Chapter 7
Portuguese Textbooks on Descriptive Geometry

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Abstract The introduction of descriptive geometry in Portuguese did not begin in Portugal itself but in what had so far been the outskirts of the Empire: in Brazil. There, a commented translation of Monge's treatise was published in 1812, but it was decades later before a descriptive geometry textbook was first published in Portugal. Furthermore most of the first descriptive geometry booklets and textbooks published in Portugal were written for secondary school students, and the publication of a comprehensive descriptive geometry textbook intended for higher education had to wait until the end of the nineteenth century. This chapter will describe the evolution of the teaching of descriptive geometry in Portugal up to the end of the nineteenth century.

Keywords Castel-Branco · Achilles Machado · Shiappa Monteiro · Descriptive geometry · Polytechnic school · Portugal · Mota Pegado · *Universidade de Coimbra* · *Escola Politécnica de Lisboa* · *Academia Politécnica do Porto*

1 Introduction

Portugal was invaded by France in 1807. As a consequence the entire royal family of Portugal, including Queen Maria I (1734–1816) and her son Prince Regent (later King) João VI (1767–1826) escaped to Brazil. King João VI returned to Portugal in

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1821, and the next year Brazil—from where Portugal had been reigned after the end of the Napoleonic invasions—became an independent country.

Before 1807, Brazil had been kept underdeveloped as a colony by the Portuguese crown. For instance, printing presses had not been allowed and were imported only by the royal fleet in 1808. Now, however, all efforts were undertaken to develop this huge country and to establish an educational system and to promote a proper economic and technological system. The first signs of this technical transformation had been the *Real Academia de Artilharia, Fortificação e Desenho* founded in 1792 following the Real Academia de Artilharia, Fortificação e Desenho founded in 1790 in Lisbon. Now, an ambitious institution to educate military and civil engineers for Brazil was created, the *Real Academia Militar* in 1811. It was there that descriptive geometry was taught for the first time in the Portuguese Empire (see Schubring, Chap. 21, this volume). The first professor was an army officer, Captain José Victorino dos Santos e Souza, who published in 1812 a translation of Monge's textbook (Souza 1812) with the title *Elementos de Geometria Descritiva; com aplicações às artes* (Elements of descriptive geometry; with applications to the arts). Its subtitle emphasized the objective of forming the students of the military institution: "Para uso dos alunos da Real Academia Militar". It was not only a translation, though as it had a lengthy introduction, in which the author explained the aim of descriptive geometry. Here is what he wrote about the subject of the book (in a single sentence):

> The goal of descriptive geometry is not only to lead us to a large number of theorems, to give us the solution to an infinite number of problems deduced from the properties possessed by points, lines and planes when we consider them placed in any way in space with respect to each other, and in reference to three dimensions, of which the planar geometry problems, which are usually dealt with in the first approaches to this subject, are nothing but particular cases, but also to a way of representing on a paper the shapes of bodies from nature, or from art, which, as we know, have three dimensions, that is to transmit these descriptions to those instructed in the conventions and methods of this geometry, or to deduce from this representation new truths about their properties, their shapes and their dimensions, in various senses.[1] (Souza 1812, pp. 206–207)

And he emphasized that his desire in publishing the translation was to help raise knowledge of the sciences and of the fine arts in this new world (see Schubring, Chap. 21, this volume).[2] The introduction also mentions Sylvestre-François Lacroix's *Essais de Géométrie sur les plans et les surfaces courbes ou Éléments de Géométrie Descriptive* (Lacroix 1795), which was published in 1795, that is 4 years before Monge's *Géométrie Descriptive* (Monge 1799) (see Barbin, Chap. 1, this volume).

King João VI returned to Portugal in 1821, and Brazil became an independent country the year after.

[1] Translated by the authors.

[2] In fact, the translation of Monge's textbook had no impact in Portugal. For instance, the book is held by neither the National Library in Lisbon nor the library of the Academia de Marinha in Lisbon.

