Chapter 2
Descriptive Geometry in France: Circulation, Transformation, Recognition (1795–1905)

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Abstract Descriptive geometry had been taught by Monge in 1794–1795 in two schools: the *École polytechnique*, intended for future engineers and officers, and the *École normale*, intended for future teachers. Monge's two proposals were preparatory teaching for various applications, but also a new teaching of geometry, which could follow (or replace) the ordinary Elements of geometry. In this chapter, our main goal is to examine the future of these two proposals in France. Indeed, the spirit of the first lessons given by Monge changed at the same time that descriptive geometry underwent a considerable dissemination into all French education and society. In relation to that dissemination, we examine the circulation of knowledge towards artists, craftsmen, and engineers. We investigate teaching given in the preparatory grades for the entrance examination to the *École polytechnique*, to the *École centrale des arts et manufactures*, and to other schools, and we analyze the process that led to teaching descriptive geometry at secondary level. We also examine the role of descriptive geometry in the teaching of geometry in the end of the nineteenth century.

Keywords Descriptive geometry · Projective geometry · Teaching of geometry · Teaching of methods · *Rebatment* · Rotation · Secondary school · School for engineers · Gaspard Monge · Jean-Nicolas Pierre Hachette · Joseph Adhémar · Théodore Olivier · Eugène Rouché · Louis Léger Vallée · *École polytechnique* · *École des ponts et chaussées* · *École centrale des arts et manufactures*

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1 The Spreading of Descriptive Geometry into Preparatory Grades (1813–1833)

In 1794–1795, Monge's first lessons on "descriptive geometry" took place in two schools. We have the text of those for the *École normale de l'an III*, but we only have the program for the *École centrale des travaux publics* (the future *École polytechnique*). The starting point of the famous *Géométrie descriptive* of 1799 (Monge 1799) was a transcription of oral teaching given in this *École normale* (Monge 1992), without the structure in lessons and without the four last lessons (Barbin, Chap. 1, this volume). The purpose of the lessons in the *École normale* was to present a new geometry in the spirit of this school (Barbin 2015b). Firstly, lessons adopted an "order of invention" which consisted in giving problems before introducing tools to solve them. So, *Leçon* 1 began with an inaugural problem: "how to determine the position of a point in space?" before introducing the method of projections in *Leçon* 2. Secondly, they presented geometric objects, not in the order of simplicity, from plane to space, but starting with the latter immediately. Thirdly, they expressed a will of generalization, especially with the general "generation of surfaces" in *Leçon* 3, where a curved surface was defined as generated by the movement of a curved line. Fourthly, they promote the union of descriptive geometry and analysis.

Jean Nicolas Pierre Hachette was the "assistant" of Monge in the *École polytechnique* in 1794 and he replaced him as professor of descriptive geometry until his dismissal in 1816. His *Traité de géométrie descriptive* contained applications to shadows, perspective, and stereotomy (Hachette 1822). He gave great importance to the general definition of surfaces introduced by Monge, as generated by the motion of a curve, and he introduced the notion of ruled surfaces (Barbin, Chap. 1, this volume). Charles-François Leroy succeeded Hachette. In 1834 he edited his *Traité de géométrie descriptive* in which he criticized Monge's textbook, because it did not offer numerous and various examples, and it did not give clear drawings (Leroy 1834, p. v). He did not adopt Monge's order: he began with problems on straight lines and planes and continued with the trihedral angle. He introduced the general generation of surfaces, but the problems on tangent planes only concern cylinders and cones. His textbook met considerable success with 15 editions from 1834 to 1910 (with additions of Émile Martelet from the fourth edition of 1855). The contents (developable surfaces, curvatures, etc.) renders clear that the students had to be familiar with descriptive geometry before they entered the *École polytechnique*.

Indeed, teaching descriptive geometry rapidly decreased in the *École polytechnique* but appeared in the entrance program to this school in 1813. From its creation, there existed an examination to enter the *École polytechnique*, which had more and more candidates over the years. The *Conseil de perfectionnement* defined the entrance program and chose the examiners every year, but most of them remained

for a long time. In 1804, the first examiners (like Monge or Jean-Baptiste Biot) had been replaced by former students of the *École polytechnique*, like Louis-Benjamin Francœur and Charles-Louis Dinet, who were also teachers in *Lycées*. From 1810, examiners began to ask questions outside the entrance program, especially in descriptive geometry. As a result, in 1813, the *Conseil de perfectionnement* decided that "the candidates would be questioned on the first six lessons of descriptive geometry concerning the straight line and the plane; and that they would construct, with the compass and the ruler, at a given scale, one figure of the elements of geometry which will be indicated by the examiner"[1] (Fourcy 1828, p. 320).

