# Chapter 5
# The Italian Subcommission of the International Commission on the Teaching of Mathematics (1908–1920): Organizational and Scientific Contributions

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**Abstract** In this paper, I will illustrate the Italian contribution to the activities of the International Commission on the Teaching of Mathematics (ICMI) from 1908 to 1920, focusing on the following aspects: the most relevant figures, with particular emphasis on their ideas on the teaching of mathematics; the influence of these activities on Italian education policies, paying attention to legislative measures, university courses, texbooks and debates; and the impact of the political situation in Italy on the collective action of the subcommission after the dissolution of ICMI.

My aim is to elucidate the role of the Italian community of mathematicians; the influence of Klein's ideas on mathematics education; the image of the Italian schools that emerges from the reports and the international discussions during the various meetings; and the effects of the devaluation of scientific teaching by the Giovanni Gentile Reform on the work of the Italian subcommission.

**Keywords** Castelnuovo · Enriques · Fascism's impact on education · ICMI · Italian subcommission · Klein · Mathematics teaching in Italy · Scorza

## 1 Historical Background

The historical roots of the Italian subcommission of the International Commission on the Teaching of Mathematics (Commission Internationale de l'Enseignement Mathématique, Internationale Mathematische Unterrichtskommission) can be traced back to the preparatory work of Section IV, dedicated to the historical,

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philosophical and educational questions of the International Congress of Mathematicians (ICM) held in Rome from the 6th to the 11th of April 1908, during which the Commission was established under the chairmanship of Felix Klein.[1]

Section IV – in particular with regard to educational issues – was much richer than in previous congresses and addressed questions that would occupy the new commission in the coming years: the programmes of the different levels and kinds of schools in the various countries, the methods of teaching geometry, the modern trends in teaching, the relationships between intuition and rigour, etc. The section was carefully prepared by Giovanni Vailati (1863–1909), a member of the Peano School of logic, with contributions from Gino Loria (1862–1954), a well-known historian of mathematics, who was informed about European reform movements, in particular that of Klein (see, e.g. Loria 1906). The Organizing Committee of the Congress, whose president was Pietro Blaserna (president of the Accademia dei Lincei) and whose secretary was Guido Castelnuovo (1865–1952), officially gave Vailati the task of looking for speakers for the didactic section in February 1907. Moreover, Castelnuovo suggested some names of scholars already agreed upon with Klein (J. Tannery, A. Gutzmer, E. H. Moore, A. R. Forsyth) and invited him to present on the teaching of mathematics in Italy. He also asked him to contact Loria who had some proposals to make.[2] In fact, during the previous year, Loria had been in contact with David Eugene Smith (1860–1944), a historian of mathematics and a mathematics educator, in order to get his advice on the didactic section of the future International Congress of Mathematicians. Smith answered him with a long letter, in which he first remarked:

> It seems to me that a pedagogical section should devote its energies to the larger questions of mathematical education, leaving the small details for discussion by associations of less importance.

Secondly, he suggested three questions that he presented in detail:

I. Is it possible to formulate a normal curriculum for a culture course in elementary mathematics, international in character, based as to difficulty on the psychological development of the child, and as to applications on the needs of the average citizen, and extending through the first 12 school years? (…)
II. What should be the basis of the applications given in regular culture courses in elementary mathematics? …
III. What is the ideal culture course in demonstrative geometry?[3]

For each of these themes, Smith also suggested the points that - in his opinion - should be treated. In particular, with regard to the second point provided above, he wondered if the applications of mathematics should be drawn from physics, from

[1] Concerning the origins and the early period of ICMI, see Schubring (2008a,b) and Donoghue (2008). Starting in the early 1950s, the commission took on the name of the International Commission on Mathematical Instruction. In what follows we will use the acronym ICMI for simplicity.

[2] G. Castelnuovo to Vailati, s.l. 16 February 1907, in FVM.

[3] D.E. Smith to G. Loria, New York, 12 January 1906, FVM; also in DESPP, Box 32. In FVM see also D. E. Smith to G. Vailati, New York, 27 May 1907 (Cart. 11, fasc.182); A. Gutzmer to G. Vailati, Halle, 17 July 1907 (Cart. 3, fasc. 87).

other sciences, or from daily life. With reference to the third point, he posed the following questions: When should the course of rational geometry begin? How much time should be devoted to it? Which and how many demonstrations is it advisable to introduce? What should be the interrelations between plane and solid geometry, between geometry and trigonometry and between geometry and algebra? Some of these issues would be submitted again, by Smith, at the end of his speech during the ICM in Rome, in the belief that "the influence of a Congress like this might greatly help many who are earnestly seeking to improve the teaching of mathematics" (Smith 1909, p. 477). A successive letter by Loria shows that he also asked for information about the mathematics programmes in the Italian secondary schools, the problems assigned at the final diploma exams, the most widely used textbooks as well as the books concerning pedagogical and methodological issues, according the comparative perspective that would characterize the actions of ICMI.

In Section IV of the 1908 ICM, there were thirteen communications on the teaching of mathematics, many more than those expected by Castelnuovo; mathematicians internationally known for their commitment to education presented a paper.[4] Nine countries were represented, and most of the talks were devoted to mathematics teaching in the secondary schools of the various countries. Among the speakers was Smith himself, who in his communication on the 9th of April proposed the creation of a standing committee to study issues related to the teaching of mathematics (*Atti* 1909 I, p. 45); this proposal was supported by F. Archenhold in the same session and was put forward again on the 11th of April, when Federigo Enriques (1871–1946), at Smith's suggestion, reopened the session, already closed. After a lively discussion involving R. Bonola, Castelnuovo, A. Conti, H. Fehr, C. Stephanos, Smith, Archenhold and Enriques himself, the following agenda was proposed by Castelnuovo and approved:

> Session Number Four, having recognised the need for a comparative study of the syllabi and the methods of mathematics teaching in secondary schools of the various nations, assigns the task of forming an International Commission, which would investigate the question and present its results at the next Congress, to Professors Klein, Greenhill and H. Fehr. (*Atti* 1909 I, p. 51, my translation)[5]

Seven daily bulletins were published during the Congress, giving news in real time on work, and the two main journals addressed to teachers - *Il Bollettino di Matematica* and *Periodico di Matematica* - published reports, sometimes very detailed, which focused on the talks of Section IV and on the proposal to establish an international commission concerned with the teaching of mathematics.[6]

After its establishment, eighteen countries from all over the world joined the commission. These included Italy, which was among the "major" countries with three delegates: Castelnuovo, Vailati and Enriques. The choice was quite obvious:

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[4] See http://www.icmihistory.unito.it/1908-1910/ICM-1908.pdf.

[5] On the backstory of this matter, see Donoghue (2008) and Schubring (2008a).

[6] See *Il Bollettino di Matematica* 1908, p. 102–120: all three steps of the proposal (Smith, Archenhold, Castelnuovo) are reported; see also *Periodico di Matematica* 1908, pp. 258–271, where only the first and the third steps of the proposal are cited.

Castelnuovo was secretary general of the Congress and together with Enriques was a member of the Italian School of algebraic geometry which was known and appreciated all over the world. Above all, both Castelnuovo and Enriques were in contact with Klein and shared his way of conceiving research, as well as his ideas on the teaching of mathematics (Giacardi 2013). The choice of Vailati, who represented secondary school teachers, was also natural, since at that time he was engaged in the work of the Royal Commission for the reform of secondary school and Klein, whom he had met in Göttingen in 1899, was one of his points of reference (Giacardi 2009a). Moreover, Italian was one of the four official languages, together with French, German and English.

## 2 The First Actions of the Italian Subcommission: The Meetings in Milan and Cambridge

The period we are dealing with, which goes from the origins of the International Commission on Mathematical Instruction to the declaration of its dissolution in 1921 (*EM*, 21, 1920–21, pp. 317–318), was rightly called the "Klein Era", because the charismatic figure of Klein, who combined a broad mathematical knowledge with a profound interest in education, marked strongly the early actions of the Commission. His influence is also evident in the work of the Italian subcommission, which closely reflected the ups and downs of ICMI.