2 Descriptive Geometry at the University of Coimbra

At that time, the only university in Portugal was the University of Coimbra, which was founded in 1290 and had a Faculty of Mathematics from 1772 onwards. At the beginning of the nineteenth century, Coimbra actively fought for its monopoly as the only Portuguese university (Carvalho 1986, p. 567), and new universities (in Lisbon and Porto) would only be established in 1911. Due to the political and social turbulence of this period, the university could not open in some academic years, in particular between 1831 and 1834 when the civil war ended, and lacked teachers until 1837 (Freire 1872, pp. 61–63). The necessary adjustments to the new achievements in mathematical theories were made only after this date.

According to Gino Loria (Loria 1921, p. 402), the French mathematicians whose treatises on descriptive geometry had the strongest influence in Portugal were Olivier (1843), Fourcy (1830), and Leroy (1834) (on these textbooks, see Barbin, Chap. 2, this volume). After 1821, the first Portuguese mathematician to write a book on descriptive geometry was Rodrigo Ribeiro de Sousa Pinto (1811–1891). He was a professor of mathematics and astronomy at the University of Coimbra (and director of the Astronomical Observatory of Coimbra) and it was there that he published the book *Complementos da Geometria Descriptiva de M. de Fourcy* (Pinto 1853). However, as the author himself states (and as the title suggests), the book is not a standalone textbook. It is mostly a collection of observations requiring the original treatise (Fourcy 1830) to be comprehensible. However, this book would have been a major reference for the teaching of descriptive geometry at the University of Coimbra. According to Sousa Pinto (besides de Fourcy's treatise) his knowledge of descriptive geometry was based on Monge's textbook and a little-known booklet by Gascheau (1828) (Barbin, Chap. 2, this volume).

A course on descriptive geometry was created in 1840 as part of the degree in Mathematical Science (Freire 1872, pp. 64–69). It was titled "Descriptive geometry, geodesy and architecture", and it was given in the 4th year of a five-year degree, and the textbook adopted was de Fourcy's treatise (Fourcy 1830). Descriptive geometry was successively merged with other subjects in 1844 and 1852 and acquired the status of a discipline on its own in 1861 with the name "Descriptive geometry; applications to stereotomy, to perspective and to shadow theory". The book *Complementos da Geometria Descriptiva de M. de Fourcy* (Pinto 1853) was adopted in 1853 and, according to (Freire 1872), Leroy's treatise on Stereotomy became the adopted textbook in 1872 (Leroy 1870).

3 The First Standalone Descriptive Geometry Textbook

At last, a descriptive geometry booklet written by an engineer (and army officer), José Frederico d'Assa Castel-Branco (1836–1912), was published in 1873 (Castel-Branco 1873). It was published in Goa, which is currently part of India, but which was part of the Portuguese State of India until 1961. It was a very short work: it was only 47 pages long (including 3 pages containing exercises), and it had

no introduction (although Monge is mentioned on the first page), bibliography or diagrams. It was written for the students of the *Instituto Profissional de Nova-Goa*, an institution of higher learning created in 1871. It so happens that of all the Portuguese colonies of that time, Goa was the one in which the school system was most developed (Pery 1875, pp. 383–384). We found no evidence of its having been used for teaching descriptive geometry in mainland Portugal.

4 The Establishment of Polytechnic Schools and of Industrial Schools

Two schools with the word "polytechnic" in their name were created in Portugal in 1837, the *Escola Polytechnica de Lisboa* and *Academia Polytechnica do Porto*. In fact, both of them were replacements for other pre-existing schools. At the political level, this was mainly due to the action of the Prime Minister Manuel da Silva Passos (better known as Passos Manuel), who felt the need to have more and better educated engineers in Portugal. In spite of the fact that his time in office as Prime Minister was less than a year, his government had a strong impact on higher education in Portugal, mainly because of the founding of these two schools. The presence of the word "polytechnic" in their names suggests that they were modelled on the *École polytechnique*, but it is more likely that they were based upon the *École Centrale* (Basto 1937, p. 152), which had been founded in 1829 by, among others, Théodore Olivier (Barbin, Chap. 2, this volume). This is in fact rather natural since the *École polytechnique* was mainly meant to prepare its students for public service, whereas the training of students at the *École Centrale* was concerned with work in industry (Schubring, Chap. 22, this volume). The first director of the *Escola Polytechnica de Lisboa* was José Feliciano da Silva Costa (1798–1866), a former student of the École des Ponts et Chaussées (Pereira 2009).[3]

