Chapter 3
Descriptive Geometry in Italy in the Nineteenth Century: Spread, Popularization, Teaching

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Abstract The spread of descriptive geometry in Italy began in the years of the French occupation (1796–1813) and continued during the Restoration, mainly through military schools and universities. At the same times, the first technical-professional schools (public and private) were founded in Italy. Elementary descriptive geometry (theoretical and/or graphical) was taught in many of these schools, which played a key role in the popularization of this subject. In the first decades of the nineteenth century, Italy, after France, "was the European country that, before all others, had provided original contributions in the field of descriptive geometry" (Loria. 1921. Storia della Geometria descrittiva. Milano: Hoepli. [p. vii]). After unification and the Founding of the Kingdom of Italy (1861), a new model for universities and secondary education was created. The tradition that began at the time of the French invasion continued in the universities even in the united Italy. In contrast, after a major attempt to reform secondary schools in 1871, the teaching of descriptive geometry was gradually neglected.

This chapter focuses attention on the main aspects of the spread, popularization, and teaching of descriptive geometry in Italy during the nineteenth century. In the first part, we consider the era before unification (1796–1861). In the second part, we analyze the period after unification (1861–1923) with particular regard to the lack of interest shown in teaching descriptive geometry at secondary education level.

Keywords Descriptive geometry · Projective geometry · School for engineers · University · Secondary school · Technical education · Teaching of geometry · Casati law · Filippo Corridi · Luigi Cremona · Vincenzo Flauti · Giuseppe Tramontini

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1 Descriptive Geometry in Italy Before Unification

1.1 The Historical Background

During the first French invasion between 1796 and 1799, the Italian territory was divided into numerous satellite states. The three largest of these were the Cisalpine Republic, situated in the north, the Roman Republic, which included Rome and the territories of the Church, and the Neapolitan Republic, situated in the south.

From 1800 to 1813 there was the second phase of the French invasion, during which the Italian territory was partly annexed by France and partly divided into new satellite states, each with its own laws but without any real political autonomy. The most extensive territorial states, as well as the most strategic for French politics, were the Italian Republic in the north, which became the Kingdom of Italy in 1805, and the Kingdom of Naples in the south.

After the defeat of Napoleon, the Congress of Vienna sanctioned the division of the Italian peninsula into seven principal states. The only ones that retained their autonomy from the Habsburg Empire were the Kingdom of Sardinia, ruled by the Savoy, and the Papal State, which was returned to the Pope. The other principal states were the Kingdom of Lombardy-Venetia, which was directly dependent on the Austrian Emperor Francis I, and the Kingdom of the Two Sicilies, which was created in 1816 after the union of the former Kingdoms of Naples and Sicily and was returned to the Bourbons.

This political division lasted until 1859 when the unification process began.

1.2 Descriptive Geometry and Higher Education in the Napoleonic Period and During the Restoration

Descriptive geometry was introduced to Italy via the new cultural channels created by the French during the first phase of their invasion. Monge sojourned twice in Italy between 1796 and 1798 to carry out the tasks related to the office of the Commissioner of the French Republic. At the start of the peace negotiations with the Austrians (1797), Napoleon requested the presence of Monge in French headquarters and was able to converse with him about various topics, including descriptive geometry (Fiocca 1992, p. 196).

It can be assumed that the presence of Monge in Italy somehow facilitated the spread of descriptive geometry. In fact, just 2 years after the French invasion began, and just 3 years after the same Monge had held his famous lectures at the *École normale* and the *École centrale des travaux publics*, descriptive geometry was introduced to Italy. In 1798, a first course was opened at the *Scuola di Artiglieria e Genio* in Modena (Cisalpine Republic), and in 1799, another course was opened in Naples, at the *Scuole teoriche di Artiglieria e Genio*, founded to replace the old military schools of the Bourbon period. Simultaneously with the start of French reformist activity, new plans for education (which remained uncompleted) were prepared in 1798, which included the creation of two engineering schools in the cities of Milan and Rome with courses in descriptive geometry. At the beginning of the nineteenth century, with the introduction of the new French educational models and the creation of the *Lycées*, the main Italian universities were reformed, and a descriptive geometry course was inaugurated for the first time at the University of Naples in 1806.

In Italy, alongside the profession of military engineer, the profession of civil engineer began to take shape in the early decades of the nineteenth century. In fact, during the French invasion, a process began that was consolidated during the Restoration and which transformed the traditional architect (who operated independently and acquired skills via training and experience) to a more modern professional belonging to special corps that were responsible for the design, construction, and maintenance of roads, canals, ports, and rivers as in the French model of the *Corps des ponts et chaussées*. This process favored the creation of the first courses for civil engineers, in which teaching was focused on subjects such as geodesy, hydraulics, structural engineering, stereotomy, and the branches of mathematics related to them like differential calculus, mechanics, and descriptive geometry. In Italy, however, the training of civil engineers took place almost exclusively in the universities, and in the period preceding unification, only two schools for engineers in Naples and Rome were founded. In Naples, the *Scuola del Corpo Reale di Ponti e Strade* was created by the French in 1811. It was re-founded by the Bourbons during the Restoration and became one of the leading engineering schools in the country after the unification of Italy. In Rome, after the first unfinished projects of 1798, the *Scuola d'ingegneri pontifici* was opened in 1817 and in 1826 was integrated into the University.