The introduction of descriptive geometry in the entrance program led to the publication of many textbooks explicitly devoted to the candidates to the *École polytechnique*, but also to the other schools of government (Military school of Saint-Cyr, Naval school of Brest, Forestry school). These candidates were students in upper grades of *Lycées* or private *Collèges*, named "special mathematics" and created in the beginning of the century, some in provinces but most in Paris (Belhoste 2001). Throughout the century, many authors of textbooks on descriptive geometry taught in these schools, they were former students of the *École polytechnique* and sometimes examiners. They constituted a network of Parisian authors, who wrote collections of textbooks on all the parts of mathematics (Barbin 2015a).

In 1828, Émile Duchesne wrote his *Éléments de géométrie descriptive*, a short and elementary textbook devoted to candidates for the *École polytechnique* and other schools. The later authors will be more prolix. Among the textbooks on descriptive geometry for the entrance examination to *École polytechnique* in the years 1820–1840, the most famous was the *Traité de géométrie descriptive* of Louis Lefébure de Fourcy, edited in 1830. The author was a teacher in the *Collège Royal Saint-Louis*, a former student of the *École polytechnique*, and an examiner during more than 30 years. This longevity explains the success of the textbook, which had been republished eight times until 1881. The first volume contains 295 pages in a small format and the second one around 150 pages of figures (Lefébure de Fourcy 1830). It began with an inaugural problem, which was to find the center of a sphere circumscribed to a triangular pyramid, and it continued with a list of problems, mixed with theorems. The order was far away from the one of Monge's lessons, with three parts: straight line and plane, curved surfaces and tangent planes, curved lines and their tangents. Lefébure de Fourcy gave Monge's general conception of a surface of revolution but he added that it was not useful for the applications. The part named "Exercises" shows that the textbook was a tool for training the students to prepare examinations. There were not any applications of descriptive geometry in the textbook, although the author considered it as "complete". The existence of new students, schools, and teachers was at the origin of a new conception of descriptive geometry, oriented not by problems but by formatted examination exercises (Barbin 2015b).

[1] All the translations of quotations are made by Évelyne Barbin.

2 Descriptive Geometry as a Part or as a Sequel of a Geometry Teaching (1812–1844)

Independently of the entrance into the *École polytechnique*, some teachers of *Lycées* proposed to introduce descriptive geometry in secondary school, as a part of geometry or as a sequel to ordinary geometry. In some sense, they followed the "Elements of descriptive geometry" edited in 1795 and 1817 by Monge's students, Sylvestre-François Lacroix (Lacroix 1795) and Jean Nicolas Pierre Hachette (Barbin, Chap. 1, this volume). The fact that some examiners or former students of the *École* became teachers of *Lycées* is an important factor for this dissemination of descriptive geometry. As in the case of Jean-Guillaume Garnier, who was an examiner of the *École*, an assistant of Lagrange until 1802, and then a teacher at the *Lycée* of Rouen. His *Éléments de géométrie* of 1812 contained a part on descriptive geometry composed of "preliminary notions" and five problems. For him, these notions "constitute a natural sequel to plane geometry, and introduce at the same time space geometry, in other words descriptive geometry" (Garnier 1812, p. 258).