The first meetings of the Italian subcommission were held in Padua on the 21st and 22nd of September 1909, on the occasion of the Congress of the Associazione Mathesis, the national association of mathematics teachers. There is a backstory that deserves to be mentioned. At that time, Francesco Severi (1879–1961), a young but renowned geometer, was president of the Mathesis. The Association's aim was precisely that of improving the school system and training mathematics teachers, so Severi attempted to become officially involved in the work of the Italian subcommission and his reasons were not groundless.

To reach his objective, in April, he wrote to the Ministry of Education. In his letter, Severi complained because the Mathesis had been left aside for the appointment of the delegates. Furthermore, he threatened to carry out his own inquiry on the teaching of mathematics in the Italian schools, an inquiry requested by ICMI, and asked for funding equal to that assigned to Italian delegation, if any. He also sought the support of Vito Volterra,[7] a famous mathematician and senator of the Italian Kingdom, and even suggested that Vailati should be encouraged to resign. He wrote: (My translation)

[7] F. Severi to V. Volterra, Padua, 13 April 1909 in Nastasi (2004, pp.176–178). In this letter a copy of the document written to the Ministry is included.

> Poor Vailati, afflicted as he is by his long illness, might do well to step down … and then much could be put to rights by having a replacement elected by the Mathesis.[8]

This behaviour on Severi's part led to the resignation of Conti, member of the Board of Directors of the Mathesis and collaborator of the Italian subcommission.[9] Severi's ambition to occupy top-level positions within the mathematics and the academic communities was well known to Castelnuovo and Enriques, who believed that it was important that the subcommission, while collaborating with the Mathesis, maintained its "freedom to act" and not be obliged to conform to the directives of this Association. Thanks to Volterra, an agreement was found, and in May, when Vailati died, his replacement was to be elected by the members of Mathesis. One condition, posed by Enriques and Castelnuovo, was that the replacement be a secondary school teacher.[10]

The Italian subcommission was formed during the Padua Congress of the Mathesis: Enrico D'Ovidio from the University of Turin, the author of a successful textbook of geometry for secondary schools, was nominated president, and Castelnuovo, professor at the University of Rome, was chosen as Secretary General. Vailati was replaced by Gaetano Scorza, at that time professor at the Technical Institute of Palermo. The other members of the commission were chosen from among both university professors with ample experience in education and secondary school teachers, who were almost all directors of journals devoted to mathematics teaching: S. Pincherle (University of Bologna); F. Severi (University of Padua and president of the Associazione Mathesis); C. Somigliana (University of Torino); G. Veronese (University of Padua); A. Conti (Scuola normale Margherita di Savoia, Rome, and director of *Il Bollettino di Matematica*); G. Fazzari (Liceo Umberto I, Palermo, and director of the journal *Il Pitagora*); G. Lazzeri (Accademia navale, Livorno, and director of *Periodico di Matematica*); and U. Scarpis (Liceo Minghetti, Bologna).[11]

In the course of these first sessions in Padua, and in keeping with the instructions of ICMI, a plan of activity was defined, assigning various members to report on mathematics teaching in primary and normal schools (Conti), in the secondary schools in Italy (Fazzari, Scarpis, Scorza e Lazzeri), on the mathematical training of engineers (Somigliana) and on the preparation of future teachers (Pincherle).[12]

To direct the work of those preparing reports, Castelnuovo illustrated the activities of ICMI and suggested the following issues to be addressed:

> What means can be used to increase the effectiveness of mathematics teaching in secondary schools?
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> Which methods of teaching, different from those ordinarily used, were adopted and how successful were they?

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[8] F. Severi to V. Volterra, Padua, 20 April 1909 in Nastasi (2004, p. 180).

[9] F. Severi to V. Volterra, Padua, 14 April 1909 in Nastasi (2004, pp. 179–180).

[10] See Commissione internazionale per l'insegnamento matematico, *Bollettino della Mathesis*, 1909, pp. 53–56.

[11] *L'Enseignemnt Mathématique* (*EM* from now on) 12, 1910, pp. 135–136.

[12] Ibidem, p. 136.

> Is it useful to experiment with new teaching programmes and methods on a small scale before imposing them on all schools?
>
> Which means are best adopted for keeping all secondary school teachers informed about the new educational or scientific views? (Castelnuovo 1909, p. 52, my translation)

In any case, the Italian subcommission, according to him, was not to "deal only with statistical data; rather, it should address the investigation in higher fields and to deal with pedagogical and psychological issues" (Castelnuovo 1909, p. 2).

It is also worth mentioning that in his opening speech of the Congress, Loria, after illustrating the vicissitudes of the secondary schools in Italy, dwelt on the reform movements in France and in Germany, paying particular attention to the proposals by Klein (Loria 1909, pp. 20–23).

In 1911,[13] four new reports were assigned: one on geometry textbooks for secondary schools (Scorza); one on proposals for improving the teaching of mathematics from primary schools to the Scuole di Magistero (Teacher Training Schools) (Alessandro Padoa); another on the evolution of the teaching of geometry at university (Severi); and one on the proposals for reforming the teaching of mathematics in secondary schools (Giovanni Vacca).[14] An investigation into the evolution of the teaching of infinitesimal analysis at university was also planned, but the report was never assigned.

All the planned studies were carried out except for the last three. Eleven reports[15] were published between 1911 and 1912, and the results of the Italian subcommission's work in the Italian journals dedicated to mathematics teaching were widely disseminated. In particular, from 1911 to 1915, the *Bollettino della Mathesis* published information on ICMI, translations or summaries of general reports and, with different page numbering, the Italian reports in the supplement *Atti della Sottocommissione italiana per l'insegnamento matematico*[16]; from 1909 to 1914 *Il Bollettino di Matematica* gave room to ICMI and to investigations carried out in Italy, both in the section "Programs and relative proposals for reform" and in the column devoted to congresses and sometimes in the special section *Atti della Sottocommissione Italiana della Commissione Internazionale dell'insegnamento matematico*.[17] It is not surprising that the reports appeared in each of the two jour-

[13] *EM* 13 1911, p. 113.

[14] See G. Castelnuovo to Giovanni Vacca, Rome 3 January 1911, in Nastasi and Scimone (1995, pp. 45–46). In this letter Castelnuovo states that Vacca should prepare a report on the reform proposals of the Royal Commission and in particular on the mathematical programmes drafted by Vailati. He also adds: "Not only you do know the needs of our secondary education very well, but you have that broad-mindedness, which is so rare in the teachers of our schools". (my translation).

[15] The summaries (by E. Chatelain) of six reports (two by Lazzeri, two by Conti, one by Padoa and one by Scorza) of the Italian subcommittee were also published in *EM* 14, 1912, pp. 249–253, pp. 416–424.

[16] See *Bollettino della Mathesis*, 1911, pp. 1–14, 15–23, 25–33, 35–48, 49–80, 81–95, 97–110, 111–178, 179–214; 1912, pp. 131, 215–234, 235–247; 1914, pp. 85–108; 1915, pp. 45–50.

[17] See *Il Bollettino di Matematica*, 1909, pp. 38–39, 272–274; 1910, pp. 236–238; 1911, pp. I, 94–96, 133, 134–157, 157–192, 203–204, 213–248, 249–318; 1912, pp. 206–208.

nals, because the first was directed mainly to mathematicians and secondary school teachers, while the second was addressed especially to middle schools, including the "normal schools" for training primary teachers. In contrast, the *Periodico di Matematica*, after having published the full agenda of ICMI in 1909, once translated into Italian, reported on it only briefly until 1914.[18]

The president of the Mathesis from 1911 to 1914 was Castelnuovo, and the directors of the other two journals were, respectively, Alberto Conti and Giulio Lazzeri; all three were members of Italian subcommission, thus making it easy to understand the particular attention given to the activities of ICMI.

## *2.1 The Meeting in Milan*

The first plenary meeting of the International Commission on Mathematical Instruction took place in Italy in Milan from the 18th to the 21st of September 1911 (*EM* 13, 1911, pp. 437–511). Two special subcommittees had been appointed at an earlier date to prepare for the meeting:

- Subcommittee A (for secondary teaching): E. Beke, Ch. Bioche, F. Klein, W. Lietzmann, G. Scorza, and J.W. Young
- Subcommittee B (for university teaching): C. Bourlet, H. Fehr, F. Klein, C. Somigliana, H. Timerding, and W. Wirtinger

Klein was included in both of them, and also members of the Italian subcommission were involved.