Unlike the *École Centrale*, which was a private school until 1857, the Portuguese *Polytechnic* schools were created by the central government and, as a consequence, depended heavily on the changing mood of whoever was in power. They soon became underfunded, as Passos Manuel himself acknowledged in 1857 (Santos 2011, p. 75).

The *Academia Polytechnica do Porto* had a course on descriptive geometry right from its foundation (Pinto 2012) and adopted the textbook by de Fourcy (Santos 2011, p. 70). The first chair from 1838 until 1850 was an engineer, General José Victorino Damásio (1807–1875) (Delgado 1877, p. 9), a remarkable teacher of engineering but with no published texts on descriptive geometry (Pinto 2012, p. 142). The *Traité de géométrie descriptive* by Leroy (1834) was the adopted textbook for almost all of the last quarter of the nineteenth century (Pinto 2012, pp. 218–223). Although the course on descriptive geometry had existed since the

[3]For more about the influence of the *Ècole des Ponts et Chaussées* on the *Escola Polytechnica de Lisboa*, see (Matos 2013).

foundation of the school, the first explicit references to its content relate to the school year of 1879–1880 due to the start of an annual publication, *Annuario da Academia Polytechnica do Porto*. Topics concerning the straight line and the plane were dealt with in a mathematic course, whereas the "descriptive geometry of the curves and tangent planes and of the curves with their tangents" were part of another course. The adopted textbook for both courses was de Fourcy's textbook, from which the exercises and problems solved in class were picked.

The next number of the *Annuario* contained the first known syllabus of descriptive geometry for the *Academia Polytechnica do Porto*. This syllabus was designed in 1880 by the Academic Council and is divided into two parts, the first with 12 lectures and the second with 9 lectures, described as follows (Anonymous 1880–1881, 1886–1887):

- Study of notable curves, especially the helix, epicycloids and involutes of the circle.
- Aim of descriptive geometry. Several projection systems. Representation of points, straight lines, and planes.
- Representation by a single projection with elevation. Problems.
- Intersection of two straight lines and the angle between them. To make a straight line pass through two given points.
- To make a plane pass through three given points. Intersection of a straight line and a plane, angle between them. Intersection of two planes.
- Distance between a point and a straight line and between two straight lines: shortest distance.
- Representation of two curved surfaces. Second degree surfaces. Tangent planes.
- Construction of the plane that is tangent to the cylinder and to the cone, given a point of the contact generatrix.
- Construction of the plane that is tangent to the ellipsoid of revolution or tri-axial, given the contact point.
- Construction of the plane that is tangent to the ellipsoid and that passes through a point outside the ellipsoid. Contact curve of the ellipsoid and of the circumscribed cone.
- Intersections of a plane and a cylinder, a cone, an ellipsoid.
- Intersection of a cylinder or cone with an ellipsoid.
- Intersection of two ellipsoids.

Second part (nine lectures):

- On the generation of surfaces.
- On developable surfaces. Developable helicoid.
- On skew surfaces.
- On the hyperboloid of one sheet.
- On the hyperbolic paraboloid.
- On the skew helicoid.
- Evolutes and involutes. Spherical involute.
- Curvature of surfaces. Lines of curvature of an ellipsoid.
- The torus and its tangent planes.