The spread of descriptive geometry throughout Italian universities took place at different times. In some states, the discipline was introduced in the first years following the Restoration, for example, in Turin (Kingdom of Sardinia). In 1801, the French devised a reform plan for the University of Turin that included the teaching of descriptive geometry, but this plan remained unrealized (Conte and Giacardi 1990, pp. 285 ff.). Monge's discipline was taught at this university only in 1824,[1] after a course in descriptive geometry had been established at the *Regia Accademia Militare* in 1816.

In other universities such as those in Pisa (Grand Duchy of Tuscany), Pavia, and Padua (Kingdom of Lombardy-Venetia), the introduction of descriptive geometry took place a few decades later. Overall, according to Fiocca (1992, p. 188), it can be said that the widespread dissemination of descriptive geometry in Italy was realized between 1798 and 1840. In 1840s, the discipline even transferred to Sardinia, the Italian region with the lowest school attendance rate and in which the Savoy had implemented policies that had slowed cultural development (Scoth 2016, p. 186).

[1]During the Napoleonic occupation, many Piedmontese students were recruited to attend the *École polytechnique* in Paris (see Id., pp. 289–296).

1.3 *The Emergence of Technical and Professional Education and the Popularization of Descriptive Geometry*

In Italy, the development of technical and scientific teaching at the pre-university level began in the early decades of the nineteenth century with the creation of a number of educational institutions that, for the first time, attributed a "popular" character to the subjects and removed them from the hegemony of military schools and universities. There are basically two reasons for this development: the growth of the economy and modern industry that required a radically different education from the one linked to the eighteenth-century models, and the orientation of Italian society towards greater popular culture. In those decades, technical and professional education in Italy was very varied and took place in many different schools, supported by patrons and entrepreneurs. The best initiatives were linked to the industrialization of northern Italy and the modernization of agriculture. There were, however, numerous examples also in the south of both schools for artisans and agricultural schools. Moreover, some nautical schools were established in several coastal cities of the peninsula and on Sicily and Sardinia.

In 1830s and 1840s, there were the first attempts to "popularize" descriptive geometry and to teach it in these new schools. One of the most famous examples was that of the *Società d'incoraggiamento d'arti e mestieri* in Milan, founded in 1838 on the initiative of a group of intellectuals and entrepreneurs, where a descriptive geometry course held by Giuseppe Colombo, one of the most famous Italian engineers of those times, was introduced. Other famous technical and professional schools were opened in 1838 in the Kingdom of Sardinia, for example, those of the *Società per l'avanzamento delle arti, dei mestieri e dell'agricoltura* in Biella, and the *Istituto di arti e mestieri* in Novara. In these schools, descriptive geometry was part of the teaching of solid geometry and had as its goal, above all, its application to mechanical and construction drawing (MAIC 1862, pp. 409 ff.).

Alongside the *Scuole di arti e mestieri* (Schools for Arts and Crafts), which were at the elementary school level, some secondary technical schools were created in Italy from public initiatives in the nineteenth century. In general, private initiative was faster than political initiatives in interpreting the needs of the local economy but, in the middle of the century and with the increase in industrial development, the creation of public technical schools as an alternative to grammar schools became could no longer be delayed, and all governments enacted special measures to create new courses of study. In Lombardy-Venetia, the *Scuole Reali* (similar to the Austrian and German *Realschulen*) were founded in Venice (1841) and Milan (1851). In Turin in 1852, the Municipal Technical Schools and the Royal Technical Institute, a school that depended directly on the Ministry of Education, were founded. In all these schools, descriptive geometry applied to the drawing of orthogonal projections was taught (Scoth 2008, pp. 33–34). In Florence, a chair of descriptive geometry at the *Istituto Tecnico*, a high-level institution was set up in 1853.

Overall, it may be concluded that in the mid-nineteenth century, especially in northern Italy, descriptive geometry was widely taught in secondary technical schools and primary schools for artisans.

1.4 The Italian Treatises of Descriptive Geometry in the First Half of the Nineteenth Century

In the first half of the nineteenth century, the introduction and the spread of descriptive geometry in Italy was supported by an original local production of treatises and manuals. In this period, almost 40 new texts, reprints, and translations (wholly or partly devoted to the descriptive geometry and its applications) were published.