Subcommittee A was in charge of dealing with the questions concerning rigour in middle school teaching and the fusion of the various branches of mathematics; subcommittee B was assigned to consider the teaching of mathematics to students of physical and natural sciences.

Castelnuovo was charged with presenting the general report on the question of rigour (*EM* 13, 1911, pp. 461–468). In order to be able to compare the methods employed in the various countries, he focused his report on a single type of school (high schools with a humanistic orientation) and a single area of mathematics (geometry). Taking a number of school textbooks by renowned mathematicians as examples, Castelnuovo divided teaching methods up into the following kinds: (A) the purely logical method (e.g. Peano, Hilbert, Halsted), (B) methods based on empirical principles and logical development (e.g. Sannia-D'Ovidio, Veronese, Enriques-Amaldi), (C) methods consisting in alternating and mixing intuitive and deductive considerations (e.g. Borel) and, finally, (D) the intuitive-experimental method (e.g. Perry). The methodological approach denoted by (B) was further subdivided into three subgroups according to whether all the necessary axioms (BA)

[18] See *Periodico di Matematica*, 1909, pp. 184–190; 1912, pp. 43; 1913, pp. 95–96; 1914, p. 190–192.

are enunciated, only one part of them is given (BB), or only those that are not evident (BC) are explicitly set out.

Castelnuovo observed that from the reports received, it appeared that the Latin countries (Italy, France, French Switzerland) preferred method B, the Germanic nations (Germany, Austria, German-speaking Switzerland) method C and that England had passed from method BB to method BC. He also illustrated the evolution of the methods in the main countries and noted that while teachers in France, Germany and England, while remaining within the scope of B, had shifted to methods where more importance is given to intuition, in Italy, the inverse trend had occurred.

At the conclusion of his report Castelnuovo mentioned some unanswered questions: What difficulties do the students encounter in following the logical development of geometry, or the development of geometry based on experience? What can be done to overcome these difficulties? What are the results in both cases, not only as regards the usefulness for the students but also their overall culture?

After an initial comment by Klein, who underlined the profound difference between the textbooks and the actual teaching, an interesting discussion concerning Castelnuovo's report took place between Veronese, D'Ovidio, Bourlet, Hobson, Dintzl, Lietzmann and Enriques. While the comments of foreign mathematicians focused primarily on the situation of their own countries, the Italians preferred methodological questions. Veronese stated that excessive rigour was to be avoided in secondary school teaching. He also maintained that experimental intuitive teaching had to pave the way for deductive teaching and that the propositions presented without proofs (axioms or not) had to be obvious and explicitly stated. Regarding this, Enriques underlined the difference between the intuitive and the experimental method and cited as an example Vailati, who opposed the intuitive approach, but believed that it was important that logical rigour be flanked by actual geometric experiments.[19]

The general lectures were given by two Italians, one being Enriques and the other Giuseppe Colombo. First, Enriques illustrated the profound links between mathematics and the theory of knowledge, drawing examples from history and showing the importance of reinvigorating the dialogue between mathematicians and philosophers. Second, Giuseppe Colombo, rector of the Politecnico di Milano, addressed the problem of the mathematical preparation of future engineers and put forward the proposal to establish a minimum programme containing all the basic subjects that an engineer had to know regardless of his particular specialty and also suggested offering the subsequent possibility of choosing complementary and special courses according to individual attitudes and preferences.

[19] *EM* 13, 1911, pp. 464–468.

### 2.1.1 The First Group of Reports for ICMI by the Italian Subcommission

During the meeting the various national subcommittees illustrated the work done. The work done by the Italian subcommission appears to have progressed less than that of the other major countries due to the delay in releasing the promised funding by the Minister of Education.[20] It presented the five published reports among those that had been planned, providing a detailed picture of the organization of secondary schools (classical and technical), of the teacher training schools and of the first 2 years of university, highlighting defects and making proposals.

In order to appreciate correctly the influence of ICMI's activities on the Italian educational system, it is useful to look briefly at each report.

Scarpis and Fazzari (Scarpis 1911 and Fazzari 1911), who dealt with classical-humanistic schools (*ginnasio-liceo*),[21] observed that from the Coppino Decree of 1867[22] until the time of writing, the mathematics in these schools had increasingly lost importance because the legislation had relegated this discipline to a secondary status, with less than half of the number of hours allotted for Latin and Italian. In order to address this situation, they put forward the following proposals: increase the number of hours allocated to mathematics; restore the final written and oral exams; abolish the option of choosing between Greek and mathematics, introduced in 1904 by the Orlando decree; adapt the teaching to the psychological and mental development of the student; establish connections between the various sectors of mathematics; start from problems or questions of practical nature; present examples of applications to other sciences; and eliminate some topics and introduce new subjects.

In his report, Pincherle (Pincherle 1911) illustrated the shortcomings of the teacher training schools and proposed to establish, after the first 2 years of university, a special school leading to an educational degree (*laurea didattica*), distinct from that in pure mathematics (*laurea scientifica*), in which emphasis was placed on elementary mathematics considered from a scientific, critical and pedagogical point of view.

The organization of the first 2 years of university, which allowed access both to the continuation of university studies in mathematics, physics or chemistry and to the schools for engineers, was discussed by Somigliana (Somigliana 1911), who identified two approaches, the classic and the applied one; this last was gradually expanding with the establishment of the Politecnico di Torino and was aimed at the basic mathematical studies of engineering and applied sciences in general. Somigliana also reviewed the various teaching sectors and cited the most important treatises related to them.

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[20] See the *Bollettino della Mathesis*, 1910, pp. 30–31.

[21] See the inquiry proposed by them in *Il Bollettino di Matematica*, 1909, pp. 272–273.

[22] In 1867 the Coppino Decree had imposed a "strange" timetable (Scarpis 1911, p. 26): of the 8 years of *ginnasio-liceo*, mathematics was only taught in 3 of them, and beginning only in the fifth year with arithmetic—without prior teaching: moreover, the last year, which prepared for the final exams, was also without mathematics (Giacardi and Scoth 2014).

Scorza (Scorza 1911) presented an extremely detailed report on schools and technical institutes, which was also the result of an inquiry carried out among about sixty teachers.[23] After a historical introduction, he provided statistical data and gave information on the programmes of the various sections of the technical institutes, pointing out that some colleagues introduced the concepts of function, limit and derivative on their own initiative, even in the absence of ministerial decisions. The main defect of these schools was the "double soul", that is, the fact that they had to prepare students, on the one hand, for administrative jobs and, on the other, for university studies. Further, Scorza criticized the teaching method, which tended towards exaggerated rigour or rigid purism, and underlined the fact that little or nothing was done to orient secondary school teaching towards university teaching or to coordinate it with that of closely related sciences. Conversely, in Europe, radical advancements had been made thanks to the reform movements of Klein, Perry and Borel.

## *2.2 The Meeting in Cambridge*

During the Congress organized the following year by ICMI at the fifth International Congress of Mathematicians held in Cambridge (United Kingdom) from the 21st to the 27th of August 1912, the contribution of the Italian subcommission was not particularly significant, even though Castelnuovo and Enriques were part of the International Committee of the ICM and eight members or collaborators of the subcommission participated in the Congress.[24]

The Italian subcommission contributed only to the first of the two inquiries proposed by ICMI, "The mathematical training of the physicist in the University"; instead it did not send any contribution to the second inquiry, "Intuition and experiment in mathematical teaching in the secondary schools". The general report on the first inquiry was presented by Carl Runge from Göttingen (*EM* 14, 1912, pp. 495–507), and that on the second was compiled by Smith (*EM* 14, 6, 1912, pp. 507–534). It is rather strange that the Italian subcommission did not intervene in the discussion on the role of intuition and experiment because the debate on that theme was lively among the Italian mathematicians. In any case, the contribution to the discussion was reduced to comments given by Enriques following Runge's report. Enriques reproached the compiler of the report for adopting a "utilitarian" approach, paying attention, above all, to what kind of mathematical instruments are necessary to physicists; Enriques considered another problem to be of greater importance, namely, how to attract young people to the study of physics. In particular, he maintained that the current instruction in mathematical physics was characterized by an excessive development of the algorithmic aspects to the detriment of intuition (*EM* 14, 1912, p. 503).

