$D_{ii'} = w_{\mathrm{o}}\delta_{ii'} + w_c d_{ii'}; \quad i, i' \in \{1, \ldots, m\} \tag{8.7}$$

where $w_o = \sum_{j \in O} w_j$ and $w_c = \sum_{j \in C} w_j$. Then, the evaluation score of alternatives is calculated by Eq. (8.8) [49, 55].

$$S_i = \left[\sum_{i'} \frac{D_{i'i}}{D_{ii'}}\right]^{-1}; \quad i, i' \in \{1, \ldots, m\} \tag{8.8}$$

8.2.5 The Final Ranking of Alternatives

The alternatives are ranked based on the evaluation scores (S_i).

8.3 Case Study

A company should choose a model for its production line among the automatic inspection machine models of A_1, A_2, and A_3. The attributes, such as cost (C_1), refinability (C_2), repeatability (C_3), inspection capacity of pieces (C_4), complexity of working with device (C_5), and warranty period in terms of years (C_6), are available for decision making. The decision matrix is as shown in Fig. 8.1. In addition, Table 8.1 indicates the weight given by the decision maker.

It aims to select the best alternative and express the final ranking of the alternatives by EVAMIX method.

❖ **Solution**

(A) **The superiority rate of alternatives**

In solution, the qualitative attributes are initially separated from the decision matrix as Fig. 8.2.

Then, the matrix $\alpha_{ii'}$ is formed using Eq. (8.2). By replacing $C = 1$, the following result is obtained:

Fig. 8.1 Decision matrix for purchasing automatic inspection machine

	−	+	+	+	−	+
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	30	Moderate	Very high	24	Very high	12.500
A_2	12	High	Moderate	25	Moderate	22
A_3	15	Very high	Low	32	Low	10

Table 8.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
w_j	0.100	0.175	0.250	0.150	0.125	0.200

Fig. 8.2 Qualitative attributes of the decision matrix

	C_2	C_3	C_5
A_1	Moderate	Very high	Very high
A_2	High	Moderate	Moderate
A_3	Very high	Low	Low

$$
\begin{aligned}
&\alpha_{11} = \alpha_{22} = \alpha_{33} : \\
&sgn(e_{11} - e_{11}) = sgn(e_{22} - e_{22}) = sgn(e_{33} - e_{33}) = 0 \\
&\alpha_{11} = \alpha_{22} = \alpha_{33} = 0 \\
&\alpha_{23} : \\
&sgn(e_{22} - e_{32}) = -1,\ sgn(e_{23} - e_{33}) = +1,\ sgn(e_{25} - e_{35}) = -1 \\
&\alpha_{23} = (0.175 \times -1) + (0.250 \times 1) + (-1 \times 0.125) = -0.050 \\
&\alpha_{13} : \\
&sgn(e_{12} - e_{32}) = -1,\ sgn(e_{13} - e_{33}) = +1,\ sgn(e_{15} - e_{35}) = -1 \\
&\alpha_{13} = (0.175 \times -1) + (0.250 \times 1) + (-1 \times 0.125) = -0.050 \\
&\alpha_{12} : \\
&sgn(e_{12} - e_{22}) = +1,\ sgn(e_{13} - e_{23}) = +1,\ sgn(e_{15} - e_{25}) = -1 \\
&\alpha_{12} = (0.175 \times -1) + (0.250 \times 1) + (-1 \times 0.125) = -0.050
\end{aligned}
$$

Now, the other values of matrix are symmetric of these values; therefore, the matrix $\alpha_{ii'}$ is as follows:

$$
a_{ii'} = \begin{pmatrix} 0 & -0.050 & -0.050 \\ 0.050 & 0 & -0.050 \\ 0.050 & 0.050 & 0 \end{pmatrix}
$$

Then, the quantitative attribute is separated from the decision matrix as shown in Fig. 8.3. Then, the matrix $\alpha_{ii'}$ is formed by Eq. (8.4) to form this matrix. By replacement of C = 1, the following result is obtained:

Fig. 8.3 Quantitative attributes of the decision matrix

	C_1	C_4	C_6
A_1	30	24	12.500
A_2	12	25	22
A_3	15	32	10

$$\alpha_{11} = \alpha_{22} = \alpha_{33}:$$
$$(e_{11} - e_{11}) = (e_{22} - e_{22}) = (e_{33} - e_{33}) = 0$$
$$\alpha_{11} = \alpha_{22} = \alpha_{33} = 0$$
$$\alpha_{12}:$$
$$\alpha_{12} = [-0.100 \times (30 - 12) + 0.150 \times (24 - 25) + 0.200 \times (12.500 - 22)] = -3.850$$
$$\alpha_{13}:$$
$$\alpha_{13} = [-0.100 \times (30 - 15) + 0.150 \times (24 - 32) + 0.200 \times (12.500 - 10)] = -2.200$$
$$\alpha_{23}:$$
$$\alpha_{23} = [-0.100 \times (12 - 15) + 0.150 \times (25 - 32) + 0.200 \times (22 - 10)] = 1.650$$

The other values of matrix are symmetric of these values; consequently, the matrix $\alpha_{ii'}$ is as follows:

$$a_{ii'} = \begin{pmatrix} 0 & -3.850 & -2.200 \\ 3.850 & 0 & 1.650 \\ 2.200 & -1.650 & 0 \end{pmatrix}$$

(B) **The differential matrix in the ordinal attributes**

The elements of the matrix $\Delta_{ii'}$ are computed according to Eq. (8.5) as following:

$$\Delta_{ii'} = \begin{pmatrix} 0.500 & 0 & 0 \\ 1 & 0.500 & 0 \\ 1 & 1 & 0.500 \end{pmatrix}$$

(C) **The differential matrix in the cardinal attributes**

The elements of the matrix $\nabla_{ii'}$ are calculated according to Eq. (8.6) as follows:

$$\nabla_{ii'} = \begin{pmatrix} 0.500 & 0 & 0.214 \\ 1 & 0.500 & 0.714 \\ 0.786 & 0.286 & 0.500 \end{pmatrix}$$

(D) **The total dominance**

The values of $D_{ii'}$ are as follows:

$$D_{11} = (0.550 \times 0.500) + (0.450 \times 0.500) = 0.500$$
$$D_{12} = (0.550 \times 0) + (0.450 \times 0) = 0$$
$$D_{13} = (0.550 \times 0) + (0.450 \times 0.214) = 0.096$$
$$D_{21} = (0.550 \times 1) + (0.450 \times 1) = 1$$
$$D_{22} = (0.550 \times 0.500) + (0.450 \times 0.500) = 0.500$$
$$D_{23} = (0.550 \times 0) + (0.450 \times 0.714) = 0.321$$
$$D_{31} = (0.550 \times 1) + (0.450 \times 0.786) = 0.904$$
$$D_{32} = (0.550 \times 1) + (0.450 \times 0.286) = 0.679$$
$$D_{33} = (0.550 \times 0.500) + (0.450 \times 0.500) = 0.500$$

Finally, the evaluation score of points is calculated:

$$S_1 = \left[\frac{D_{11}}{D_{11}} + \frac{D_{21}}{D_{12}} + \frac{D_{31}}{D_{13}}\right]^{-1} = \left[1 + 0 + \frac{0.904}{0.096}\right]^{-1} = 0.096$$
$$S_2 = \left[\frac{D_{12}}{D_{21}} + \frac{D_{22}}{D_{22}} + \frac{D_{32}}{D_{23}}\right]^{-1} = \left[0 + 1 + \frac{0.679}{0.321}\right]^{-1} = 0.321$$
$$S_3 = \left[\frac{D_{13}}{D_{31}} + \frac{D_{23}}{D_{32}} + \frac{D_{33}}{D_{33}}\right]^{-1} = \left[\frac{0.096}{0.904} + \frac{0.321}{0.679} + 1\right]^{-1} = 0.633$$

(E) **The final ranking of alternatives**

Subsequently, the third automatic inspection machine (A_3) is the best alternative, and the alternatives are ranked as follows:

$$A_3 > A_2 > A_1$$

8.4 Conclusion

The EVAMIX method with the characteristic of the ordinal and cardinal attributes has relative superiority to other methods. On the other hand, the lack of need to convert the quantitative attributes into quantitative ones shortens the process of obtaining result. The EVAMIX method has a short process, and its steps are summarized in Fig. 8.4. Thus, these features have led to use of this technique until recent years.

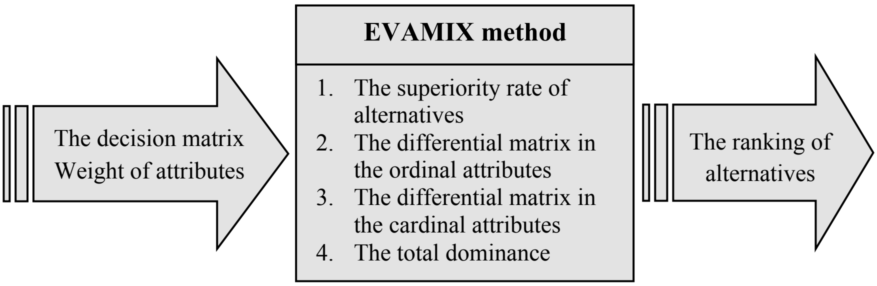


Fig. 8.4 A summary of EVAMIX method

Chapter 9
ARAS Method

9.1 Introduction

The Additive Ratio ASsessment (ARAS) method was introduced by Zavadskas and Turskis in 2010 [56–58], which aims to select the best alternative based on a number of attributes and the final ranking of alternatives is made by determining the utility degree of each alternative. Considering these issues, this technique has various applications such as recruitment and selection of personnel [59–61], and ranking of factoring companies [62, 63]. This new method has the following features:

- It is one of the compensatory methods;
- The qualitative attributes should be converted into the quantitative attributes;
- Attributes are independent.

Also, the decision matrix is utilized based on the information received from the decision maker, according to Eq. (9.1).

$$X = \begin{bmatrix} r_{o1} & \cdots & r_{oj} & \cdots & r_{on} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} ; \quad i = o, 1, \ldots, m,\ j = 1, \ldots, n \qquad (9.1)$$

where r_{ij} illustrates the element of decision matrix for ith alternative in jth attribute and r_{oj} indicates the optimal value of jth attribute. If the value of r_{oj} is unknown, Eq. (9.2) is used for positive attribute and Eq. (9.3) for negative attribute [64].

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$$r_{oj} = \max_i r_{ij}; \quad i = o, 1, \ldots, m,\ j = 1, \ldots, n \tag{9.2}$$

$$r_{oj} = \min_i r_{ij}; \quad i = o, 1, \ldots, m,\ j = 1, \ldots, n \tag{9.3}$$

On the other hand, the decision maker presents weight of the attribute $[w_1, w_2, \ldots, w_n]$.

9.2 Description of ARAS Method

9.2.1 The Normalized Decision Matrix

Eq. (9.4) is used to make the normalized decision matrix [64].

$$r_{ij}^* = \frac{r_{ij}}{\sum_{i=0}^{m} r_{ij}}; \quad j = 1, \ldots, n \tag{9.4}$$

9.2.2 The Weighted Normalized Decision Matrix

Given the weight of attributes $[w_1, w_2, \ldots, w_n]$, the weighted normalized values of each attribute are obtained by Eq. (9.5) [64].

$$\hat{r}_{ij} = r_{ij}^* \cdot w_j; \quad i = o, 1, \ldots, m,\ j = 1, \ldots, n \tag{9.5}$$

9.2.3 The Optimality Function

The optimality function (S_i) is value which is regarded as the larger the better, which is specified through Eq. (9.6) for ith alternative [64].

$$S_i = \sum_{j=1}^{n} \hat{r}_{ij}; \quad i = o, 1 \ldots, m \tag{9.6}$$

9.2.4 The Utility Degree

The utility degree is used for final ranking of alternatives. The utility degree is in the interval (0, 1). The utility degree (k_i) for ith alternative is obtained by Eq. (9.7) [64].

$$k_i = \frac{S_i}{V_o}; \quad i = \mathrm{o}, 1, \ldots, m \tag{9.7}$$

In Eq. (9.7), V_o is the optimality value of S_i [58, 64].

9.2.5 The Final Ranking of Alternatives

In the final ranking, the k_i values are arranged in descending order, and the alternative with the highest k_i value is selected as the best alternative.

9.3 Case Study

Three contractors (A_1, A_2, and A_3) were proposed for construction and maintenance during the operation of an automotive parts manufacturing plant. Experts provided the attributes such as construction management cost (C_1), construction period (C_2), strength (C_3), complexity of maintenance (C_4), work hardness (C_5), and duration of maintenance contract (C_6), and after converting the qualitative attributes into quantitative attributes, the decision matrix is presented as shown in Fig. 9.1. In addition, the weight of each attribute is determined according to Table 9.1.

It is desirable is to select the best alternative and express the final ranking of alternatives by the ARAS method.

Fig. 9.1 Decision matrix of the plant construction and maintenance

$$\begin{array}{c} \\ \\ A_o \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{cccccc} - & - & + & - & - & + \\ C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \end{array}$$

$$\begin{matrix} A_o \\ A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 0.710 & 4.100 & 0.740 & 0.310 & 0.420 & 0.830 \\ 0.710 & 4.100 & 0.180 & 0.720 & 0.990 & 0.250 \\ 1.330 & 5.900 & 0.740 & 0.310 & 0.420 & 0.830 \\ 1.450 & 4.900 & 0.270 & 0.650 & 0.420 & 0.440 \end{bmatrix}$$

Table 9.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
w_j	0.171	0.185	0.177	0.225	0.157	0.085

Table 9.2 Normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_o	0.169	0.216	0.383	0.156	0.187	0.353
A_1	0.169	0.216	0.093	0.362	0.440	0.106
A_2	0.317	0.310	0.383	0.156	0.187	0.353
A_3	0.345	0.258	0.140	0.327	0.187	0.187

Table 9.3 Weighted normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_o	0.029	0.040	0.068	0.035	0.029	0.030
A_1	0.029	0.040	0.016	0.081	0.069	0.009
A_2	0.054	0.057	0.068	0.035	0.029	0.030
A_3	0.059	0.048	0.025	0.074	0.029	0.016

❖ Solution

(A) **The normalized decision matrix**

In this example, the normalized values of decision matrix are according to Table 9.2.

(B) **The weighted normalized decision matrix**

With respect to the weight values of each attribute in Table 9.2, the weighted normalized values of each attribute are calculated as shown in Table 9.3.

(C) **The optimality function**

The values of the optimality function for each alternative are computed as follows:

$$s_0 = 0.029 + 0.040 + 0.068 + 0.035 + 029 + 0.030 = 0.231$$

$$s_1 = 0.029 + 0.040 + 0.016 + 0.081 + 0.069 + 0.009 = 0.244$$

$$s_2 = 0.054 + 0.057 + 0.068 + 0.035 + 0.029 + 0.030 = 0.273$$

$$s_2 = 0.059 + 0.048 + 0.025 + 0.074 + 0.029 + 0.016 = 0.251$$

(D) **The degree utility**

Given that the value of V_0 equals 0.273, the utility degree of each alternative is determined as follows:

$$k_1 = 0.894, \quad k_2 = 1, \quad k_3 = 0.919$$

(E) **The final ranking of alternatives**

Finally, the second contractor (A_2) is the best alternative and the final ranking is as follows:

$$A_2 > A_3 > A_1$$

9.4 Conclusion

The utility degree is considered as the ranking index of each alternative in the ARAS method. The presentation of this technique has led to its various utilization in recent years and has been used in fuzzy and gray environments in combination with other methods. The simplicity and having short steps, as summarized in Fig. 9.2, are other features of the ARAS method.

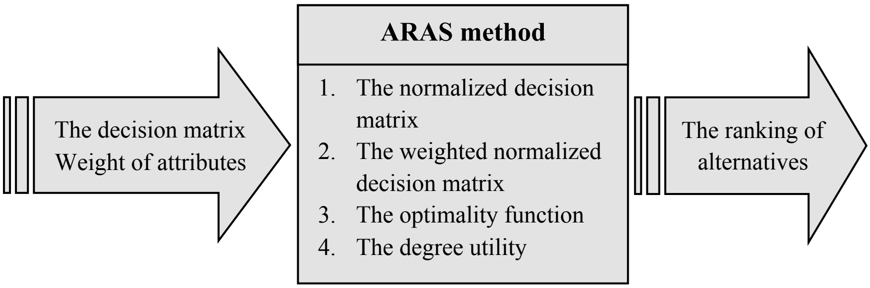

Fig. 9.2 A summary of ARAS method

Chapter 10
Taxonomy Method

10.1 Introduction

The taxonomy method was introduced by Adanson in 1763 and expanded by a group of mathematicians from Poland in 1950. In 1968, Zyegnant Hellwing from the Wroclaw high school introduced this method as a means of classifying and determining the degree of development [65, 66]. This method is very appropriate for grading, classifying, and comparing different activities with respect to their advantages and utility degree from studied attributes. This method is applied in typological division of countries [67–69] and evaluation of manpower [70, 71]. This method has the following features:

- It is one of the compensatory methods;
- The qualitative attributes are converted into the quantitative attributes;
- Attributes are independent.

The decision matrix is used for input information of the taxonomy method. In this matrix, the alternatives and attributes are expressed based on the information received from the decision maker, as Eq. (10.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{10.1}$$

In Eq. (10.1), r_{ij} is the element of the decision matrix for ith alternative in jth attribute.

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10.2 Description of Taxonomy Method

10.2.1 The Mean and Standard Deviation of Attributes

In order to calculate the mean and standard deviation of attributes, Eqs. (10.2) and (10.3) are used, respectively.

$$\bar{r}_j = \frac{1}{m}\sum_{i=1}^{m} r_{ij}; \quad j = 1, \ldots, n \tag{10.2}$$

$$S_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(r_{ij} - \bar{r}_j)^2}; \quad j = 1, \ldots, n \tag{10.3}$$

10.2.2 The Standard Matrix

In the decision matrix, the alternatives are expressed based on the attributes which have different measurement scales and this stage tries to equalize their different units, and Eq. (10.4) is used for this purpose [67–71].

$$Z_{ij} = \frac{r_{ij} - \bar{r}_j}{S_j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{10.4}$$

In Eq. (10.4), z_{ij} denotes the standardized element for ith alternative in jth attributes and the standard matrix is as Eq. (10.5).

$$\mathrm{Z} = \begin{bmatrix} z_{11} & \cdots & z_{1j} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{i1} & \cdots & z_{ij} & \cdots & z_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{m1} & \cdots & z_{mj} & \cdots & z_{mn} \end{bmatrix}_{m\times n}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{10.5}$$

10.2.3 The Composite Distance Matrix

Firstly, the distance of each alternative from the other alternatives compared to each of attributes is calculated using Eq. (10.6) [67–71].

$$C_{ab} = \sqrt{\sum_{j=1}^{n} (z_{aj} - z_{bj})^2} \tag{10.6}$$

In Eq. (10.6), a and b are evaluated alternatives for pairwise comparison between two alternatives. The composite distance matrix between alternatives is as Eq. (10.7).

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{10.7}$$

10.2.4 Homogenizing the Alternatives

In this step, firstly, the minimum distance rate of each row of composite distance matrix is determined between alternatives. Then, the mean and standard deviation of the minimum distance values of each row are calculated according to Eqs. (10.8) and (10.9), respectively.

$$\overline{O} = \frac{1}{m} \sum_{i=1}^{m} O_i \tag{10.8}$$

$$S_O = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (O_i - \overline{O})^2} \tag{10.9}$$

where, o_i is the minimum distance of ith row. Eq. (10.10) is utilized to determine the homogeneity range of composite distance matrix.

$$O = \overline{O} \pm 2S_O \tag{10.10}$$

If the minimum distance values of each row are not situated in the interval Eq. (10.10), they are inhomogeneous and eliminated, and again the mean and standard deviation of the values are calculated.

10.2.5 The Development Pattern

By homogenizing the alternatives, the attribute development pattern is determined through Eq. (10.11) using the matrix Z obtained in the second step [67–71].

$$C_{io} = \sqrt{\sum_{j=1}^{n} (z_{ij} - z_{oj})^2}; \quad i = 1, \ldots, m \tag{10.11}$$

where, z_{oj} represents the ideal value for the jth standard attribute, depending on the positive or negative type of attribute, z_{ij} indicates the standardized value of jth attribute for ith alternative, and C_{io} illustrates the development pattern for ith attribute.

10.2.6 The Final Ranking of Alternatives

At this step, the high limit of development (C_O) is initially calculated according to Eq. (10.12) [67–71].

$$C_O = \overline{C}_{io} + 2S_{C_{io}}; \quad i = 1, \ldots, m \tag{10.12}$$

Then, the attributes are ranked by the development attribute F_i, which is for ith alternative, and is obtained from Eq. (10.13) [67–71].

$$F_i = \frac{C_{io}}{C_o}; \quad i = 1, \ldots, m \tag{10.13}$$

The amount of F_i is between zero and one, and any close value to zero indicates a greater development of the alternative (the highest rank) and any close value to one demonstrates the lack of development of that alternative (the lowest rank).

10.3 Case Study

The organizational transportation department intends to purchase a number of buses, which should choose a model among the models of a bus with LPG fuel (A_1), a bus with CNG fuel (A_2), and diesel bus (A_3). Experts provided attributes such as air pollution (C_1), price (C_2), facilities and equipment (C_3), and qualitative attributes became quantitative attributes. The decision matrix is as Fig. 10.1.

The purpose is to choose the best bus model using taxonomy method.

Fig. 10.1 Decision matrix of bus purchase

$$
\begin{array}{c}
 \\ \\ A_1 \\ A_2 \\ A_3
\end{array}
\begin{array}{ccc}
- & - & + \\
C_1 & C_2 & C_3
\end{array}
$$

$$
\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array}
\begin{bmatrix}
0.710 & 4.100 & 0.180 \\
1.330 & 5.900 & 0.740 \\
1.450 & 4.900 & 0.270
\end{bmatrix}
$$

❖ **Solution**

(A) **The mean and standard deviation of attributes**

Table 10.1 indicates the mean and standard deviation values of each attribute.

(B) **The standard matrix**

With respect to the mean and standard deviation values obtained from Table 10.1, the standard matrix (Z) is as Fig. 10.2.

(C) **The composite distance matrix**

For instance, according to the decision matrix, the first alternative distance compared to the other alternatives is computed as follows:

$$C_{12} = \sqrt{(-1.141 - 0.420)^2 + (-0.961 - 1.035)^2 + (-0.720 - 1.142)^2} = 3.145$$

$$C_{13} = \sqrt{(-1.141 - 0.722)^2 + (-0.961 - (-0.074))^2 + (-0.720 - (-0.421))^2} = 2.085$$

$$C_{23} = \sqrt{(0.420 - 0.722)^2 + (1.035 - (-0.074))^2 + (1.142 - (-0.421))^2} = 1.940$$

$$C_{11} = C_{22} = C_{33} = 0$$

Table 10.1 Mean and standard deviation values

	C_1	C_2	C_3
$\bar{r}$	1.163	4.967	0.397
S	0.397	0.902	0.301

Fig. 10.2 Standard matrix

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}
\begin{array}{c}
\begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\
\begin{bmatrix}
-1.141 & -0.961 & -0.720 \\
0.420 & 1.035 & 1.142 \\
0.722 & -0.074 & -0.421
\end{bmatrix}
\end{array}
$$

Fig. 10.3 Composite distance matrix

	A_1	A_2	A_3
A_1	-	3.145	2.085
A_2	3.145	-	1.940
A_3	2.085	1.940	-

The composite distance matrix (D) is as Fig. 10.3.

(D) **Homogenization the alternatives**

First, the shortest distance values of each row (o_i) of the composite distance matrix are obtained as follows:

$$o_1 = 2.085, \quad o_2 = 1.940, \quad o_3 = 1.940$$

Then, by calculating the mean and standard deviation of the shortest distance values of each row, the homogeneity range of the values of composite distance matrix is as follows:

$$O = \overline{O} \pm 2S_O = 1.988 \pm 2(0.084) = 1.988 \pm 0.168$$

Therefore, all values of composite distance matrix are in this range, and the alternatives are homogeneous.

(E) **The development pattern**

Regarding the positive or negative type of attributes, the ideal values of alternatives according to the homogeneous standard matrix are as follows:

$$Z_{o1} = -1.141, \quad Z_{o2} = -0.961, \quad Z_{o3} = 1.142$$

Thus, the development pattern for each alternative is determined as follows:

$$C_{1o} = \sqrt{(-1.141 - (-1.141))^2 + (-0.961 - (-0.961))^2 + (-0.72 - 1.142)^2} = 1.862$$
$$C_{2o} = \sqrt{(0.42 - (-1.141))^2 + (1.035 - (-0.961))^2 + (1.142 - 1.142)^2} = 2.534$$
$$C_{3o} = \sqrt{(0.722 - (-1.141))^2 + (-0.074 - (-0.961))^2 + (-0.421 - 1.142)^2} = 2.589$$

(F) **The final ranking of alternatives**

The high limit of development (C_o) equals 3.138, and the development attribute values (F_i) are calculated as follows:

$$F_1 = 0.593, \quad F_2 = 0.808, \quad F_3 = 0.825$$

Finally, the bus with LPG fuel (A_1) is the best alternative and the alternatives are ranked as follows:

$$A_1 > A_2 > A_3$$

10.4 Conclusion

Given that the taxonomy method is considered as a tool for classifying and determining the degree of development, the development index is used for alternatives ranking, which has many applications. Although this technique is long and its steps are summarized in Fig. 10.4, it cannot be considered as the weak point of this method. Rather, it can be regarded a more accurate and more efficient method. On the other hand, attributes are independent and compensatory and qualitative attributes need to be quantified.

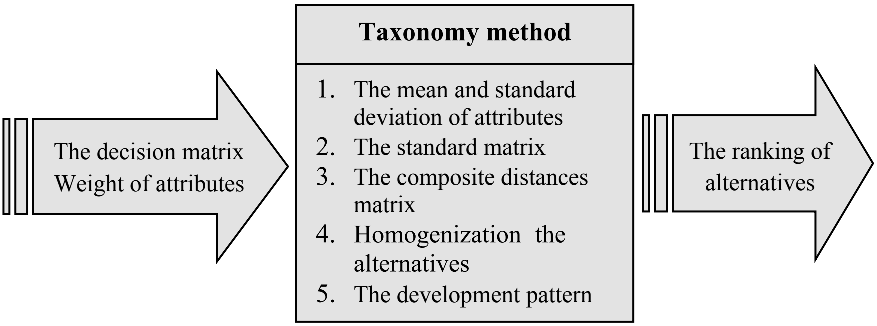

Fig. 10.4 A summary of the taxonomy method

Chapter 11
MOORA Method

11.1 Introduction

The Multi-Objective Optimization Ratio Analysis (MOORA) method was introduced by Brauers in 2004 [72–76], which is considered as an objective (non-subjective) method. Moreover, desirable and undesirable criteria are used simultaneously for ranking to select a superior or higher alternative among different alternatives. This technique has a large number of applications such as contractor selection [77, 78], optimization of machinery process parameters [79–81], and supplier selection [82]. The MOORA method has the following features:

- It belongs to the compensatory methods;
- Attributes are independent;
- The qualitative attributes are converted into the quantitative attributes.

In addition, the decision matrix is used to determine the input information of method as in Eq. (11.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1,\ldots,m, \quad j = 1,\ldots,n \qquad (11.1)$$

In decision matrix of Eq. (11.1), r_{ij} denotes the element of the decision matrix for ith alternative in jth attribute. The decision maker provides the weight of attribute $[w_1, w_2, \ldots, w_n]$, regarding the normalized property $\left(\sum_{j=1}^{n} w_j = 1\right)$.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_11

11.2 Description of MOORA Method

11.2.1 The Normalized Decision Matrix

Eq. (11.2) is used to the normalize decision matrix [83].

$$r_{ij}^{*} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}; \quad j = 1, \ldots, n \tag{11.2}$$

Accordingly, r_{ij}^{*} illustrates the normalized value of decision matrix of *i*th alternative in *j*th attribute.

11.2.2 The Reference Points

Considering the positive or negative state of each attribute, the reference points for the negative attributes are minimum values and for the positive attributes are maximum values.

11.2.3 The Assessment Values

With respect to the weight of attribute $[w_1, w_2, \ldots, w_n]$, the assessment values of each attribute are obtained through Eq. (11.3) [83].

$$\hat{y}_j = \sum_{j=1}^{g} r_{ij}^{*}.w_j - \sum_{j=g+1}^{n} r_{ij}^{*}.w_j; \quad i = 1, \ldots, m \tag{11.3}$$

where g represents the number of positive attributes and n-g displays the number of negative attributes, and according to the type of attribute, the ideal points of the *j*th attribute are deducted from all values of the *j*th attribute [83].