At the *Escola Polytechnica de Lisboa*, descriptive geometry started to be taught only in 1860, 6 years after such a course was proposed (Sequeira 1937, p. 5). The syllabus of that course was (in a summarized version):

- Aims of descriptive geometry.
- Representation of points, straight lines, and planes on two orthogonal planes.
- Rabatment of the vertical projection plane.
- Notation.
- Representation of polyhedra.
- To find the length of a straight line segment.
- Change of the projection planes.
- Rotation of a shape around an axis.
- Intersection of planes, straight lines and planes.
- To find planes that are perpendicular to given straight lines, or planes, under several conditions.
- Angles between straight lines and planes and study of several particular cases.
- Problems concerning the definition of straight lines and planes given different sets of data concerning angles, distances, points, projections, etc.
- Solid angles and resolution of several particular problems. General considerations concerning curved lines, tangent asymptotes, singular points, curvature, etc.
- To find tangents and their contact points.
- General definition of a surface.
- Curves on a surface and their tangents.
- Tangent plane and normal.
- The cylinder and the cone, their generatrices, apparent contours, projections, nets, tangent planes and sections.
- Several problems concerning the determination of generatrices and tangent planes, given different sets of conditions.
- General problem of cone and cylinder planar sections.
- Surfaces of revolution, axes, tangent planes and sections.
- Main properties of the hyperboloid of revolution of one sheet, sections and its asymptotic cone.
- The intersection of curved surfaces.
- Problems concerning curves with double curvature.
- Several special cases of the intersection of cones, cylinders and other surfaces of revolution.
- The helix, its projection, generatrices, tangents and subtangents.
- The developable helicoid, its tangent plane, sections and nets.
- The epicycloid.
- Definition of ruled surfaces.
- Considerations concerning the properties of developable and skew ruled surfaces.
- Definition of conoids and skew surfaces of the second degree.
- The hyperbolic paraboloid and the hyperboloid of one sheet and their double generation by straight lines, tangent planes, planar sections, and other properties.
- Particular conditions for the intersection of skew surfaces.

Fig. 7.1 Luiz Porfírio da Mota Pegado (1831–1903) (Sequeira 1937, pp. 6–7)

- Tangent planes and normals to skew surfaces.
- Study of several skew surfaces: conoid, bias passé, skew helicoid, screw surface, groin vault.
- Problems concerning planes that are tangent to several surfaces, in particular to two or three spheres, or to a sphere and a cone.

The first teacher was a former student of the Military Academy and of the *Escola Polytechnica de Lisboa*, General Luiz Porfírio da Mota Pegado (1831–1903), who taught there until 1902 (Fig. 7.1). Near the end of this long period, he published a textbook on descriptive geometry (Pegado 1899).

Before examining Mota Pegado's textbook, it should be noted that he published original research (although not in descriptive geometry) in two Portuguese scientific journals: *Jornal de Sciencias Mathematicas* and *Physicas e Naturais* and *Jornal de Sciencias Mathematicas e Astronomicas*. We shall have more to say about the second one below.

Fortunately for us, Mota Pegado not only tells us which works influenced his textbook, but he also tells us from where he learned descriptive geometry. He learned it from two textbooks. Not surprisingly, one of them was José Victorino dos Santos e Souza's translation of Monge's textbook (Souza 1812). The other one was a booklet that he described this way: it was only 28 pages long, it was divided into 39 sections and it mentioned 32 figures, which were absent from the copy that he studied from (and he did not know whether they had ever existed). Mota Pegado tells his readers that, according to an old rumour, the author was José Monteiro Feio (1787–1884), a former teacher at the *Escola Polytechnica de Lisboa*. He complains about the fact that no other Portuguese textbooks existed from which one could learn descriptive geometry. Therefore, he was presumably unaware of the existence of the booklet by Castel-Branco (Castel-Branco 1873).