[23] See also *Il Bollettino di Matematica*, 1911, p. 184.

[24] They were Castelnuovo, Enriques, Padoa, Pincherle, Severi, Somigliana, Vacca and Veronese.

### 2.2.1 The Second Group of Reports for ICMI by the Italian Subcommission

Notwithstanding the poor contribution, the Italian subcommission had carried out six other reports - by Lazzeri, Scorza, Conti and Padoa - that Castelnuovo presented briefly during the meeting (*EM* 14, 1912, pp. 488–489). Lazzeri presented two reports: in the first (Lazzeri 1911a), he considered the teaching of mathematics in industrial, professional and commercial schools that had sprung up throughout Italy by local initiative and, therefore, displayed an uneven organization and different curricula. For this reason he limited himself to exposing the aims and the programmes of the secondary commercial school in Florence and of the commerce high school in Bari. The report also contains statistical data related to the period between 1908 and 1909 and concludes with some reflections by Corrado Ciamberlini on industrial schools, which showed that the main defects of these schools were due to the lack of adequate textbooks. The second report (Lazzeri 1911b) concerns the teaching of mathematics in the Royal Naval Academy in Livorno, where Lazzeri himself taught, and in the Royal Military Academy in Turin, which since 1896 had assumed the character of a university. In both of these, algebra (determinants, linear systems, complex numbers, etc.), infinitesimal calculus, analytical and projective geometry (as well as descriptive geometry in Turin) and mechanics were taught with few differences and with a curriculum somewhat reduced, compared to those of the first 2 years of university.

In his report on the normal schools for the training of primary school teachers (Conti 1911a), Conti, a professor in the Royal Normal School "Margherita di Savoia" in Rome, traced the history of the legislative measures starting from the establishment of the first *Scuole di Metodo* at the beginning of the nineteenth century and examined the various programmes. He pointed out the defects of these schools, making reference to a report by his colleague Ersilia Bisson-Minio, and presented the recent reform proposals. In his second report, on the kindergartens and elementary schools (Conti 1911b), he underlined the adoption of the Frobelian method in the former and of the intuitive-experimental method in the latter.

The report by Padoa (Padoa 1912) presents a comprehensive plan to reform the teaching of mathematics from primary schools to the teacher training schools. In particular, as far as secondary schools are concerned, he maintained that mathematics teaching should be divided into three phases: preparatory (3 years), deductive (3 years) and complementary (2 years). In the preparatory phase the teacher should provide the basic notions of arithmetic and geometry by using intuition and experiment, while in the deductive phase the approach should be exclusively deductive. Depending on the type of school, the complementary phase should offer different themes such as reflections on the foundations of mathematics, notions on functions, complements of algebra, etc. Padoa, although not part of the Italian subcommission, had been involved in its activities as one of the best representatives of the Italian School of mathematical logic with experience of teaching in secondary schools, as Castelnuovo explicitly pointed out (*EM* 14 1912, p. 489).

Scorza, who at the time still taught in secondary schools, reviewed the most important textbooks of geometry for upper secondary schools, analysing the approach of each to the theories of congruence, equivalence and proportions (Scorza 1912). In particular, he underlined the fusionist approach of Lazzeri and Bassani, the introduction of the group of motions by Michele De Franchis and Faifofer's adoption of the theory of equivalence of Duhamel.

Reading all the reports, some remarks arise naturally. First, they show two different positions in the debate on intuition and rigour in mathematics teaching in Italy. For example, Padoa affirms that in the upper secondary school, the teaching of mathematics must be exclusively deductive without ever recourse to intuition or experience. Instead, Scorza criticizes a teaching method inspired by a rigid purism; he prefers those authors of texbooks who are able to successfully combine scientific and didactic needs, balancing rigour and intuition, such as Giuseppe Veronese and Enriques and Amaldi, while recognizing that the majority of Italian textbooks were oriented towards a logically rigorous presentation of elementary geometry. Moreover, we can notice that even though at the beginning of the nineteenth century the Italian secondary schools had many defects, there was a group of outstanding university professors and talented secondary school teachers, who were deeply interested in educational questions and tried to improve mathematics teaching through different channels: textbooks, journals, information on European reform movements and legislative measures. Finally, the role of the Italian School of algebraic geometry - Castelnuovo, Enriques, Loria and Scorza were members of it – is evident throughout the entire period in question.

## 3 The Paris Congress in 1914 and the Dynamic Participation of the Italian Subcommission

At the Heidelberg meeting (the 21st–23rd of July 1913), the Central Committee of ICMI decided to increase its membership from four to seven by adding three members, including Castelnuovo.[25] On that occasion the Commission resolved to focus its attention on the two following questions:

A. The results obtained by the introduction of differential and integral calculus into the upper years of middle school
B. The place and role of mathematics in higher technical instruction (*EM* 15, pp. 394–395)

To this end, two questionnaires were drawn up in the four official languages (French, German, English, Italian), with the aim of presenting the general reports during a congress to be held in 1914 in Paris.

[25] The others were E. Czuber from Vienna and J. Hadamard from Paris. See *EM* 15, 5, 1913, pp. 394–412.

The Paris Congress of ICMI (the 1st–4th of April 1914)[26] can be considered the first international conference on the teaching of mathematics. The two debated themes reflected the main interests of Klein, who was, however, unable to attend the meeting for health reasons.

Klein invited Castelnuovo to give the opening address in his stead, and together they drew up the outline. This choice is no coincidence. In fact, Klein knew that Castelnuovo shared his point of view on education and thus would have accepted his advice.[27] In his speech, after illustrating the aims of ICMI, Castelnuovo mentioned the important reform movement that, starting in 1902, had introduced the elements of infinitesimal analysis in French secondary schools. This was just one of the reasons for choosing Paris as the venue of the Congress. He concluded his talk by underlining how important it was that the mathematicians working in pure research also concern themselves with the problems related to teaching:

> We sometimes wonder if the time we devote to teaching issues would not have been better used in scientific research. Well, we answer that it's a social duty that forces us to deal with these problems. (Castelnuovo 1914a, p. 191, my translation)

The general reports on the two questions, which summarized the national reports, were given, respectively, by the Hungarian mathematician, Emanuel Beke, and by the German geometer, Paul Staeckel. Beke's report (Beke 1914) shows that the countries where the elements of infinitesimal calculus were included in the official syllabuses or in the curricula drawn up by the schools themselves were the following: some German territories (Bavaria, Württemberg, Baden, Hamburg), Austria, Denmark, France, the British Isles, Italy, Romania, Russia, Sweden and Switzerland. Beke then dealt with the following points: the scope of the teaching of infinitesimal calculus; its applications (geometric, physical, etc.); the question of rigour; the interactions between calculus and other subjects; and, finally, the reactions from secondary and university teachers. With regard to this point, Beke identified a singular fact: while secondary school teachers were generally enthusiastic about this innovation, university professors expressed a certain degree of coolness, if not hostility. According to Klein, the reason for this behaviour was due to the lack of rigour of the textbooks of infinitesimal calculus and to the gulf that existed between teachers and academics.

The report on the Italian situation was prepared by Castelnuovo, who took this occasion to illustrate the syllabus of the new *liceo moderno* that he had just designed, officially introducing for the first time the fundamental concepts of infinitesimal calculus in an Italian secondary school. He also stressed that, according to him, concepts that are too difficult for an average student (such as the Taylor series) should be left aside and that the teacher should likewise avoid the rough empiricism which conceals the logical character of mathematics and the subtle criticism that the

[26] *EM* 16, 3, 1914, pp. 165–226; 16, 4–5, 1914, pp. 245–356.