11.2.4 The Final Ranking of Alternatives

Based on the previous step, the obtained maximum values of $(\hat{y}_i)$ are determined for *i*th alternative, and then, the values are ranked in a descending order, and the highest amount has the highest rank.

11.3 Case Study

A board of directors of a factory plans to select the best alternative among the four maintenance contractors (A_1, A_2, A_3, and A_4). The attributes such as the number of required workforce (C_1), machinery maintenance cost (C_2), overall cost reduction (C_3), contractor contract cost (C_4), and contract duration (C_5) are specified by experts. Fig. 11.1 indicates the decision matrix.

Additionally, the weights of attributes are equal. The purpose is to select the best contractor and express the final ranking of alternatives.

❖ **Solution**

(A) **The normalized decision matrix**

Table 11.1 indicates the normalized values of decision matrix.

(B) **The reference points**

According to the negative or positive state of attribute, the reference points are determined as shown in Table 11.2.

Fig. 11.1 Decision matrix of choosing contractor

$$
\begin{array}{c}
 \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4
\end{array}
\begin{array}{ccccc}
- & - & + & - & + \\
C_1 & C_2 & C_3 & C_4 & C_5 \\
\end{array}
$$
$$
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
5 & 54 & 600 & 90 & 80 \\
1 & 97 & 200 & 58 & 65 \\
7 & 72 & 400 & 60 & 83 \\
10 & 75 & 1000 & 80 & 40
\end{bmatrix}
$$

Table 11.1 Normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.378	0.355	0.480	0.614	0.579
A_2	0.076	0.638	0.160	0.396	0.470
A_3	0.529	0.473	0.320	0.410	0.600
A_4	0.756	0.493	0.801	0.546	0.289

Table 11.2 Reference points

Attribute	C_1	C_2	C_3	C_4	C_5
Value	0.076	0.355	0.801	0.396	0.600

(C) The assessment values

The assessment values are calculated according to the reference points and the weight of each attribute. For example, the first attribute values for each alternative are calculated as follows:

$$\hat{y}_1 = \frac{1}{5}(0.378 - 0.076) = 0.060$$
$$\hat{y}_2 = \frac{1}{5}(0.076 - 0.076) = 0$$
$$\hat{y}_3 = \frac{1}{5}(0.529 - 0.076) = 0.091$$
$$\hat{y}_4 = \frac{1}{5}(0.756 - 0.076) = 0.136$$

The assessment values of all attributes are as shown in Table 11.3.

(D) The final ranking of alternatives

By specifying the maximum amount of $(\hat{y}_i)$, the rank of each alternative is also determined as shown in Table 11.4.

Therefore, the forth contractor (A_4) is the best alternative, and the final ranking is as follows:

$$A_4 > A_2 > A_3 > A_1$$

Table 11.3 Assessment values

	C_1	C_2	C_3	C_4	C_5
A_1	0.060	0	0.064	0.044	0.004
A_2	0	0.057	0.128	0	0.026
A_3	0.091	0.024	0.096	0.003	0
A_4	0.136	0.028	0	0.030	0.062

Table 11.4 Final ranking of alternatives

	Value	Rank
A_1	0.064	4
A_2	0.128	2
A_3	0.096	3
A_4	0.136	1

11.4 Conclusion

Recently, the MOORA method has been used in many studies, due to its simplicity. The steps are summarized in Fig. 11.2, and the development of this method has increased its application. Furthermore, the assessment value of each alternative is used for ranking. This is a compensatory method, and the qualitative attributes are converted into the quantitative attributes.

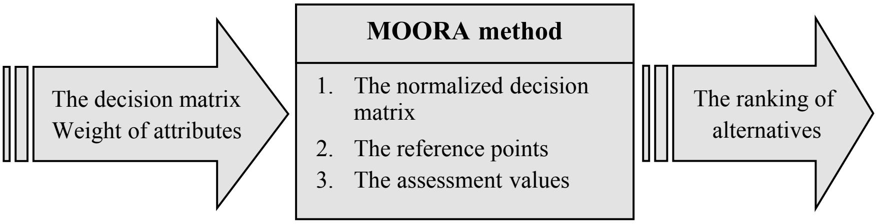

Fig. 11.2 A summary of the MOORA method

Chapter 12
COPRAS Method

12.1 Introduction

The COmplex PRoportional ASsessment (COPRAS) method was introduced by Zavadskas, Kaklauskas, and Sarka in 1994 [84–87]. This method is used to assess the maximizing and minimizing index values, and the effect of maximizing and minimizing indexes of attributes on the results assessment is considered separately. The COPRAS method is applied in some areas such as risk assessment [88, 89], investment project selection [90], and material selection [91]. Accordingly, the following features are considered for this method:

- It is a compensatory method;
- Attributes are independent;
- The qualitative attributes are converted into the quantitative attributes.

In this technique, the decision matrix is formed based on the information received from decision maker in Eq. (12.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \qquad (12.1)$$

In Eq. (12.1), r_{ij} is the element of decision matrix for ith alternative in jth attribute. On the other hand, decision maker provides the weight of the attribute $[w_1, w_2, \ldots, w_n]$.

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12.2 Description of COPRAS Method

12.2.1 The Normalized Decision Matrix

Eq. (12.2) is used to normalize the decision matrix [92].

$$r_{ij}^{*} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}; \quad j = 1, \ldots, n \tag{12.2}$$

Here, r_{ij}^{*} indicates the normalized value of the decision matrix of ith alternative in jth attribute.

12.2.2 The Weighted Normalized Decision Matrix

Eq. (12.3) is used to determine the values of weighted normalized decision matrix [92].

$$\hat{r}_{ij} = r_{ij}^{*}.w_j; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{12.3}$$

In Eq. (12.3), w_j is the weight of attribute $[w_1, w_2, \ldots, w_n]$.

12.2.3 The Maximizing and Minimizing Indexes

Given the negative or positive type of attributes, the maximizing and minimizing indexes of each attribute are obtained by Eqs. (12.4) and (12.5) [92].

$$S_{+i} = \sum_{j=1}^{g} \hat{r}_{ij}; \quad i = 1, \ldots, m \tag{12.4}$$

$$S_{-i} = \sum_{j=g+1}^{n} \hat{r}_{ij}; \quad i = 1, \ldots, m \tag{12.5}$$

where g indicates the number of positive attributes and n-g represents the number of negative attributes, and S_i describes the maximizing and minimizing indexes of ith at tribute, according to the type of it.

12.2.4 The Relative Significance Value

The relative significance value of each alternative is calculated through Eqs. (12.6) or (12.7) [92].

$$Q_i = S_{+i} + \frac{\min_i S_{-i} \sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{\min_i S_{-i}}{S_{-i}}} \tag{12.6}$$

$$Q_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{1}{S_{-i}}} \tag{12.7}$$

12.2.5 The Final Ranking of Alternatives

The relative significance values of alternatives are ranked in descending order, and the highest final value has the highest rank.

12.3 Case Study

The construction company wants to choose a prefabricated wall model among models of A_1, A_2, and A_3 for its project. In the same vein, the experts represented some attributes such as the weight of each square meter (C_1), cost of materials per square meter (C_2), strength (C_3), heat transfer attribute (C_4), labor cost (C_5), and the average score of experts evaluation in the range [0,1] (C_6). After converting the qualitative attributes into quantitative attributes, the decision matrix is as shown in Fig. 12.1. In addition, the weight of each attribute is determined as in Table 12.1.

The purpose is to choose the best prefabricated wall.

Fig. 12.1 Decision matrix for prefabricated wall purchase

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{cccccc} - & - & + & - & - & + \\ C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \end{array}$$

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \begin{bmatrix} 0.710 & 4.100 & 0.180 & 0.720 & 0.990 & 0.250 \\ 1.330 & 5.900 & 0.740 & 0.310 & 0.420 & 0.830 \\ 1.450 & 4.900 & 0.270 & 0.650 & 0.420 & 0.440 \end{bmatrix}$$

Table 12.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
w_j	0.171	0.185	0.177	0.225	0.157	0.085

❖ Solution

(A) The normalized decision matrix

Table 12.2 indicates the normalized values of decision matrix.

(B) The weighted normalized decision matrix

Table 12.3 indicates the weighted normalized values of decision matrix.

(C) The maximizing and minimizing indexes

In this step, the maximizing and minimizing indexes of each attribute are determined as follows:

$$\begin{aligned}
S_1^- &= 0.035 + 0.051 + 0.096 + 0.085 = 0.267 \\
S_1^+ &= 0.027 + 0.014 = 0.041 \\
S_2^- &= 0.065 + 0.073 + 0.042 + 0.036 = 0.216 \\
S_2^+ &= 0.046 + 0.110 = 0.156 \\
S_3^- &= 0.071 + 0.061 + 0.087 + 0.036 = 0.255 \\
S_3^+ &= 0.040 + 0.025 = 0.065
\end{aligned}$$

Table 12.2 Normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.203	0.275	0.151	0.429	0.541	0.165
A_2	0.381	0.396	0.622	0.185	0.230	0.546
A_3	0.415	0.329	0.227	0.387	0.230	0.289

Table 12.3 Weighted normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.035	0.051	0.027	0.096	0.085	0.014
A_2	0.065	0.073	0.110	0.042	0.036	0.046
A_3	0.071	0.061	0.040	0.087	0.036	0.025

(D) **The relative significance value**

The relative significance value of each alternative is also obtained as follows:

$$Q_1 = 0.041 + \frac{0.738}{0.267 \times 12.295} = 0.266$$
$$Q_2 = 0.156 + \frac{0.738}{0.216 \times 12.295} = 0.434$$
$$Q_3 = 0.065 + \frac{0.738}{0.255 \times 12.295} = 0.300$$

(E) **The final ranking of alternatives**

Regarding the relative significance values:

$$Q_2 > Q_3 > Q_1$$

The second prefabricated wall model (A_2) is the best alternative, and the alternatives are ranked as follows:

$$A_2 > A_3 > A_1$$

12.4 Conclusion

In the COPRAS method, the full and final ranking of alternatives is performed using the value evaluation of maximizing and minimizing indexes. Thus, according to the method steps summarized in Fig. 12.2, and the features such as compensatory attributes, this technique is used in various areas. Recently, the COPRAS method has been used in combination with other methods in the fuzzy and gray environments.

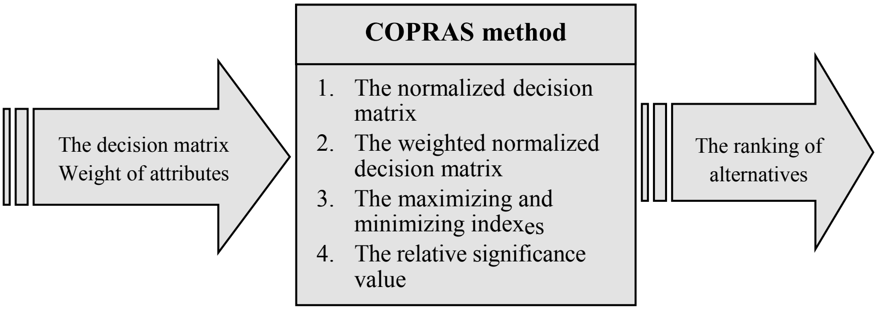

Fig. 12.2 A summary of the COPRAS method

Chapter 13
WASPAS Method

13.1 Introduction

The Weighted Aggregates Sum Product Assessment (WASPAS) method was introduced by Zavadskas, Turskis, Antucheviciene, and Zakarevicius in 2012 [93–96]. This method is a combination of Weighted Sum Model (WSM) and Weighted Product Model (WPM) [97]. Thus, the relative importance of each attribute is simply determined, and then, the alternatives are evaluated and prioritized. This technique is applied in the personal selection [98–100], analysis of machining processes [101, 102], and material selection [103, 104]. The features of the WASPAS method are as follows

- It is regarded as a compensatory method;
- The attributes are independent;
- The qualitative attributes are converted into the quantitative attributes.

The input information of the method is expressed in terms of the matrix of alternatives and attributes, which is based on information received from the decision maker, as shown in Eq. (13.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{13.1}$$

where r_{ij} represents the decision matrix for ith alternative in jth attribute. Also, the decision maker provides the weight of attribute $[w_1, w_2, \ldots, w_n]$.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_13

13.2 Description of WASPAS Method

13.2.1 The Normalized Decision Matrix

Eq. (13.2) is used to normalize the positive attributes and normalization the negative attribute is calculated through Eq. (13.3) [97].

$$r_{ij}^{*} = \frac{r_{ij}}{\max\limits_{i} r_{ij}}; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{13.2}$$

$$r_{ij}^{*} = \frac{\min\limits_{i} r_{ij}}{r_{ij}}; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{13.3}$$

where r_{ij}^{*} illustrates the normalized value of the decision matrix of ith alternative in jth attribute.

13.2.2 The Additive Relative Importance

Eq. (13.4) is used to determine the additive relative importance in the weighted normalized data of each alternative [97].

$$Q_i^{(1)} = \sum_{j=1}^{n} r_{ij}^{*}.w_j; \quad i = 1, \ldots, m \tag{13.4}$$

where w_j indicates the weight of attribute $[w_1, w_2, \ldots, w_n]$ and $Q_i^{(1)}$ indicates the additive relative importance in the ith alternative.

13.2.3 The Multiplicative Relative Importance

Eq. (13.5) is used to determine the multiplicative relative importance of the weighted normalized data of each alternative [97].

$$Q_i^{(2)} = \prod_{j=1}^{n} (r_{ij}^{*})^{w_j}; \quad i = 1, \ldots, m \tag{13.5}$$

where $Q_i^{(2)}$ demonstrates the multiplicative relative importance of the ith alternative.

13.2.4 The Joint Generalized Criterion (Q)

The joint generalized criterion (Q) was proposed for generalizing and integrating additive and multiplicative methods, defined as Eq. (13.6) [97].

$$Q_i = \frac{1}{2}\left(Q_i^{(1)} + Q_i^{(2)}\right) = \frac{1}{2}\left(\sum_{j=1}^{n} r_{ij}^{*}.w_j + \prod_{j=1}^{n}(r_{ij}^{*})^{w_j}\right); \quad i = 1, \ldots, m \tag{13.6}$$

In addition, Eq. (13.7) was proposed to increase the ranking accuracy [97].

$$Q_i = \lambda\sum_{j=1}^{n} r_{ij}^{*}.w_j + (1-\lambda)\prod_{j=1}^{n}(r_{ij}^{*})^{w_j}; \quad i = 1, \ldots, m, \ \lambda \in [0, 1] \tag{13.7}$$

13.2.5 The Final Ranking of Alternatives

The joint generalized criterion (Q) values obtained from Eqs. (13.6) and (13.7) are ranked in a descending order, and the highest amount of joint generalized criterion has the highest rank. In Eq. (13.7), if the λ value equals to 1, the equation is converted into the WSM model and if the λ value equals to zero, the equation is converted into the WPM model.

13.3 Case Study

A company intends to choose the best type of cutting fluid among four types (A_1, A_2, A_3, and A_4) for its milling machines, according to the defined attributes. Experts defined the attributes such as the friction (C_1), stoning temperature (C_2), recycling capability (C_3), work surface roughness (C_4), and stability (C_5), and the qualitative attributes are converted into the quantitative attributes. The decision

Fig. 13.1 Decision matrix for choosing the cutting fluid

	−	−	+	−	+
	C_1	C_2	C_3	C_4	C_5
A_1	0.035	847	0.335	1.760	0.590
A_2	0.027	834	0.335	1.680	0.665
A_3	0.037	808	0.590	2.400	0.500
A_4	0.028	821	0.500	1.590	0.410

matrix is shown in Fig. 13.1. The weight of each attribute is determined in Table 13.1.

It aims to select the best type of cutting fluid for milling machines.

❖ **Solution**

(A) **The normalized decision matrix**

Table 13.2 indicates the normalized values of decision matrix.

(B) **The additive relative importance**

The additive relative importance of each alternative is calculated as follows:

$$Q_1^{(1)} = (0.771 \times 0.331) + (0.954 \times 0.181) + (0.568 \times 0.369) + (0.903 \times 0.072) + (0.887 \times 0.047) = 0.744$$

$$Q_2^{(1)} = (1 \times 0.331) + (0.969 \times 0.181) + (0.568 \times 0.369) + (0.946 \times 0.072) + (1 \times 0.047) = 0.831$$

$$Q_3^{(1)} = (0.730 \times 0.331) + (1 \times 0.181) + (1 \times 0.369) + (0.663 \times 0.072) + (0.752 \times 0.047) = 0.875$$

$$Q_4^{(1)} = (0.964 \times 0.331) + (0.984 \times 0.181) + (0.848 \times 0.369) + (1 \times 0.072) + (0.617 \times 0.047) = 0.911$$

(C) **The multiplicative relative importance**

The multiplicative relative importance of each alternative is computed as follows:

Table 13.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5
w_j	0.331	0.181	0.369	0.072	0.047

Table 13.2 Normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.771	0.954	0.568	0.903	0.887
A_2	1	0.969	0.568	0.946	1
A_3	0.730	1	1	0.663	0.752
A_4	0.964	0.984	0.848	1	0.617

$$
\begin{aligned}
Q_1^{(2)} &= (0.771)^{0.331} \times (0.954)^{0.181} \times (0.568)^{0.369} \\
&\times (0.903)^{0.072} \times (0.887)^{0.047} = 0.729 \\
Q_2^{(2)} &= (1)^{0.331} \times (0.969)^{0.181} \times (0.568)^{0.369} \\
&\times (0.946)^{0.072} \times (1)^{0.047} = 0.804 \\
Q_3^{(2)} &= (0.730)^{0.331} \times (1)^{0.181} \times (1)^{0.369} \times (0.663)^{0.072} \\
&\times (0.752)^{0.047} = 0.863 \\
Q_4^{(2)} &= (0.964)^{0.331} \times (0.984)^{0.181} \times (0.847)^{0.369} \\
&\times (1)^{0.072} \times (0.617)^{0.047} = 0.906
\end{aligned}
$$

(D) **The joint generalized criterion (Q)**

In this step, Eq. (13.6) is used for determining the joint generalized criterion as follows:

$$
\begin{aligned}
Q_1 &= \frac{1}{2}(0.744 + 0.729) = 0.737 \\
Q_2 &= \frac{1}{2}(0.831 + 0.804) = 0.818 \\
Q_3 &= \frac{1}{2}(0.875 + 0.863) = 0.869 \\
Q_4 &= \frac{1}{2}(0.911 + 0.906) = 0.909
\end{aligned}
$$

Further, in order to determine the effect of λ values on the joint generalized criterion, according to Eq. (13.7), the λ values should be defined in the range [0, 1]. For instance, with a value of 0.200, the joint generalized criterion values for each alternative are as follows:

$$
\begin{aligned}
Q_1 &= (0.200 \times 0.744) + (0.800 \times 0.729) = 0.732 \\
Q_2 &= (0.200 \times 0.831) + (0.800 \times 0.804) = 0.809 \\
Q_3 &= (0.200 \times 0.875) + (0.800 \times 0.863) = 0.865 \\
Q_4 &= (0.200 \times 0.911) + (0.800 \times 0.906) = 0.907
\end{aligned}
$$

The effects of the other λ values on the joint generalized criterion with 0.200 distance value in the range [0, 1] are calculated as shown in Table 13.3.

(E) **The final ranking of alternatives**

Considering the effects of different λ values on the joint generalized criterion of each alternative, the values reduce and actually move toward the WSM model if the λ value approaches to 1. In addition, the values increase and move toward the WPM

Table 13.3 Effect of λ values on the joint generalized criterion (Q)

	$\lambda = 0$	$\lambda = 0.200$	$\lambda = 0.400$	$\lambda = 0.600$	$\lambda = 0.800$	$\lambda = 1$
A_1	0.729	0.732	0.735	0.738	0.741	0.744
A_2	0.804	0.809	0.815	0.820	0.826	0.831
A_3	0.863	0.865	0.868	0.870	0.873	0.875
A_4	0.906	0.907	0.908	0.909	0.910	0.911

model if the λ value approaches zero. In general, given the joint generalized criterion values obtained from Eqs. (13.6) and (13.7), the fourth type of cutting fluid (A_4) is the best alternative. Then, the final ranking is as follows:

$$A_4 > A_3 > A_2 > A_1$$

13.4 Conclusion

The WASPAS method is considered as one of the newest methods of MADM including the combination of the WSM and WPM models. The additive and multiplicative relative importance of each method is determined based on the steps of the method, summarized in Fig. 13.2, the alternatives are ranked by specifying the joint generalized criterion of the aforementioned cases, and the best alternative is selected. The WASPAS method is used increasingly, due to the features such as compensatory attributes, the need for converting the quantitative attributes into the quantitative attributes by experts, as well as the shorter stages.

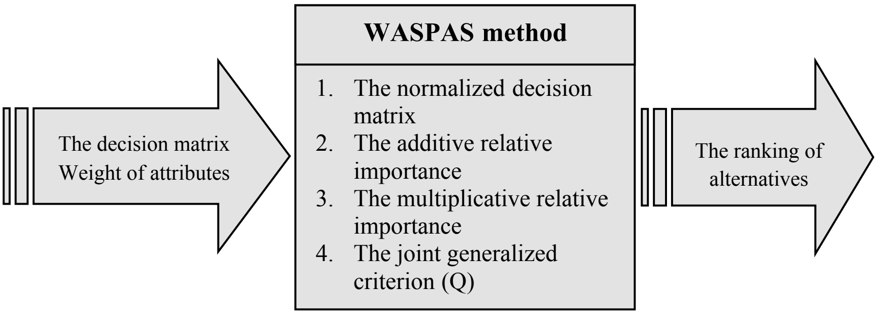

Fig. 13.2 A summary of the WASPAS method

Chapter 14
SWARA Method

14.1 Introduction

The Stepwise Weight Assessment Ratio Analysis (SWARA) method was introduced by Kersuliene, Zavadskas, and Turskis in 2010 [105–108]. In this method, done by the weighting method, the relative importance and the initial prioritization of alternatives for each attribute are determined by the opinion of the decision maker, and then, the relative weight of each attribute is determined. Finally, the final priority and ranking the attributes are done according to the following characteristics of the technique:

- The attributes are compensatory;
- The attributes are independent of each other.

This technique is applied in the supplier selection [109, 110], the selection of a packaging design [111], and evaluating and choosing the R&D project [112].

In SWARA method, the relative importance (S_j) of the jth attribute is determined as the input information based on the idea of the decision makers. Considering the evaluation of attributes, this method is applied in evaluating attributes for choosing the location of the factory and the dam construction, the ERP system selection, assessing the attributes for improving the performance of employees.

14.2 Description of SWARA Method

14.2.1 The Initial Prioritization of Attributes

First, the attributes are prioritized in terms of relative importance, determined by decision makers.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_14

14.2.2 The Coefficient (K)

Eq. (14.1) is used to determine the coefficient (K) of an attribute for each decision maker [113].

$$K_j = \begin{cases} 1 & if\ j = 1 \\ S_j + 1 & if\ j > 1 \end{cases}; \quad j = 1, \ldots, n \tag{14.1}$$

14.2.3 The Initial Weight

At this stage, Eq. (14.2) is applied to compute the initial weight of an attribute for each decision maker [113].

$$q_j = \begin{cases} 1 & if\ j = 1 \\ \frac{q_j}{K_j} & if\ j > 1 \end{cases}; \quad j = 1, \ldots, n \tag{14.2}$$

14.2.4 The Relative Weight

Eq. (14.3) is applied to determine the relative weight of an attribute for each decision maker [113].

$$w_j = \frac{q_j}{\sum_{j=1}^{n} q_j} \tag{14.3}$$

14.2.5 The Final Ranking of Attributes

By determining the relative weight of each attribute, the values are arranged in a descending order and the final ranking takes place.

14.3 Case Study

An organization intends to improve the performance of its employees by examining the attributes such as the quality of working (C_1), quantity of working (C_2), work satisfaction (C_3), and knowledge and skills (C_4), according to the experts' opinions in the order of importance. The decision makers provide the relative importance of each attribute as presented in Table 14.1.

The purpose is to prioritize and evaluate the employees' performance evaluation attributes.

❖ **Solution**

(A) **The initial prioritization of attributes**

Table 14.1 represents the initial prioritization of attributes based on the experts' opinions, respectively.

(B) **The coefficient (K)**

In order to determine the coefficient (K), a unit is added to the amount of the attributes, respectively, and the new values are obtained as shown in Table 14.2.

(C) **The initial weight**

The initial weight of an attribute for each decision maker is calculated by dividing the initial weight of the $i - 1$ attribute by the coefficient value (k) of ith attribute in the same decision maker, which is as follows for the first attribute:

$$q_1 = 1$$

$$q_2 = \frac{1}{1.480} = 0.676$$

$$q_3 = \frac{0.676}{1.350} = 0.501$$

$$q_4 = \frac{0.501}{1.220} = 0.411$$

Table 14.1 Relative importance values

Attribute	Relative importance
C_1	
C_2	0.480
C_3	0.350
C_4	0.220

Table 14.2 Coefficient (K) of attributes

	Coefficient (K)
C_1	1
C_2	1.48
C_3	1.35
C_4	1.22

(D) **The relative weights**

At this stage, the relative weight values are determined with the initial weights of each attribute as follows:

$$w_1 = \frac{1}{1 + 0.676 + 0.501 + 0.411} = 0.386$$
$$w_2 = \frac{0.676}{1 + 0.676 + 0.501 + 0.411} = 0.261$$
$$w_3 = \frac{0.501}{1 + 0.676 + 0.501 + 0.411} = 0.194$$
$$w_4 = \frac{0.411}{1 + 0.676 + 0.501 + 0.411} = 0.159$$

(E) **The final ranking of attributes**

With respect to the relative weight of each attribute, the quality of working (C_1) has a higher priority than the other attributes, and the attributes are ranked as follows:

$$C_1 > C_2 > C_3 > C_4$$

14.4 Conclusion

The SWARA method is known as one of the methods for evaluating attributes in which the experts' opinions are highly preferred and even are clear in the first stage as shown in Fig. 14.1. Given the evaluation of the attributes and presented studies, this method is obviously used in combination with other methods, which is considered as an advantage of this technique. Therefore, it is considered as one of the most widely used methods in the MADM, which was presented in 2010.

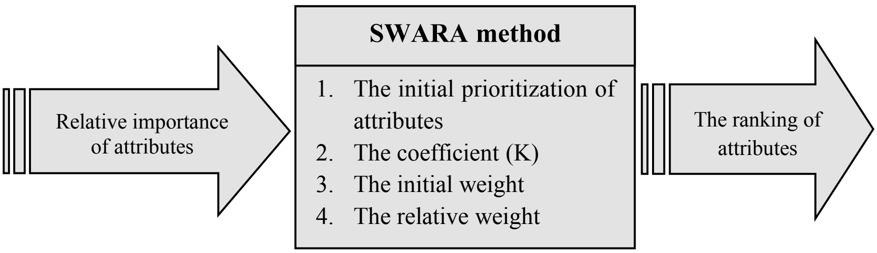

Fig. 14.1 A summary of the SWARA method

Chapter 15
DEMATEL Method

15.1 Introduction

The DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) method was introduced by Fonetla and Gabus in 1971 [114–117], mainly used to study very complex global issues. The DEMATEL method is applied to construct a network relation design in order to examine the internal relation among the attributes. This technique is successfully applied in many situations such as the analysis of barriers of waste recycling [118], project selection [119, 120], and evolution of e-learning programs [121]. The DEMATEL method has the following features:

- It is a compensatory method;
- No need for the independence of attributes;
- The qualitative attributes are converted into the quantitative attributes.

In this method, the direct relation matrix from the arithmetic mean of the pair-wise comparison matrices related to each attribute is used and determined by the decision maker, as shown in Eq. (15.1).

$$\mathrm{M} = \begin{bmatrix} m_{11} & \cdots & m_{1j} & \cdots & m_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ m_{i1} & \cdots & m_{ij} & \cdots & m_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nj} & \cdots & m_{nn} \end{bmatrix}_{n\times n} \quad ; \quad i,j \in \{1,\ldots,n\} \tag{15.1}$$

In Eq. (15.1), m_{ij} represents the element of the direct relation matrix for ith attribute in jth attribute.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_15

15.2 Description of DEMATEL Method

15.2.1 The Normalized Direct Relation Matrix

Eq. (15.2) is used to normalize the direct relation matrix, and the k index value is obtained using Eq. (15.3) [122].

$$D = k \times M \tag{15.2}$$

$$k = \max_{i,j}\left\{\frac{1}{\max\limits_{i}\sum\limits_{j=1}^{n}\left|m_{ij}\right|}, \frac{1}{\max\limits_{j}\sum\limits_{i=1}^{n}\left|m_{ij}\right|}\right\} \tag{15.3}$$

15.2.2 The Total Relation Matrix

Eq. (15.4) is used to compute the full penetration value of attribute. When $\lim X^l = [0]_{n \times n}$ and $l \to \infty$ as:

$$\begin{aligned} T &= D + D^2 + D^3 + \ldots + D^l \\ &= D\left(I + D + D^2 + D^3 + \ldots + D^{l-1}\right)(I - D)(I - D)^{-1} \\ &= D\left(I - D^l\right)(I - D)^{-1} \end{aligned} \tag{15.4}$$

In Eq. (15.4), (I) is the identity matrix. The total relation matrix is represented by Eq. (15.5) [122].

$$T = D(I - D)^{-1} \tag{15.5}$$

where t_{ij} indicates the direct relation amount for ith attribute in jth attribute.