The lectures given at the *École normale de l'an III* were translated into Italian by Carlo Lauberg, a Neapolitan philosopher and revolutionary who moved to the Cisalpine Republic for political reasons. These translations (*Lezioni ad uso delle scuole normali di Francia raccolte per mezzo dei Stenografi e rivedute dai Professori*, Milano, Raffaele Netti, 1798) are very rare, and only the first two volumes are known. They contain lessons by Garat, Sicard, Berthollet, Vandermonde, Bauche de la Neuville, Mentelle, La Harpe, Volney, Daubenton, and also five mathematics lessons by Laplace, four physics lessons by Hauy, and the first three lessons of descriptive geometry by Monge (Pepe 2003, pp. 332–333).

The first Italian translation of Monge's *Géométrie descriptive* (Placci 1805) was published in Bologna by Giuseppe Placci, a former student of the Military School in Modena. This version, which contains some notes added by the translator, probably had a low circulation (Fiocca 1992, p. 205). In fact a second translation by the mathematician Filippo Corridi was printed in Florence in 1838 (Corridi 1838), with the subtitle "first Italian edition" (Fig. 3.1).

Instead, the first Italian treatise of descriptive geometry (Flauti 1807) was published in Naples by the mathematician Vincenzo Flauti for the students of the *Scuole teoriche di Artiglieria e Genio*. This treatise, as the author explains, follows the classic setting of Monge's work although it contains new examples and different construction methods (Id., pp. 212 ff.). A second work (Flauti 1815, and subsequent eds.) contains a personal re-working of descriptive geometry. The originality lies mostly in the fact that the language used was that of the ancient Greek geometers with references to *Book of the Euclid's Data* (Id., pp. 219–220). Flauti, in fact, was an exponent of the so-called *Scuola sintetica napoletana* (Neapolitan synthetic school), namely a then philosophical movement that favored the model of Euclid's Geometry in contrast to analytical methods (see also Mazzotti 1998, pp. 675 ff.).

One of the descriptive geometry teachers of the *Scuola di Artiglieria e Genio* in Modena, Giuseppe Tramontini, published a treatise (Tramontini 1811) that shows several important aspects (Fiocca 1992, pp. 199 ff.):

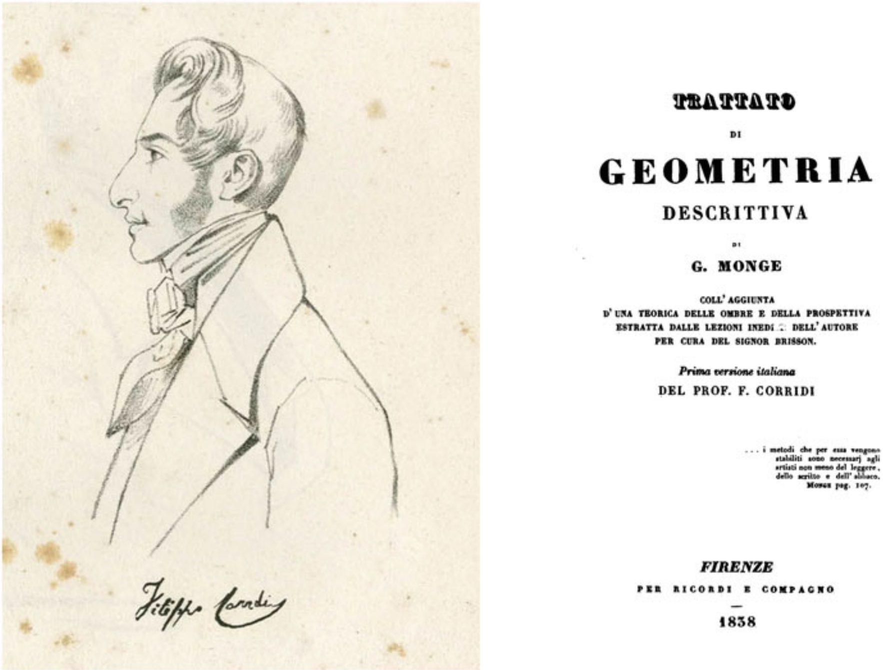

Fig. 3.1 Mathematician Filippo Corridi, translator of the most famous Italian edition of Monge's *Géométrie descriptive*

- for the first time, he introduced the technique of the change of projection planes in the most general form, making use of non-orthogonal planes between them[2];
- in some cases, he used three projection planes since in general there is no bijective correspondence between all the points of an object and their representation on two orthogonal planes;
- the second section of his work was given to applications, in particular prospective and theory of shadows.[3] This section was necessary because school courses in Modena were divided into a theoretical and an applied part such as those of Monge at the *École Centrale des Travaux Publics*.