[27] See the letter by G. Castelnuovo to F. Klein, Rome, 3 March 1914: "Following your instructions, I have prepared the speech (rather short) that I am supposed to give in Paris in your place" in Luciano, Roero (2012, p. 209, my translation).

minds of students cannot appreciate at that age (*EM* 16, 1914, p. 295).[28] In this regard, Beke noted:

> We await with keen interest how the principles of the Calculus will be presented to the students in the country of mathematical criticism where Dini, Genocchi and Peano have treated these principles in a masterly way. We can be sure that if the work is done by the same men who, in their geometry textbooks, so interesting but so difficult to read in other countries, have sought with ability to reconcile a rigorous scientific treatment with the aims of the secondary teaching, our reform movement will be infinitely beholden to our Italian counterparts. (Beke 1914, p. 255, my translation)

In his report on the place and role of mathematics in higher technical instruction (Staeckel 1914), Staeckel dealt with a number of questions: (1) what kind of teaching should be provided; (2) which subjects, methods and books should be used; and (3) who should teach mathematics, whether mathematicians or engineers themselves. The prevailing opinion in the various national reports was that an engineer should receive a solid mathematical training, which, however, should forgo questions that are too specialized and marked by a purely theoretical interest.

A very lively discussion followed the two reports (*EM* 16, 1914, pp. 290–306, 328–356). The comments of the representatives of the Italian group were numerous and repeated on both themes and show once again two different points of view on the teaching methods. Padoa stressed the need to avoid the infinitesimal pseudo-intuition: a rigorous approach does not exclude appeals to intuition, but refuses that these appeals are surreptitiously made in definitions and demonstrations. He also maintained that the course of mathematics for engineers should not be distinguished from that for mathematicians by rigour but rather by the choice of topics to be treated, because without rigour there is no mathematics. Castelnuovo, on the other hand, was convinced that in addressing future engineers, the teacher should not give the illusion that the rigorous development of a theory is sufficient to carry the results into the applications because there is an abyss between theory and practice. For this reason he steered the debate towards the following two themes: (1) the place of mathematics in the syllabus for future engineers and (2) the style of teaching that should be adopted for the two categories of engineers, the technicians, who apply the already established science, and the theoreticians, who build the science of the engineer. Gino Fano, Enriques and Loria underlined above all the importance of not separating theory from practice and the utility for mathematics students to share some courses with engineers so as not to lose contact with real-world applications.[29]

All the above shows quite clearly the constructive participation of the Italian subcommission in the Paris Congress and the role of Castelnuovo and his driving force. He not only combined his own mathematical prestige with a vision of mathematics teaching very close to that of Klein but he also had a strong democratic spirit. From the beginning this attitude had driven him to involve both secondary

[28] See also Castelnuovo (1914b).

[29] See the debate on the same theme during the third Congress of the Mathesis Association held in Genoa in 1912, in *Atti*, 1913, p. 78–87.

school teachers and mathematicians such as Vailati and Padoa, whose point of view differed from his own in many respects but who shared his deep and sincere interest in the problems of education.

## 4 The Impact of the Earliest ICMI Activities on Italian Education Policies: Legislative Measures, Debates, Courses and Textbooks

In the years that preceded the establishment of ICMI, the Italian secondary education presented evident deficiencies that emerged from the poor results obtained by students in the final mathematics examinations and from the various inquiries made by the Ministry of Education. The causes were mainly due to the low level of preparation of teachers, the inappropriate teaching methods, the strong pre-eminence of humanities in the curricula over scientific disciplines and the persistence of large disparities between the different Italian regions.

To deal with this situation, on the 19th of November 1905, the Minister of Education, Leonardo Bianchi, appointed a Royal Commission for the Reform of Secondary Education, which included university professors, secondary school teachers and ministerial inspectors. The work of the Commission was completed in 1909; however, the proposed reform was never approved by the Ministry of Education (Giacardi and Scoth 2014). The curricula for mathematics and the methodological instructions had been prepared by Vailati, who introduced the concepts of function and the elements of calculus in the upper secondary schools following Klein's ideas (Giacardi 2009a).[30]

Klein's vision on education was well known in Italy by the members of the Italian School of algebraic geometry (Giacardi 2013), which included Castelnuovo and Enriques. However, the first activities of ICMI contributed to the spread of his ideas among teachers and also fostered an international perspective. In particular, Klein's thoughts on education inspired some changes in the curricula, influenced the debate on teaching methods and on teacher training and favoured the publication of new textbooks and the introduction of new university courses.

In this process, Castelnuovo, who was also president of the Mathesis Association from 1911 to 1914, played a key role. He was supported not only by a very active group of mathematicians, such as Enriques and Loria, but also by some talented secondary school teachers such as Vailati, Scorza, Scarpis, Padoa, Fazzari and Conti.[31] This is not surprising because at that time the best secondary school teachers

[30] In the Vailati archives, various documents give evidence that, even before the establishment of ICMI, he gathered a lot of information about the curricula in the European countries and the recent reform movements. See FVM, Cart. 41, fasc. 346, Cart. 31, fasc. 272.

[31] All of them had taught – and in most cases they continued to teach – in secondary schools, but they also had contacts with university. Vailati gave university courses at the end of the nineteenth century, when he was in Turin; in 1912 Scorza obtained a professorship at the University of

often also held classes at the university, and university professors often began their careers teaching in secondary schools. This fostered dialogue between the two.

In 1911, the Minister of Education, Luigi Credaro, established the *Liceo moderno* (modern secondary school), which differed from the *Liceo classico* from the second year. In the programs of the new kind of school, Greek was replaced by a modern language (German or English), the development of scientific subjects was broadened and elements of economic and juridical sciences were added. It was Castelnuovo who, together with the ministerial inspector Mineo Chini, designed the syllabus of mathematics and related instructions.[32] He accepted a number of Klein's proposals by introducing the notion of function and the concepts of derivative and integral, attaching a greater importance to numerical approximations and coordinating mathematics and physics teaching. He wrote:

> But if we truly wish the middle school student to feel an inspiring breeze in this modern mathematics, and perceive something of the grandeur of its whole structure, it is necessary to speak to him of the concept of function and show him, even summarily, the two operations that constitute the foundation of infinitesimal calculus. In this way, if he will have a scientific spirit, he will acquire a more correct and balanced idea of the exact sciences nowadays. (Castelnuovo 1919, p. 5, my translation)

On that occasion Castelnuovo wrote to Klein:

> With regard to teaching, sure that you will accept with pleasure the news, I am going to inform you that the (modern) programs of mathematics which I have adopted in the *licei moderni*, have been so well received that the Ministry of Education now thinks of introducing them even in the *licei classici* and in the technical institutes, by developing further, in these latter schools, the infinitesimal calculus. (My translation)[33]

Among the various textbooks written for this new kind of school, those worthy of note are Enriques and Amaldi's *Nozioni di matematica ad uso dei licei moderni* (1914–1915) and Sebastiano Catania's *Corso di algebra elementare per i licei classici e moderni secondo i nuovi programmi* (1914). Although both of these textbooks seek to follow the guidelines laid down by Castelnuovo, they show two different methodological approaches. While the second is strongly influenced by Peano's School of logic, the first is inspired by the reform movement "which is reflected in the work of the International Commission for the reform of the teaching of Mathematics" (Enriques and Amaldi 1914–1915, I p. IV). In fact, the authors of *Nozioni di matematica* open the textbook with a chapter on approximate measures and irrational numbers, discuss the calculation of areas and volumes from an

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Cagliari; besides teaching in secondary schools, Scarpis was *libero docente* (free lecturer) at the Bologna University; and since 1911 Padoa held several mathematical courses at the Genoa Naval School. Fazzari and Conti, as mentioned above, were directors of journals devoted to mathematics teaching.

[32] See "Ginnasio – Liceo Moderno. Orario – Istruzioni – Programmi", *Bollettino Ufficiale del Ministero dell'Istruzione Pubblica*, XL, 45, 30 ottobre 1913, pp. 2791–2795, also in http://www.subalpinamathesis.unito.it/storiains/uk/liceomod.pdf. Loria illustrated the new programmes in a German journal, see Loria (1914).