15.2.3 The Cause and Effect Values

At this stage, the calculations are made using r and c values, representing the sum of row and column values, determined from Eqs. (15.6) and (15.7) [122].

$$r = [r_i]_{n \times 1} = \left[\sum_{j=1}^{n} t_{ij}\right]_{n \times 1} \quad ; \quad i = 1, \ldots, n \tag{15.6}$$

$$c = \left[c_j\right]'_{1\times n} = \left[\sum_{i=1}^{n} t_{ij}\right]'_{1\times n} ; \quad j = 1, \ldots, n \tag{15.7}$$

In Eq. (15.7), $\left[c_j\right]'$ indicates the transpose value of the jth column, c_j represents the sum of the jth column values, indicating of the effect of attribute jth on other attributes, and r_i denotes the sum of values of ith row, indicating the cause of ith attribute on other attributes.

If $i = j$, the horizontal vector $(\mathrm{r}_i + \mathrm{c}_i)$ indicates the effect and impact of the ith attribute. In other words, when the value $(\mathrm{r}_i + \mathrm{c}_i)$ is greater, the attribute has more interaction with other attributes. The vertical vector $(\mathrm{r}_i + \mathrm{c}_i)$ indicates the influence power of ith attribute. If $(\mathrm{r}_i + \mathrm{c}_i)$ is positive, it is a causal variable and it is considered as an effect if it is negative.

15.2.4 The Threshold Value (a)

The threshold value should be determined to draw the interrelationship map. Only the equations are plotted in the interrelationship map in which their values in the full penetration matrix are greater than the threshold value. The average of elements of the full penetration matrix is calculated using Eq. (15.8) [122].

$$a = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}}{N} \tag{15.8}$$

15.2.5 The Interrelationship Map

At this stage, the interrelationship map is drawn for appropriate analysis of the final solution with the values $(\mathrm{r}_i + \mathrm{c}_j)$ and $\left(\mathrm{r}_i - \mathrm{c}_j\right)$ for each attribute with respect to the threshold value.

15.2.6 The Final Ranking of Attributes

A possible structure and ranking of the factors are obtained using the interrelationship map and arranging the values $(\mathrm{r}_i + \mathrm{c}_j)$ and $\left(\mathrm{r}_i - \mathrm{c}_j\right)$ in a descending order.

15.3 Case Study

A home appliance manufacturing company intends to examine and prioritize the attributes such as the innovation management ability (C_1), general learning capability (C_2), sourcing capability (C_3), technology development capability (C_4), the ability in designing the product process (C_5), and technology commercialization ability (C_6), according to the experts' opinions to improve competition and marketing for its products. The direct relation matrix is shown in Fig. 15.1.

It is desirable is to prioritize attributes and determine the most preferred attribute.

❖ **Solution**

(A) **The normalized direct relation matrix**

First, the k index value is determined as follows:

$$k = max\left\{\frac{1}{12.274}, \frac{1}{11.364}\right\} = 0.088$$

Then, the equation matrix values are multiplied by k index. Finally, the normalized values of direct relation matrix are computed as shown in Table 15.1.

(B) **The total relation matrix**

The total relation matrix is obtained using the normalized values of direct relation matrix. The total relation matrix is shown in Fig. 15.2.

(C) **The cause and effect values**

By calculating the amount of c_j and r_i representing the sum of the values of jth column and the ith row, respectively, the cause and effect values are obtained as shown in Table 15.2.

(D) **The threshold value (a)**

By determining the mean values of total relation matrix, the threshold value is calculated as follows:

Fig. 15.1 Direct relation matrix

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	2	2.273	2.273	2.273	3.455
C_2	1.909	0	2.545	3	2.545	1.727
C_3	1.182	2.182	0	1.727	2	1.364
C_4	1.727	3.364	1.727	0	1.818	1.909
C_5	1.727	1.636	1.818	1.818	0	2.909
C_6	1.727	1.273	1.636	1.636	1.909	0

Table 15.1 Normalized values of direct relation matrix

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	0.176	0.200	0.200	0.200	0.304
C_2	0.168	0	0.224	0.264	0.224	0.152
C_3	0.104	0.192	0	0.152	0.176	0.120
C_4	0.152	0.296	0.152	0	0.160	0.168
C_5	0.152	0.144	0.160	0.160	0	0.256
C_6	0.152	0.112	0.144	0.144	0.168	0

Fig. 15.2 Total relation matrix

$$
\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\
\begin{bmatrix}
1.198 & 1.602 & 1.564 & 1.619 & 1.634 & 1.790 \\
1.315 & 1.431 & 1.552 & 1.637 & 1.619 & 1.645 \\
0.984 & 1.242 & 0.038 & 1.210 & 1.238 & 1.253 \\
1.220 & 1.558 & 1.404 & 1.326 & 1.471 & 1.543 \\
1.129 & 1.334 & 1.300 & 1.346 & 1.220 & 1.497 \\
0.988 & 1.143 & 1.126 & 1.165 & 1.193 & 1.110
\end{bmatrix}
\end{array}
$$

Table 15.2 Cause and effect values

	r_i	c_j	$r_i + c_j$	$r_i - c_j$
C_1	9.407	6.834	16.240	2.573
C_2	9.199	8.310	17.509	0.888
C_3	6.968	7.984	14.952	−1.015
C_4	8.522	8.307	16.829	0.215
C_5	7.827	8.375	16.202	−0.547
C_6	6.726	8.838	15.564	−2.113

$$a = \frac{48.648}{36} = 1.351$$

(E) **The interrelationship map**

The interrelationship map is illustrated in Fig. 15.3.

(F) **The final ranking of attributes**

Considering the interrelationship map as well as the cause and effect values of the attributes, the general learning capability (C_2) has a higher priority than the other attributes, and the final ranking is as follows:

$$C_2 > C_4 > C_1 > C_5 > C_6 > C_3$$

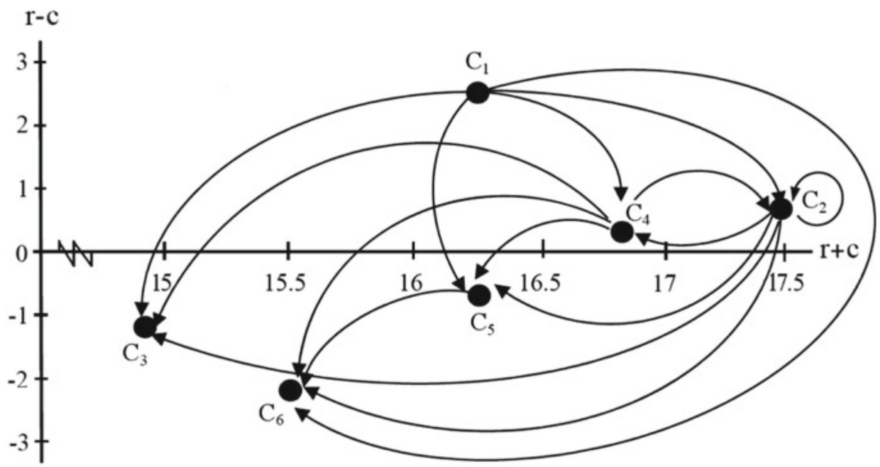

Fig. 15.3 Interrelationship map

15.4 Conclusion

The DEMATEL method is one of the attribute evaluation methods, in which the attributes are evaluated based on the experts' opinions on the direct relation matrix, as one of the main differences of the DEMATEL with the other methods according to the steps summarized in Fig. 15.4. Similar to the other attribute evaluation methods, the DEMATEL is combined with other techniques, resulting in using the wider application of the method. Also, qualitative attributes should be converted into the quantitative attributes by forming a direct relation matrix.

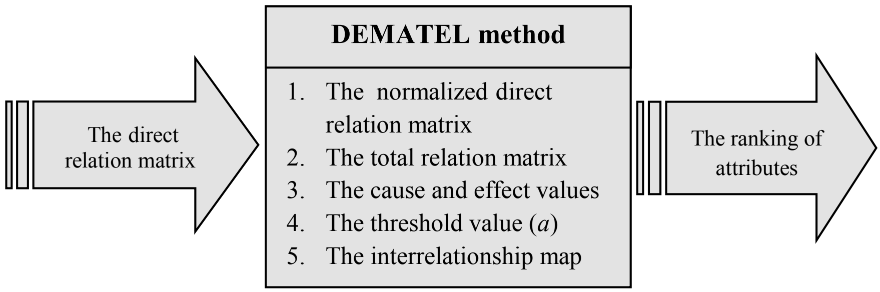

Fig. 15.4 A summary of the DEMATEL method

Chapter 16
MACBETH Method

16.1 Introduction

The Measuring Attractiveness by a Categorical Based Evaluation TecHnique (MACBETH) method, introduced by Bana e Costa and Vansnick in 1990 [123–125], examines the alternatives with multi-attributes and opposite objectives. In fact, this interactive method is appropriate for examining and ranking of alternatives with respect to a wide range of qualitative and quantitative attributes. Therefore, the MACBETH method is applied in many cases such as performance analysis of online bookstores [126], selection of manufacturing systems [127], and evaluation of supplier [128, 129]. Also, this technique has the following features:

- It is a compensatory method;
- The attributes are independent of each other;
- There is no need to convert the qualitative attributes into the quantitative attributes.

Further, the decision matrix is used to collect the input information based on the information received from the decision maker, as shown in Eq. (16.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} \quad ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{16.1}$$

where r_{ij} is the element of the decision matrix for ith alternative in jth attribute. In addition, the decision maker provides the weight of attribute $[w_1, w_2, \ldots, w_n]$.

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16.2 Description of MACBETH Method

16.2.1 Converting of Semantic Scale into Numerical Scale

According to the positive or negative type of the attributes, each semantic scale is first turned into the numerical scale and subsequently the negative attributes are turned into the positive attributes according to Table 16.1.

16.2.2 The Reference Levels

Eqs. (16.2) and (16.3) are used to determine the reference levels of each attribute [130].

$$r_j^- = min\, r_{ij}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{16.2}$$

$$r_j^+ = max\, r_{ij}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{16.3}$$

16.2.3 The MACBETH Score (V)

The MACBETH score (V) is calculated through Eq. (16.4) for each attributes [130, 131].

$$v(r_{ij}) = v\left(r_j^-\right) + \frac{\left(r_{ij} - r_j^-\right)}{\left(r_j^+ - r_j^-\right)}\left[v\left(r_j^+\right) - v\left(r_j^-\right)\right]; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{16.4}$$

where the $v\left(r_j^-\right)$ value equals to zero and the $v\left(r_j^+\right)$ value equals to 100.

Table 16.1 Seven-point semantic scale [129]

Semantic scale	Equivalent numerical scale (negative attribute)	Equivalent numerical scale (positive attribute)
Null	6	0
Very weak	5	1
Weak	4	2
Moderate	3	3
Strong	2	4
Very strong	1	5
Extreme	0	6

16.2.4 The Overall Score

The overall score of each alternative is determined by Eq. (16.5) [130, 131].

$$V_i = \sum_{j=1}^{n} v\left(r_{ij}\right).w_j; \quad i = 1, \ldots, m \tag{16.5}$$

16.2.5 The Final Ranking of Alternatives

Regarding the final ranking of alternatives, the overall scores are arranged in descending order and ranked.

16.3 Case Study

In order to construct a chemical production factory, locating facilities should be selected among the alternatives of A_1, A_2, A_3, and A_4. The attributes such as the flexibility and development capability (C_1), the amount of storage area of raw materials and products (C_2), the quantitative flow of material (C_3), the general risk-taking assessment of departments (C_4), and the overall assessment of working environment conditions (C_5) are determined by experts in order of priority. Further, the decision matrix is presented in Fig. 16.1. Additionally, the weight of attributes is indicated in Table 16.2.

The purpose is to select the best alternative for the facility location.

❖ **Solution**

(A) **Converting of semantic scale into numerical scale**

First, according to Table 16.1, the qualitative attributes are turning into the quantitative attributes as shown in Table 16.3.

	+	+	+	−	+
	C_1	C_2	C_3	C_4	C_5
A_1	Strong	3000	200	Strong	Very weak
A_2	Moderate	1800	140	Moderate	Strong
A_3	Very weak	2200	230	Very strong	Moderate
A_4	Weak	2500	180	Weak	Weak

Fig. 16.1 Decision matrix to choose the facility location

Table 16.2 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5
w_j	0.116	0.207	0.242	0.340	0.095

Table 16.3 Converting of semantic scale into numerical scale

	C_1	C_4	C_5
A_1	4	2	1
A_2	3	3	4
A_3	5	1	3
A_4	2	4	2

Table 16.4 Values of the reference levels

	C_1	C_2	C_3	C_4	C_5
r^-	2	1800	140	1	1
r^+	5	3000	230	4	4

(B) The reference levels

The values of the reference levels are obtained as shown in Table 16.4.

(C) The MACBETH score (V)

For example, the MACBETH scores (V) for the first attribute are calculated as follows:

$$v_1 = 0 + \frac{(4-2)}{(5-2)} \times (100-0) = 66.667$$

$$v_2 = 0 + \frac{(3-2)}{(5-2)} \times (100-0) = 33.333$$

$$v_3 = 0 + \frac{(5-2)}{(5-2)} \times (100-0) = 100$$

$$v_4 = 0 + \frac{(2-2)}{(5-2)} \times (100-0) = 0$$

Other MACBETH scores are determined as indicated in Table 16.5.

Table 16.5 MACBETH scores (V)

	C_1	C_2	C_3	C_4	C_5
A_1	66.667	100	66.667	33.333	0
A_2	33.333	0	0	66.667	100
A_3	100	33.333	100	0	66.667
A_4	0	58.333	44.444	100	33.333

Table 16.6 Overall scores

	C_1	C_2	C_3	C_4	C_5	V
A_1	7.733	20.700	16.133	11.333	0	55.899
A_2	3.867	0	0	22.667	9.500	36.034
A_3	11.600	6.900	24.200	0	6.333	49.033
A_4	0	12.075	10.755	34	3.167	59.997

(D) The overall score

During this stage, the overall score of each alternative is obtained from the weight amounts of each attribute, as shown in Table 16.6.

(E) The final ranking of alternatives

At this stage, the overall scores are arranged in a descending order:

$$V_4 > V_1 > V_3 > V_2$$

Therefore, the fourth alternative (A_4) is the best alternative for locating the facilities and the final ranking is obtained as follows:

$$A_4 > A_1 > A_3 > A_2$$

16.4 Conclusion

The lack of the need for converting the qualitative attributes into the quantitative attributes by experts and the use of conversion standard by decision makers has increased the reliability and accuracy of the MACBETH method. Therefore, given the low number of steps in the method (Fig. 16.2), only receiving the decision matrix, ranking alternative, and choosing the best alternative are based on the overall global score. During the last years, the MACBETH method has been combined with other methods by expanding its application. On the other hand, this technique is considered as one of the methods with high flexibility.

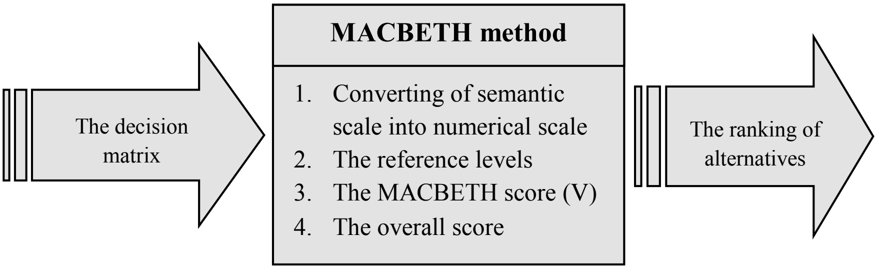

Fig. 16.2 A summary of the MACBETH method

Chapter 17
ANP Method

17.1 Introduction

Analytic Network Process (ANP) method was introduced by Saaty in 1996 [132–137]. In this method, a decision-making problem is analyzed into several different levels, and the sum of these decision-making levels forms a hierarchy. The ANP can consider all types of dependencies and solve the problems of interdependence, and the feedback among attributes and alternatives in the real world by considering the following features:

- It is considered as a compensatory method;
- No need for the independence of attributes.

Recently, a lot of studies have been conducted on this model and its various applications such as risk assessment [138], supplier selection [129], and selection of logistics service provider [140]. In this technique, the network structure is first specified. The decision maker determines the objective, attributes, sub-attributes, and alternatives as a network structure, which has relation among attributes, sub-attributes, and alternatives (external dependence) as well as internal relation of attributes, sub-attributes, and alternatives (internal dependence), presented in Fig. 17.1.

In addition, the sub-attributes are considered in the network structure and the form of the structure changes into Fig. 17.2.

In Fig. 17.2, W_{21} indicates the target effects on the attributes, W_{22} represents the internal effects of the attributes, W_{32} denotes the effects of the attributes on the sub-attributes, and W_{33} shows the internal effects of the sub-attributes, determined by the decision maker. Then, the pairwise comparisons matrix for attributes, sub-attributes, and alternatives are specified by the decision maker in general form of Eq. (17.1).

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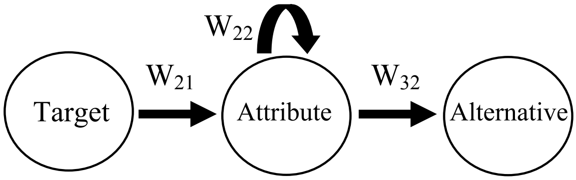

Fig. 17.1 Network structure [135]

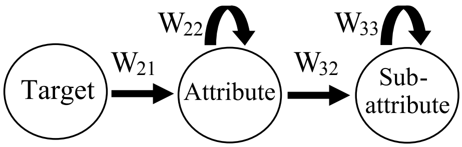

Fig. 17.2 Network structure using the sub-attributes [135]

$$\begin{array}{c} \\ C_1 \\ C_j \\ C_n \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & & C_j & & C_n \end{array} \\ \begin{bmatrix} m_{11} & \cdots & m_{1j} & \cdots & m_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ m_{i1} & \cdots & m_{ij} & \cdots & m_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nj} & \cdots & m_{nn} \end{bmatrix} \end{array}; \quad i,j \in \{1, \ldots, n\} \tag{17.1}$$

17.2 Description of ANP Method

17.2.1 The Priority Vectors

At first, the pairwise comparison matrices are normalized by Eq. (17.2) to determine the priority vectors of the attributes, sub-attributes, and alternatives.

$$m_{ij}^* = \frac{m_{ij}}{\sum_{j=1}^{n} m_{ij}}; \quad i = 1, \ldots, n \tag{17.2}$$

Then, the priority vector is determined by the arithmetic mean of each row, using Eq. (17.3).

$$\bar{m}_{ij} = \frac{1}{n}\sum_{i=1}^{n} m_{ij}^*; \quad j = 1, \ldots, n \tag{17.3}$$

17.2.2 The Super Matrix

The super matrix is formed by determining the priority vectors as the matrix of Eq. (17.4) [135].

$$W = \begin{array}{c} \\ \text{Target} \\ \text{Attribute} \\ \text{Sub-attribute} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Target} & \text{Attribute} & \text{Sub-attribute} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & w_{22} & 0 \\ 0 & w_{32} & w_{33} \end{bmatrix} \end{array} \tag{17.4}$$

17.2.3 The Cluster Matrix

The cluster matrix determines the impact of each cluster of attributes and sub-attributes on target achievement. The decision maker compares the attributes and sub-attributes in pairs to determine the cluster matrix, and by placing the obtained priority vector, as the sum of each column equals to one, the cluster matrix is defined as the matrix of Eq. (17.5) [135].

$$T = \begin{array}{c} \\ \text{Target} \\ \text{Attribute} \\ \text{Sub-attribute} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Target} & \text{Attribute} & \text{Sub-attribute} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & t_{22} & 0 \\ 0 & t_{32} & 0 \end{bmatrix} \end{array} \tag{17.5}$$

17.2.4 The Weighted Super Matrix

At this stage, the weighted super matrix is formed as the matrix of Eq. (17.6) by multiplying the cluster matrix values in the primary super matrix [135].

$$\widehat{W} = \begin{array}{c} \\ \text{Target} \\ \text{Attribute} \\ \text{Sub-attribute} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Target} & \text{Attribute} & \text{Sub-attribute} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ \widehat{w}_{21} & \widehat{w}_{22} & 0 \\ 0 & \widehat{w}_{32} & \widehat{w}_{33} \end{bmatrix} \end{array} \tag{17.6}$$

17.2.5 The Limit Super Matrix

At this stage, based on the Markov chain method, the weighted super matrix is powered to the extent that its rows move toward a fixed numbers and the limit super matrix is formed according to Eq. (17.7) [135, 141].

$$\mathrm{L} = \lim_{K\to\infty}\left(\widehat{\mathrm{W}}\right)^{2\mathrm{K}+1} \tag{17.7}$$

Each row of the matrix represents the final importance vector (I_j) of the sub-attributes.

17.2.6 The Utility Index

The utility index of each alternative is determined by comparing the alternatives and sub-attributes in pairs by Eq. (17.8) [135, 141].

$$U_i = \sum_{j=1}^{n} I_j.P_{ij}; \quad i = 1, \ldots, m \tag{17.8}$$

where P_{ij} indicates the score obtained from the pairwise comparison of ith alternative from the jth sub-attributes.

17.2.7 The Final Ranking of Alternatives

By determining the utility index of each alternative, the values are arranged in a descending order and the final ranking of alternatives is made.

17.3 Case Study

A company in the area of steel production intends to select an alternative as a supplier among three alternatives (A_1, A_2, and A_3). The attributes of cost (C_1), quality (C_2), and delivery function (C_3) are determined. In the same vein, the experts determine the sub-attributes of the product price (S_1) and transportation cost (S_2) for the cost attribute, the sub-attributes of the reliability of the product (S_3) and the ability to deliver products with high quality (S_4) for the quality attribute, and the sub-attributes of delivery time (S_5) and timely delivery (S_6) for the third attribute. Fig. 17.3 illustrates the supplier selection network structure.

The experts present the pairwise comparisons of relation among attributes, as well as the separate pairwise comparisons of internal relation among attributes, as indicated in Tables 17.1 and 17.2.

The paired comparisons of sub-attributes for each attribute and the paired comparisons of internal relation among attributes are presented in Tables 17.3, 17.4, 17.5, 17.6, 17.7, and 17.8.

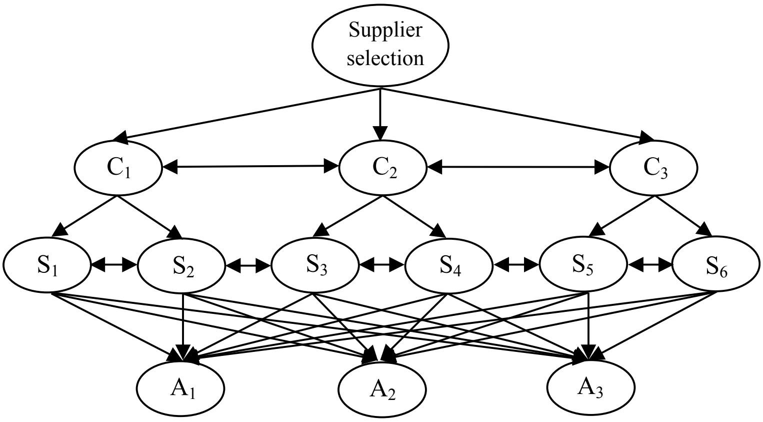

Fig. 17.3 Supplier selection network structure

Table 17.1 Pairwise comparisons of the attribute

	C_1	C_2	C_3
C_1	1	5	3
C_2	$\frac{1}{5}$	1	$\frac{1}{2}$
C_3	$\frac{1}{3}$	2	1

Table 17.2 Pairwise comparisons of internal relation among attributes

	C_1			C_2			C_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
C_1	–	–	–	1	–	$\frac{1}{2}$	1	4	–
C_2	–	1	3	–	–	–	$\frac{1}{4}$	1	–
C_3	–	$\frac{1}{3}$	1	2	–	1	–	–	–

Table 17.3 Pairwise comparisons of sub-attributes C_1

	S_1	S_2
S_1	1	2
S_2	$\frac{1}{2}$	1

Table 17.4 Pairwise comparisons of sub-attributes C_2

	S_3	S_4
S_3	1	$\frac{1}{4}$
S_4	4	1

Further, experts provided the pairwise comparisons' values among the alternatives in each sub-attributes as in Tables 17.9, 17.10 and 17.11.

Table 17.5 Pairwise comparisons of sub-attributes C_3

	S_5	S_6
S_5	1	3
S_6	$\frac{1}{3}$	1

Table 17.6 Pairwise comparisons of sub-attributes S_1 and S_2

	S_1						S_2					
	S_1	S_2	S_3	S_4	S_5	S_6	S_1	S_2	S_3	S_4	S_5	S_6
S_1	–	–	–	–	–	–	1	–	$\frac{1}{3}$	$\frac{1}{2}$	2	$\frac{1}{4}$
S_2	–	1	$\frac{1}{3}$	4	$\frac{1}{4}$	2	–	–	–	–	–	–
S_3	–	3	1	3	$\frac{1}{5}$	$\frac{1}{4}$	3	–	1	2	4	4
S_4	–	$\frac{1}{4}$	$\frac{1}{3}$	1	$\frac{1}{4}$	4	2	–	$\frac{1}{2}$	1	2	$\frac{1}{4}$
S_5	–	4	5	4	1	$\frac{1}{3}$	$\frac{1}{2}$	–	$\frac{1}{4}$		1	3
S_6	–	$\frac{1}{2}$	4	$\frac{1}{4}$	3	1	4	–	$\frac{1}{4}$	4	$\frac{1}{3}$	1

Table 17.7 Pairwise comparisons of sub-attributes S_3 and S_4

	S_3						S_4					
	S_1	S_2	S_3	S_4	S_5	S_6	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1	4	–	$\frac{1}{3}$	2	2	1	2	$\frac{1}{3}$	–	$\frac{1}{5}$	$\frac{1}{4}$
S_2	$\frac{1}{4}$	1	–	$\frac{1}{4}$	$\frac{1}{3}$	2	$\frac{1}{2}$	1	2	–	1	4
S_3	–	–	–	–	–	–	$\frac{1}{3}$	$\frac{1}{2}$	1	–	2	4
S_4	3	4	–	1	$\frac{1}{5}$	1	–	–	–	–	–	–
S_5	$\frac{1}{2}$	3	–	5	1	$\frac{1}{2}$	5	1	$\frac{1}{2}$	–	1	5
S_6	$\frac{1}{2}$	$\frac{1}{2}$	–	1	2	1	4	$\frac{1}{4}$	$\frac{1}{4}$	–	$\frac{1}{5}$	1

Table 17.8 Pairwise comparisons of sub-attributes S_5 and S_6

	S_5						S_6					
	S_1	S_2	S_3	S_4	S_5	S_6	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1	1	5	$\frac{1}{2}$	–	$\frac{1}{4}$	1	$\frac{1}{4}$	3	$\frac{1}{2}$	2	–
S_2	1	1	$\frac{1}{3}$	$\frac{1}{3}$	–	2	4	1	2	$\frac{1}{5}$	5	–
S_3	$\frac{1}{5}$	3	1	$\frac{1}{3}$	–	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	4	–
S_4	2	3	$\frac{1}{3}$	1	–	2	2	5	$\frac{1}{2}$	1	2	–
S_5	–	–	–	–	–	–	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	1	–
S_6	4	$\frac{1}{2}$	3	$\frac{1}{2}$	–	1	–	–	–	–	–	–

Table 17.9 Pairwise comparisons of alternatives in S_1 and S_2

	S_1			S_2		
	A_1	A_2	A_3	A_1	A_2	A_3
A_1	1	2	4	1	$\frac{1}{2}$	$\frac{1}{3}$
A_2	$\frac{1}{2}$	1	2	2	1	$\frac{1}{5}$
A_3	$\frac{1}{4}$	$\frac{1}{2}$	1	3	5	1

Table 17.10 Pairwise comparisons of alternatives in S_3 and S_4

	S_3			S_4		
	A_1	A_2	A_3	A_1	A_2	A_3
A_1	1	2	5	1	4	$\frac{1}{2}$
A_2	$\frac{1}{2}$	1	2	$\frac{1}{4}$	1	$\frac{1}{4}$
A_3	$\frac{1}{5}$	$\frac{1}{2}$	1	2	4	1

Table 17.11 Pairwise comparisons of alternatives in S_5 and S_6

	S_5			S_6		
	A_1	A_2	A_3	A_1	A_2	A_3
A_1	1	2	3	1	2	$\frac{1}{3}$
A_2	$\frac{1}{2}$	1	4	$\frac{1}{2}$	1	$\frac{1}{5}$
A_3	$\frac{1}{3}$	$\frac{1}{4}$	1	3	5	1

Table 17.12 Pairwise comparisons among attributes and sub-attributes

	Attribute	Sub-attribute
Attribute	1	$\frac{1}{4}$
Sub-attribute	4	1

Table 17.12 indicates the pairwise comparisons among attributes and sub-attributes. It aims to choose the best supplier for the company.