The Military school in Modena, in spite of its short life (it was closed in 1815), gave rise to an important tradition in the field of descriptive geometry. Another

[2]In particular in chapter IV of his work (see also Loria 1921, p. 191 ff.; Torelli 1875; Fiocca 1992, pp. 202–203). The question of the change of the projection planes (the *Méthode d'Olivier*) was also studied by Francesco Paolo Tucci, a lecturer at the School for engineers in Naples Tucci (1823). In France, this topic was considered a few years later by Adhémar and Olivier (Barbin, Chap. 2, this volume).

[3]The treatise of Tramontini was published before Monge's lessons about the perspective and the theory of shadows was printed in France by Bernabé Brisson.

former student of this school, Carlo Sereni, was a professor for almost 50 years of this discipline in the *Scuola d'ingegneri pontifici* in Rome and published two treatises, which were very popular in Italy, one theoretical and one applied (Sereni 1826 and subsequent eds.; Sereni 1846).

Quintino Sella, one of the most famous statesmen in the united Italy and teacher of geometry at the Royal Technical Institute in Turin, was the first to introduce isometric drawing to Italy (Càndito 2003, pp. 62 ff.). He published a pamphlet on the principal methods of representation, which contain his lessons from 1856 (Sella 1856). Half of this treatise is dedicated to axonometric drawing and the methods proposed by William Farish (see also Lawrence, Chap. 17, this volume) and perfected by Julius Weisbach.

Among the various works printed before the unification of Italy, it is worth mentioning (Bellavitis 1851), which was written by Giusto Bellavitis, professor at the University of Padua. In this text, and for the first time in Italy, there is reference to modern theories of central projection.

Overall, we can say that in almost all pre-unification Italian states the diffusion of descriptive geometry was supported by the production of original local treatises.[4] Works on this subject were not published only in Piedmont. At the University of Turin in 1830, manual written by Sereni for engineering schools of Rome was adopted (Fiocca 1992, p. 222).

Currently, there are no studies regarding the transfer and circulation of treatises of descriptive geometry in Italy at that time, and so it is not possible to make definitive assertions. It is likely that the production of Italian texts hindered the translation of foreign treatises. In fact, in addition to Monge's *Géométrie descriptive*, only three other works of descriptive geometry translated into Italian in the first half of the nineteenth century are known.[5]

Another important aspect is that of didactic transposition of descriptive geometry in "schools of arts and crafts". The model of the *Conservatoire des arts et métiers* had a major impact, especially in northern Italy, thanks to the activity of disclosure made by Italian scientists that were sent to France (and to other advanced nations) to study the processes of industrialization and thanks to the existence of various technical and scientific information magazines. The *Géométrie et méchanique des arts et métiers et des beaux-arts*, the work summarizing the lessons given

[4]Since the 1840s, the treaties written by the professor of descriptive geometry were used at the University of Pavia in the Kingdom of Lombardy-Venetia. (Carlo Pasi, *Sunto di lezioni di geometria descrittiva*, 1843; *Saggio di applicazioni della geometria descrittiva*, 1844 and subsequent eds.)

[5]Two of these were the works of Sylvestre-François Lacroix, *Saggio di Geometria riguardante le superficie piane e curve o sia Elementi della Geometria descrittiva di S. F. Lacroix. Prima traduzione italiana fatta sopra la terza edizione francese*, 1829, and Charles-François-Antoine Leroy, *Trattato di geometria descrittiva con una collezione di disegni. Prima versione dal francese con note di Salvatore D'Ayala e Paolo Tucci*, 1838. The third was a treaty of Georg Schaffnit adopted in the Polytecnic of Wien: *Scienza Geometrica delle Costruzioni ovvero Geometria Descrittiva. Versione dal tedesco con illustrazioni e aggiunte di Vincenzo Tuzzi*, 1841.

to the working classes by Charles Dupin, contained an entire volume devoted to elementary geometry and its applications, including descriptive geometry. The work was twice translated Italian[6] and was an important reference for elementary technical teaching. Following the example of the famous text by Dupin, similar books were printed in Italy, which contained, in more or less expanded form, an elementary discussion of descriptive geometry.[7]

2 Descriptive Geometry in Italy After Unification

2.1 A New Educational and Academic Model

The most crucial phase in the unification of Italy began in 1859 with the annexation of Lombardy to the Kingdom of Sardinia and ended in 1861 with the annexation of the remaining territories and the birth of the Kingdom of Italy.[8] At that time, the Italian educational system was based on the 1859 Casati law (named after the Minister Gabrio Casati), created to reform education in the Kingdom of Sardinia and was subsequently extended to other areas of the new Kingdom of Italy. The educational system was divided into: Elementary, Secondary (classical, technical, and normal), and Superior (university).