[33] See the letter of G. Castelnuovo to F. Klein, Rome 10 March 1915, in Luciano, Roero (2012, pp. 212–213).

elementary point of view by establishing connections between geometry and algebra, introduce the concept of function with ample use of grid paper, present elementary functions and trigonometry with particular attention to practical problems and introduce the concepts of limit, derivative and integral. They try to show the unity of mathematics, making evident the connections between the various branches; they "abolish the boundary" (Enriques and Amaldi 1914, vol. 1, p. III) between elementary and higher mathematics and between mathematics and the other sciences, from which are drawn problems, exercises and examples, especially to illustrate the concept of function.

The whole programme of the third Congress of the Mathesis Association (Genoa, 21–24 October 1912) also showed the influence of ICMI discussions and, in particular, of Klein, including the choice of the theme to be debated (ordering of the scientific and technical studies leading to the diploma of engineer) and the repeated references to the German mathematician in the lectures. Moreover the meeting was organized in connection with that of the Italian Society for the Progress of Sciences and with the support of the Italian Electrotechnical Association and the Physics Society, thus favouring the exchange of views between pure and applied sciences.

First, in the various sessions of the Congress, space was made for debate on the reports presented by the Italian subcommission of ICMI, which had not been previously discussed, as can be inferred from the complaints of Veronese (*Atti* 1913, p. 82). Moreover, Castelnuovo, in his opening speech as president of the Mathesis (Castelnuovo 1913), illustrated the basic tenets of his vision of the teaching of mathematics, which was very close to that of Klein: to avoid excessive specialism; to break down the barriers between the various observational sciences; to support theory with experience; not to disregard approximations; and to bring theory closer to applications. He also cited ICMI work throughout the Congress.[34] In his plenary lecture, Vincenzo Reina, professor of geodesy at the School of Engineering in Rome, dealt with "mathematics of precision and mathematics of approximation" (Reina 1913) and explicitly cited the lessons on this subject given by Klein in 1901 (Klein 1902), maintaining that it was important to complement theory with applications, because the former can provide a solid foundation for applied research and the latter can be a source of new theoretical research.

During this Congress, Castelnuovo also illustrated the innovations he had begun to introduce in editing the *Bollettino della Mathesis* in order to transform it in "a journal of mathematical culture in the broader sense".[35] In particular, he inserted into the *Bollettino* summaries of the activities of the Italian subcommission, translations of lectures and inquiries concerning problems related to mathematics teaching in the various orders of schools. He also encouraged debates concerning method. In this regard, he wrote to Giovanni Vacca:

> Almost unexpectedly and against my will, I have been elected president of the Mathesis. I accept the nomination only because I think that it might be helpful for the affairs of the Italian Commission for mathematics teaching, for which the Bollettino of the Math[esis]

[34] See, for example, *Atti* (1913), p. 71 and 82.

[35] See *Atti* (1913), p. 94.

> will become the publishing organ. I would like keep the level of the Bollettino high, reducing to a minimum the Byzantine discussions in which secondary teachers too often delight. I am therefore very much counting on your cooperation. (My translation)[36]

The influence of the international contacts favoured by ICMI can also be perceived in the discussions concerning teacher training, a topic that was addressed in many of the congresses and meetings of the Mathesis Association (Furinghetti and Giacardi 2012). In 1909, during the Congress in Padua, Castelnuovo explicitly proposed following Klein's example with regard to teacher training:

> At Klein's suggestion, during the spring holidays a number of German universities hold short courses for Middle school teachers. Couldn't we too set up similar courses in our universities? (Castelnuovo 1909, p. 4, my translation)

The points of views emerging from the debates were essentially two. Some, such as Pincherle - supported by Castelnuovo and Enriques - proposed the institution, after the first 2 years of university, of a special school leading to a degree in education (*laurea didattica*) to be attended by all those who intended to pursue a career in secondary teaching; a distinct degree in pure mathematics was instead intended for those who decided to pursue a career in research.[37] Others, such as Padoa, Loria and Giuseppe Peano, disapproved this solution and proposed instituting a 2-year course of Mathematical Methodology. This alternative course would make it possible to present not only topics of arithmetic, algebra and geometry useful for the future teacher but also to include an analysis of the teaching methods and the school textbooks, as well as to use the history of mathematics to reconstruct the development of each theory:

> The new university course we are suggesting would serve, in our opinion, to fill the deplorable abyss that separates university teaching from secondary teaching today, … which F. Klein has recently referred to as 'a system of double forgetting' (*doppelte Diskontinuität*): the university student's forgetting what he studied in secondary school, and the secondary school teacher's forgetting all that he studied while he was at university. (Loria and Padoa 1909, pp. 3-4, my translation)

Like Klein, both groups upheld the importance of elementary mathematics from an advanced standpoint for the preparation of future teachers. Precisely in this light, in 1900, Enriques published the *Questioni riguardanti la geometria elementare*. In the preface, he wrote:

> These topics have recently been developed in a series of conferences held by Mr. Klein, to which we are at least partially indebted for the idea of writing this volume. (Enriques 1900, p. VII, my translation)

This book was augmented and enriched in the successive editions under the new title *Questioni riguardanti le matematiche elementari* (2nd ed. 1912–1914, 3rd ed. 1924–1927).

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[36] See the letter of G. Castelnuovo to G. Vacca, Roma, 27 January 1911, in Nastasi and Scimone (1995, p. 46).

[37] See also the report Pincherle (1911).

For the same purpose, in 1909 during the Congress of Mathesis in Padua, Roberto Bonola presented a project of an encyclopaedia of elementary mathematics addressed explicitly to mathematics teachers and to students of the teacher training schools. This project would be completed only in 1950, but nevertheless shows the spreading of Klein's ideas throughout Italy.

In the years immediately following, Castelnuovo began to include in his courses in higher geometry in Rome a number of topics designed specifically for the scientific and methodological training of future mathematics teachers. From this perspective, the following notebooks are significant: *Geometria non-euclidea* (1910–1911), *Matematica di precisione e matematica di approssimazione* (1913–1914), *Indirizzi geometrici* (1915–1916), *Equazioni algebriche* (1918–1919) and *Geometria non-euclidea* (1919–1920).[38] In the introduction to the 1913–1914 course on the relationship between precise and approximate mathematics, Castelnuovo explicitly discusses the various ways in which future teachers can be trained and quotes Klein:

> The educational value of mathematics would be much enriched if, in addition to the logical procedures needed to deduce theorems from postulates, teachers included brief digressions on how these postulates derive from experimental observations and indicated the coefficients with which theoretical results are verified in real experience … The relationship between problems pertaining to pure mathematics and those pertaining to applied mathematics is very interesting and instructive. Klein, who dedicated a series of lectures to the subject (1901), describes the first of these as problems of 'precise mathematics' and the second as problems of 'approximate mathematics'. In this course we will … more or less follow the general outline of Klein's course. (My translation)[39]

It is no coincidence that the other two delegates, Enriques and Scorza, introduced in Roma and in Naples, respectively, courses of elementary mathematics from an advanced standpoint,[40] and Pincherle, another member of the Italian subcommission, did the same thing for the teacher training school at the University of Bologna from 1899–1900 to 1920–1921.[41] Other mathematicians followed their example, such as Corrado Segre in Turin.[42] Moreover, in 1910 and 1911, Scorza reviewed Klein's work, *Elementarmathematik vom höheren Standpunkte aus*, very carefully and in depth in the journal of the Mathesis Association,[43] so addressing mathematics teachers above all others.

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[38] See http://operedigitali.lincei.it/Castelnuovo/Lettere_E_Quaderni/menuQ3.htm

[39] See http://operedigitali.lincei.it/Castelnuovo/Lettere_E_Quaderni/quaderni/nC913_14A/mostracom.htm, fols. 3–4.

[40] See "R. Università di Roma. Argomento dei corsi superiori di matematica", *Bollettino della Unione Matematica Italiana*, 2, 1923, p. 116; "Università di Napoli: corsi del secondo biennio", *Bollettino della Unione Matematica Italiana*, 1, 1922, p. 35.

[41] All of his annual reports show the importance he attached to elementary mathematics from an advanced standpoint and his growing interest in questions regarding the principles and foundations of mathematics (ASUB, Scuole di Magistero, pos. 53/b, busta 3 (1880–1921)).