❖ Solution

(A) The priority vectors

At this stage, the priority vectors are determined according to the pairwise comparisons values provided by decision makers. For example, in determining the priority vector of the pairwise comparisons, the values are normalized as Table 17.13 and arithmetic mean is computed for each row.

Table 17.13 Normalized values of pairwise comparisons

	C_1	C_2	C_3	Mean
C_1	0.652	0.625	0.667	0.648
C_2	0.130	0.125	0.111	0.122
C_3	0.217	0.250	0.222	0.230

Therefore, the priority vector of the pairwise comparisons (w_{21}) is formed as follows:

$$w_{21} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \text{Target} \\ \left[\begin{array}{c} 0.648 \\ 0.122 \\ 0.230 \end{array}\right] \end{array}$$

Additionally, the other priority vectors are determined as follows:

$$w_{22} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \left[\begin{array}{ccc} 0 & 0.333 & 0.800 \\ 0.750 & 0 & 0.200 \\ 0.250 & 0.667 & 0 \end{array}\right] \end{array}$$

$$w_{32} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \left[\begin{array}{ccc} 0.667 & 0 & 0 \\ 0.333 & 0 & 0 \\ 0 & 0.200 & 0 \\ 0 & 0.800 & 0 \\ 0 & 0 & 0.750 \\ 0 & 0 & 0.250 \end{array}\right] \end{array}$$

$$w_{33} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \left[\begin{array}{cccccc} 0 & 0.105 & 0.234 & 0.115 & 0.180 & 0.166 \\ 0.168 & 0 & 0.094 & 0.235 & 0.149 & 0.254 \\ 0.153 & 0.384 & 0 & 0.263 & 0.123 & 0.193 \\ 0.153 & 0.142 & 0.225 & 0 & 0.306 & 0.326 \\ 0.298 & 0.158 & 0.281 & 0.172 & 0 & 0.061 \\ 0.229 & 0.211 & 0.166 & 0.115 & 0.242 & 0 \end{array}\right] \end{array}$$

(B) **The super matrix**

At this stage, the following super matrix is formed by the priority vectors:

$$W = \begin{array}{c} \\ \text{Target} \\ C_1 \\ C_2 \\ C_3 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{cccccccccc} \text{Target} & C_1 & C_2 & C_3 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \left[\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.648 & 0 & 0.333 & 0.800 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.122 & 0.750 & 0 & 0.200 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.230 & 0.250 & 0.667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.667 & 0 & 0 & 0 & 0.105 & 0.234 & 0.115 & 0.180 & 0.166 \\ 0 & 0.333 & 0 & 0 & 0.168 & 0 & 0.094 & 0.235 & 0.149 & 0.254 \\ 0 & 0 & 0.200 & 0 & 0.153 & 0.384 & 0 & 0.263 & 0.123 & 0.193 \\ 0 & 0 & 0.800 & 0 & 0.153 & 0.142 & 0.225 & 0 & 0.306 & 0.326 \\ 0 & 0 & 0 & 0.750 & 0.298 & 0.158 & 0.281 & 0.172 & 0 & 0.061 \\ 0 & 0 & 0 & 0.250 & 0.229 & 0.211 & 0.166 & 0.115 & 0.242 & 0 \end{array}\right] \end{array}$$

(C) The cluster matrix

The cluster matrix is defined based on the pairwise comparisons among the attributes and the sub-attributes so that the sum of each column equals to 1:

T =

	Target	Attribute	sub-attribute
Target	0	0	0
Attribute	1	0.200	0
Sub-attribute	0	0.800	1

(D) The weighted super matrix

At this stage, the weighted super matrix is determined by super matrix and the cluster matrix. Thus, the weighted super matrix is formed by multiplying two matrices in each other as follows:

$\widehat{W}$ =

	Target	C_1	C_2	C_3	S_1	S_2	S_3	S_4	S_5	S_6
Target	0	0	0	0	0	0	0	0	0	0
C_1	0.648	0	0.067	0.160	0	0	0	0	0	0
C_2	0.122	0.150	0	0.040	0	0	0	0	0	0
C_3	0.230	0.050	0.133	0	0	0	0	0	0	0
S_1	0	0.534	0	0	0	0.105	0.234	0.115	0.180	0.166
S_2	0	0.266	0	0	0.168	0	0.094	0.235	0.149	0.254
S_3	0	0	0.160	0	0.153	0.384	0	0.263	0.123	0.193
S_4	0	0	0.640	0	0.153	0.142	0.225	0	0.306	0.326
S_5	0	0	0	0.600	0.298	0.158	0.281	0.172	0	0.061
S_6	0	0	0	0.200	0.229	0.211	0.166	0.115	0.242	0

(E) The limit super matrix

In order to specify the final importance vector, the weighted super matrix is converted into the following limit super matrix using the Markov chain:

L =

	Target	C_1	C_2	C_3	S_1	S_2	S_3	S_4	S_5	S_6
Target	0	0	0	0	0	0	0	0	0	0
C_1	0	0	0	0	0	0	0	0	0	0
C_2	0	0	0	0	0	0	0	0	0	0
C_3	0	0	0	0	0	0	0	0	0	0
S_1	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
S_2	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
S_3	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
S_4	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
S_5	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
S_6	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020

(F) The utility index

In order to determine the utility index of alternatives, the pairwise comparisons' score of the alternatives (*P*) is initially calculated as follows:

Table 17.14 Utility index values

	S_1	S_2	S_3	S_4	S_5	S_6	U
A_1	0.011	0.003	0.018	0.010	0.010	0.005	0.057
A_2	0.006	0.004	0.008	0.003	0.007	0.002	0.030
A_3	0.003	0.013	0.004	0.016	0.002	0.013	0.051

$$P = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} 0.571 & 0.147 & 0.595 & 0.345 & 0.517 & 0.330 \\ 0.286 & 0.196 & 0.276 & 0.108 & 0.358 & 0.122 \\ 0.143 & 0.657 & 0.128 & 0.547 & 0.124 & 0.648 \end{bmatrix} \end{array}$$

The final priority vector of sub-attributes *I* is determined using the limit super matrix as follows:

$$\begin{array}{ccccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ \mathrm{I} = \ \text{Target} & [0.020 & 0.020 & 0.030 & 0.030 & 0.020 & 0.020] \end{array}$$

Therefore, the utility index of alternatives is determined by multiplying the final priority vector of the sub-attributes and the pairwise comparisons score of alternatives and the sum of rows of the resulting values as Table 17.14.

(G) **The final ranking of alternatives**

The utility index values are arranged in a descending order as follows:

$$U_1 > U_3 > U_2$$

Thus, the first supplier (A_1) is the best alternative and the alternatives are ranked as follows:

$$A_1 > A_3 > A_2$$

17.4 Conclusion

The ANP method, presented by Saaty, has been largely considered due to its different structures and steps (Fig. 17.4). After many years of presenting The ANP method, many papers and books have considered using this method. Given the high flexibility of the ANP, it is used in combination with other methods and in uncertainty environments. The inputs of this technique are the pairwise comparisons matrix and network structures which consider the attributes and sub-attributes of different alternatives. Therefore, this method is applied in many areas, due to its high accuracy in evaluating alternatives based on the opinions of experts as well as the features such as being compensatory.

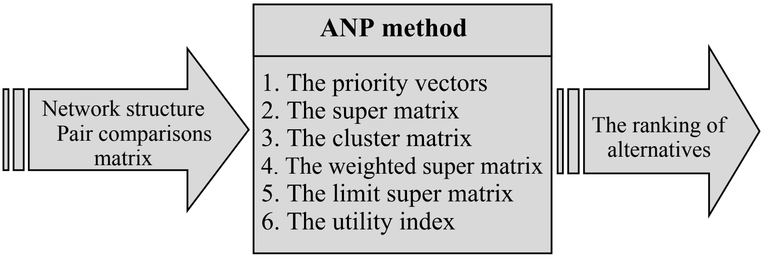

Fig. 17.4 A summary of the ANP method

Chapter 18
MAUT Method

18.1 Introduction

The Multi-Attribute Utility Theory (MAUT) method was introduced by Keeney and Raiffa in 1976 [27, 142–144]. The simplicity in solving multiple attribute decision-making problems is one of the advantages of this technique, and it gives abundant freedom of action to the decision makers to make the result more accurate and realistic. This method is applicable in areas such as assessment of industry firms [145] and selecting a project portfolio [146]. Further, this method has the following features:

- This method belongs to the compensatory methods;
- The attributes are independent of each other;
- The qualitative attributes are converted into the quantitative attributes.

The input information of the MAUT method is determined using the decision matrix. In this matrix, the alternatives and attributes are expressed based on the information received from the decision maker, as shown in Eq. (18.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \quad (18.1)$$

In Eq. (18.1), r_{ij} is the element of the decision matrix for ith alternative in jth attribute. Then, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$.

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18.2 Description of MAUT Method

18.2.1 The Normalized Decision Matrix

First, the decision matrix values are normalized depending on the positive or negative type of attributes. Eq. (18.2) is used to normalize the positive attributes, and Eq. (18.3) is used to normalize the values of negative attributes [147].

$$r_{ij}^{*} = \frac{r_{ij} - min(r_{ij})}{max(r_{ij}) - min(r_{ij})}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{18.2}$$

$$r_{ij}^{*} = 1 + \left(\frac{min(r_{ij}) - r_{ij}}{max(r_{ij}) - min(r_{ij})} \right); \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{18.3}$$

Obviously, r_{ij}^{*} is the normalized amount of the decision matrix of *i*th alternative in *j*th attribute.

18.2.2 The Marginal Utility Score

Eq. (18.4) is used to determine the marginal utility score [147, 148].

$$u_{ij} = \frac{e^{(r_{ij}^{*})^2} - 1}{1.71}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{18.4}$$

where u_{ij} indicates the marginal utility score of *i*th alternative in *j*th attribute [148].

18.2.3 The Final Utility Score

At this stage, the final utility score of each alternative is calculated using Eq. (18.5) by considering the weight of each attribute [147].

$$U_i = \sum_{j=1}^{n} u_{ij} \cdot w_j; \quad i = 1, \ldots, m \tag{18.5}$$

18.2.4 The Final Ranking of Alternatives

The final utility score of alternatives is arranged in a descending order for the final ranking, and the alternative with the highest final utility score is the best alternative.

18.3 Case Study

The organizational procurement department plans to buy a desktop phone among the models of A_1, A_2, A_3, and A_4. Experts determined attributes such as price (C_1), required space (C_2), ease of use (C_3), and general assessment of facilities (C_4). After converting the qualitative attributes into the quantitative attributes, the decision matrix is represented in Fig. 18.1. In addition, the weight of each attribute is illustrated in Table 18.1.

The purpose is to choose the best desktop phone model.

❖ **Solution**

(A) **The normalized decision matrix**

The normalized values of the decision matrix are determined based on the positive or negative type of attributes. For example, the normalized values of the first attribute are computed as follows:

$$r_{11}^{*} = 1 + \frac{(419 - 429)}{(649 - 419)} = 0.957$$

$$r_{21}^{*} = 1 + \frac{(419 - 649)}{(649 - 419)} = 0$$

$$r_{31}^{*} = 1 + \frac{(419 - 459)}{(649 - 419)} = 0.826$$

$$r_{41}^{*} = 1 + \frac{(419 - 419)}{(649 - 419)} = 1$$

The other values are illustrated in Table 18.2.

Fig. 18.1 Decision matrix of choosing a desktop phone

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} - & - & + & + \\ C_1 & C_2 & C_3 & C_4 \\ \end{array}$$

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} 429 & 0.600 & 5 & 4 \\ 649 & 0.700 & 4 & 5 \\ 459 & 0.400 & 1 & 1 \\ 419 & 0.500 & 2 & 2 \end{bmatrix}$$

Table 18.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4
w_j	0.345	0.350	0.155	0.150

Table 18.2 Normalized values of decision matrix

	C_1	C_2	C_3	C_4
A_1	0.957	0.333	1	0.750
A_2	0	0	0.750	1
A_3	0.826	1	0	0
A_4	1	0.667	0.250	0.250

(B) The marginal utility score

The marginal utility scores are obtained according to the normalized values of the decision matrix. For instance, the marginal utility scores of the first attribute are determined as follows:

$$u_{11} = \frac{e^{(0.957)^2} - 1}{1.71} = 0.877$$
$$u_{21} = \frac{e^{(0)^2} - 1}{1.71} = 0$$
$$u_{31} = \frac{e^{(0.826)^2} - 1}{1.71} = 0.572$$
$$u_{41} = \frac{e^{(1)^2} - 1}{1.71} = 1.005$$

The other marginal utility scores of attributes are presented in Table 18.3.

(C) The final utility score

The final utility score of alternatives is calculated as follows:

$$\begin{aligned} U_1 &= (0.877 \times 0.345) + (0.069 \times 0.350) \\ &\quad + (1.005 \times 0.155) + (0.442 \times 0.150) = 0.548 \\ U_2 &= (0 \times 0.345) + (0 \times 0.350) + (0.442 \times 0.155) + (1.005 \times 0.150) = 0.219 \\ U_3 &= (0.572 \times 0.345) + (1.005 \times 0.350) + (0 \times 0.155) + (0 \times 0.150) = 0.549 \\ U_4 &= (1.005 \times 0.345) + (0.328 \times 0.350) \\ &\quad + (0.038 \times 0.155) + (0.038 \times 0.150) = 0.473 \end{aligned}$$

Table 18.3 Marginal utility scores

	C_1	C_2	C_3	C_4
A_1	0.877	0.069	1.005	0.442
A_2	0	0	0.442	1.005
A_3	0.572	1.005	0	0
A_4	1.005	0.328	0.038	0.038

(D) **The final ranking of alternatives**

At this step, the final utility score of alternatives is arranged in a descending order:

$$U_3 > U_1 > U_4 > U_2$$

Therefore, the third alternative (A_3) is the best desktop phone model to buy. Then, the final ranking is as follows:

$$A_3 > A_1 > A_4 > A_2$$

18.4 Conclusion

The different simple and short models are observed in the MADM models. The MAUT method is used abundantly and even in everyday decision making, due to being short (Fig. 18.2) and its simplicity. Therefore, after passing many years of its presentation, the MAUT method is still considered and even used in combination with other methods. On the other hand, according to the experts' opinions, the weight of attributes is determined, the qualitative attributes are converted into the quantitative attributes and then the decision matrix is formed.

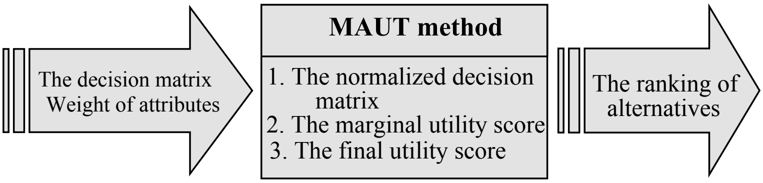

Fig. 18.2 A summary of the MAUT method

Chapter 19
IDOCRIW Method

19.1 Introduction

The Integrated Determination of Objective CRIteria Weights (IDOCRIW) method was introduced by Zavadskas and Podvezko in 2016 [149–151]. This technique benefits from the Entropy and Criterion Impact LOSs (CILOS) methods to determine a relative impact loss as well as the weight of attributes in a combination with two methods. Given the presentation of the IDOCRIW method in recent years, it is applied in areas such as analysis of rotor systems [152], assessing the performance of the construction sectors [153]. In addition, this method has the following features:

- It is one of the compensatory methods;
- Attributes are independent;
- The qualitative attributes should be converted into the quantitative attributes.

In addition, the input information is determined according to the decision matrix, as Eq. (19.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nj} & \cdots & r_{nn} \end{bmatrix}_{n \times n} \quad ; \quad i,j \in \{1, \ldots, n\} \tag{19.1}$$

In this decision matrix, r_{ij} is the element of the decision matrix for ith alternative in jth attribute.

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19.2 Description of IDOCRIW Method

19.2.1 The Normalized Decision Matrix

The normalized values of the decision matrix are calculated using Eq. (19.2) [154].

$$\bar{r}_{ij} = \frac{r_{ij}}{\sum_{i=1}^{n} r_{ij}}; \quad j = 1, \ldots, n \tag{19.2}$$

where $\bar{r}_{ij}$ demonstrates the normalized value of the decision matrix of alternative ith in the attribute jth.

19.2.2 The Degree of Entropy

Eq. (19.3) is used to determine the degree of entropy [154].

$$E_j = -\frac{1}{\ln n} \sum_{i=1}^{n} \bar{r}_{ij}.\ \ln \bar{r}_{ij}; \quad j = 1, \ldots, n, \quad 0 \leq E_j \leq 1 \tag{19.3}$$

19.2.3 The Entropy Weight (W)

At this stage, the deviation rate of the degree of the entropy is initially computed by Eq. (19.4) [154].

$$d_j = 1 - E_j; \quad j = 1, \ldots, n \tag{19.4}$$

Then, the entropy weight is obtained from Eq. (19.5) [154].

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \tag{19.5}$$

19.2.4 The Square Matrix

First, Eq. (19.6) is used to make positive the negative attributes of the decision matrix [154].

$$\hat{r}_{ij} = \frac{\min_i r_{ij}}{r_{ij}}; \quad i, j \in \{1, \ldots, n\} \tag{19.6}$$

Then, the normalized values of the decision matrix are calculated using Eq. (19.2), and square matrix values are determined according to Eq. (19.7) [154].

$$a_j = \max_i \overline{r}_{ij} = a_{k_i j}; \quad i, j \in \{1, \ldots, n\} \tag{19.7}$$

where $a_{k_i j}$ specifies the maximum values of jth criteria, are taken from decision matrix with k_i rows to form a square matrix and $a_{ij} = a_{k_i} j$ and $a_{jj} = a_j$ [149]. The ith row of square matrix contains the elements of the row k_i of decision matrix. It should be noted that some rows in square matrix can be the same as those in decision matrix; if the highest values of different criteria are in the same row, they belong to the same alternative [154].

19.2.5 The Relative Impact Loss Matrix

With respect to the values obtained from the previous step, the relative impact loss matrix is determined by Eq. (19.8) [149].

$$p_{ij} = \frac{a_{jj} - a_{ij}}{a_{jj}}; \quad i, j \in \{1, \ldots, n\}, \quad p_{jj} = 0 \tag{19.8}$$

p_{ij} represents the relative impact loss of the jth attribute, if selected as the best value.

19.2.6 The Weight System Matrix

Given the values of p_{ij}, the weight system matrix is formed in Eq. (19.9) [154].

$$\mathrm{F} = \begin{pmatrix} -\sum_{i=1}^{n} P_{i1} & P_{12} & \cdots & P_{1\mathrm{n}} \\ P_{21} & -\sum_{i=1}^{n} P_{i2} & \cdots & P_{2\mathrm{n}} \\ \vdots & \vdots & \ddots & \vdots \\ P_{\mathrm{n}1} & P_{\mathrm{n}2} & \cdots & -\sum_{i=1}^{n} P_{i\mathrm{n}} \end{pmatrix}_{n \times n} \tag{19.9}$$

19.2.7 The Criterion Impact Loss Weight (Q)

At this stage, Eq. (19.10) is first established [154].

$$Fq^T = 0 \tag{19.10}$$

The weight of the attributes $[q_1, q_2, \ldots, q_n]$ is determined through the solution of Eq. (19.10).

19.2.8 The Aggregate Weight (ω)

Considering the entropy weight (q) and CILOS weight (W), the aggregate weight value of the attributes is determined by Eq. (19.11) [154].

$$\omega_j = \frac{q_j . w_j}{\sum_{j=1}^{n} q_j . w_j} \tag{19.11}$$

19.2.9 The Final Ranking of Attributes

Regarding the final ranking of attributes, the aggregate weights of attributes are arranged in a descending order and are ranked accordingly.

19.3 Case Study

An organization intends to build a new administrative building among A_1, A_2, A_3, and A_4 plans. The authorities of the company intend to examine and prioritize the preference of attributes such as cost (C_1), the building area (C_2), the distance from organizational home (C_3), and building quality (C_4), according to experts' opinions. The decision matrix is shown in Fig. 19.1.

Fig. 19.1 Decision matrix of constructing building

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} - & + & - & + \\ C_1 & C_2 & C_3 & C_4 \\ \end{array}$$

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} 3 & 100 & 10 & 7 \\ 2.500 & 80 & 8 & 5 \\ 1.800 & 50 & 20 & 11 \\ 2.200 & 70 & 12 & 9 \end{bmatrix}$$

It is desirable is to prioritize attributes and determine the preferred attribute.

❖ **Solution**

(A) **The normalized decision matrix**

At first, the decision matrix values are normalized in Table 19.1.

(B) **The degree of entropy**

The degree of entropy for each attribute is determined as follows:

$$E_1 = -\frac{1}{\ln 4}(0.316 \times \ln 0.316 + 0.263 \times \ln 0.263 + 0.189 \times \ln 0.189 + 0.232 \times \ln 0.232) = 0.988$$

$$E_2 = -\frac{1}{\ln 4}(0.333 \times \ln 0.333 + 0.267 \times \ln 0.267 + 0.167 \times \ln 0.167 + 0.233 \times \ln 0.233) = 0.979$$

$$E_3 = -\frac{1}{\ln 4}(0.200 \times \ln 0.200 + 0.160 \times \ln 0.160 + 0.400 \times \ln 0.400 + 0.240 \times \ln 0.240) = 0.955$$

$$E_4 = -\frac{1}{\ln 4}(0.219 \times \ln 0.219 + 0.156 \times \ln 0.156 + 0.344 \times \ln 0.344 + 0.281 \times \ln 0.281) = 0.971$$

(C) **The entropy weight (W)**

Initially, the deviation rate of the degree of entropy is calculated as follows:

$$d_1 = 1 - 0.988 = 0.012$$
$$d_2 = 1 - 0.979 = 0.021$$
$$d_3 = 1 - 0.955 = 0.045$$
$$d_4 = 1 - 0.971 = 0.029$$

Table 19.1 Normalized values of decision matrix

	C_1	C_2	C_3	C_4
A_1	0.316	0.333	0.200	0.219
A_2	0.263	0.267	0.160	0.156
A_3	0.189	0.167	0.400	0.344
A_4	0.232	0.233	0.240	0.281

Then, the entropy weights are obtained as follows:

$$w_1 = \frac{0.012}{0.012 + 0.021 + 0.045 + 0.029} = 0.112$$
$$w_2 = \frac{0.021}{0.012 + 0.021 + 0.045 + 0.029} = 0.196$$
$$w_3 = \frac{0.045}{0.012 + 0.021 + 0.045 + 0.029} = 0.421$$
$$w_4 = \frac{0.029}{0.012 + 0.021 + 0.045 + 0.029} = 0.271$$

(D) The square matrix

The values of the negative attributes for the direct relation matrix are maximized in Table 19.2 to make all attributes positive. Then, all values are normalized according to Table 19.3.

Additionally, the maximum amount of each column is specified as follows:

$$a_{11} = 0.319, a_{22} = 0.333, a_{33} = 0.349, a_{44} = 0.344$$

Finally, the square matrix is formed in Fig. 19.2.

(E) The relative impact loss matrix

The values of the relative impact loss matrix are determined according to the square matrix values. For example, the first attribute values are obtained as follows:

$$p_{11} = 0$$
$$p_{21} = \frac{0.319 - 0.191}{0.319} = 0.401$$
$$p_{31} = \frac{0.319 - 0.229}{0.319} = 0.282$$
$$p_{41} = 0$$

Fig. 19.3 indicates the relative impact loss matrix.

(F) The weight system matrix

The weight system matrix is also illustrated in Fig. 19.4.

Table 19.2 Maximum values of decision matrix

	C_1	C_2	C_3	C_4
A_1	0.600	100	0.800	7
A_2	0.720	80	1	5
A_3	1	50	0.400	11
A_4	0.818	70	0.667	9

Table 19.3 Normalized values of decision matrix

	C_1	C_2	C_3	C_4
A_1	0.191	0.333	0.279	0.219
A_2	0.229	0.267	0.349	0.156
A_3	0.319	0.167	0.140	0.344
A_4	0.261	0.233	0.233	0.281

Fig. 19.2 Square matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0.319 & 0.167 & 0.140 & 0.344 \\ 0.191 & 0.333 & 0.279 & 0.219 \\ 0.229 & 0.267 & 0.349 & 0.156 \\ 0.319 & 0.167 & 0.140 & 0.344 \end{bmatrix} \end{array}$$

Fig. 19.3 Relative impact loss matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 0.498 & 0.598 & 0 \\ 0.401 & 0 & 0.201 & 0.363 \\ 0.282 & 0.198 & 0 & 0.546 \\ 0 & 0.498 & 0.599 & 0 \end{bmatrix} \end{array}$$

Fig. 19.4 Weights system matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} -0.683 & 0.498 & 0.598 & 0 \\ 0.401 & -1.194 & 0.201 & 0.363 \\ 0.282 & 0.198 & -1.398 & 0.546 \\ 0 & 0.498 & 0.599 & -0.909 \end{bmatrix} \end{array}$$

(G) The criterion impact loss weight (q)

At this stage, the following Eq. (19.10) is formed by the weight vector $[q_1, q_2, q_3, q_4]$:

$$\begin{cases} -0.683q_1 + 0.498q_2 + 0.598q_3 = 0 \\ 0.401q_1 - 1.194q_2 + 0.201q_3 + 0.363q_4 = 0 \\ 0.282q_1 + 0.198q_2 - 1.398q_3 + 0.546q_4 = 0 \\ 0.498q_2 + 0.599q_3 - 0.909q_4 = 0 \end{cases}$$

The weight value of each attribute is determined as follows:

$$q_1 = 0.334,\ q_2 = 0.220,\ q_3 = 0.196,\ q_4 = 0.250$$

(H) **The aggregate weight (ω)**

The aggregate weight of attributes is computed with respect to the entropy weight (q) and criterion impact loss weight (W), as follows:

$$\omega_1 = \frac{0.334 \times 0.112}{0.334 \times 0.112 + 0.220 \times 0.196 + 0.196 \times 0.421 + 0.250 \times 0.271} = 0.162$$
$$\omega_2 = \frac{0.220 \times 0.196}{0.334 \times 0.112 + 0.220 \times 0.196 + 0.196 \times 0.421 + 0.250 \times 0.271} = 0.187$$
$$\omega_3 = \frac{0.196 \times 0.421}{0.334 \times 0.112 + 0.220 \times 0.196 + 0.196 \times 0.421 + 0.250 \times 0.271} = 0.358$$
$$\omega_4 = \frac{0.250 \times 0.271}{0.334 \times 0.112 + 0.220 \times 0.196 + 0.196 \times 0.421 + 0.250 \times 0.271} = 0.293$$

(I) **The final ranking of attributes**

In this step, the aggregate weights of each attribute are arranged in a descending order:

$$\omega_3 > \omega_4 > \omega_2 > \omega_1$$

Therefore, the third attribute (C_3) has a higher priority than the other attributes, and the attributes are ranked as follows:

$$C_3 > C_4 > C_2 > C_1$$

19.4 Conclusion

The IDOCRIW method is a new method in MADM for evaluating the attributes. This technique includes the methods of entropy and CILOS with eight stages (Fig. 19.5). Also, a specific and unit weight is presented by integrating the weight of attributes obtained from the entropy and CILOS methods. Considering the previous techniques of evaluating attributes, the IDOCRIW is the first method, which combines two different methods in the area of the attributes evaluation, increasing the accuracy and reliability of the technique. In addition, the qualitative attributes should be converted into quantitative attributes.