Although he had been teaching descriptive geometry since 1861, Mota Pegado kept in touch with developments in the discipline, as reflected in his book. He

Fig. 7.2 Alfredo Augusto Schiappa Monteiro de Carvalho (1838–1919) (Sequeira 1937, pp. 10–11)

quotes an article by Amédée Mannheim published in 1882 as well as the edition of Mannheim's textbook (Mannheim 1886) published in 1886 and the third edition of Félix Chomé's textbook (Chomé 1898). From these texts, Mota Pegado extracted the idea of teaching descriptive geometry without using the concept of ground line. Mota Pegado states that he was influenced by La Gournerie's textbook (Gournerie 1860) in the context of the interaction between descriptive geometry and projective geometry, and he also mentions two textbooks written by Michel Chasles: his geometry textbook (Chasles 1880) and his textbook on the history of geometry (Chasles 1875). It is perhaps for this reason that Mota Pegado's textbook starts with a 42-page long chapter about modern geometry. Finally, he also mentions textbooks by Luigi Cremona (Menghini, Chap. 4, this volume), G. F. Monteverde and Ferdinando Aschier; all of them published no more than 20 years before the publication of his textbook.

General Alfredo Augusto Schiappa Monteiro de Carvalho (usually known as Schiappa Monteiro) was an assistant to Mota Pegado from 1870 until Mota Pegado retired in 1903, and then he replaced him until his own retirement in 1911 (Fig. 7.2). Unlike Mota Pegado, Schiappa Monteiro did publish articles about descriptive geometry. Most of these, if not all, were published in French in the *Jornal de Sciencias Mathematicas e Astronomicas*. These articles were usually short notes about very specific aspects of descriptive geometry. The descriptive geometry textbooks that he cites are all by French authors: Gournerie (1860), Fourcy (1830) (he cites the 1842 edition) and (Poncelet 1865–1866).

Schiappa Monteiro was one of the two people that Mota Pegado thanked concerning the writing of his textbook.

The *Jornal de Sciencias Mathematicas e Astronomicas*, a mathematical journal, was founded in 1877 by the Portuguese mathematician Francisco Gomes Teixeira (1851–1933). The first issue of the journal had a short list of topics in applied mathematics that were considered for publication, and one of them was stereotomy. As we saw above, it did indeed publish articles on that subject.

Gomes Teixeira was a professor at the University of Coimbra and then a professor at the *Academia Polytechnica do Porto*, where he taught descriptive geometry for a short period although he published nothing about it. From 1911 on, he was a professor and the first rector of the University of Porto, and he was a towering figure in the Portuguese mathematical community in the late 19th and early twentieth century.

Minister Fontes Pereira de Melo (1819–1887), a military engineer who studied at the *Escola Polytechnica de Lisboa*, had a good understanding of industrial and scientific matters, as opposed to the vast majority of politicians, who had a background in law (Carvalho 1986, p. 587). Fontes Pereira de Melo proceeded with several reforms in factories and public infrastructures. The need for professional expertise in industrial arts became apparent, and two new institutions were created by a decree published in the Diério do Governo on December 30, 1852 (pp. 864–870)—the *Escola Industrial do Porto* and the *Instituto Industrial de Lisboa*. These were very important in the spread of descriptive geometry in Portugal. With their focus on the education of industrial workers and craftsmen, descriptive geometry was established from the beginning as a fundamental discipline, articulated with linear drawing and machine drawing. These institutions had a course on descriptive geometry from their foundation in 1852. The first syllabus that we found for the industrial school in Porto was from 1887, when the course is designated as "Descriptive geometry and stereotomy—topography". This follows on from a decree in 1886 that reformulates industrial and commercial teaching in Portugal and is very similar to the syllabus from the Lisbon institute dating back to 1872. This syllabus, apart from the topography section, has two parts: the first one that we list below and the second part called "Applications of descriptive geometry" with the subsections shadows and wash drawing, machine drawing,[4] stone cutting and wood cutting.

The first part of the descriptive geometry syllabus begins with some preliminaries followed by the sections "Method of orthogonal projections" with six topics and "Method of projections with elevation" with two topics (Anonymous 1886):

- Introduction.
- Aims and usefulness of descriptive geometry. Projections: several methods.
- Method of orthogonal projections.