[42] See, for example, his 1916–1917 course of advanced geometry *Vedute superiori sulla geometria elementare (1916–1917)* (BMP Fondo Segre, Quaderni. 30: http://www.corradosegre.unito.it/Quaderni/Quad30/1_30.php)

[43] See *Bollettino della Mathesis*, II, 1910, p. 130–146, III, 1911, pp. 48–54. On Scorza's vision of

When the Minister of Education Benedetto Croce abolished the teacher training schools with the Royal Decree dated the 8th of October 1920, some of the most vigorous oppositions came from the Mathesis Association and the two members of the Italian school of geometry, Loria and Fano. In particular, in 1921, during the Mathesis congress in Naples, Fano, who had spent a period of post-graduate work in Göttingen with Klein, energetically suggested the establishment of university courses of elementary mathematics from an advanced standpoint, with an emphasis on the historical, critical, methodological and didactical aspects, citing the lessons of Corrado Segre and Enriques as examples. He also invited faculties to accept dissertations for degree theses in complementary mathematics (*matematiche complementari*), that is, concerning those sectors of mathematics more strictly connected to elementary mathematics (Fano 1922).

The proposals were accepted, at least in part, by the Minister for Education, Orso Mario Corbino, who in 1921 established the "combined" degrees (*lauree miste*) in physical and mathematical sciences[44] aimed at qualifying young people to teach scientific subjects in secondary schools and in 1922 instituted a course in complementary mathematics, accompanied by didactic and methodological exercises.[45]

## 5 A Caesura in the History of the Italian Subcommission

The reasons for the ICMI crisis which, after World War I (1914–1918), led to its subsequent dissolution, have been already clarified by referring also to unpublished documents (Schubring 2008b, pp. 120–127). For this reason we limit ourselves to saying that the attempt by Henri Fehr, ICMI secretary general, to induce Klein to resign and to cede the presidency to Smith at the end of 1914 failed. In fact, Klein had signed the deplorable document, *Aufruf an die Kulturwelt*, which denied the war crimes of the German army, and only much later did it become known that many of the scholars who signed that document, including Klein, were completely unaware of its real content (Schubring 2008a, pp. 18–22). Most of the Central Committee, in particular Smith himself and Castelnuovo, supported him, so ultimately he remained in his office as the ICMI president. Castelnuovo wrote:

> All my colleagues in the Commission, I am sure recognize the admirable work of organization you have accomplished and the mark you have given to our work; they know that no one in this respect can replace you. Moreover, the serious moment that the international institutions are undergoing advises to introduce no change in their organization lest these weak organisms should succumb. On the contrary, we must endeavour to make them survive until the achievement of peace, so that they can ease the resumption of normal relations between peoples, as soon as the war is over … For the moment it is enough that every

mathematics teaching, see (Giacardi 2008b).

[44] See http://www.associazionesubalpinamathesis.it/storia-insegnamento/formazione-degli-insegnanti/#1513288613646-2be1ce59-d822

[45] See http://www.associazionesubalpinamathesis.it/storia-insegnamento/formazione-degli-insegnanti/#1513288614475-cd072737-4285

> country that has the strength to do so, continues the work it has planned. For my part, I will tell you that just now I have agreed with Mr Loria and Mr Pincherle (reporter for Italy) to draft the answers to the questionnaire on the training of teachers. (My translation)[46]

During the war and in the years immediately following, ICMI did not meet, and its activities suffered a sharp slowdown. In 1915, a questionnaire for the "Inquiry into the training of teachers of mathematics in secondary schools in different countries" was published in the four official languages. Loria was entrusted with the general report, and it was Pincherle who was supposed to be concerned with the Italian situation, as it appears from the above letter by Castelnuovo to Klein and also from a letter by Loria to Smith.[47]

From 1916 to 1922, the reports by some national subcommissions appeared in *L'Enseignement mathématique.* Italy was not included among them; the report by Germany was presented by Lietzmann.[48] Loria commented on this and wrote to Smith: "Have you received the German Report related to the Questionnaire? It's almost unbelievable that with so much on their plate, they find time to take care of IMUK!" (My translation).[49]

Meanwhile, on the 20th of September 1920, the International Mathematical Union (IMU) was formed. For political reasons, the former Central Powers (Germany, Bulgaria, Austria and Hungary) were excluded and were not allowed to participate in the International Congress of Mathematicians held immediately afterwards in Strasbourg (22–30 September 1920) (Lehto 1998, Section 2.6). Behind this campaign were French mathematicians, and Fehr supported them. During the congress, the ICMI mandate was not renewed (Schubring 2008a, pp. 23–27). ICMI was dissolved, but the subcommissions were permitted to continue with their activities and were instructed to send the product of their work to *L'Enseignement Mathématique* for publication. Echoes of these events are found in the letter from Klein to Enriques on the 13th of August 1920, in which he regretted that the work of ICMI had come to an end, but welcomed the fact that it could be continued unofficially through the subcommissions. Doing so would make possible the reunification of the Commission, when better times came.[50] In any case, Enriques maintained his contacts with Klein, and, likewise, Castelnuovo was a strong supporter of international cooperation.

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[46] See G. Castelnuovo to F. Klein, Rome, 10 March 1915 in Luciano and Roero 2012, p. 212. See also the letter by Klein to Castelnuovo, Göttingen, 4 March 1915 at p. 211 and in http://www.icmihistory.unito.it/LetteraKleinLAST.pdf

[47] See G. Loria to D. E. Smith, Gênes, 28 April 1915, Smith, D.E. Professional, Special Collections, cit.

[48] *EM* 17, 1,1915, pp. 60–65, 129–145.

[49] G. Loria to D.E. Smith, Gênes, 29 November 1915, DESPP, Box 32. The international events led to the general report being presented only in 1932 during the International Congress of Mathematicians in Zurich (Loria 1933). The Italian report was prepared by the ministerial supervisor Alfredo Perna and not by Pincherle, who in the 1920s had to face bigger problems.

[50] F. Klein to F. Enriques, 13 August 1920, in Luciano and Roero (2012, p. 215)

In connection with the international events, the Italian Mathematical Union was constituted in 1922, with Pincherle as its first president. In the presentation of the programme of the new society, Pincherle hoped for the re-establishment of international collaboration:

> From now on, we can and must firmly hope that the echoes of the world conflagration having died away, the International Union shall fully live up to its name, gathering, in a not distant future, the scientists from all over the world: this is one of the vows of the Italian Mathematical Union. (Pincherle 1922, p. 1, my translation)

In that same year, 1922, following the march on Rome, Benito Mussolini became head of the Italian government, and gradually the Fascist dictatorship was established.

In 1923, Giovanni Gentile, Minister of Education, implemented a complete and organic reform of the Italian school system according to the lines of neo-idealistic philosophy. Secondary education was divided into two main branches: (1) the classical-humanistic and (2) the technical-scientific. The first branch, intended to train the future ruling class, was considered absolutely pre-eminent over the second branch. Moreover, the old programmes, which provided the teacher with detailed outlines of the subjects and also with helpful instructions on methodology, were substituted with examination programmes that indicated the objective, but not the way to achieve it. Protests were of no avail: neither those of the mathematicians nor those of the Mathesis Association and its president Enriques (Enriques 1923) and nor those of the Accademia dei Lincei, which expressed its dissent in a report edited by Castelnuovo (Castelnuovo 1923).

Castelnuovo, who had been invited by Gentile to collaborate in the drafting of the programmes, refused and was replaced by Scorza. Scorza tried to follow those tenets that had inspired the work of ICMI and its president, Felix Klein. In fact, he included elements of differential and integral calculus in the syllabus of the newly established *liceo scientifico* (scientific upper secondary school),[51] but he was constrained by the general framework of the reform, which did not recognize the formative value of mathematics and science, a fact that was in contrast with his own ideas.[52] In 1921 he had expressed his doubts about the neo-idealistic philosophical movement, behind which he seemed to "glimpse a hardly seductive nihilism" (Scorza 1921, p. 19). In 1923, he explicitly stated that "the philosopher who hasn't had a good education in mathematics pays bitterly for his deficiency", and he maintained the importance of a mathematics teaching which was not "passive or dogmatic" but which used an "active and heuristic method", developing both the creative imagination and an acute critical sense (Scorza 1923, p. 83).