New *Facoltà di Fisica, Matematica e Scienze naturali* were created in the universities. Moreover, two engineering schools were opened, called respectively *Scuola d'applicazione per ingegneri* and *Istituto tecnico superiore* in Turin and Milan. Only students who had done two first years of mathematical studies in universities were allowed into these schools. Consequently, the training of engineers was divided into two phases: a first phase of mathematical studies in the Faculty of Science (which also included descriptive geometry) and a second phase of application studies in the engineering schools. Even when schools for engineers were gradually established in other major Italian cities (Palermo, Naples, Bologna, Rome) this structure was maintained and remained unchanged for the rest of the nineteenth century.[9]

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[6] Anon. (actually Antonio Cioci), *Geometria e meccanica delle arti, dei mestieri e delle belle arti, ad uso degli artisti, e direttori d'officine e manifatture, del Barone Carlo Dupin*, 1829, and Giacomo Laderchi, *Geometria e meccanica delle arti e mestieri e delle belle arti del Barone Carlo Dupin. Prima versione italiana*, 1829–1830.

[7] Luigi Poletti, *Geometria applicata alle arti belle e alle arti meccaniche*, 1829; Giovanni Alessandro Majocchi, *Manuale di geometria per le arti e pei mestieri*, 1832.

[8] Venice was annexed in 1866 and the city of Rome in 1870.

[9] The courses for architects and engineers were reformed during the Fascist rule (see also Menghini, Chap. 4, this volume).

Under the Casati law, Secondary classical (humanistic) instruction consisted in the *Ginnasio-Liceo* axis. Technical instruction constituted a separate sector and was divided into two levels called *Scuola tecnica* and *Istituto tecnico*, respectively. The *Scuola tecnica* started after elementary school; the *Istituto tecnico* started after *Scuola tecnica* and was divided into different sections, one of which—called *Sezione Fisico Matematica (SFM)*—was dedicated to mathematics and scientific studies and, unlike the others, provided access to university. Descriptive geometry was never taught in the *Ginnasio-Liceo*. It was instead taught in the *SFM* and in the two new sections (industrial and for surveyors) created after 1859. In fact, the regulations, the structure, and the syllabi of technical institutes were changed eight times between 1859 and 1891 (Scoth 2010). Moreover, descriptive geometry was taught in nautical schools during the entire nineteenth century.

2.2 *Descriptive Geometry in the Universities and Engineering Schools*

As regards higher education, one of the effects of the Casati law was that descriptive geometry was taught only at the beginning of university mathematics courses (Fig. 3.2) except for the engineering schools where its applications were taught

Anno Primo

Primo Semestre	*Secondo Semestre*
Introduzione al calcolo. Fisica Geometria descrittiva. Disegno	Come nel primo Semestre meno la Geometria descrittiva

Anno Secondo

Calcolo differenziale ed integrale Geometria descrittiva. Fisica Disegno	Come nel primo Semestre meno la Geometria descrittiva

Anno Terzo

Meccanica razionale Geodesia Chimica. Disegno	Come nel primo Semestre

13. Il Professore di disegno sarà coadiuvato da Assistenti, a cui potrà essere affidato l'insegnamento di qualche parte del disegno. Questi saranno in tal numero che a ciascun di loro ed al Professore tocchino circa trenta studenti.

Gli studi del disegno avranno per oggetto più specialmente l'*architettura*, le *macchine* e la *topografia*.

14. Gli esami orali speciali che gli studenti di Matematica debbono sostenere per conseguire la laurea, appaiono nel seguente specchio:

N.° d'ordine	ESAMI	Durata dell' Esame
1	Introduzione al calcolo	45′
2	Calcolo differenziale ed integrale	45′
3	Meccanica razionale	45′
4	Geodesia	30′
5	Fisica sperimentale	30′
6	Chimica	30′
7	Geometria descrittiva	30′
8	Disegno	30′

15. Prima degli esami orali di Geometria descrittiva e Disegno, si eseguiranno due componimenti, l'uno sulla Geometria descrittiva, e l'altro sul Disegno.

I temi di questi componimenti saranno tratti a sorte da quelli proposti ne' programmi degli esami; si faranno a porte chiuse in una sessione non maggiore di sei ore per la Geometria descrittiva, ed in due sessioni ciascuna non maggiore di sei ore pel Disegno.

16. I componimenti, di cui all'articolo precedente, saranno sottoposti alla Commissione deputata all'esame orale; della Commissione per l'esame di disegno farà parte l'Assistente che vegliò il candidato nell'esecuzione del lavoro.

Il candidato presenterà pure alla Commissione per l'esame di disegno i lavori fatti durante il corso.