(i) Preliminary notions:

1. Representation of points
2. Representation of lines in general
3. Representation of straight lines
4. Representation of the planes[5]

[4]In the Porto school, machine drawing is not included and is replaced by linear perspective. However, the study of screws, gears and serpentines appears in the first part of the course in the Porto school syllabus.

[5]In the Porto school syllabus, the notion of rabatment of planes is added here.

(ii) Problems concerning straight lines and planes:

1. Intersection of straight lines and planes
2. Straight lines and planes defined by different conditions
3. Perpendicular straight lines and planes
4. Angles of straight lines and planes

(iii)
1. Representation of right and oblique prisms
2. Representation of right and oblique pyramid;
3. Representation of frustums of prisms and pyramids

(iv) Curved surfaces:

1. Generation of surfaces and their graphic representation
2. Tangent planes in general, apparent contours, normal
3. On the several kinds of surfaces and their main properties

(v) Problems concerning tangent planes:

1. Tangent plane through a given point on the surface
2. Tangent plane parallel to a given straight line

(vi) Intersection of surfaces:

1. General principles
2. Planar sections of curved surfaces
3. Intersection of curved surfaces

– Method of projections with elevation.

(i) Preliminary notions:

1. Representation of points;
2. Representation of lines in general
3. Representation of straight lines
4. Representation of the planes
5. Representation of curved surfaces and especially of the terrain surface

(ii) Problems:

1. Problems concerning straight lines
2. Problems concerning planes

5 Descriptive Geometry for Secondary School Teaching

Let us now see which other descriptive geometry books were written by Portuguese authors in the nineteenth century.

Already in 1840, Maurício José Sendim (1790–1870), a painter and a teacher at the Casa Pia (which was, and still is, an educational institution dedicated to helping youngsters at risk of social exclusion or without parental support) announced the

publication of such a book. It was to be part of a series of booklets published under the general title *O Estudante de Desenho e Pintura* (The student of drawing and painting). Unfortunately, although other booklets in the collection were published, the one about descriptive geometry never was Rodrigues (2001) and Pimenta (2003).

In 1878, a descriptive geometry booklet written by António Augusto Gonçalves (1848–1932) was published (Gonçalves 1878). António Augusto Gonçalves studied at the Universidade de Coimbra, but he never graduated. In spite of that, after having been a teacher of drawing in several schools for many years, he became professor of drawing at the University de Coimbra in 1902 (Rodrigues 1992). His book is only 39 pages of text followed by 6 pages of diagrams. It is divided into 93 very short sections; many of which consist of a single sentence. It has no bibliography, references to other authors or practical applications of what it teaches. The word "rebatimento" (which is the Portuguese word for "rabatment") is already mentioned in it. The booklet has a short two-page introduction, in which the author claims that the contents of the book follow the official syllabus. That is quite likely to be the secondary school syllabus, although he doesn't state that explicitly. Indeed, descriptive geometry was taught in secondary schools in Portugal around 1880. In this period, the syllabus for the *Lyceus* (secondary schools) had descriptive geometry in the 6th year, but it only addressed elementary notions related to points, straight lines and planes (Anonymous 1880, 1886). More precisely, the students were exposed to the following topics:

- Method of projections
- Representation of points, straight lines and planes
- Problems concerning straight lines and planes
- Intersection between a straight line and a plane
- Orthogonal straight lines and planes
- Method of rabatments
- Angle between two straight lines; angle between two planes
- Rotation around a vertical axis

By the end of the nineteenth century, descriptive geometry was no longer an independent course although it was still taught in drawing classes. More details concerning the teaching of descriptive geometry in secondary schools can be found in Palaré (2013).

Another descriptive geometry book published in 1883, which was explicitly meant for secondary schools, was *Elementos de Geometria no Espaço e de Geometria Descriptiva para uso nos Lyceus*, written by two army officers and teachers of Physics at the *Escola Polytechnica de Lisboa* (Vidal and Almeida 1883). This was an expanded edition of an earlier book, which did not mention descriptive geometry at all. Actually, the part of the book which deals with descriptive geometry is very short; of the 150 pages, only 27 are about descriptive geometry, and the rest of the book is about geometry in space.