The actors were still the Italian delegates of ICMI. Despite this, Gentile's reform, probably even more than the First World War, marked a break in the activity of the Italian subcommission which saw its efforts thwarted. Moreover, Klein, who had been the driving force of ICMI, passed away in 1925.

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[51] See http://www.associazionesubalpinamathesis.it/wp-content/uploads/2017/12/pl26.pdf

[52] See Giacardi 2008b: http://www.icmihistory.unito.it/portrait/scorza.php

The International Commission on Mathematical Instruction - as well as the Italian subcommission - remained stagnant until 1928, when once again an Italian city, Bologna, was the theatre of important events: the International Congress of Mathematicians took place here from the 3rd to the 10th of September 1928. It was organized by Pincherle who, by serving in the double role as President of the Italian Mathematical Union and President of the International Mathematical Union, succeeded in re-establishing international collaboration, removing the previous restrictions due to political reasons (Lehto 1998, section 2.6; Giacardi 2009b).

At the Bologna Congress, on the 4th of September 1928, during the Section VI devoted to elementary mathematics, under the chairmanship of Castelnuovo and at the prompting of Fehr, the mandate of ICMI was "prolonged", requiring that all countries present at the Congress be represented in such a way that would ensure international scientific cooperation. Smith was elected the new president, while Castelnuovo and Jacques Hadamard were the two vice presidents.[53] This rebirth, however, was short-lived because the re-established working group was incapable of producing new ideas and limited itself to finishing up projects that had already been started. Between 1929 and 1933, the reports of the various countries on the modifications made in mathematics teaching since 1910 and on the training of mathematics teachers were published in *L'Enseignement mathématique.*

The Italian reports offer a picture, even though somewhat mitigated, of the situation in Italy after the Gentile Reform. Enriques's account of this reform (Enriques 1929) appears less critical than might be expected: he limited himself to pointing out the reduction in the number of hours devoted to mathematics and the unsolved problem of teacher training. Instead, he gave ample space to the flourishing of new textbooks and presented his many initiatives (Giacardi 2012) aimed at teacher training: in addition to the book dedicated to elementary mathematics, *Questioni riguardanti le matematiche* (the second edition had just been published), he cited the school for specialization in history of the sciences annexed to the Istituto Nazionale per la Storia delle Scienze, which he had created after the reform, and the book series on the history of mathematics, entitled *Per la storia e la filosofia delle matematiche*, which he began in 1925 and which was expressly intended for teacher training.

Equally veiled are the criticisms of the Gentile Reform on the part of Loria in his general report on the theoretical and practical training of mathematics teachers for secondary schools (Loria 1933). The national report prepared by the ministerial supervisor, Alfredo Perna, on teacher training in Italy (Perna 1933) appears to be less ambiguous; in fact, it lists the institutional shortcomings in this area: there were no institutions for professional teacher training; there were no courses on methodology and pedagogy at the universities; there were no scholarships designated for teacher training; life-long learning was not compulsory; and training courses for in-service teachers were left up to individuals.

[53] See *Atti* (1929–1932), I, p. 113; *Il Bollettino di Matematica*, 24, 1928, pp. 153–154; *Bollettino della Unione Matematica Italiana*, 7, 1928, p. 273.

Even if Enriques and Loria prepared the reports for ICMI, the Italian subcommission actually no longer existed: no collective and coordinated activities were undertaken; rather, it was up to individual members to promote initiatives aimed at improving mathematics teaching, which had been undermined by the Gentile Reform. Echoes of the past ICMI activity continued to persist after its dissolution. Here I will cite only a few examples. In 1921 an article by Smith on the reforms of the mathematics teaching in the United States appeared in the *Periodico di Matematiche*, translated into Italian by Alma Enriques, one of the daughters of Federigo (Smith 1921). The following year Loria gave some news on the ICMI work in the *Bollettino di Matematica.*[54] In 1924, Dionisio Gambioli, professor of the Royal Technical Institute in Rome, translated into Italian the work of J. Wesley Young, ICMI delegate for the United States, entitled *The Teaching of Mathematics in the Elementary and the Secondary School* (1908). In the additional bibliography related to the period spanning 1906 to 1913, there is a digression on the activities of ICMI (Young 1924, pp. 519–526). Many years later, Enriques, in the entry "Matematica" of the *Enciclopedia Italiana* (Enriques 1934) and in the book *Le matematiche nella storia e nella cultura* (1938), cited the work of ICMI (Enriques 1938, p. 188).

This situation reflected both the disorientation within ICMI and the political situation in Italy: Fascism, with a series of so-called *leggi fascistissime* (extremely Fascist laws), promulgated between 1925 and 1928, gradually turned into a totalitarian regime, strongly limiting the freedom of the press, of teaching and of association, and ultimately - in 1938 - enacted the shameful racial laws. Castelnuovo and Enriques were targeted by these laws. Further, Scorza passed away in 1939.

The actors of the first phase of the story of the Italian subcommission were out.

## 6 Conclusions

From this historical overview, some concluding remarks can be drawn.

First, it appears that the first period of the Italian subcommission, from its founding up to World War I, was dominated by a charismatic personality, paralleling what took place in the early history of ICMI. Castelnuovo was in fact the driving force of the Italian group; he was the intermediary between ICMI and the educational world, supported by a group of secondary school teachers and university professors, who, in spite of occasional differences in points of view, worked together willingly. In his activity within the Italian subcommission and as president of the Mathesis Association, he succeeded in building an international network and in promoting an increased exchange of information on new movements for reform in Europe, in particular those reforms proposed by Klein, whose methodological approach he wholeheartedly endorsed. He made the *Bollettino della Mathesis* the spokesperson for the work of the International Commission on Mathematical Instruction and the

[54] See *Bollettino di Matematica*, 1922, p. XCIII.

Italian subcommission, introducing reports on their activities, translations of lectures, surveys and debates. He designed the innovative programmes for the *liceo moderno*, devoted some of the university courses to teacher training and dedicated various articles to issues relating to mathematics teaching (Gario 2006, Giacardi 2008a).

Moreover, we clearly see the role of the Italian School of algebraic geometry in the work of the Italian subcommission. Castelnuovo, Enriques, Loria and Scorza were well-known geometers, and all of them were in Turin to discuss or collaborate with Corrado Segre, the leader of this School, who was strongly influenced by Klein both in scientific research and in a vision of mathematics teaching (Conte and Giacardi 2016). Castelnuovo, Enriques and Loria were in contact with him even before the foundation of ICMI[55] and shared his ideas on education, albeit with some differences. In addition, they occupied prominent institutional positions, allowing them to spread information through journals, courses, lectures and reviews. Thus, it seems to me that, if on the one hand it is true that ICMI's activities stimulated the debate about certain themes among teachers and university professors in Italy, on the other hand their impact on the Italian education system went through the influence of Klein on Italian geometers.

Nevertheless, from the reports and – most importantly – from the debates during the ICMI meetings, it is evident that, in Italy, there were two different positions concerning the methodological approach to the teaching of mathematics: that of the Italian School of algebraic geometry was contrasted by that of the Peano School of mathematical logic. Despite the differing views, Castelnuovo always made room for different opinions and even involved Vailati and Padoa, members of the Peano school, in the activities of the Italian subcommission, because of their competence.

Another final remark. In the period between the two World Wars, the activities of the Italian subcommission reflected both the disorientation within ICMI and the political situation in Italy. Its mission continued through some of its representatives, in particular Enriques, Scorza and Loria, but the role of the teachers within the group gradually became increasingly less important, so the connective tissue that bound academic environments with secondary schools was lacking.

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[55] They collaborated on the *Encyklopädie der Mathematischen Wissenschaften* an international publishing enterprise promoted by Klein and other German mathematicians and corresponded with him (Luciano and Roero 2012).

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ASUB: *Archivio Storico dell'Università di Bologna*
FVM: *Fondo Vailati*, Biblioteca di Filosofia, Università di Milano
DESPP: *David Eugene Smith Professional Papers*, Columbia University, Rare Book & Manuscript Library, Correspondence

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