17. Gli esami speciali debbono farsi entro un quinquennio comin-

Fig. 3.2 A timetable of the mathematical courses in Italian Universities (1860). Descriptive geometry was mandatory in the first 2 years (Decreto 1860, pp. 2174–2175)

ORARIO DELLA SCUOLA D' APPLICAZIONE

1ª Classe

	8 – 9½	9½ – 10½	10½ – 12	12 – 1	1 – 4
Lunedi	Meccanica razionale	Applicazioni di geometria descrittiva		Statica grafica	Disegno di statica grafica
Martedi	Disegno di applicazioni di geometria descrittiva			Mineralog.ª e geologia applicate	Chimica docimastica
Mercoledi	Meccanica razionale	Geodesia		Statica grafica	Disegno di statica grafica
Giovedi	Meccanica razionale	Applicazioni di geometria descrittiva		Mineralog.ª e geologia applicate	Chimica docimastica
Venerdi	Disegno di applicazioni di geometria descrittiva			Statica grafica	Disegno di statica grafica
Sabato	Meccanica razionale	Geodesia		Mineralog.ª e geologia applicate	Chimica docimastica

Fig. 3.3 Subjects and lesson timetables in the Engineering School of Rome (1877/78). Descriptive geometry was limited to the applications (Programma 1877, p. 20)

exclusively (Fig. 3.3). This situation also concerned other branches of mathematics and was a consequence of the fact that engineering schools were not considered high school alternatives to university faculties but as university post-training schools. In these conditions, the teaching of descriptive geometry in universities was assimilated into other branches of pure mathematics. The courses were almost always held by mathematicians who were more interested in investigating the properties of three-dimensional geometric bodies rather than developing the drawing of orthogonal projections and practical applications. Luigi Cremona, one of the most famous Italian mathematicians of the time, who taught descriptive geometry at the University of Bologna, wrote: "I think I am unfit to teach descriptive geometry; I am not a designer, drawing figures makes me uncomfortable and I am not an expert in applications" (Gatto 1996, p. 26).

Besides Cremona, many famous Italian mathematicians were professors of descriptive geometry, for example, Gino Fano, Gino Loria, and later Beppo Levi, Federigo Enriques and Francesco Severi.

In 1875, a compulsory course in projective geometry separate from that of descriptive geometry was introduced into the mathematical degree courses at Italian universities. Research into projective geometry had many development prospects, and mathematicians followed them with great attention. At that time in Italian engineering schools, the teaching of graphic statics was introduced, and this made preparatory teaching of projective geometry at universities even more important. The introduction of graphic statics had already been tested at the end of 1860s in the *Istituto tecnico superiore* in Milan, where the director (the mathematician Francesco Brioschi) and Cremona had introduced new methods devised by Karl Culmann and

Theodor Reye at the Polytechnic of Zurich (see also Benstein, Chap. 9, this volume) into Italy.

In schools for engineers, applications of descriptive geometry were focused on classic topics: contour lines, perspective, shadows, stereotomy, and photogrammetry. In some cases, applications were taught in the mathematics faculty but were generally limited to the more purely geometrical topics, perspective, theory of shadows, or gnomonic projections.

New textbooks of descriptive geometry were produced in Italy in the second half of the nineteenth century and the first decades of the 20th. As well as those written by Fano (1910) and Loria (1909) the books written by Ferdinando Aschieri, professor of descriptive geometry at the University of Padoa (Aschieri 1884) and by Federigo Enriques (1902) were quite successful. All these texts were reprinted and revised several times. Those of Loria and Fano, in particular, considered the two methods of central projection and orthogonal projection, following a trend that had developed since the introduction of projective geometry course at Italian universities.

2.3 *Descriptive Geometry in Secondary Technical Education: the SFM*

After the Casati law, the first syllabi for technical institutes were introduced in 1860. Descriptive geometry was taught in the *SFM* and was a part of the course in pure and applied mathematics.[10] These syllabi were changed a few years later. In fact, a law of 1861 shifted the technical institutes from the Ministry of Public Education to Ministry of Industry. Consequently, technical schools remained isolated from the rest of the school environment and the regulations were changed several times in search of an optimal structure.

In 1865, the mathematics course was modified and descriptive geometry became an independent subject. The syllabus was divided (according to the teaching tradition that dated back to the time of Monge) into a theoretical part and a more practical one, which also included stereotomy. The changes of the projection planes and the rotations were added to the theoretical part.

In 1871, the *SFM* was transformed into a specific preparatory course for higher education in engineering. The mathematics syllabi were expanded and modernized by Cremona and projective geometry was introduced (see Menghini, Chap. 4, this volume; Menghini 2006), and the parallel course in descriptive geometry was assigned to a professor other than mathematics. This reform, however, was

[10] All mathematics syllabi cited in this chapter are available at http://www.associazionesubalpinamathesis.it/storia-insegnamento/provvedimenti-legislativi/.

unsuccessful for various reasons (Scoth 2011, pp. 276 ff.), but partly because the curricula were far too innovative.