Finally, an anonymous booklet (Machado 1885) about descriptive geometry was published in 1885 (Fig. 7.3). It was part of a collection called *Biblioteca do Povo e das Escolas* (Library for the People and the Schools), which was published both in

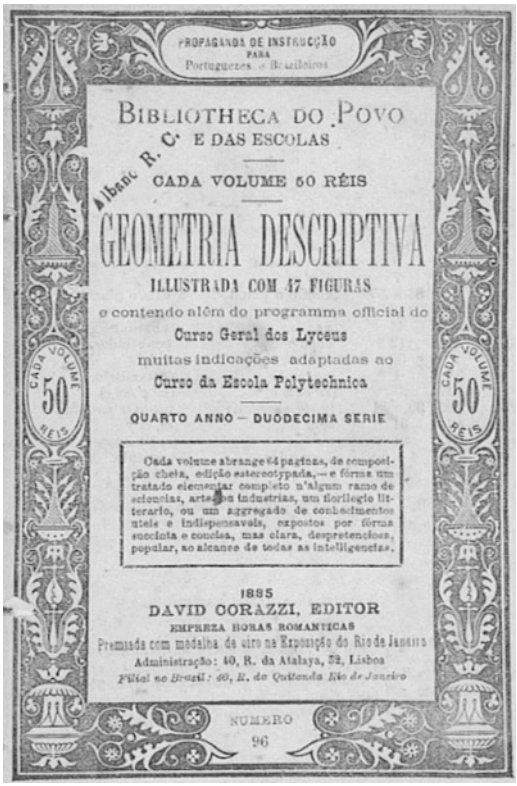

Fig. 7.3 Cover of *Geometria Descriptiva* (Machado 1885, cover)

Portugal and in Brazil. This collection was very wide-ranging with booklets about many varied subjects such as history, biology, gymnastics, geography, agriculture, medicine and so on. In particular, there were three other booklets on geometry, two of which (about plane geometry and space geometry) were written by Carlos Adolfo Marques Leitão (1855–1938); the third one (about spherical geometry, as well as spherical trigonometry) was written by Rodolphe Guimarães (1866–1918). Although the collection had several books published anonymously, in most cases the names of the authors were provided. The author of this booklet was Achilles Alfredo da Silveira Machado (1862–1942), who, at the time he wrote this text, had just graduated from the *Escola Polytechnica*, where he must have been a student of Mota Pegado. He published two other booklets in the collection (under his name) about gunpowder and railways. We know that he is the author of this booklet from two sources; the editor of the collection says so in the foreword of its twelfth series[6] and in (Anonymous 1942) it is stated that Achilles Machado published a book on descriptive geometry for the general public.

[6]Personal communication from Nabo (2012).

The long title of the booklet states that not only its contents follow the official secondary schools syllabus but also that part of its contents correspond to what is taught at the *Escola Polytechnica*. In spite of being only 58 pages long, the author spends some time explaining what descriptive geometry is. He states that it "can be seen both as an art and a science" and that it was "only after the works of the famous Monge at the end of the last century that descriptive geometry started to be seen as a science". Also, he describes descriptive geometry as "the science which allows us to represent objects placed in space precisely in a plane, and to solve, using drawings on the plane, the problems related to these objects". Therefore, even outside the academic world, by this time the idea that descriptive geometry is a useful and important science was firmly established in Portugal.

6 Conclusions

In Portugal, as far as universities and similar institutions were concerned, the teaching of descriptive geometry, followed closely the way that this subject was being taught in France, mainly at the *École polytechnique* and the *École Centrale*. This, together with the fact that knowledge of French was expected from students at institutions of higher learning in Portugal, is possibly the reason why it took so long before a Portuguese textbook on descriptive geometry was published. However, it was felt that there was a need for descriptive geometry at the secondary school level, in part in the context of drawing. So, in Portugal, books on descriptive geometry at the secondary school level were published before they were published at the university level.

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