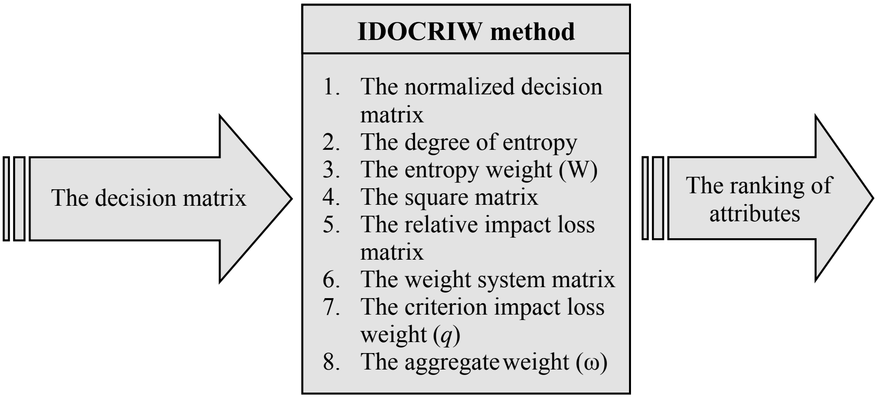

Fig. 19.5 A summary of the IDOCRIW method

Chapter 20
TODIM Method

20.1 Introduction

The TODIM method was introduced by Gomes and Lima in 1992 [155–159]. The main idea is to measure the dominance degree of each alternative over the other alternatives using the overall value, and then, the alternatives are evaluated and ranked according to the following features:

- This method belongs to the compensatory methods;
- The attributes are independent of each other;
- The qualitative attributes should be converted into the quantitative attributes.

The TODIM method is applicable in the areas such as medical treatment selection [160], evaluation of elective courses [161], and choosing the best ERP software [162].

The input information of this technique is determined based on the decision matrix. In this matrix, the alternatives and attributes are expressed according to the decision maker's opinion, as shown in Eq. (20.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} \quad ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \qquad (20.1)$$

where r_{ij} represents the element of the decision matrix for ith alternative in jth attribute. The decision maker presents the weight of attributes $[w_1, w_2, \ldots, w_n]$ as well as the attenuation factor of losses (θ) in determining the dominance degree.

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20.2 Description of TODIM Method

20.2.1 The Normalized Decision Matrix

Eqs. (20.2) and (20.3) are used to normalize the decision matrix for positive and negative attributes, respectively [163, 164].

$$r_{ij}^{*} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}; \quad j = 1, \ldots, n \tag{20.2}$$

$$r_{ij}^{*} = \frac{\frac{1}{r_{ij}}}{\sum_{i=1}^{m} \frac{1}{r_{ij}}}; \quad j = 1, \ldots, n \tag{20.3}$$

r_{ij}^{*} indicates the normalized value of decision matrix of *i*th alternative in *j*th attribute.

20.2.2 The Relative Weight

Eq. (20.4) is used to determine the relative weight [163, 164].

$$\tilde{w}_j = \frac{w_j}{\hat{w}}; \quad j = 1, \ldots, n \tag{20.4}$$

where $\hat{w}$ indicates the reference weight and the maximum amount of the attributes' weight is determined as the reference weight.

20.2.3 The Dominance Degree

The dominance degree of the alternative A_i over the alternative $A_{i'}$, representing the dominance degree of each alternative, is calculated as Eq. (20.5) [163, 164].

$$\delta_i = \delta(A_i, A_{i'}) = \sum_{j=1}^{n} \emptyset_j(A_i, A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{20.5}$$

In Eq. (20.5), the preference index value is computed as Eq. (20.6) [163, 164].

$$\emptyset_j(A_i, A_{i'})\begin{cases} -\frac{1}{\theta}\sqrt{\frac{\left(\sum_{j=1}^{n}\tilde{w}_j\right)\left|r_{ij}^*-r_{ji}^*\right|}{\tilde{w}_j}} & if \quad \left(r_{ij}^*-r_{ji}^*\right)<0 \\ 0 & if \quad \left(r_{ij}^*-r_{ji}^*\right)=0 \\ \sqrt{\frac{\tilde{w}_j\left(r_{ij}^*-r_{ji}^*\right)}{\sum_{j=1}^{n}\tilde{w}_j}} & if \quad \left(r_{ij}^*-r_{ji}^*\right)>0 \end{cases} \tag{20.6}$$

where θ indicates the attenuation factor of the losses.

20.2.4 The Overall Dominance Degree

The overall dominance degree of each alternative is determined using Eq. (20.7) [163, 164].

$$\zeta_i = \frac{\delta_i - min\delta_i}{max\delta_i - min\delta_i}; \quad i = 1, \ldots, m \tag{20.7}$$

20.2.5 The Final Ranking of Alternatives

The overall dominance degree of alternatives is arranged in a descending order for the final ranking of alternatives.

20.3 Case Study

An educational institution intends to evaluate three software training courses (A_1, A_2, and A_3). The attributes such as the number of course holding (C_1), course duration (C_2), teacher's knowledge level in the course (C_3), course content applicability (C_4), and the average score of the participants' evaluation (C_5) were determined in this regard. Fig. 20.1 demonstrates the decision matrix. Table 20.1 indicates the weight of attributes.

Fig. 20.1 Decision matrix of evaluating the software training course

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccccc} + & + & + & + & + \\ C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 5 & 3 & 4 & 2 & 3 \\ 2 & 4 & 1 & 4 & 2 \\ 5 & 2 & 4 & 3 & 3 \end{bmatrix} \end{array}$$

Table 20.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5
w_j	0.260	0.160	0.100	0.420	0.060

The attenuation factor of the losses equals to 1, and the purpose is to choose the best software training course.

❖ **Solution**

(A) **The normalized decision matrix**

Given the positivity of all attributes, the normalized values of decision matrix are determined as indicated in Table 20.2.

(B) **The relative weight**

Considering the maximum relative weight of attributes, the reference-level values are obtained as follows:

$$\tilde{w}_1 = \frac{0.260}{0.420} = 0.619$$
$$\tilde{w}_2 = \frac{0.160}{0.420} = 0.381$$
$$\tilde{w}_3 = \frac{0.100}{0.420} = 0.238$$
$$\tilde{w}_4 = \frac{0.420}{0.420} = 1$$
$$\tilde{w}_5 = \frac{0.060}{0.420} = 0.143$$

(C) **The dominance degree**

At this stage, for example, the preference index values of the first alternative for the first attribute are calculated as follows:

$$\emptyset_1(A_1, A_2) = \sqrt{\frac{0.619 \times (0.417 - 0.167)}{2.381}} = 0.255$$
$$\emptyset_1(A_1, A_3) = 0$$

Table 20.2 Normalized values of decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.417	0.333	0.444	0.222	0.375
A_2	0.167	0.444	0.111	0.444	0.250
A_3	0.417	0.222	0.444	0.333	0.375

The other values for each alternative are calculated as shown in Tables 20.3, 20.4, and 20.5.

The dominance degree of each alternative equals the sum of the preference indexes of each alternative as follows:

$$\delta_1 = -1.417,\ \delta_2 = -7.656,\ \delta_3 = -1.785$$

(D) **The overall dominance degree**

The overall dominance degree of each attribute is obtained as follows:

$$\zeta_1 = \frac{-1.417 - (-7.656)}{-1.417 - (-7.656)} = 1$$
$$\zeta_2 = \frac{-7.656 - (-7.656)}{-1.417 - (-7.656)} = 0$$
$$\zeta_3 = \frac{-1.785 - (-7.656)}{-1.417 - (-7.656)} = 0.941$$

(E) **The final ranking of alternatives**

At this step, the overall dominance degree of each alternative is arranged in a descending order:

$$\zeta_1 > \zeta_3 > \zeta_2$$

Table 20.3 Preference index values for the first alternative

	$\emptyset_1(A_i, A_{i'})$	$\emptyset_2(A_i, A_{i'})$	$\emptyset_3(A_i, A_{i'})$	$\emptyset_4(A_i, A_{i'})$	$\emptyset_5(A_i, A_{i'})$
(A_1, A_2)	0.255	−0.833	0.182	−0.727	0.087
(A_1, A_3)	0	0.133	0	−0.514	0

Table 20.4 Preference index values for the second alternative

	$\emptyset_1(A_i, A_{i'})$	$\emptyset_2(A_i, A_{i'})$	$\emptyset_3(A_i, A_{i'})$	$\emptyset_4(A_i, A_{i'})$	$\emptyset_5(A_i, A_{i'})$
(A_2, A_1)	−0.981	0.133	−1.825	0.305	−1.443
(A_2, A_3)	−0.981	0.188	−1.825	0.216	−1.443

Table 20.5 Preference index values for the third alternative

	$\emptyset_1(A_i, A_{i'})$	$\emptyset_2(A_i, A_{i'})$	$\emptyset_3(A_i, A_{i'})$	$\emptyset_4(A_i, A_{i'})$	$\emptyset_5(A_i, A_{i'})$
(A_3, A_1)	0	−0.833	0	0.216	0
(A_3, A_2)	0.255	−1.178	0.182	−0.514	0.087

Therefore, the first software training course (A_1) has the best rank and the final ranking is determined as follows:

$$A_1 > A_3 > A_2$$

20.4 Conclusion

The main and distinguishing idea of the TODIM method, presented by Gomes and Lima, is to measure the dominance degree of each alternative over the other alternatives using the dominance degree value. This is a new method in the MADM, which evaluates and ranks the alternatives, by considering its short steps (Fig. 20.2) and converting the qualitative attributes into the quantitative attributes. Recently, this technique has been used in combination form and has been considered by researchers.

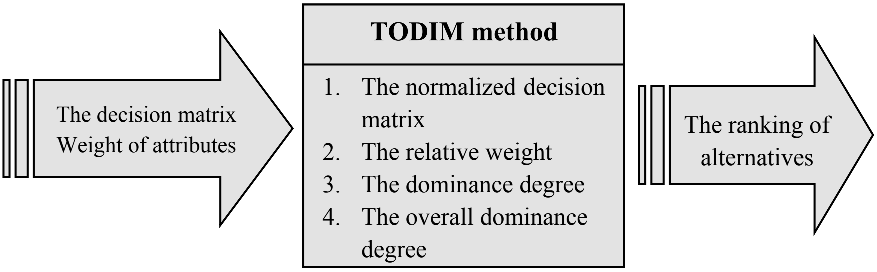

Fig. 20.2 A summary of the TODIM method

Chapter 21
EDAS Method

21.1 Introduction

The Evaluation based on Distance from Average Solution (EDAS) method was introduced by Keshavarz Ghorabaee, Zavadskas, Olfat, and Turskis in 2015 [165–168]. This method is very practical in conditions with the contradictory attributes, and the best alternative is chosen by calculating the distance of each alternative from the optimal value. The EDAS method is applied in the evaluation of airline services [166], solving air traffic problems [169], and personnel selection [170], and has the following features:

- This method belongs to the compensatory methods;
- The attributes are independent of each other;
- The qualitative attributes should be converted into quantitative attributes.

Further, the input information is determined as the decision matrix, as shown in Eq. (21.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{21.1}$$

where r_{ij} is the element of the decision matrix for ith alternative in jth attribute. In addition, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$.

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21.2 Description of EDAS Method

21.2.1 The Average Solution

Eq. (21.2) is used to determine the average solution of each attribute [171].

$$AV_j = \frac{\sum_{i=1}^{m} r_{ij}}{m}; \quad j = 1, \ldots, n \tag{21.2}$$

21.2.2 The Positive and Negative Distances from Average Solution

According to the positive and negative types of attributes, the positive distances from average (PDA) and negative distances from average (NDA) of the positive attributes are calculated by Eqs. (21.3) and (21.4), respectively [171].

$$PDA_{ij} = \frac{max\big(0, (r_{ij} - AV_j)\big)}{AV_j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{21.3}$$

$$NDA_{ij} = \frac{max\big(0, (AV_j - r_{ij})\big)}{AV_j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{21.4}$$

In addition, the PDA and NDA values of the negative attributes are determined using Eqs. (21.5) and (21.6) [171].

$$PDA_{ij} = \frac{max\big(0, (AV_j - r_{ij})\big)}{AV_j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{21.5}$$

$$NDA_{ij} = \frac{max\big(0, (r_{ij} - AV_j)\big)}{AV_j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{21.6}$$

21.2.3 The Weighted PDA and NDA

Considering the weight of the attributes, Eqs. (21.7) and (21.8) are used to determine the values of the weighted PDA and weighted NDA of each alternative, respectively [171].

$$SP_i = \sum_{j=1}^{n} PDA_{ij}.w_j; \quad i = 1, \ldots, m \tag{21.7}$$

$$SN_i = \sum_{j=1}^{n} NDA_{ij}.w_j; \quad i = 1, \ldots, m \tag{21.8}$$

21.2.4 Weighted Normalized PDA and NDA

Eqs. (21.9) and (21.10) are used to normalize the values of the weighted PDA and weighted NDA, respectively [171].

$$NSP_i = \frac{SP_i}{\max_i(SP_i)}; \quad i = 1, \ldots, m \tag{21.9}$$

$$NSN_i = \frac{SN_i}{\max_i(SN_i)}; \quad i = 1, \ldots, m \tag{21.10}$$

21.2.5 The Appraisal Score

The appraisal score for each alternative is computed as Eq. (21.11) [171].

$$AS_i = \frac{1}{2}(NSP_i + NSN_i); \quad i = 1, \ldots, m \tag{21.11}$$

21.2.6 The Final Ranking of Alternatives

For the final ranking of alternatives, the appraisal scores of alternatives are arranged in a descending order and the final ranking is made.

21.3 Case Study

A road construction company plans to purchase an excavator among A_1, A_2, and A_3 models proposed by experts. Experts provided attributes such as annual maintenance cost (C_1), price (C_2), working weight (C_3), fuel consumption rate (C_4), the complexity level of working with excavator by the operator (C_5), and bucket capacity (C_6). The decision matrix is shown in Fig. 21.1. Further, the weight of each attribute is determined in Table 21.1.

The purpose is to choose the best excavator model.

❖ **Solution**

(A) **The average solution**

The average solution of each attribute is obtained as follows:

$$AV_1 = \frac{0.710 + 1.330 + 1.450}{3} = 1.163$$
$$AV_2 = \frac{4.100 + 5.900 + 4.900}{3} = 4.967$$
$$AV_3 = \frac{0.180 + 0.740 + 0.270}{3} = 0.397$$
$$AV_4 = \frac{0.720 + 0.310 + 0.650}{3} = 0.560$$
$$AV_5 = \frac{0.990 + 0.420 + 0.420}{3} = 0.610$$
$$AV_6 = \frac{0.250 + 0.830 + 0.440}{3} = 0.507$$

(B) **The positive and negative distances from average solution**

For example, the values of the positive distance from average solution for the first attribute are determined as follows:

$$PDA_{11} = \frac{max(0, (1.163 - 0.710))}{1.163} = 0.390$$
$$PDA_{21} = \frac{max(0, (1.163 - 1.330))}{1.163} = 0$$
$$PDA_{31} = \frac{max(0, (1.163 - 1.450))}{1.163} = 0$$

Table 21.2 indicates the other values of the positive distance from average solution.

Fig. 21.1 Decision matrix for purchasing the excavator

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{cccccc} - & - & + & - & - & + \\ C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \left[\begin{array}{cccccc} 0.710 & 4.100 & 0.180 & 0.720 & 0.990 & 0.250 \\ 1.330 & 5.900 & 0.740 & 0.310 & 0.420 & 0.830 \\ 1.450 & 4.900 & 0.270 & 0.650 & 0.420 & 0.440 \end{array}\right] \end{array}$$

Table 21.1 Weight attributes

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
w_j	0.171	0.185	0.177	0.225	0.157	0.085

Table 21.2 Values of the positive distance from average solution

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.390	0.175	0	0	0	0
A_2	0	0	0.864	0.446	0.311	0.637
A_3	0	0.013	0	0	0.311	0

For example, the values of the negative distance from average solution of alternatives for the first attribute are determined as follows:

$$NDA_{11} = \frac{max(0, (0.710 - 1.163))}{1.163} = 0$$
$$NDA_{21} = \frac{max(0, (1.330 - 1.163))}{1.163} = 0.144$$
$$NDA_{31} = \frac{max(0, (1.450 - 1.163))}{1.163} = 0.247$$

Table 21.3 indicates the other values of the negative distance from average solution.

(C) **The weighted PDA and NDA**

According to the weight of attributes, the weighted positive distances from average solution are determined in Table 21.4.

Table 21.5 demonstrates the weighted negative distances from average solution.

(D) **Weighted normalized PDA and NDA**

The values of weighted normalized PDA of each alternative are obtained as follows:

Table 21.3 Values of the negative distance from average solution

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0	0	0.547	0.286	0.623	0.507
A_2	0.144	0.188	0	0	0	0
A_3	0.247	0	0.320	0.161	0	0.132

Table 21.4 Values of the weighted positive distances

	C_1	C_2	C_3	C_4	C_5	C_6	Sum
A_1	0.067	0.032	0	0	0	0	0.099
A_2	0	0	0.153	0.100	0.049	0.054	0.356
A_3	0	0.002	0	0	0.049	0	0.051

Table 21.5 Values of the weighted negative distances

	C_1	C_2	C_3	C_4	C_5	C_6	Sum
A_1	0	0	0.097	0.064	0.098	0.043	0.302
A_2	0.025	0.035	0	0	0	0	0.060
A_3	0.042	0	0.057	0.036	0	0.011	0.146

$$NSP_1 = \frac{0.099}{0.356} = 0.278$$
$$NSP_2 = \frac{0.356}{0.356} = 1$$
$$NSP_3 = \frac{0.051}{0.356} = 0.143$$

In addition, the values of the weighted normalized NDA of each alternative are determined as follows:

$$NSN_1 = \frac{0.302}{0.302} = 1$$
$$NSN_2 = \frac{0.060}{0.302} = 0.197$$
$$NSN_3 = \frac{0.146}{0.302} = 0.483$$

(E) **The appraisal score**

The appraisal score of each alternative is calculated as follows:

$$AS_1 = \frac{1}{2}(0.278 + 1) = 0.639$$
$$AS_2 = \frac{1}{2}(1 + 0.197) = 0.599$$
$$AS_3 = \frac{1}{2}(0.143 + 0.483) = 0.313$$

(F) **The final ranking of alternatives**

At this stage, the appraisal scores of alternatives are arranged in descending order:

$$AS_1 > AS_2 > AS_3$$

Therefore, the first excavator model (A_1) is the best alternative for the prefabricated wall and the alternatives are ranked as follows:

$$A_1 > A_2 > A_3$$

21.4 Conclusion

The EDAS method is considered as one of the new methods in MADM, which has been considered in a short time. This technique evaluates different alternatives in the presence of contradictory attributes and has four stages (Fig. 21.2). On the other hand, the best alternative is selected by determining the positive and negative distances from the optimal amount. The optimal value is determined from the average values of decision matrix. Further, this is the compensatory method and the qualitative attributes should be converted into quantitative attributes.

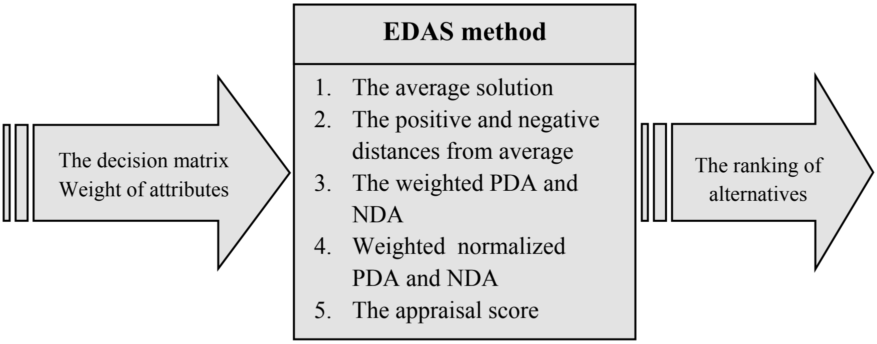

Fig. 21.2 A summary of EDAS method

Chapter 22
PAMSSEM I & II

22.1 Introduction

The PAMSSEM methods were introduced by Martel, Kiss, and Rousseau in 1996 [172–175]. This method patterns the preferences of decision maker to choose the best alternative using an outranking approach, according to the ordinal or cardinal of each attribute. Also, only the entering and leaving flows are examined and alternatives are ranked partially. However, in the PAMSSEM II method, the net flow is determined as final values and alternatives are ranked completely. This technique is used in asset management [176] and resource consolidation management in clouds [177]. The PAMSSEM method has the following features:

- It is a compensatory method;
- The qualitative attributes should be converted into quantitative attributes;
- There is no need for independence of attributes.

The input information is determined based on the information received from the decision maker in the form of decision matrix as Eq. (22.1).

$$\mathrm{F}=\begin{bmatrix} k_1(A_1) & \cdots & k_j(A_1) & \cdots & k_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ k_1(A_i) & \cdots & k_j(A_i) & \cdots & k_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ k_1(A_m) & \cdots & k_j(A_m) & \cdots & k_n(A_m) \end{bmatrix}_{m\times n} ;\quad i=1,\ldots,m,\ j=1,\ldots,n \tag{22.1}$$

In Eq. (22.1), $B = \{A_1, A_2, \ldots, A_m\}$ is a finite set of alternatives, and $C = \{k_1(0), k_2(0), \ldots, k_n(0)\}$ is a set of evaluation attributes of the alternatives of the set B. In addition, the decision maker presents the weight of attributes $[w_1, w_2, \ldots, w_n]$ and

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specifies the indifference threshold parameters (q), the preference threshold (p), and the veto threshold (v).

22.2 Description of PAMSSEM Methods

22.2.1 The Local Outranking Index

The local outranking index for the ordinal attributes is computed using Eq. (22.2) [178, 179].

$$\delta(A_i, A_{i'}) = \sum_{A_{i'}} \left(\sum_{A_i} \bar{\delta}_j(A_i, A_{i'}).f_j(A_i) \right).f_j(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{22.2}$$

$f_j(A_{i'})$ and $f_j(A_i)$ are probability density functions (discrete) which are assumed to be equal to one. $\bar{\delta}_{\mathrm{j}}(A_i, A_{i'})$ is index computed according to Eq. (22.3) [178, 179].

$$\bar{\delta}_j(A_i, A_{i'}) = \begin{cases} 0 & \textit{if } \Delta_j \leq -p_j \\ \frac{\Delta_j - P_j}{P_j - q_j} & \textit{if } -p_j < \Delta_j - q_j; \quad p_j \geq q_j \geq 0 \\ 1 & \textit{if } \Delta_j \geq -q_j \end{cases} \tag{22.3}$$

In Eq. (22.3), the value of $\Delta_j = k_{\mathrm{j}}(A_i) - k_{\mathrm{j}}(A_{i'})$ and the indifference threshold (q), the preference threshold (p) is the values determined by the decision maker for each attribute. Further, the local outranking index for the cardinal attributes is obtained from Eq. (22.4) [178, 179].

$$\delta_j(A_i, A_{i'}) = \begin{cases} 0 & \textit{if } \Delta_j < -1 \\ \frac{1}{2} & \textit{if } -1 \leq \Delta_j < 0; \quad i, i' \in \{1, \ldots, m\}, \quad j = 1, \ldots, n \\ 1 & \textit{if } \Delta_j \geq 0 \end{cases} \tag{22.4}$$

where Δ_j indicates the difference between the levels [178].

22.2.2 The Concordance Index

Regarding the local outranking index and weight of attributes, Eq. (22.5) is used to determine the concordance index [178, 179].

$$\mathrm{C}(A_i, A_{i'}) = \sum_{j=1}^{n} \delta_j(A_i, A_{i'}).w_j; \quad i, i' \in \{1, \ldots, m\} \tag{22.5}$$

22.2.3 The Local Discordance Index

At this stage, the local discordance index is calculated using Eq. (22.6) [178, 179].

$$\mathrm{D}(A_i, A_{i'}) = \sum_{A_i}\left(\sum_{A_{i'}} \bar{D}_j(A_i, A_{i'}).f_j(A_{i'})\right).f_j(A_i); \quad i, i' \in \{1, \ldots, m\} \tag{22.6}$$

$f_j(A_{i'})$ and $f_j(A_i)$ are probability density functions (discrete) which are assumed to be equal to one. $\bar{D}_j(A_i, A_{i'})$ for the ordinal attributes are computed using Eq. (22.7) [178, 179].

$$\bar{D}_j(A_i, A_{i'}) = \begin{cases} 1 & \textit{if } \Delta_j \le -v_j \\ -\left(\frac{\Delta_j + p_j}{v_j - p_j}\right) & \textit{if } -v_j < \Delta_j < -p_j; \quad v_j > p_j \\ 0 & \textit{if } \Delta_j \ge -p_j \end{cases} \tag{22.7}$$

where the veto threshold (v) is determined by the decision maker. In addition, $\bar{D}_j(A_i, A_{i'})$ for the cardinal attributes is determined by Eq. (22.8) [178, 179].

$$\bar{D}_j(A_i, A_{i'}) = \begin{cases} min\left\{1, \xi(w_j).\Delta_j + \frac{\gamma_j + 1}{2}\right\} & \textit{if } \Delta_j < -\left[\frac{\gamma_j + 1}{2}\right] \\ 0 & \textit{if } \Delta_j \ge -\left[\frac{\gamma_j + 1}{2}\right] \end{cases} \tag{22.8}$$

where γ_j is the number of measurement scale levels of the jth attribute $(\gamma_j > 3)$ and $\xi(w_j)$ is calculated using Eq. (22.9) [178, 179].

$$\xi(w_j) = 0.2(1 + \frac{w_j}{2}); \quad j = 1, \ldots, n \tag{22.9}$$

22.2.4 The Outranking Degree

The outranking degree is computed by Eq. (22.10) [178, 179].

$$\varphi(A_i, A_{i'}) = \mathrm{C}(A_i, A_{i'}).\prod_{j=1}^{n}\left[1 - D_j^3(A_i, A_{i'})\right]; \quad 0 \le \varphi(A_i, A_{i'}) \le 1 \tag{22.10}$$

22.2.5 The Entering and Leaving Flows

The entering flow (φ^+) and the leaving flow (φ^-) are obtained using Eqs. (22.11) and (22.12) [178, 179].

$$\varphi^+(A_i) = \sum_{A_i \in A} \varphi(A_i, A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{22.11}$$

$$\varphi^-(A_i) = \sum_{A_i \in A} \varphi(A_{i'}, A_i); \quad i, i' \in \{1, \ldots, m\} \tag{22.12}$$

22.2.6 The Net Flow

Initially, the net flow of each alternative is computed through Eq. (22.13) for full ranking of alternatives [178, 179].

$$\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i); \quad i = 1, \ldots, m \tag{22.13}$$

22.2.7 The Final Ranking of Alternatives (PAMSSEM I Method)

In the PAMSSEM I method, the partial ranking is done using the entering and leaving flows. As alternative A_i is better than alternative $A_{i'}$ if:

$$A_i P A_{i'} \quad if \begin{cases} A_i P^+ A_{i'} & and \quad A_i P^- A_{i'} \\ A_i P^+ A_{i'} & and \quad A_i I^- A_{i'}; \quad i, i' \in \{1, \ldots, m\} \\ A_i I^+ A_{i'} & and \quad A_i P^- A_{i'} \end{cases} \tag{22.14}$$

And, the alternatives A_i and $A_{i'}$ are indifferent to each other if:

$$A_i I A_{i'} \; if \; A_i I^- A_{i'} \; and \; A_i I^+ A_{i'}; \quad i, i' \in \{1, \ldots, m\} \tag{22.15}$$

22.2.8 The Final Ranking of Alternatives (PAMSSEM II Method)

In the PAMSSEM II method, given the net flow, the alternative A_i is better than alternative $A_{i'}$ if:

$$A_i\,P^{II}A_{i'}\ if\ \varphi(A_i) > \varphi(A_{i'});\quad i, i' \in \{1, \ldots, m\} \tag{22.16}$$

In addition, the alternatives A_i and $A_{i'}$ are indifferent to each other if:

$$A_i\,I^{II}A_{i'}\ if\ \varphi(A_i) = \varphi(A_{i'});\quad i, i' \in \{1, \ldots, m\} \tag{22.17}$$

Accordingly, all alternatives are ranked.

22.3 Case Study

Organizational experts offered attributes of the price (C_1), storage capacity in gigabyte (C_2), and CPU speed (C_3) for purchasing a computer among three different models (A_1, A_2, and A_3), and the decision matrix of Fig. 22.1 was specified to the board for evaluating. Further, the weights of attributes are equal and Table 22.1 indicates the parameters q, p, and v of each attribute.

The purpose is to determine and compare the best alternative for buying a computer.