In 1876, technical institutes were back under the control of the Ministry of Education. In the *SFM*, mathematics syllabi were gradually reduced in 1876, in 1885, and in 1891, and the teaching of descriptive geometry experienced a steady decline.

In 1885, projective geometry was removed and descriptive geometry, in the context of orthogonal projections, was absorbed into the mathematics course. In 1891, the syllabus was restricted to the representation of points, lines, and planes and to a limited portion on the representation of solids. In the early twentieth century, descriptive geometry took up such an irrelevant role in teaching to the point that two famous mathematicians, Enriques and Severi, proposed eliminating it from the teaching programs of *SFM* "because it is taught more effectively in the Universities, after a good introductory course of projective geometry" (Enriques et al. 1903, p. 55).

This situation had important repercussions on the production of elementary treatises. In contrast with the important production of other mathematics treatises for secondary schools, only a dozen textbooks on descriptive geometry were published in Italy between 1860 and 1900.[11] Among the most important examples are the text for adoption in the *Istituto tecnico* in Florence (Peri 1869) (Fig. 3.4), the Italian translation (reprinted five times) of *Leçons nouvelles de Géométrie descriptive* by the French mathematician Antoine Amiot[12] (Mazzitelli 1875), and the treatise written by Salvatore Ortu Carboni (1894–1895), a mathematician who was very active in the Italian debate on the teaching of mathematics in secondary schools.[13] Even the handbooks of geometry that had a section devoted to descriptive geometry were very few, and this overall shortage forced teachers and students in some cases to use university treatises or old texts from the first half of the nineteenth century that were unsuitable for elementary teaching.

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[11]The list is available at http://www.associazionesubalpinamathesis.it/storia-insegnamento/libri-di-testo/#1513285344390-8eed2f07-ed22.

[12]About Amiot see also (Barbin, Chap. 2, this volume).

[13]Ortu Carboni was one of the few Italian mathematicians who at the time had dealt with the problems of secondary teaching of descriptive geometry. He believed in the educational value of this discipline and during his speech at the congress of the Association of Italian mathematics teachers (the *Mathesis*) in 1901, he said: "Even in Italy descriptive geometry should become a subject of elementary mathematics courses. It helps to broaden and deepen geometric ideas, to arouse interest in the examination of figures, to understand many simple representations of everyday life and to present to people a broader view of the territories of mathematics. These few reasons are enough to show how important it is." (Ortu Carboni 1902, p. 110).

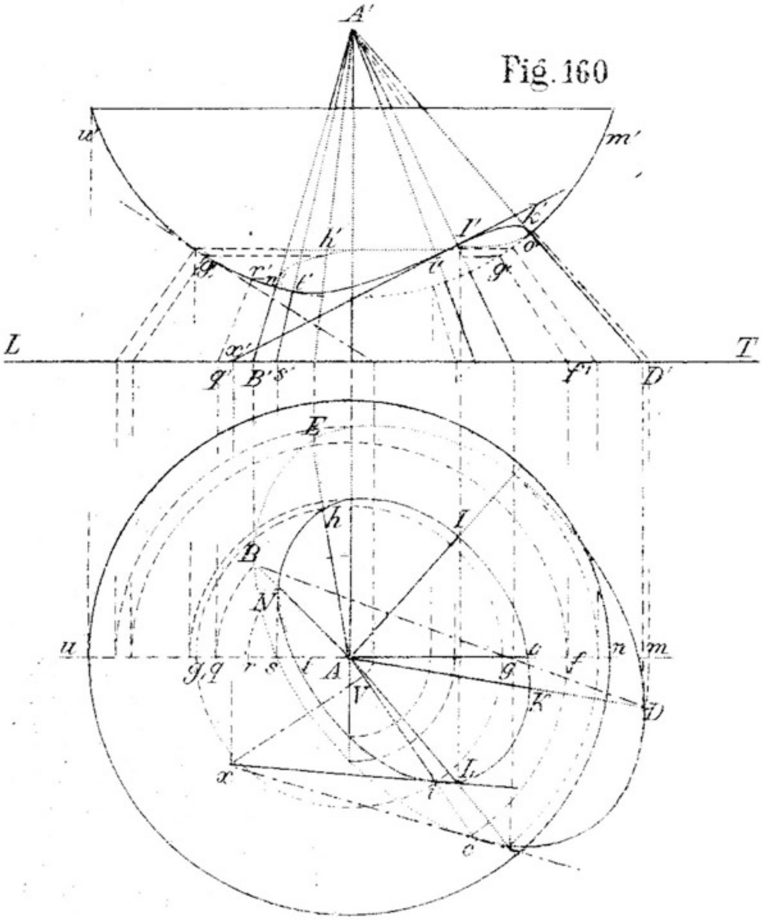

Fig. 3.4 A classical problem from descriptive geometry: the intersection between conic and spherical surfaces (Peri 1869, vol. II p. XVI)