❖ **Solution**

(A) **The local outranking index**

Given that the values of all attributes are the ordinal, for example, $\bar{\delta}(A_1, A_2)$ are calculated for all attributes as follows:

$$\bar{\delta}_1(A_1, A_2) = 0$$
$$\bar{\delta}_2(A_1, A_2) = 1$$
$$\bar{\delta}_3(A_1, A_2) = 1$$

Fig. 22.1 Computer purchase decision matrix

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{ccc} - & + & + \\ C_1 & C_2 & C_3 \\ \end{array}$$
$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array}\begin{bmatrix} 80 & 90 & 5 \\ 65 & 58 & 2 \\ 83 & 60 & 7 \end{bmatrix}$$

Table 22.1 Parameters values

Parameter	C_1	C_2	C_3
q	5	15	1
p	12	25	2
v	18	32	3

Table 22.2 Values of the local outranking index

	A_1	A_2	A_3
A_1	–	2	2
A_2	1	–	2
A_3	2	2	–

The value of the local outranking index is determined as follows:

$$\delta(A_1, A_2) = (0 \times 1 \times 1) + (1 \times 1 \times 1) + (1 \times 1 \times 1) = 2$$

Table 22.2 illustrates the other values of local outranking index.

(B) **The concordance index**

According to the weight of attributes, for example, the concordance index value for the first and second alternatives is computed as follows:

$$\mathrm{C}(A_1, A_2) = \frac{1}{3}(0 + 1 + 1) = 0.667$$

Table 22.3 demonstrates the other values of the concordance index.

(C) **The local discordance index**

First, $\bar{\mathrm{D}}(A_i, A_{i'})$ are determined for each attribute. For example, the amounts of $\bar{\mathrm{D}}(A_1, A_2)$ for all attributes are calculated as follows:

$$\bar{\mathrm{D}}_1(A_1, A_2) = -\left(\tfrac{-15+12}{18-12}\right) = 0.500$$
$$\bar{\mathrm{D}}_2(A_1, A_2) = 0$$
$$\bar{\mathrm{D}}_3(A_1, A_2) = 0$$

Finally, the local discordance index is obtained as follows:

$$\mathrm{D}_1(A_1, A_2) = (0.500 \times 1 \times 1) + (0 \times 1 \times 1) + (0 \times 1 \times 1) = 0.500$$

The other local discordance index values are computed as indicated in Table 22.4.

(D) **The outranking degree**

For example, the value of $\varphi(A_1, A_2)$ is calculated as follows:

Table 22.3 Concordance index values

	A_1	A_2	A_3
A_1	–	0.667	0.667
A_2	0.333	–	0.667
A_3	0.667	0.667	–

Table 22.4 Local discordance index values

	A_1	A_2	A_3
A_1	–	0.500	0
A_2	2	–	1
A_3	0.714	1	–

Table 22.5 Outranking degree values

	A_1	A_2	A_3
A_1	–	0.584	0.667
A_2	0	–	0
A_3	0.424	0	–

$$\varphi(A_1, A_2) = 0.667 \times (1 - (0.500)^3) \times (1 - (0)^3) \times (1 - (0)^3) = 0.584$$

The other outranking degree values are determined as shown in Table 22.5.

(E) **The entering and leaving flows**

For example, the entering and leaving flows for alternative A_1 are determined as follows:

$$\varphi^+(A_1) = 0.584 + 0.667 = 1.251$$
$$\varphi^-(A_1) = 0 + 0.424 = 0.424$$

Table 22.6 demonstrates the other entering and leaving flows.

(F) **The net flow**

By calculating the entering and leaving flows, the net flow of all alternatives is computed as follows:

$$\varphi_1 = 1.251 - 0.424 = 0.827$$
$$\varphi_2 = 0 - 0.584 = -0.584$$
$$\varphi_3 = 0.424 - 0.667 = -0.243$$

(G) **The final ranking of alternatives (PAMSSEM I method)**

The preference rate of alternatives over each other is determined based on the entering and leaving flows as indicated in Table 22.7.

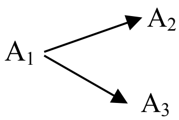
Table 22.6 Entering and leaving flows

	φ^+	φ^-
A_1	1.251	0.424
A_2	0	0.584
A_3	0.424	0.667

Table 22.7 Preference rate values

	A_1	A_2	A_3
A_1	I	P	P
A_2	–	I	–
A_3	–	–	I

The ranking of the alternatives is as follows:



(H) **The final ranking of alternatives (PAMSSEM II method)**

At this stage, the net flows of each alternative are initially arranged in descending order:

$$\varphi_1 > \varphi_3 > \varphi_2$$

Therefore, the first model of computer (A_1) is the best alternative for buying a computer, and the final ranking of alternatives is as follows:

$$A_1 > A_3 > A_2$$

22.4 Conclusion

The PAMSSEM method was presented in accordance with the PROMETHEE method, as well as the changes and differences occurred in the steps of the method (Figs. 5.4 and 22.2). On the other hand, the partial and full ranking of the various alternatives and the best alternative selection are done using the PAMSSEM I & II methods based on the inputs of the model including the parameters of functions, the weight of attributes, and the decision matrix, determined by the experts. Furthermore, it is not necessary to convert the qualitative attributes into the quantitative attributes and attributes are dependent on each other.

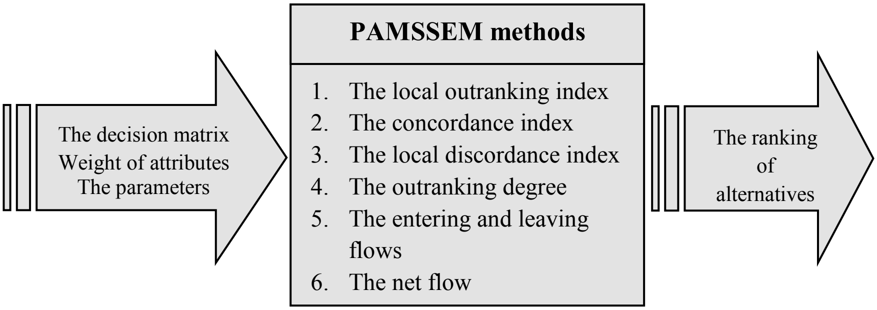


Fig. 22.2 A summary of the PAMSSEM I & II methods

Chapter 23
ELECTRE I–II–III Methods

23.1 Introduction

ELimination Et Choix Traduisant la REalite (ELECTRE) method was first introduced by Roy in 1990 [34, 180–182], which evaluates all alternatives using outranking comparisons, and ineffective and eliminates low-attractive alternatives. Therefore, the final ranking of alternatives is more likely problematic and the ELECTRE II and ELECTRE III methods are presented to solve this problem. Considering the development of the method, the scope of its applications has also increased. The ELECTRE methods are applied in some areas such as network selection [183], evaluation of solid waste management system [184, 185], power distribution system planning [186], and automated inspection device selection [187]. This technique has the following features:

- This method belongs to the compensatory methods;
- There is no need for independence of attributes;
- The qualitative attributes are converted into the quantitative attributes.

Further, the decision matrix is used in the ELECTRE method as Eq. (23.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} \quad ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \qquad (23.1)$$

where r_{ij} is the element of the decision matrix for ith alternative in jth attribute.

Furthermore, the decision maker provides the weight of attributes $[w_1, w_2, \ldots, w_n]$. The parameter θ in the ELECTRE II method and the parameters α, β, p, v and q in the ELECTRE III method are specified by the decision maker.

A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, International Series in Operations Research & Management Science 277, https://doi.org/10.1007/978-3-030-15009-9_23

23.2 Description of ELECTRE Methods

23.2.1 The Normalized Decision Matrix

Eq. (23.2) is used to normalize the decision matrix.

$$r_{ij}^{*} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^{2}}}; \quad j = 1, \ldots, n \tag{23.2}$$

Obviously, r_{ij}^{*} indicates the normalized amount of the decision matrix of ith alternative in jth attribute.

23.2.2 The Weighted Normalized Decision Matrix

Eq. (23.3) is used to determine the values of the weighted normalized decision matrix [188, 189].

$$\hat{r}_{ij} = r_{ij}^{*}.w_{j}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{23.3}$$

where w_j is the weight of the attributes $[w_1, w_2, \ldots, w_n]$.

23.2.3 The Dominant Matrix

At this stage, the dominant set is first obtained as Eq. (23.4) [188, 189].

$$C_{i,k} = \left\{ j | \hat{r}_{ij} \geq \hat{r}_{kj} \right\}; \quad i, k \in \{1, \ldots, m\} \quad j = 1, \ldots, n \tag{23.4}$$

In addition, Eq. (23.5) is indicated the dominant matrix [180–182].

$$G_{m \times m} = (g_{ik})_{m \times m}; \quad g_{ik} = \sum_{j \in C_{i,k}} w_j, \quad 0 \leq g_{ik} \leq 1 \tag{23.5}$$

23.2.4 The Dominated Matrix

First, the dominated set is calculated using Eq. (23.6) [188, 189].

$$d_{i,k} = \left\{ j | \hat{r}_{ij} < \hat{r}_{kj} \right\} = j - C_{i,k}; \quad i, k \in \{1, \ldots, m\}, \quad j = 1, \ldots, n \tag{23.6}$$

Further, the dominated matrix is determined by Eq. (23.7) [188, 189].

$$D_{m\times m} = \left(d'_{ik}\right)_{m\times m}; \quad d'_{ik} = \frac{\max\limits_{j\in d_{i,k}} \left|\hat{r}_{ij} - \hat{r}_{kj}\right|}{\max\limits_{j\in J} \left|\hat{r}_{ij} - \hat{r}_{kj}\right|}, \quad 0 \le d'_{ik} \le 1 \tag{23.7}$$

23.2.5 The Concordance Matrix

The concordance matrix is formed as Eq. (23.8) [188, 189].

$$F_{m\times m} = (f_{ik}); \quad f_{ik} \begin{cases} 0 & if \quad g_{ik} < \bar{g} \\ 1 & if \quad g_{ik} \ge \bar{g} \end{cases}, \quad i, k \in \{1, \ldots, m\} \tag{23.8}$$

$\bar{g}$ represents the average of dominant matrix elements, computed as Eq. (23.9) [180–182, 188,189].

$$\bar{g} = \sum_{k=1}^{m} \sum_{i=1}^{m} \frac{g_{ik}}{m(m-1)} \tag{23.9}$$

23.2.6 The Discordance Matrix

The discordance matrix is computed by Eq. (23.10) [188, 189].

$$E_{m\times m} = (e_{ik}); \quad e_{ik} \begin{cases} 1 & if \quad D_{ik} \le \bar{D} \\ 0 & if \quad D_{ik} > \bar{D} \end{cases}, \quad i, k \in \{1, \ldots, m\} \tag{23.10}$$

$\bar{D}$ indicates the average of dominated matrix elements, calculated as Eq. (23.11) [188, 189].

$$\bar{D} = \sum_{k=1}^{m} \sum_{i=1}^{m} \frac{D_{ik}}{m(m-1)} \tag{23.11}$$

23.2.7 The Aggregate Dominant Matrix

The aggregate dominant matrix is formed using Eq. (23.12) [180–182, 188, 189].

$$P_{m\times m} = (p_{ik}); \quad p_{ik} = f_{ik}.e_{ik}, \quad i,k \in \{1, \ldots, m\} \tag{23.12}$$

23.2.8 The Final Ranking of Alternatives (ELECTRE I Method)

The low-attractive alternatives are eliminated in the final ranking of alternatives using the integration dominance matrix and then ranked.

23.2.9 The Final Ranking of Alternatives (ELECTRE II Method)

First, with respect to the weighed normalized values obtained from Eq. (23.3), the dominant set is determined by Eqs. (23.13) and (23.14) [180–182, 188, 189].

$$\begin{cases} C_{i,k}^{-} = \{j|\hat{r}_{ij} < \hat{r}_{kj}\} \\ C_{i,k}^{=} = \{j|\hat{r}_{ij} = \hat{r}_{kj}\} \\ C_{i,k}^{+} = \{j|\hat{r}_{ij} > \hat{r}_{kj}\} \end{cases}; \quad i,k \in \{1, \ldots, m\}, \quad j = 1, \ldots, n \tag{23.13}$$

$$\begin{cases} g_{ik}^{-} = \sum\limits_{j\in C_{i,k}^{-}} w_j \\ g_{ik}^{=} = \sum\limits_{j\in C_{i,k}^{=}} w_j \\ g_{ik}^{+} = \sum\limits_{j\in C_{i,k}^{+}} w_j \end{cases}; \quad i,k \in \{1, \ldots, m\}, \quad j = 1, \ldots, n, \quad 0 \le g_{ik} \le 1 \tag{23.14}$$

Then, the dominant matrix is formed using Eq. (23.15) [188, 189].

$$G_{m\times m} = (g_{ik})_{m\times m}; \quad g_{ik} = \frac{g_{ij}^{+} + g_{ij}^{=}}{g_{ij}^{+} + g_{i}^{-} + g_{ij}^{=}} \quad , 0 \le g_{ik} \le 1 \tag{23.15}$$

Additionally, the dominated matrix is determined by Eq. (23.16) [188, 189].

$$D_{m\times m} = \left(d'_{ik}\right)_{m\times m}; \quad d'_{ik} = \frac{\max\limits_{j\in C^-_{i,k}}\left|\hat{r}_{ij} - \hat{r}_{kj}\right|}{\max\limits_{j\in J}\left|\hat{r}_{ij}, \theta_i\right|}, \quad 0 \le d'_{ik} \le 1 \tag{23.16}$$

where the parameter θ is specified by the decision maker. The concordance matrix, discordance matrix, and aggregate dominant matrix are determined using Eqs. (23.8), (23.10), and (23.12), respectively, and the final ranking is made by eliminating low-attractive alternatives.

23.2.10 The Final Ranking of Alternatives (ELECTRE III Method)

First, the values of negative attributes are reversed to make the attribute positive and then, the normalized values are obtained using Eq. (23.2) and the weighted normalized values by Eq. (23.3). According to the weighted normalized values, the dominant set is determined from Eq. (23.17) [180–182, 188, 189].

$$C_j(i,k) = \begin{cases} 1 & \textit{if} \quad \hat{r}_{kj} \le q_j + \hat{r}_{ij} \\ 0 & \textit{if} \quad \hat{r}_{kj} \ge p_j + \hat{r}_{ij} \\ \frac{\hat{r}_{ij} + P_j - \hat{r}_{kj}}{P_j - q_j}, & \text{otherwise} \end{cases} \tag{23.17}$$

where the parameters p and q are presented by the decision maker. The dominant matrix is obtained by Eq. (23.18) [182–182, 188, 189].

$$G_{m\times m} = (g_{ik})_{m\times m}; \quad g_{ik} = \frac{\sum\limits_{j=1}^{n} C_j(i,k).w_j}{\sum\limits_{j=1}^{n} w_j}, \quad 0 \le g_{ik} \le 1 \tag{23.18}$$

Furthermore, the dominated set is calculated using Eq. (23.19) [180–182, 188, 189].

$$d_j(i,k) = \begin{cases} 0 & \textit{if} \quad \hat{r}_{kj} \le p_j + \hat{r}_{ij} \\ 1 & \textit{if} \quad \hat{r}_{kj} \le v_j + \hat{r}_{ij} \\ \frac{\hat{r}_{kj} - \hat{r}_{ij} - P_j}{V_j - P_j}, & \text{otherwise} \end{cases} \tag{23.19}$$

where the parameters p and v are specified by the decision maker. The dominated matrix is as Eq. (23.20) [180–182].

$$D_{m\times m} = \left(d'_{ik}\right)_{m\times m}; \quad d'_{ik} = \frac{\sum_{j=1}^{n} d_j(i,k).w_j}{\sum_{j=1}^{n} w_j}, \quad 0 \le d'_{ik} \le 1 \tag{23.20}$$

Then, the credibility matrix is formed as Eq. (23.21) [188, 189].

$$S_{m\times m} = (s_{ik}); \quad s_{ik}\begin{cases} g_{ik} & if \quad d'_{ik} \le g_{ik} \\ g_{ik}\cdot \prod_{j\in J_{ik}} \frac{1-d'_{ik}}{1-g_{ik}}, & \text{otherwise} \end{cases} \tag{23.21}$$

where J_{ik} represents those attributes which are $d'_{ik} > g_{ik}$. The aggregate dominance matrix is determined as Eq. (23.22) [180–182].

$$P_{m\times m} = (p_{ik}); \quad p_{ik}\begin{cases} 1 & if \quad s_{i,k} \ge \lambda - S(\lambda), \\ 0 & \text{otherwise} \end{cases} \tag{23.22}$$

where the values of λ and S (λ) are specified by Eq. (23.23) [180–182, 188, 189].

$$\begin{cases} \lambda = max\ s_{ik} \\ S(\lambda) = \alpha + \beta\lambda \end{cases}; \quad i,k \in 1,\ldots,m \tag{23.23}$$

The final ranking is done with respect to the aggregate dominance matrix.

23.3 Case Study

The company is active in the field of residential building and plans to choose a building project among the alternatives (A_1, A_2, and A_3). Experts determined the attributes such as the cost (C_1), strength (C_2), beauty of building (C_3), infrastructure area (C_4), and construction time (C_5) and presented the decision matrix as Fig. 23.1. In addition, the weight of attributes is indicated in Table 23.1.

Further, the parameters α and β equal 0.010 and 0.030, respectively, and the parameter θ for all alternatives equals 0.500, and the parameters p, v, and q for each attribute are determined as Table 23.2.

The purpose is to choose the best building project.

Fig. 23.1 Decision matrix of choosing building project

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccccc} - & + & + & + & - \\ C_1 & C_2 & C_3 & C_4 & C_5 \\ \end{array}$$

	C_1 (−)	C_2 (+)	C_3 (+)	C_4 (+)	C_5 (−)
A_1	3	5	9	24000	7
A_2	1.200	7	5	25000	3
A_3	1.500	9	3	32000	1

Table 23.1 Weight of attributes

Attribute	C_1	C_2	C_3	C_4	C_5
w_j	0.179	0.062	0.211	0.017	0.531

Table 23.2 Parameters' values

Parameter	C_1	C_2	C_3	C_4	C_5
v	0.090	0.020	0.110	0.001	0.350
p	0.080	0.015	0.100	0.001	0.330
q	0.020	0.005	0.050	0	0.150

❖ Solution

(A) The normalized decision matrix

The normalized values of decision matrix are as Table 23.3.

(B) The weighted normalized decision matrix

The values of the weighted normalized decision matrix are obtained as Table 23.4.

(C) The dominant matrix

The dominant sets are determined as follows:

$$C_{1,2} = \{3, 4\}$$
$$C_{1,3} = \{3\}$$
$$C_{2,1} = \{1, 2, 4, 5\}$$
$$C_{2,3} = \{1, 3\}$$
$$C_{3,1} = \{1, 2, 4, 5\}$$
$$C_{3,2} = \{2, 4, 5\}$$

Table 23.3 Normalized values of the decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.842	0.402	0.839	0.509	0.911
A_2	0.337	0.562	0.466	0.530	0.390
A_3	0.421	0.723	0.280	0.678	0.130

Table 23.4 Values of the weighted normalized decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.151	0.025	0.177	0.009	0.484
A_2	0.060	0.035	0.098	0.009	0.207
A_3	0.075	0.045	0.059	0.011	0.069

Fig. 23.2 Dominant matrix

	A_1	A_2	A_3
A_1	-	0.228	0.211
A_2	0.789	-	0.390
A_3	0.789	0.610	-

For example, given the weight of attributes and the value of $C_{1,2} = \{3, 4\}$, the value of g_{12} is computed as follows:

$$g_{12} = w_3 + w_4 = 0.211 + 0.017 = 0.228$$

In addition, the dominant matrix is as Fig. 23.2.

(D) **The dominated matrix**

At this stage, the dominated sets are as follows:

$$\begin{aligned} d_{1,2} &= \{1, 2, 5\} \\ d_{1,3} &= \{1, 2, 4, 5\} \\ d_{2,1} &= \{3\} \\ d_{2,3} &= \{2, 4, 5\} \\ d_{3,1} &= \{3\} \\ d_{3,2} &= \{1, 3\} \end{aligned}$$

For example, considering the weight of attributes and the value of $d_{1,2} = \{1, 2, 5\}$, the amount of d'_{12} of the dominated matrix is calculated as follows:

$$d'_{12} = \frac{max(0.091,\ 0.010,\ 0.277)}{max(0.091,\ 0.010,\ 0.079,\ 0.277)} = \frac{0.277}{0.277} = 1$$

Fig. 23.3 illustrates the dominated matrix.

(E) **The concordance matrix**

$\bar{g}$ indicates the average of dominant matrix elements, calculated as follows:

$$\bar{g} = \frac{0.228 + 0.211 + 0.390 + 0.789 + 0.789 + 0.610}{3 \times 2} = 0.503$$

Then, the concordance matrix is formed as Fig. 23.4.

Fig. 23.3 Dominated matrix

	A_1	A_2	A_3
A_1	-	1	1
A_2	0.285	-	1
A_3	0.284	0.283	-

Fig. 23.4 Concordance matrix

	A1	A2	A3
A1	-	0	0
A2	1	-	0
A3	1	1	-

(F) **The discordance matrix**

$\bar{D}$ demonstrates the average of dominated matrix elements, obtained as follows:

$$\bar{D} = \frac{1+1+1+0.285+0.284+0.283}{3 \times 2} = 0.642$$

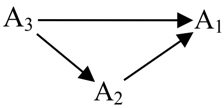
Then, the discordance matrix is as Fig. 23.5.

(G) **The aggregate dominant matrix**

The aggregate dominant matrix is specified as Fig. 23.6.

(H) **The final ranking of alternatives (ELECTRE I method)**

First, with respect to the aggregate dominant matrix, the low-attractive alternatives are eliminated as follows:



The final ranking of alternatives is as follows:

$$A_3 > A_2 > A_1$$

Fig. 23.5 Discordance matrix

	A1	A2	A3
A1	-	0	0
A2	1	-	0
A3	1	1	-

Fig. 23.6 Aggregate dominant matrix

	A1	A2	A3
A1	-	0	0
A2	1	-	0
A3	1	1	-

(I) The final ranking of alternatives (ELECTRE II method)

Initially, the dominant set is obtained as follows:

$$\begin{array}{lll}
C^{+}_{1,2} = \{3\}, & C^{-}_{1,2} = \{1,2,5\}, & C^{=}_{1,2} = \{4\} \\
C^{+}_{1,3} = \{3\}, & C^{-}_{1,3} = \{1,2,4,5\}, & C^{=}_{1,3} = \emptyset \\
C^{+}_{2,1} = \{1,2,5\}, & C^{-}_{2,1} = \{3\}, & C^{=}_{2,1} = \{4\} \\
C^{+}_{2,3} = \{1,3\}, & C^{-}_{2,3} = \{2,4,5\}, & C^{=}_{2,3} = \emptyset \\
C^{+}_{3,1} = \{1,2,4,5\}, & C^{-}_{3,1} = \{3\}, & C^{=}_{3,1} = \emptyset \\
C^{+}_{3,2} = \{2,4,5\}, & C^{-}_{3,2} = \{1,3\}, & C^{=}_{3,2} = \emptyset
\end{array}$$

For example, g_{12} is determined as follows:

$$g_{12} = \frac{0.211 + 0.017}{0.211 + 0.179 + 0.062 + 0.531 + 0.017} = 0.228$$

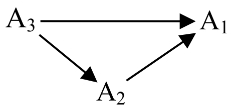
Fig. 23.7 demonstrates the dominant matrix. For example, according to the weight of attributes and the value of $C^{-}_{1,2} = \{1,2,5\}$, the amount of d'_{12} of the dominant matrix is as follows:

$$d'_{12} = \frac{max(0.091,\ 0.010,\ 0.277)}{max(0.151,\ 0.025,\ 0.177,\ 0.009,\ 0.484,\ 0.500)} = \frac{0.277}{0.500} = 0.554$$

Fig. 23.8 indicates the dominated matrix. Given the mean value of dominant matrix rows, which equals 0.503, the concordance matrix is formed as Fig. 23.9.

As the mean value of dominated matrix rows equals 0.355, the discordance matrix is as Fig. 23.10.

Fig. 23.11 illustrates the aggregate dominant matrix. Initially, the low-attractive alternatives are eliminated as follows:



The final ranking of alternatives is as follows:

$$A_3 > A_2 > A_1$$

Fig. 23.7 Dominant matrix

	A_1	A_2	A_3
A_1	-	0.228	0.211
A_2	0.789	-	0.390
A_3	0.789	0.610	-

Fig. 23.8 Dominated matrix

	A_1	A_2	A_3
A_1	-	0.554	0.830
A_2	0.158	-	0.276
A_3	0.236	0.078	-

Fig. 23.9 Concordance matrix

	A_1	A_2	A_3
A_1	-	0	0
A_2	1	-	0
A_3	1	1	-

Fig. 23.10 Concordance matrix

	A_1	A_2	A_3
A_1	-	0	0
A_2	1	-	1
A_3	1	1	-

Fig. 23.11 Aggregate dominant matrix

	A_1	A_2	A_3
A_1	-	0	0
A_2	1	-	0
A_3	1	1	-

(J) **The final ranking of alternatives (ELECTRE III method)**

At first, by converting the negative attributes into the positive attributes, the values of the weighted normalized attributes are as Table 23.5.

Then, the dominant set is determined and, for instance, for the value of $C(1, 2)$, all attributes are as follows:

Table 23.5 Weighted normalized values

	C_1	C_2	C_3	C_4	C_5
A_1	0.053	0.025	0.177	0.009	0.071
A_2	0.133	0.035	0.098	0.009	0.166
A_3	0.107	0.045	0.059	0.011	0.499

$$C_1(1,2) = 0$$
$$C_2(1,2) = \frac{0.025 + 0.015 - 0.035}{0.015 - 0.005} = 0.500$$
$$C_3(1,2) = 1$$
$$C_4(1,2) = 1$$
$$C_5(1,2) = 1$$

For example, according to the weight of attributes and the value of $C(1,2)$ for each attribute, g_{12} is obtained as follows:

$$g_{12} = \frac{0 + (0.500 \times 0.062) + (1 \times 0.211) + (1 \times 0.017) + (1 \times 0.531)}{0.179 + 0.062 + 0.211 + 0.017 + 0.531} = 0.790$$

Fig. 23.12 indicates the dominant matrix. Then, the dominated set is calculated and, e.g., for the value $d(1,3)$, all attributes are as follows:

$$d_1(1,3) = 0$$
$$d_2(1,3) = 1$$
$$d_3(1,3) = 0$$
$$d_4(1,3) = 1$$
$$d_5(1,3) = 1$$

For instance, according to the weight of attributes and the $d(1,3)$ value of each attribute, d'_{13} is as follows:

$$d'_{13} = \frac{0 + (1 \times 0.062) + 0 + (1 \times 0.017) + (1 \times 0.531)}{0.179 + 0.062 + 0.211 + 0.017 + 0.531} = 0.610$$

Fig. 23.13 displays the dominated matrix. Additionally, the credibility matrix is formed as Fig. 23.14. S (λ) and λ are obtained as follows:

$$\begin{cases} \lambda = 1 \\ S(\lambda) = 0.010 + 0.030(1) = 0.040 \end{cases}$$

Fig. 23.12 Dominant matrix

	A_1	A_2	A_3
A_1	-	0.790	0.289
A_2	1	-	0.421
A_3	0.789	0.982	-

Fig. 23.13 Dominated matrix

	A_1	A_2	A_3
A_1	-	0	0.610
A_2	0	-	0.097
A_3	0.211	0	-

Fig. 23.14 Credibility matrix

	A_1	A_2	A_3
A_1	-	0.790	0.159
A_2	1	-	0.421
A_3	0.789	0.982	-

Fig. 23.15 Aggregate dominant matrix

	A_1	A_2	A_3
A_1	-	0	0
A_2	1	-	0
A_3	0	1	-

Fig. 23.15 indicates the aggregate dominant matrix. The low-attractive alternatives are initially eliminated as follows:

$$\mathrm{A}_3 \rightarrow \mathrm{A}_2 \rightarrow \mathrm{A}_1$$

Then, the third building project (A_3) is the best alternative and the final ranking of the alternatives is as follows:

$$A_3 > A_2 > A_1$$

23.4 Conclusion

The ELECTRE method is one of the most widely used methods in MADM, which has been used by a lot of researchers and experts due to its development. Further, this method is available in the form of software, and it is easy to calculate the seven steps (Fig. 23.16) and determine the final result. The qualitative attributes should be converted into the quantitative attributes in this technique, belonging to the compensatory and outranking methods. Accordingly, the ELECTRE method ranks the alternatives and selects the best alternative by eliminating low-attractive alternatives. Therefore, the purpose of developing this method and presenting ELECTRE II and ELECTRE III methods is to increase the accuracy and expands its application scope. Now, it is also used in combination and in uncertainty condition.

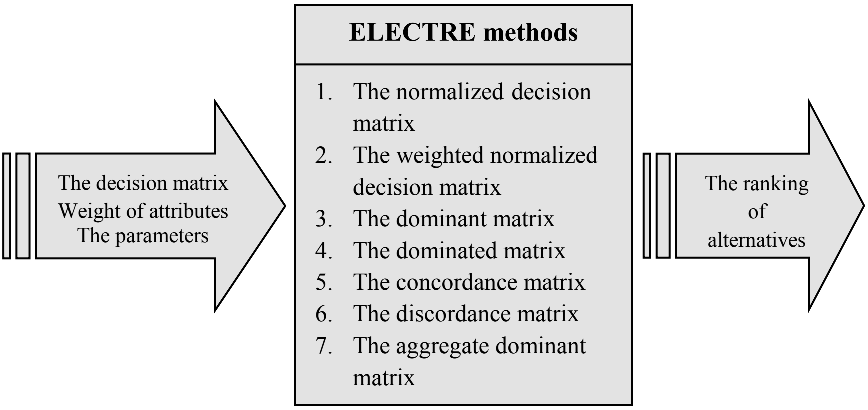


Fig. 23.16 A summary of the ELECTRE I-II-III methods

Chapter 24
EXPROM I & II Method

24.1 Introduction

The EXtension of the PROMethee (EXPROM) methods were first introduced by Diakoulaki and Koumoutsos in 1991 [190–192] and seek to find a solution for evaluating alternatives and rank the alternatives more accurately using widely available information. In the EXPROM I method, only the entering and leaving flows are examined and a partial ranking is done. However, in the EXPROM II method, the net flow is determined as the final value, and the full ranking of the alternatives is performed with respect to the following features:

- This method belongs to the compensatory methods;
- The qualitative attributes are converted into the quantitative attributes;
- There is no need for independence of attributes.

This method is used in some areas such as the country market selection [193], sustainable water resources planning [194], and material selection [195, 196]. The matrix of alternatives and attributes of Eq. (24.1) is formed based on the information received from the decision maker and consists of the input information of the method.

$$\mathrm{F} = \begin{bmatrix} f_1(A_1) & \cdots & f_j(A_1) & \cdots & f_n(A_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_i) & \cdots & f_j(A_i) & \cdots & f_n(A_i) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_1(A_m) & \cdots & f_j(A_m) & \cdots & f_n(A_m) \end{bmatrix}_{m\times n} \quad ; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{24.1}$$

where $\mathrm{B} = \{A_1, A_2, \ldots, A_m\}$ is a finite set of the alternatives and $\mathrm{C} = \{f_1(0), f_2(0), \ldots, f_n(0)\}$ is the set of the evaluation attributes of the set B.

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Furthermore, the decision maker provides the weight of the attributes $[w_1, w_2, \ldots, w_n]$. In addition, all the parameters of the weak preference function such as l in the quasi-criterion, m in the V-shaped criterion, s and r in the linear criterion, q and p in the level criterion, and σ in the Gaussian criterion are specified by the decision maker.

24.2 Description of EXPROM Methods

24.2.1 The Weak Preference Function

In order to determine the amount of weak preference function, the difference between the pair of alternatives is first obtained from Eq. (24.2) [197].

$$d_j(A_i, A_{i'}) = f_j(A_i) - f_j(A_{i'}); \quad i, i' \in \{1, \ldots, m\}, \quad j = 1, \ldots, n \tag{24.2}$$

Therefore, the value of the weak preference function is calculated based on the function (24.3) [197].

$$\mathrm{P}_j(A_i, A_{i'}) = f_j[d_j(A_i, A_{i'})], \quad 0 \le \mathrm{P}_j(A_i, A_{i'}) \le 1 \tag{24.3}$$

The type of function should first be specified to determine the values of the weak preference function. Therefore, the type of each function is determined according to the type of attributes evaluated through Tables 24.1, 24.2, 24.3, 24.4, 24.5, and 24.6.

24.2.2 The Weak Preference Index

With respect to the weight of attributes, the weak preference index is computed as shown in Eq. (24.4) [197].

Table 24.1 Usual criterion (Type I) [32]

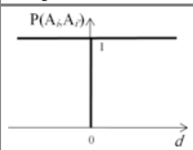
Description	Graph	Function	Condition	Parameter
Absence of difference in interval (d $\le$ 0), the existence of a complete priority of an alternative in interval (d > 0)		$f(d) = \begin{cases} 0 & d \le 0 \\ 1 & d > 0 \end{cases}$	Impacts and the issues related to the ecology	–

Table 24.2 Quasi-criterion (Type II) [32]

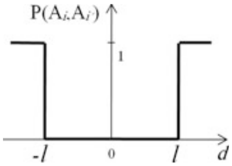
Description	Graph	Function	Condition	Parameter
Lack of differences in the interval (d $\le$ *l*) the existence of a complete priority of an alternative in interval (d > *l*)		$f(d)=\begin{cases}0 & d\le l\\ 1 & d>l\end{cases}$	Attributes related to the discrete sources	*l*

Table 24.3 V-shape criterion (Type III) [32]

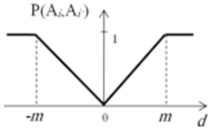
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d $\le$ *m*), the existence of a complete priority of an alternative in interval (d > m)		$f(d)=\begin{cases}\frac{d}{m} & d\le m\\ 1 & d>m\end{cases}$	Operational attributes, purchase costs	*m*

Table 24.4 Level criterion (Type IV) [32]

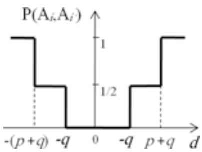
Description	Graph	Function	Condition	Parameter
Lack of difference in interval (d $\le$ q), change in priority value of alternative linearly in the interval ($q < d \le q + p$), the existence of a complete priority of an alternative in the interval (d > $q + p$)		$f(d)=\begin{cases}0 & d\le \mathrm{q}\\ \frac{1}{2} & \mathrm{q}<d\le \mathrm{q}+\mathrm{p}\\ 1 & d>q+p\end{cases}$	Long-term benefit, maintenance cost, lifetime cost	*q, p*

Table 24.5 Linear criterion (Type V) [32]

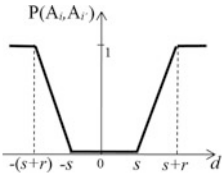
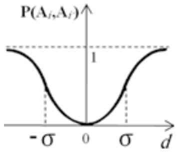
Description	Graph	Function	Condition	Parameter
Absence of difference in the interval (d ≤ s), change in the priority value of alternative linearly in the interval (s < d ≤ s + r), the existence of the full priority of an alternative in interval (d > s + r)		$f(d) = \begin{cases} 0 & d \leq s \\ \frac{d-s}{r} & s < d \leq s+r \\ 1 & d > s+r \end{cases}$	Exploration cost, short-term profit, constructing cost	*s, r*

Table 24.6 Gaussian criterion (Type VI) [32]

Description	Graph	Function	Condition	Parameter
lack of difference in the interval (d ≤ 0), an increase in the priority rate of alternative in the interval (d > 0)		$f(d) = \begin{cases} 0 & d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}} & d > 0 \end{cases}$	Appearance, quality, and safety	σ

$$WP(A_i, A_{i'}) = \sum_{j=1}^{n} P_j(A_i, A_{i'}).w_j / \sum_{j=1}^{n} w_j; \quad i, i' \in \{1, \ldots, m\} \tag{24.4}$$

24.2.3 The Strict Preference Function

At this stage, the ideal and anti-ideal solutions of each attribute are obtained by Eqs. (24.5) and (24.6), respectively [197].

$$f_j(\dot{x}) = max\{f_j(\mathrm{x}_1), f_j(\mathrm{x}_2), \ldots, f_j(\mathrm{x}_n)\}; \quad j = 1, \ldots, n \tag{24.5}$$

$$f_j(\ddot{x}) = min\{f_j(\mathrm{x}_1), f_j(\mathrm{x}_2), \ldots, f_j(\mathrm{x}_n)\}; \quad j = 1, \ldots, n \tag{24.6}$$

Then, the maximum spreading of the strict preference function is calculated using Eq. (24.7) [197].

$$dm_j = f_j(\dot{x}) - f_j(\ddot{x}); \quad j = 1, \ldots, n \tag{24.7}$$

Eq. (24.8) displays the strict preference function [197].

$$\mathrm{P}'_j(A_i, A_{i'}) = max\{0, (d_j(A_i, A_{i'}) - L_j)/(dm_j - L_j)\} \tag{24.8}$$

In Eq. (24.8), the range of the strict preference function (L) for ordinary functions is equal to zero and equal to an infinite number for the Gaussian criterion.

24.2.4 The Strict Preference Index

The strict preference index is calculated in Eq. (24.9) [197].

$$SP(A_i, A_{i'}) = \sum_{j=1}^{n} \mathrm{P}'_j(A_i, A_{i'}).w_j / \sum_{j=1}^{n} w_j; \quad i, i' \in \{1, \ldots, m\} \tag{24.9}$$

24.2.5 The Entering and Leaving Flows

First, the values of the total preference index are determined by Eq. (24.10) [197].

$$TP(A_i, A_{i'}) = min\{1, WP(A_i, A_{i'}) + SP(A_i, A_{i'})\}; \quad i, i' \in \{1, \ldots, m\} \tag{24.10}$$

Then, the entering and leaving flows are obtained as shown in Eqs. (24.11) and (24.12) [197].

$$\varphi^+(A_i) = \frac{1}{m-1} \sum_{A_i \epsilon A} TP(A_i, A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.11}$$

$$\varphi^-(A_i) = \frac{1}{m-1} \sum_{A_i \epsilon A} TP(A_{i'}, A_i); \quad i, i' \in \{1, \ldots, m\} \tag{24.12}$$

24.2.6 The Net Flow

In this method, the full ranking, including $\left(\mathrm{P}^{\mathrm{II}}, \mathrm{I}^{\mathrm{II}}\right)$, is done. The net flow values are calculated by Eq. (24.13) and then, the alternatives are rank [197].

$$\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i); \quad i = 1, \ldots, m \tag{24.13}$$

24.2.7 The Final Ranking of Alternatives (EXPROM I Method)

Initially, Eqs. (24.14) to (24.17) are considered [197].

$$A_i P^+ A_{i'} \text{ } if \quad \varphi^+(A_i) > \varphi^+(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.14}$$

$$A_i I^+ A_{i'} \text{ } if \quad \varphi^+(A_i) = \varphi^+(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.15}$$

$$A_i P^- A_{i'} \quad if \quad \varphi^-(A_i) < \varphi^-(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.16}$$

$$A_i I^- A_{i'} \text{ } if \quad \varphi^-(A_i) = \varphi^-(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.17}$$

On the other hand, the alternative A_i is better than the alternative $A_{i'}$, if:

$$A_i P A_{i'} \quad if \quad \begin{cases} A_i P^+ A_{i'} & and \quad A_i P^- A_{i'} \\ A_i P^+ A_{i'} & and \quad A_i I^- A_{i'} \\ A_i I^+ A_{i'} & and \quad A_i P^- A_{i'} \end{cases}; \quad i, i' \in \{1, \ldots, m\} \tag{24.18}$$

And, the alternatives A_i and $A_{i'}$ are indifferent to each other, if:

$$A_i I A_{i'} \text{ } if \quad A_i I^- A_{i'} \quad and \quad A_{i'} I^- A_i; \quad i, i' \in \{1, \ldots, m\} \tag{24.19}$$

Accordingly, all alternatives are ranked.

24.2.8 The Final Ranking of Alternatives (EXPROM II Method)

In this method, the alternative A_i is better than the alternative $A_{i'}$, if:

$$A_i P^{II} A_{i'} \text{ } if \quad \varphi(A_i) > \varphi(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.20}$$

And, the alternatives A_i and $A_{i'}$ are indifferent to each other, if:

$$A_i I^{II} A_{i'} \quad if \quad \varphi(A_i) = \varphi(A_{i'}); \quad i, i' \in \{1, \ldots, m\} \tag{24.21}$$

Consequently, all alternatives are ranked.

24.3 Case Study

A production system should be selected among the six production systems proposed by the experts for a new pharmacy company. In the same vein, experts specified the decision-making attributes such as the volume of workforce (C_1), overall cost reduction (C_2), equipment cost (C_3), annual cost of maintenance (C_4), environmental destructive impact (C_5), and security level (C_6). After successive sessions, the qualitative attributes were converted into the quantitative attributes, and the matrix of Fig. 24.1 was determined.

In addition, the weights of attributes are equal. Table 24.7 indicates the type and values of the preference function parameters. The purpose is to determine the best production system using the EXPROM I & II methods.

❖ Solution

(A) The weak preference function

At first, the difference between the values of the pair of alternatives is computed. For example, the pair of the alternatives A_2 and A_3 is calculated in the attribute six:

$$d_6(A_2, A_3) = 1 - 7 = -6$$

$$d_6(A_3, A_2) = 7 - 1 = 6$$

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{cccccc} - & + & - & - & - & + \\ C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \end{array}$$

$$\begin{array}{c|cccccc} & - & + & - & - & - & + \\ & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \hline A_1 & 80 & 90 & 600 & 54 & 8 & 5 \\ A_2 & 65 & 58 & 200 & 97 & 1 & 1 \\ A_3 & 83 & 60 & 400 & 72 & 4 & 7 \\ A_4 & 40 & 80 & 1000 & 75 & 7 & 10 \\ A_5 & 52 & 72 & 600 & 20 & 3 & 8 \\ A_6 & 94 & 96 & 700 & 36 & 5 & 6 \end{array}$$

Fig. 24.1 Decision matrix of the production system selection

Table 24.7 Parameters of the preference function

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
Type	II	III	V	IV	I	VI
Value	$l = 10$	m = 30	$s = 50$ r = 450	$q = 10$ $p = 50$	–	$\sigma = 5$

Thus, the values of the preference function are calculated according to its type and parameters as follows:

$$P_6(A_2, A_3) = 0$$

$$P_6(A_3, A_2) = 1 - e^{-\left(\frac{(6)^2}{2(5)^2}\right)} = 0.513$$

Finally, other values of preference function are shown in Table 24.8.

(B) **The weak preference index**

Regarding the amounts obtained for the attributes, for example, the value of the weak preference index for the alternatives A_2 and A_3 is as follows:

$$\pi(A_2, A_3) = \frac{1}{6}(1 + 0 + 0.333 + 0 + 1 + 0) = 0.389$$

$$\pi(A_3, A_2) = \frac{1}{6}(0 + 0.067 + 0 + 0.500 + 0 + 0.513) = 0.180$$

The other values are obtained in Table 24.9.

(C) **The strict preference function**

The ideal and anti-ideal solutions, as well as the spreading range of strict preference function are shown in Table 24.10.

Therefore, the values of the strict preference function are shown in Table 24.11.

Table 24.8 Values of the weak preference function

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	1.776	1.500	1.608	0.600	1.110
A_2	2.772	–	2.388	1.998	1.776	3.000
A_3	1.416	1.080	–	1.998	0.336	2.574
A_4	2.394	3.030	1.830	–	1.338	1.272
A_5	2.640	3.090	2.922	2.380	–	2.688
A_6	1.716	2.394	1.500	2.592	0.798	–

Table 24.9 Values of the weak preference index

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	0.296	0.250	0.268	0.100	0.185
A_2	0.462	–	0.389	0.333	0.296	0.500
A_3	0.236	0.180	–	0.333	0.056	0.429
A_4	0.399	0.505	0.305	–	0.223	0.212
A_5	0.444	0.515	0.487	0.380	–	0.448
A_6	0.286	0.399	0.250	0.432	0.133	–

Table 24.10 Ideal and anti-ideal solutions and the spreading range

	C_1	C_2	C_3	C_4	C_5	C_6
$\dot{x}$	40	96	200	20	1	10
$\ddot{x}$	94	58	1000	97	8	1
dm	−54	38	−800	−77	−7	9

Table 24.11 Values of the strict preference function

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	3.064	1.610	1.147	2.148	0.663
A_2	0.559	–	0.325	0.749	1.241	0.792
A_3	0.511	1.732	–	0.835	1.392	0.578
A_4	1.329	3.436	2.038	–	2.217	1.611
A_5	0.333	1.932	0.677	0.222	–	0.222
A_6	0.653	3.289	1.669	1.421	2.029	–

Table 24.12 Values of the strict preference index

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	0.511	0.268	0.191	0.358	0.110
A_2	0.559	–	0.054	0.125	0.207	0.132
A_3	0.085	0.289	–	0.139	0.232	0.096
A_4	0.221	0.573	0.340	–	0.369	0.268
A_5	0.055	0.322	0.113	0.037	–	0.037
A_6	0.109	0.548	0.278	0.237	0.338	–

(D) The strict preference index

Given the amounts obtained for the alternatives in the previous step, the values of the strict preference index are obtained in Table 24.12.

(E) The entering and leaving flows

First, the values of the total preference index are determined in Table 24.13.

For instance, the entering and leaving flows for alternative A_1 are calculated as follows:

$$\varphi^{+}(A_1) = \frac{1}{6-1}(0.807 + 0.518 + 0.459 + 0.458 + 0.295) = 0.507$$

$$\varphi^{-}(A_1) = \frac{1}{6-1}(1 + 0.321 + 0.620 + 0.499 + 0.395) = 0.567$$

Table 24.14 indicates the other values of the entering and leaving flows.

Table 24.13 Values of the total preference index

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	–	0.807	0.518	0.459	0.458	0.295
A_2	1	–	0.443	0.458	0.503	0.632
A_3	0.321	0.469	–	0.472	0.288	0.525
A_4	0.620	1	0.645	–	0.592	0.480
A_5	0.499	0.837	0.600	0.417	–	0.485
A_6	0.395	0.947	0.528	0.669	0.471	–

Table 24.14 Entering and leaving flows

	φ^-	φ^+
A_1	0.507	0.567
A_2	0.607	0.651
A_3	0.415	0.547
A_4	0.667	0.495
A_5	0.568	0.462
A_6	0.602	0.483

(F) **The net flow**

The net flow of all alternatives is specified by calculating the entering and leaving flows, as follows:

$$\varphi_1 = -0.060,\ \varphi_2 = -0.044,\ \varphi_3 = -0.132,\ \varphi_4 = 0.172,\ \varphi_5 = 0.106,\ \varphi_6 = 0.119$$

(G) **The final ranking of alternatives (EXPROM I method)**

The priority of the alternatives on each other is determined based on the entering and leaving flows, as shown in Table 24.15.

The ranking of alternatives is as follows:

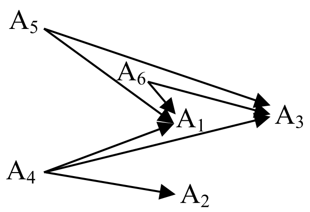


Table 24.15 Priority of the alternatives

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	I	–	–	–	–	–
A_2	–	I	–	–	–	–
A_3	–	–	I	–	–	–
A_4	P	P	P	I	–	–
A_5	P	–	P	–	I	–
A_6	P	–	P	–	–	I

(H) **The final ranking of alternatives (EXPROM II method)**

Given the net flow of alternatives, the fourth alternative (A_4) is the best production system. The other alternatives are ranked as follows:

$$A_4 > A_6 > A_5 > A_2 > A_1 > A_3$$

24.4 Conclusion

Given the similarities between the EXPROM and PROMETHEE methods, the steps of the EXPROM method are as Fig. 24.2. In this method, the entering and leaving flows and the net flow are determined based on the strict and weak preference indexes. The partial ranking of EXPROM I is done based on the entering and leaving flows and the full ranking of EXPROM II is done based on the net flow. The preferred functions are as the PROMETHEE method. Therefore, the similarity between the methods has led to its use in various fields. Further, in the EXPROM method, the qualitative attributes should be converted into the quantitative attributes.

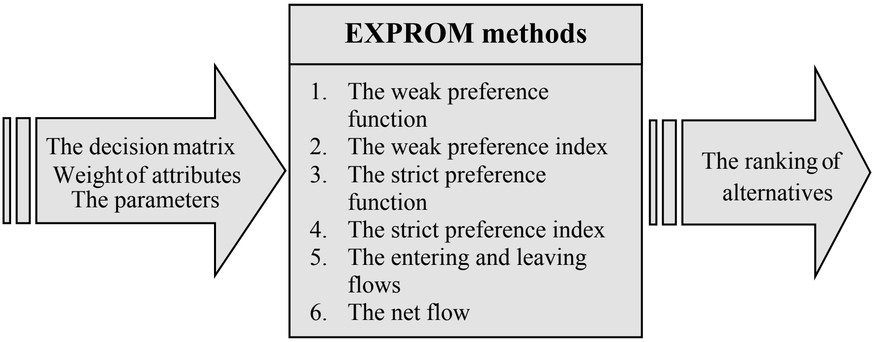


Fig. 24.2 A summary of the EXPROM I & II methods

Chapter 25
MABAC Method

25.1 Introduction

The Multi-Attributive Border Approximation area Comparison (MABAC) method was introduced by Pamucar and Cirovic in 2015 [198–201]. The basic assumption in this method is to define the distance of the alternatives from the border approximation area. In fact, each alternative is evaluated and ranked by specifying the difference between the distances. This method is mostly used in some areas such as evaluation of railway stations [202–204], selecting the location of wind farms [205], and site energy generation technology [206]. This method has the following features:

- This is one of the compensatory methods;
- The attributes are independent of each other;
- The qualitative attributes are converted into the quantitative attributes.

In addition, the input information is determined according to the decision matrix, as in Eq. (25.1).

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} ;\quad i = 1,\ldots,m, \quad j = 1,\ldots,n \qquad (25.1)$$

where r_{ij} is the element of the decision matrix for ith alternative in jth attribute. In this method, the decision maker provides the weight of the attributes $[w_1, w_2, \ldots, w_n]$.

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25.2 Description of MABAC Method

25.2.1 The Normalized Decision Matrix

Eqs. (25.2) and (25.3) are used to normalize the positive and negative attributes of the decision matrix, respectively [207].

$$r_{ij}^{*} = \frac{r_{ij} - r_i^{-}}{r_i^{+} - r_i^{-}}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{25.2}$$

$$r_{ij}^{*} = \frac{r_{ij} - r_i^{+}}{r_i^{-} - r_i^{+}}; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{25.3}$$

where r_{ij}^{*} indicates the normalized value of the decision matrix of ith alternative in jth attribute. Additionally, $r_i^{+} = max(r_1, r_2, \ldots, r_m)$ and $r_i^{-} = min(r_1, r_2, \ldots, r_m)$ [207].

25.2.2 The Weighted Normalized Decision Matrix

Given the normalized values of the decision matrix and the weight of the attributes $[w_1, w_2, \ldots, w_n]$, the weighted normalized values of each attribute are obtained from Eq. (25.4) [207].

$$\hat{r}_{ij} = w_j + r_{ij}^{*} w_j; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{25.4}$$

25.2.3 The Border Approximation Area Matrix

The values of the border approximation area matrix are obtained from Eq. (25.5) [207]

$$g_j = (\prod_{i=1}^{m} \hat{r}_{ij})^{1/m}; \quad j = 1, \ldots, n \tag{25.5}$$

By determining the values of the border approximation area matrix, a $n \times 1$ matrix is obtained.

25.2.4 The Distance from the Border Approximation Area

With respect to the amounts of the border approximation area matrix and the weighted normalized values of each attribute, the distance of the alternatives from the border approximation area is determined as in Eq. (25.6) [207]

$$q_{ij} = \hat{r}_{ij} - g_j; \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \tag{25.6}$$

25.2.5 The Total Distances from the Border Approximate Area

The total distances of each alternative from the border approximate area is determined as in Eq. (25.7) [207].

$$S_i = \sum_{j=1}^{n} q_{ij}; \quad i = 1, \ldots, m \tag{25.7}$$

25.2.6 The Final Ranking of Alternatives

The amounts of the total distances of the alternatives from the border approximate area are determined from the previous stage in a descending order and the final ranking of the alternatives is made.

25.3 Case Study

The board of directors of a steel production factory is looking for selecting the best alternative among the four alternatives (A_1, A_2, A_3, and A_4) proposed by experts for designing a rebar rolled line. In this regard, experts specified attributes such as annual maintenance cost (C_1), equipment prices (C_2), production capacity (C_3), and the required operations reduction for production (C_4). After converting the qualitative indices into the quantitative attributes, the decision matrix is shown as in Fig. 25.1.

Further, the weight of the attributes is considered equal. The purpose is to select the best alternative and express the final ranking of the alternatives.

Fig. 25.1 Decision matrix of designing rebar rolled line

$$
\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{cccc} - & - & + & + \\ C_1 & C_2 & C_3 & C_4 \end{array}
$$

$$
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix} 5 & 54 & 600 & 80 \\ 1 & 97 & 200 & 65 \\ 7 & 72 & 400 & 83 \\ 10 & 75 & 1000 & 40 \end{bmatrix}
$$

❖ Solution

(A) The normalized decision matrix

Given the positive or negative type of the attributes, the normalized values of the decision matrix are shown as in Table 25.1.

(B) The weighted normalized decision matrix

For example, the weighted normalized values of the first attribute are as follows:

$$
\begin{aligned}
\hat{r}_{11} &= w_1 + r^*_{11} w_1 = \frac{1}{4}(1 + 0.556) = 0.389 \\
\hat{r}_{21} &= w_1 + r^*_{21} w_1 = \frac{1}{4}(1 + 1) = 0.500 \\
\hat{r}_{31} &= w_1 + r^*_{31} w_1 = \frac{1}{4}(1 + 0.333) = 0.333 \\
\hat{r}_{41} &= w_1 + r^*_{41} w_1 = \frac{1}{4}(1 + 0) = 0.250
\end{aligned}
$$

The other weighted normalized values of the attributes are determined in Table 25.2.

(C) The border approximate area matrix

The amounts of the border approximate area matrix are as follows:

$$
g = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ [0.357 \quad 0.370 \quad 0.348 \quad 0.393] \end{array}
$$

Table 25.1 Normalized values of the decision matrix

	C_1	C_2	C_3	C_4
A_1	0.556	1	0.500	0.930
A_2	1	0	0	0.581
A_3	0.333	0.581	0.250	1
A_4	0	0.512	1	0

Table 25.2 Values of the weighted normalized decision matrix

	C_1	C_2	C_3	C_4
A_1	0.389	0.500	0.375	0.483
A_2	0.500	0.250	0.250	0.395
A_3	0.333	0.395	0.313	0.500
A_4	0.250	0.378	0.500	0.250

Table 25.3 Distance from the border approximate area

	C_1	C_2	C_3	C_4
A_1	0.032	0.130	0.027	0.090
A_2	0.143	−0.120	−0.098	0.002
A_3	−0.024	0.025	−0.035	0.107
A_4	−0.107	0.008	0.152	−0.143

(D) **The distance from the border approximate area**

The distance of each alternative from the border approximate area, defined in the previous step, is shown in Table 25.3.

(E) **The total distances from the border approximate area**

According to the previous step, the total distances of the alternatives from the border approximate area are calculated as follows:

$$S_1 = 0.032 + 0.130 + 0.027 + 0.090 = 0.279$$
$$S_2 = 0.143 - 0.120 - 0.098 + 0.002 = -0.073$$
$$S_3 = -0.024 + 0.025 - 0.035 + 0.107 = 0.073$$
$$S_4 = -0.107 + 0.008 + 0.152 - 0.143 = -0.090$$

(F) **The final ranking of alternatives**

Therefore, A_1 is the best alternative, and the final ranking of alternatives is as follows:

$$A_1 > A_3 > A_2 > A_4$$

25.4 Conclusion

The main idea in the new MABAC method is to determine the distance amount of alternatives from the border approximate area. Accordingly, after determining the decision matrix and the weight of the attributes, and converting the qualitative attributes into the quantitative attributes, the alternatives rank in five steps

(Fig. 25.2). In fact, by specifying the 1 × n matrix of the border approximate area, the distance of alternatives from the border approximate area is determined and the alternatives are ranked by calculating the total distance of each alternative.

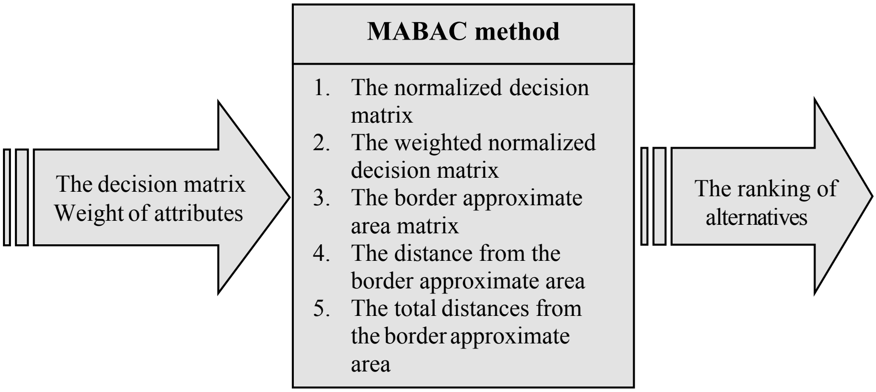

Fig. 25.2 A summary of MABAC method

Chapter 26
CRITIC Method

26.1 Introduction

The CRiteria Importance Through Intercriteria Correlation (CRITIC) method, which was proposed by Diakoulaki, Mavrotas, and Papayannakis in 1995 [208–211], is mainly used to determine the weight of attributes. In the present method, the attributes aren't in contradiction with each other, and the attributes weights are determined using the decision matrix. It is used for the automatic areal feature matching [212, 213], medical quality assessment [214], and ranking of machining processes [215]. In addition, the CRITIC method includes the following features:

- No need for the independence of attributes;
- The qualitative attributes are transformed into quantitative attributes.