2.4 *The Teaching of Descriptive Geometry in Italian Secondary Schools After Unification: A Case Study*

At this point it is worth asking a question: why did Italy, the country that first imported the tradition of Monge, that had many scholars and has produced new ideas and books in the field of descriptive geometry, gradually neglect secondary teaching of this discipline after unification? It is not easy to give an exact answer. We should first of all observe that the decline of descriptive geometry since the second half of the nineteenth century has not been a specifically Italian trend. As you can read in this volume, descriptive geometry in secondary education at that time did not adequately developed also in other European countries. In the case of

Italy, however, we need to consider particular factors that may have led to this state of affairs and make some preliminary comments.

– After unification, the establishment of effective secondary technical education was a difficult problem in Italy. The Casati law was designed for a small country like the Kingdom of Sardinia, but the speed with which unification took place forced the authorities to extend it throughout Italy. Therefore, this law was not the best solution to harmonize the different school systems of the pre-unification states. Moreover, according to the ideology of the time, humanistic studies were privileged and technical education was considered less important (Giacardi and Scoth 2014, p. 217). In Italy, humanistic education was financed directly by the state, while technical education was financed by the municipalities that did not have sufficient resources for scientific labs, materials, and skilled teachers. For this reason, technical education suffered significantly in those years.
– Another problem was that of creating an efficient model of public technical education. In Italy, there were no previous examples to refer to, and, for many years, governments proceeded with uncertainty by looking at the experience of the main European countries. For this reason, there were constant changes of syllabi and regulations, and the technical institutes were placed under the supervision of the Ministry of Industry.

    Other considerations relate more specifically to the teaching of mathematics.
– At that time, the teaching of mathematics in Italian secondary schools was characterized by a rigorist approach, excessive use of deductive reasoning, dogmatism, and recourse to mnemonic study.
– In general, the syllabi were poorly designed, not finalized to inter-disciplinarity and in some cases over-extended.
– The methods used to recruit and train teachers were inadequate. Only a small number of them had a degree in mathematics or related disciplines, and, especially in the lower grade levels, they were recruited without possessing the necessary qualifications.

Moreover, the syllabi were often superficiality designed and without taking account of educational needs. Those of 1865 for the *SFM*, for example, anticipated the teaching of drawing of orthogonal projections by 1 or 2 years compared to that of descriptive geometry.[14] Those of 1871 proved to be too demanding for of the secondary technical education in those years and for the preparation of teachers (Scoth 2011, p. 276).

The insufficient number of hours devoted to the study of mathematics in relation to the content of the curriculum forced teachers reduce the hours taught. In technical institutes, descriptive geometry was the most frequently neglected

[14]We should consider that in Italian technical institutes the method of orthogonal projections was taught also in courses of drawing that included a part of artistic and a part of industrial drawing. This teaching, however, was detached from that of descriptive geometry and was often assigned to teachers who came from the world of art and had no competence in the field of industrial drawing (MAIC 1869, p. 180).

discipline. Besides, a survey done at the beginning of the twentieth century by *Mathesis* revealed that it was not either taught, it was reduced to a few concepts with some drawings by more than half of the teachers interviewed (Scorza 1911, p. 74).

These conditions not only certainly had an impact on the teaching of descriptive geometry but also especially on its training value as a means to develop spatial intuition. Furthermore, the didactic importance of descriptive geometry for the study of three-dimensional Euclidean geometry was not considered,[15] and this pedagogical conception was the fruit of the rigorist and rational trend in Italian teaching. Consequently, descriptive geometry was always seen as an atypical science which was located half way between pure mathematics and engineering sciences and was only taught in technical institutes and nautical schools, where it was thought to be useful for applications. The structure of the technical institutes, however, did not allow an optimal teaching of descriptive geometry. In the *SFM*, where the scientific subjects were predominant, this discipline was considered redundant because it had no links to applicative topics, and for these reasons, mathematicians like Enriques and Severi requested its abolition. In other sections of the technical institutes, a utilitarian purpose prevailed and descriptive geometry was reduced to a simple graphic exercise.

After the fascist reform of education (1923), all scientific teaching was weakened in Italian secondary schools. Technical institutes were reformed and the *SFM* was closed. Descriptive geometry was taught only in industrial and nautical schools and was restricted only to the drawing of orthogonal projections.

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[15] This aspect was highlighted during the congress of the *Mathesis* in 1919, organized to standardize the programs of the former Austrian schools assigned to Italy after World War I. In Austria descriptive geometry "had got a lot of attention on the part of school authorities", while in Italy "it had been kept away from middle school because it was not considered necessary to provide students with a good education on conception of space" (Nordio 1920, p. 40).

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