The decision matrix is based on entering the method and expressing the alternatives and attributes are based on the information received from the decision maker, as shown in Eq. (26.1).

$$\mathrm{X} = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m \times n} ; \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{26.1}$$

where r_{ij} indicates the element of the decision matrix for ith alternative in jth attribute.

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26.2 Description of CRITIC Method

26.2.1 The Normalized Decision Matrix

In order to normalize the positive and negative attributes of the decision matrix, Eqs. (26.2) and (26.3) are used, respectively [216].

$$x_{ij} = \frac{r_{ij} - r_i^-}{r_i^+ - r_i^-}; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{26.2}$$

$$x_{ij} = \frac{r_{ij} - r_i^+}{r_i^- - r_i^+}; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{26.3}$$

where x_{ij} represents a normalized value of the decision matrix for ith alternative in jth attribute and $r_i^+ = max(r_1, r_2, \ldots, r_{\mathrm{m}})$ and $r_i^- = min(r_1, r_2, \ldots, r_{\mathrm{m}})$.

26.2.2 The Correlation Coefficient

The correlation coefficient among attributes is determined by Eq. (26.4) [216].

$$\rho_{jk} = \sum_{i=1}^{m} \left(x_{ij} - \bar{x}_j\right)(x_{ik} - \bar{x}_k) / \sqrt{\sum_{i=1}^{m} \left(x_{ij} - \bar{x}_j\right)^2 \sum_{i=1}^{m} (x_{ik} - \bar{x}_k)^2} \tag{26.4}$$

where $\bar{x}_j$ and $\bar{x}_k$ display the mean of jth and kth attributes. $\bar{x}_j$ is computed from Eq. (26.5). Similarly, it is obtained for $\bar{x}_k$. Also, ρ_{jk} is the correlation coefficient between jth and kth attributes [216].

$$\bar{x}_j = \frac{1}{n} \sum_{j=1}^{n} x_{ij}; \quad i = 1, \ldots, m \tag{26.5}$$

26.2.3 The Index (C)

First, the standard deviation of each attribute is estimated by Eq. (26.6) [216].

$$\sigma_j = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (x_{ij} - \bar{x}_j)^2}; \quad i = 1, \ldots, m \tag{26.6}$$

Then, the index (C) is calculated using Eq. (26.7) [216].

$$C_j = \sigma_j \sum_{k=1}^{n} \left(1 - \rho_{jk}\right); \quad j = 1, \ldots, n \tag{26.7}$$

26.2.4 The Weight of Attributes

The weights of attributes are determined by Eq. (26.8) [216].

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j}; \quad j = 1, \ldots, n \tag{26.8}$$

26.2.5 The Final Ranking of Attributes

The weights of attributes are arranged in descending order for the final ranking of attributes.

26.3 Case Study

A company intends to evaluate four different cutting machines of A_1, A_2, A_3, and A_4 to determine the best cutting machine. In this regard, the attributes such as the maximum piece thickness (C_1), the minimum cutting width (C_2), cutting edge quality (C_3), and maintenance cost (C_4) were specified by experts and qualitative attributes were converted to quantitative ones. The decision matrix is shown in Fig. 26.1.

The importance of the attributes should be determined to select the machines, which aims to rank the attributes.

Fig. 26.1 Decision matrix for evaluating the cutting machines

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} - & - & + & - \\ C_1 & C_2 & C_3 & C_4 \\ \end{array}$$

$$\begin{array}{c|cccc} & C_1\,(-) & C_2\,(-) & C_3\,(+) & C_4\,(-) \\ \hline A_1 & 30 & 0.100 & 1 & 20 \\ A_2 & 100 & 0.700 & 1 & 40 \\ A_3 & 50 & 1 & 2 & 10 \\ A_4 & 300 & 2 & 3 & 35 \end{array}$$

❖ **Solution**

(A) The normalized decision matrix

The normalized values of the decision matrix are computed with respect to the positive attributes or negative attributes as shown in Table 26.1.

(B) The correlation coefficient

Given the normalized values of decision matrix, the mean values of each attribute are as follows:

$$\bar{x}_1 = 0.667, \quad \bar{x}_2 = 0.553, \quad \bar{x}_3 = 0.375, \quad \bar{x}_4 = 0.459$$

The correlation coefficient between the attributes is according to Fig. 26.2.

(C) The index (C)

The standard deviation of each attribute is calculated as follows:

$$\sigma_1 = 0.458, \quad \sigma_2 = 0.418, \quad \sigma_3 = 0.479, \quad \sigma_4 = 0.459$$

The index (C) for all attributes is as follows:

$$C_1 = 1.064, \quad C_2 = 1.139, \quad C_3 = 2.295, \quad C_4 = 0.979$$

(D) The weight of attributes

The final weight of attributes is determined as follows:

Table 26.1 Normalized values of decision matrix

	C_1	C_2	C_3	C_4
A_1	1	1	0	0.667
A_2	0.741	0.684	0	0
A_3	0.926	0.526	0.500	1
A_4	0	0	1	0.167

Fig. 26.2 Correlation coefficient between the attributes

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 1 & 0.907 & -0.817 & 0.587 \\ 0.907 & 1 & -0.943 & 0.312 \\ -0.817 & -0.943 & 1 & -0.031 \\ 0.587 & 0.312 & -0.031 & 1 \end{bmatrix} \end{array}$$

$$w_1 = \frac{1.064}{1.064 + 1.139 + 2.295 + 0.979} = 0.194$$
$$w_2 = \frac{1.139}{1.064 + 1.139 + 2.295 + 0.979} = 0.208$$
$$w_3 = \frac{2.295}{1.064 + 1.139 + 2.295 + 0.979} = 0.419$$
$$w_4 = \frac{0.979}{1.064 + 1.139 + 2.295 + 0.979} = 0.179$$

(E) **The Final ranking of attributes**

The weights of the attributes are arranged in descending order:

$$w_3 > w_2 > w_1 > w_4$$

Therefore, it is concluded that the third attribute (C_3) is highly preferred than the other attributes and the final ranking of attributes is as follows:

$$C_3 > C_2 > C_1 > C_4$$

26.4 Conclusion

In 1995, the CRITIC method was applied to obtain the attributes weights in the decision matrix. The present method, with four different stages for specifying the weight and ranking attributes (Fig. 26.3), uses the correlation coefficient between the attributes to determine the relation among of attributes. After determining the decision matrix and converting the qualitative attributes to quantitative ones by the experts, the superior attribute is eventually specified.

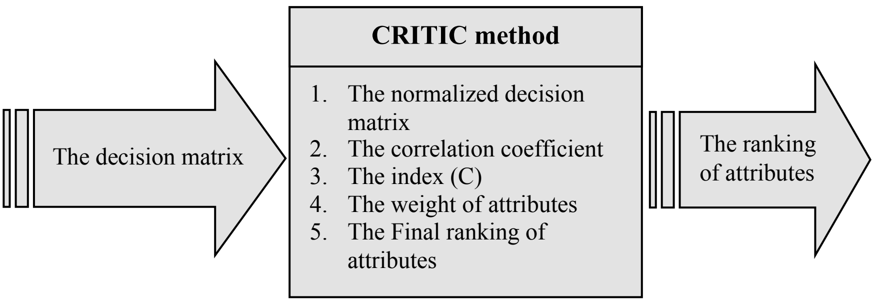

Fig. 26.3 A summary of the CRITIC method

Chapter 27
KEMIRA Method

27.1 Introduction

The KEmeny Median Indicator Ranks Accordance (KEMIRA) method was introduced by Krylovas, Zavadskas, Kosareva, and Dadelo in 2014 [217, 218]. In this method, the final ranking of alternatives is done after determining the priority and weight of attributes in two different groups and in the form of the decision matrix specified by the experts. It is used for the personal evaluation and selection [219, 220], and evaluating the sustainability of transportation systems [221]. The KEMIRA method includes the following features:

- It is one of the compensatory methods;
- The qualitative attributes should be converted into the quantitative attributes.

The decision matrix is based on entering this method, including two different groups of attributes for evaluating the alternatives based on the information received from the decision maker, as shown in Eq. (27.1).

$$\mathrm{X} = \left[\begin{array}{ccccc|ccccc} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} & t_{11} & \cdots & t_{1j} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} & t_{i1} & \cdots & t_{ij} & \cdots & t_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} & t_{m1} & \cdots & t_{mj} & \cdots & t_{mn} \end{array}\right] \tag{27.1}$$

where r_{ij} indicates the first group element of the decision matrix for ith alternative in the jth attribute and t_{ij} represents the second group element of the decision matrix for ith alternative in the jth attribute. Further, the decision matrix attributes should be reviewed and prioritized by experts.

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27.2 Description of KEMIRA Method

27.2.1 The Normalized Decision Matrix

At first, the negative attribute values are reversed and the negative attributes are converted into the positive ones. In order to normalize, the positive attributes of two groups of the decision matrix are used Eqs. (27.2) and (27.3), respectively [222].

$$x_{ij}^{*} = \frac{r_{ij} - r_i^{-}}{r_i^{+} - r_i^{-}}; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{27.2}$$

$$y_{ij}^{*} = \frac{t_{ij} - t_i^{-}}{t_i^{+} - t_i^{-}}; \quad i = 1, \ldots, m, \ j = 1, \ldots, n \tag{27.3}$$

In Eq. (27.2), x_{ij}^{*} demonstrates the normalized value of decision matrix for ith alternative in jth attribute based on $r_i^{+} = max(r_1, r_2, \ldots, r_m)$ and $r_i^{-} = min(r_1, r_2, \ldots, r_m)$.

where, in Eq. (27.3), y_{ij}^{*} indicates the normalized value of the decision matrix for ith alternative in jth attribute based on $t_i^{+} = max(t_1, t_2, \ldots, t_m)$ and $t_i^{-} = min(t_1, t_2, \ldots, t_m)$.

27.2.2 The Median Matrix

In order to determine the median matrix, the optimal value obtained from the difference between the different matrices obtained from the prioritization of the experts is first calculated by Eq. (27.4) [222].

$$R_A = arg \min_{R} \sum_{j=1}^{n} \rho\left(R_j, R_{j'}\right); \quad j' = 1, \ldots, n \tag{27.4}$$

where $\rho\left(R_j, R_{j'}\right)$ in the form of expert matrix is computed as Eq. (27.5) [222].

$$\rho\left(R_j, R_{j'}\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left| a_{ij}^{j} - a_{ij}^{j'} \right|; \quad j' = 1, \ldots, n \tag{27.5}$$

In the expert matrix $\left(R = \left(a_{ij}\right)_{n \times n}\right)$, the amount of each element is determined by experts considering the prioritization of the attributes (x,y). For example, Eq. (27.6) is considered for attributes (x) and (y) (a_{ii} = 0) [222].

$$a_{ij} = \begin{cases} 0 & if\ x_j < x_{j'} \\ 1 & if\ x_j > x_{j'} \end{cases}; \quad j, j' \in \{1, \ldots, n\} \tag{27.6}$$

27.2.3 The Set of Attribute Weights

According to the median matrix and the priorities obtained for each group of decision matrix attributes, various sets of the attribute weights $\left(w_{x_j}, w_{y_j}\right)$ are determined with respect to the normalized property.

27.2.4 The Final Weight of Attributes

The value of F(X,Y) should be determined to obtain the final weight of attributes. Initially, the values of each alternative and two groups of decision matrix attributes for the different sets of weights are determined by Eqs. (27.7) and (27.8) [222].

$$X_i = \sum_{j=1}^{n} x_{ij}^{*}.w_{x_j}; \quad i = 1, \ldots, m \tag{27.7}$$

$$Y_i = \sum_{j=1}^{n} y_{ij}^{*}.w_{y_j}; \quad i = 1, \ldots, m \tag{27.8}$$

Then, the value of F(X,Y) is obtained using Eq. (27.9) [222].

$$\mathrm{F}(X, Y) = \min_{X_i, Y_i} \sum_{\mathrm{A}} |X_i - Y_i|; \quad i = 1, \ldots, m \tag{27.9}$$

Given the values of F(X,Y), the best priority is specified for determining the final weight of attributes.

27.2.5 The Final Value of Alternatives

By determining the weights of attributes, the final value of alternatives is calculated by Eq. (27.10) [222].

$$X_i + Y_i = \sum_{j=1}^{n} x_{ij}^{*}.w_{x_j} + \sum_{j=1}^{n} y_{ij}^{*}.w_{y_j}; \quad i = 1, \ldots, m \tag{27.10}$$

27.2.6 The Final Ranking of Alternatives

In the final ranking of alternatives, the values obtained from Eq. (27.10) are arranged in descending order, and an alternative with the highest value is selected as the best alternative.

27.3 Case Study

For the construction of a hospital, seven plans (A_1, A_2, A_3, A_4, A_5, A_6, and A_7) were specified by experts in group (a) engineering factors such as the distance to city center (x_1), the distance to high pressure gas pipeline (x_2), the distance to high voltage power network (x_3), and the distance to water supply system (x_4), and group (b) social and urban factors such as the distance to industrial settlements (y_1), predicting average annual number of referrals (y_2), and number of residents in designated areas (y_3). Further, the decision matrix is shown in Fig. 27.1.

In addition, the prioritization of both groups of attributes was determined by five experts as follows:

$$\begin{aligned} \text{First expert}: x_1 > x_4 > x_2 > x_3, &\quad y_3 > y_1 > y_2 \\ \text{Second expert}: x_1 > x_4 > x_2 > x_3, &\quad y_1 > y_2 > y_3 \\ \text{Third expert}: x_1 > x_2 > x_3 > x_4, &\quad y_2 > y_3 > y_1 \\ \text{Fourth expert}: x_1 > x_2 > x_4 > x_3, &\quad y_2 > y_3 > y_1 \\ \text{Fifth expert}: x_1 > x_2 > x_4 > x_3, &\quad y_1 > y_3 > y_2 \end{aligned}$$

It aims to select the best plan and express the final ranking of the alternatives.

$$\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} - & - & - & - & + & + & + \\ x_1 & x_2 & x_3 & x_4 & y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1.500 & 0.600 & 2.500 & 1.370 & 9.260 & 3188.600 & 55269 \\ 3.500 & 1.200 & 4.500 & 0.500 & 8.640 & 497.500 & 9327 \\ 0.800 & 0.500 & 3 & 0.100 & 6.440 & 2484 & 50798 \\ 4.800 & 1.200 & 1.600 & 2 & 11.190 & 2676 & 56206 \\ 5.500 & 1 & 1.600 & 0.300 & 5.900 & 3291 & 66807 \\ 0.600 & 0.700 & 2 & 0.600 & 6.090 & 6490 & 132136 \\ 0.300 & 0.400 & 2 & 0.600 & 5.720 & 5496.700 & 123314 \end{bmatrix} \end{array}$$

Fig. 27.1 Decision matrix of the hospital construction

❖ **Solution**

(A) **The normalized decision matrix**

After reversing the values of the negative attributes related to the engineering factors and converting the negative-to-positive attributes, the normalized values of the decision matrix are presented in Table 27.1.

(B) **The median matrix**

Given the priority of attributes by five experts, the expert matrices for the engineering factor attributes are as follows:

$$R_1 = R_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_4 = R_5 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The expert matrices for attributes related to social and urban factors are as follows:

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad R_3 = R_4 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$R_5 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\rho(R_j, R_{j'})$ are initially determined to specify the amount of R_A. For example, the amount of $\rho(R_1, R_3)$, related to the engineering factors, is as follows:

Table 27.1 Normalized values of the decision matrix

	x_1	x_2	x_3	x_4	y_1	y_2	y_3
A_1	0.154	0.500	0.441	0.024	0.647	0.449	0.374
A_2	0.033	0	0	0.158	0.534	0	0
A_3	0.339	0.700	0.276	1	0.132	0.331	0.338
A_4	0.008	0	1	0	1	0.364	0.382
A_5	0	0.100	1	0.298	0.033	0.466	0.468
A_6	0.471	0.357	0.690	0.123	0.068	1	1
A_7	1	1	0.690	0.123	0	0.909	0.928

$$\rho(R_1, R_3) = |0-0| + |1-1| + |1-1| + |1-1| + |0-0| + |0-0| \\ + |1-1| + |0-1| + |0-0| + |0-0| + |0-0| + |0-1| \\ + |0-0| + |1-0| + |1-0| + |0-0| = 4$$

On the other hand, 4! or 24 possible ranking should be examined. After computing $\rho(R_j, R_{j'})$ for all states and by limiting the number of solutions, the median matrix is as follows according to Eq. (27.4):

$$R_A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & a_{24} \\ 0 & 0 & 0 & a_{34} \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix}$$

In this matrix, the states containing $a_{24} = 1 - a_{42}$ and $a_{34} = 1 - a_{43}$ should be determined for unknown values according to the following matrices:

$$\mathrm{R}'_\mathrm{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{R}''_\mathrm{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathrm{R}'''_\mathrm{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathrm{R}''''_\mathrm{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

The values $\sum_{j=1}^{n} \rho(R_j, R_{j'})$ are as follows:

$$\sum_{j=1}^{5} \rho(R'_A, R_j) = 4 + 4 + 0 + 2 + 2 = 12$$

$$\sum_{j=1}^{5} \rho(R''_A, R_j) = 2 + 2 + 2 + 0 + 0 = 6$$

$$\sum_{j=1}^{5} \rho(R'''_A, R_j) = 2 + 2 + 2 + 4 + 4 = 14$$

$$\sum_{j=1}^{5} \rho(R''''_A, R_j) = 0 + 0 + 4 + 2 + 2 = 8$$

According to Eq. (27.4), the matrix $\mathrm{R}''_{\mathrm{A}}$ with the lowest value as the median matrix is determined for the engineering factor attributes. The resulting priority is $x_1 > x_2 > x_4 > x_3$. The median matrix for attributes related to the social and urban factors is as follows:

$$\mathrm{R}'_{\mathrm{A}} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{R}''_{\mathrm{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad \mathrm{R}'_{\mathrm{A}} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

The values $\sum_{j=1}^{n} \rho\left(R_j, R_{j'}\right)$ are equal to 14 for all three matrices. Thus, the priorities are $y_1 > y_2 > y_3$, $y_3 > y_1 > y_2$, and $y_2 > y_3 > y_1$ and $y_3 > y_1 > y_2$ is selected among the priorities.

(C) The set of attribute weights

According to the priorities specified for the engineering factor attributes, all possible sets for the attribute weights are determined based on $w_{x_1} > w_{x_2} > w_{x_4} > w_{x_3}$ as shown in Table 27.2.

Table 27.2 Set of attribute weights for engineering factors

	w_{x_1}	w_{x_2}	w_{x_3}	w_{x_4}
1	1	0	0	0
2	0.900	0.100	0	0
3	0.800	0.200	0	0
4	0.800	0.100	0	0.100
5	0.700	0.300	0	0
6	0.700	0.200	0	0.100
7	0.700	0.100	0.100	0.100
8	0.600	0.400	0	0
9	0.600	0.300	0	0.100
10	0.600	0.200	0	0.200
11	0.600	0.200	0.100	0.100
12	0.500	0.500	0	0
13	0.500	0.400	0	0.100
14	0.500	0.300	0	0.200
15	0.500	0.300	0.100	0.100
16	0.500	0.200	0.100	0.200
17	0.400	0.400	0	0.200
18	0.400	0.400	0.100	0.100
19	0.400	0.300	0	0.300
20	0.400	0.300	0.100	0.200
21	0.400	0.200	0.200	0.200
22	0.300	0.300	0.100	0.300
23	0.300	0.300	0.200	0.200

Table 27.3 Set of attribute weights for social and urban factors

	w_{y_1}	w_{y_2}	w_{y_3}
1	0	0	1
2	0.100	0	0.900
3	0.200	0	0.800
4	0.100	0.100	0.800
5	0.300	0	0.700
6	0.200	0.100	0.700
7	0.400	0	0.600
8	0.300	0.100	0.600
9	0.200	0.200	0.600
10	0.500	0	0.500
11	0.400	0.100	0.500
12	0.300	0.200	0.500
13	0.400	0.200	0.400
14	0.300	0.300	0.400

Also, according to the priorities specified for the attributes based on the social and urban factors, all possible sets for the attribute weights are obtained based on $w_{y_3} > w_{y_1} > w_{y_2}$ as shown in Table 27.3.

(D) **The final weight of attributes**

Regarding the set of the attribute weights, the state 322 should be examined to determine F(X,Y). Therefore, the weighted normalized values for each set are separately calculated using Eqs. (27.7) and (27.8) and then F(X,Y) for state 322 of the attribute weights is determined using Eq. (27.9).

All the obtained states are determined for the priorities $x_1 > x_2 > x_4 > x_3$ and $y_3 > y_1 > y_2$, and the minimum amount of F(X,Y) is equal to 1.285. Further, the minimum amount of F(X,Y) is equal to 1.547 for the priority $x_1 > x_2 > x_4 > x_3$ and $y_2 > y_3 > y_1$, and 1.317 for the priority $x_1 > x_2 > x_4 > x_3$ and $y_1 > y_2 > y_3$. Therefore, the priorities $x_1 > x_2 > x_4 > x_3$ and $y_3 > y_1 > y_2$ have a lower value of F(X,Y).

Given the set of the attribute weights, the values of F(X,Y) for the priorities $x_1 > x_2 > x_4 > x_3$ and $y_3 > y_1 > y_2$ are indicated in Table 27.4.

According to Table 27.4, the final weights of attributes for the value of 1.285 of function F (X, Y) are as follows:

$$w_{x_1} = 0.400, \quad w_{x_2} = w_{x_3} = w_{x_4} = 0.200,$$
$$w_{y_1} = 0.200, \quad w_{y_2} = 0, \quad w_{y_3} = 0.800$$

Table 27.4 Values of F(X,Y)

	1	2	3	4	5	6	7
1	1.696	1.750	1.869	1.756	1.988	1.877	2.107
2	1.696	1.757	1.876	1.765	1.995	1.884	2.114
3	1.697	1.764	1.883	1.772	2.002	1.891	2.121
4	1.738	1.742	1.861	1.750	1.980	1.869	2.099
5	1.698	1.771	1.890	1.779	2.009	1.898	2.128
6	1.739	1.749	1.868	1.757	1.987	1.876	2.106
7	1.510	1.458	1.577	1.466	1.696	1.585	1.825
8	1.698	1.778	1.897	1.786	2.016	1.905	2.135
9	1.739	1.756	1.875	1.746	1.994	1.883	2.113
10	1.924	1.755	1.852	1.759	1.972	1.860	2.091
11	1.511	1.465	1.584	1.473	1.703	1.592	1.822
12	1.698	1.785	1.904	1.793	2.023	1.912	2.142
13	1.740	1.763	1.882	1.771	2	1.890	2.120
14	1.924	1.762	1.859	1.766	1.979	1.867	2.098
15	1.511	1.472	1.591	1.480	1.710	1.599	1.829
16	1.696	1.533	1.569	1.537	1.688	1.577	1.807
17	1.925	1.769	1.866	1.773	1.986	1.874	2.105
18	1.511	1.479	1.598	1.487	1.717	1.606	1.836
19	2.110	1.936	1.856	1.941	1.963	1.860	2.083
20	1.696	1.540	1.576	1.544	1.695	1.584	1.814
21	1.467	1.311	**1.285**	1.316	1.404	1.293	1.523
22	1.881	1.708	1.634	1.712	1.680	1.638	1.799
23	1.468	1.318	1.292	1.322	1.411	1.300	1.530
	8	9	10	11	12	13	14
1	1.996	1.885	2.227	2.115	2.004	2.123	2.012
2	2.003	1.892	2.234	2.122	2.011	2.130	2.019
3	2.010	1.899	2.241	2.130	2.018	2.137	2.026
4	1.988	1.877	2.218	2.107	1.996	2.115	2.004
5	1.017	1.906	2.248	2.136	2.025	2.114	2.033
6	1.995	1.884	2.225	2.114	2.003	2.122	2.011
7	1.704	1.593	1.935	1.823	1.712	1.831	1.720
8	2.024	1.913	2.255	2.143	2.032	2.151	2.040
9	2.002	1.891	2.232	2.121	2.010	2.129	2.018
10	1.980	1.868	2.210	2.100	1.988	2.107	1.996
11	1.711	1.600	1.942	1.183	1.719	1.838	1.727
12	2.031	1.920	2.262	2.151	2.039	2.158	2.047
13	2.009	1.898	2.239	2.128	2.017	2.136	2.025
14	1.987	1.875	2.217	2.106	1.995	2.114	2.003
15	1.718	1.607	1.949	1.837	1.726	1.845	1.734
16	1.696	1.585	1.926	1.815	1.704	1.823	1.712

(continued)

Table 27.4 (continued)

	8	9	10	11	12	13	14
17	1.994	1.882	2.224	2.113	2.002	2.121	2.010
18	1.725	1.614	1.956	1.844	1.733	1.852	1.741
19	1.971	1.864	2.202	2.091	1.979	2.099	1.987
20	1.703	1.592	1.933	1.822	1.711	1.830	1.917
21	1.412	1.301	1.701	1.531	1.420	1.539	1.428
22	1.688	1.642	1.918	1.807	1.696	1.815	1.704
23	1.419	1.308	1.728	1.538	1.427	1.546	1.435

Table 27.5 Normalized values of the weights in the decision matrix

	x_1	x_2	x_3	x_4	y_1	y_2	y_3
A_1	0.062	0.100	0.088	0.005	0.129	0	0.299
A_2	0.013	0	0	0.032	0.107	0	0
A_3	0.136	0.140	0.055	0.200	0.026	0	0.270
A_4	0.003	0	0.200	0	0.200	0	0.306
A_5	0	0.020	0.200	0.060	0.007	0	0.374
A_6	0.188	0.071	0.138	0.025	0.014	0	0.800
A_7	0.400	0.200	0.138	0.025	0	0	0.742

(E) The final value of alternatives

By determining the final weights of attributes, according to Table 27.1, the normalized values of the decision matrix are calculated as shown in Table 27.5.

The X + Y values are determined for each alternative according to Table 27.6. Given the X + Y values for each alternative, the seventh plan (A_7) is the best alternative and the final ranking is as follows:

$$A_7 > A_6 > A_3 > A_4 > A_1 > A_5 > A_2$$

Table 27.6 X + Y values of alternatives

	X	Y	X + Y
A_1	0.255	0.428	0.683
A_2	0.045	0.107	0.152
A_3	0.531	0.296	0.827
A_4	0.203	0.506	0.709
A_5	0.280	0.381	0.661
A_6	0.422	0.814	1.236
A_7	0.763	0.742	1.505

27.4 Conclusion

In the KEMIRA method, which was presented in 2014, the weight of the attributes is first specified and then the best alternative is determined. By following the five steps of the method (Fig. 27.2), the final value of alternatives is obtained and then the final ranking of alternatives is made. In this technique, the decision matrix including two groups of the different and independent attributes, along with the initial prioritization of the attributes, is determined by the experts. On the other hand, the decision matrix includes the quantitative attributes and the qualitative attributes should be quantitative. Further, the KEMIRA method does not need to specify the attribute weights by the decision maker, which is one of the main advantages of this method. The final weight is obtained by determining the possible sets for the attribute weights, and ultimately, the final ranking of the alternatives and the choice of the best alternative are done.

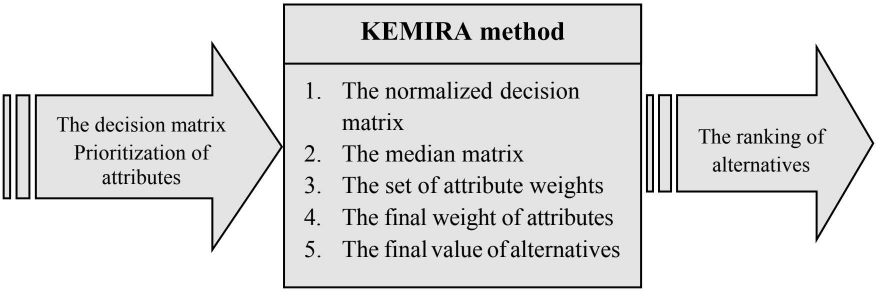

Fig. 27.2 A summary of the KEMIRA method

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