Antoine-André-Louis Reynaud, who entered in the *École* in 1796, became a teacher in a *Lycée* in 1800 and an examiner of the *École* in 1809. As soon as 1812, he introduced a part with around 80 pages entitled "Elements of descriptive geometry" into his *Notes sur la géométrie*, which followed a new edition of Bézout's *Cours de mathématiques*. He wrote: "The principal purpose of Descriptive Geometry is to provide the means to exactly represent bodies in a plane. Scholars and artists invented more or less ingenious methods to solve this problem and thanks to research we reached to give constructions with the degree of simplicity that we have to make known" (Bézout and Reynaud 1812, p. 130). It is remarkable that he gave many theorems on projections of a point, a straight line, and a curve on only one plane, before he treated the case of two planes of projections, and finally the better case, where these planes are perpendicular. He ended by showing the simplicity of the solution of problems in space geometry using descriptive geometry. In *Problèmes et développements sur diverses parties des mathématiques*, written with Jean-Marie Duhamel in 1823, the authors did not introduce descriptive geometry but used the notion of projection. On the contrary, the *Théorèmes et problèmes de géométrie* of the "Baron Reynaud" of 1833 contained an important part on descriptive geometry, similar to the one of the *Notes* of 1812. At this period, Reynaud was still the examiner and his textbook was now intended for candidates to enter the *École polytechnique* and the other schools of government (Reynaud 1833).

But we have to remark that Olry Terquem, in his *Manuel de géométrie* of 1829, written to present the "writings of contemporary geometers" to beginners, did not mention Monge and the descriptive geometry and preferred to introduce the theories of projections and polars of Poncelet and Gergonne to teach conics (Terquem 1829, pp. 347–350). He entered the *École polytechnique* in 1801, was a "répétiteur" of this school and a teacher at the *Lycée* of Mayence. Later, in 1842, he created mathematical journals with Camille Gerono: *Nouvelles Annales mathématiques and Journal des candidats aux Écoles polytechnique et normale*. In

papers of these journals, he appeared an attentive commentator on the teaching of descriptive geometry (Barbin 2015b).

Hyppolite Sonnet was assistant of mechanics in the *École centrale des arts et manufactures* created in 1829 (see further) and author of many textbooks. In 1839 he edited his *Géométrie théorique et pratique*, intended for the *Écoles normales primaire*, where the teachers of primary schools and industrial schools were trained. The edition of 1848 was also intended for special teaching given in the *Facultés*. It contained applications of geometry to drawing, architecture, perspective, and "the first elements of descriptive geometry", which covered 25 pages. Sonnet motivated this geometry with three inaugural problems on trihedrals, which had been solved in the plane just before. From the beginning of his textbook, he introduced the *rebatment* and its properties, as a direct method to solve problems, like Adhémar in 1832 (see further). A chapter concerned "the curved surfaces in general, and the cylindrical and conical surfaces in particular" (Sonnet 1848, pp. 251–264). The general surfaces were divided into ruled surfaces and surfaces of revolution with applications of cylindrical surfaces to the vaults, and of conical surfaces to a machine. Paul-Louis Cirodde was also author of many textbooks on various matters. He introduced "elementary notions on descriptive geometry" in the second edition of his *Leçons de géométrie* of 1844, in a period when he taught in the *Collège Royal Henri IV*. It essentially consisted in a collection of 26 problems and their solutions, those on surfaces concerning cylinders, conics, and spheres only (Cirrodde 1844, pp. 50–61).

3 Descriptive Geometry for Civil Engineers and Artists (1819–1841)

The teaching of descriptive geometry spread quickly in schools for engineers and technicians. For instance, Gabriel Gascheau was a teacher at the *École des arts et métiers* de Macon when in 1828 he edited his short *Géométrie descriptive. Traité des surfaces réglées* (Gascheau 1828), in which he introduced the ruled surfaces of Hachette (Barbin, Chap. 1, this volume).

Until the Revolution, the *École des ponts et chaussées* was a school for engineers, organized around lessons given by engineers and projects made by the students. In 1795, it became a school of application of the *École polytechnique*. Three professors were appointed in civil architecture, mechanics applied to the art of construction, and cutting of stones (Michel 1981). The third one was Léon Bruyère, a former student of this school, architect and engineer, who attended the lectures of Monge. Joseph Mathieu Sganzin started as teacher of descriptive geometry in 1797. In 1825–1826, Barnabé Brisson, a teacher of construction, who reedited Monge's textbook in 1820, promoted descriptive geometry in the school (Picon 1989). Two authors, related to this school, introduced novelties into the teaching of descriptive geometry, often adopted by their successors: Vallée and Adhémar.

Louis Léger Vallée was a former student of the *École polytechnique* and an engineer of the *École des Ponts et Chaussées*. He edited his *Traité de géométrie descriptive* intended for artists in 1819, where he compared constructions in a plane and in space with two problems: (1) to construct the center of a circle where three points are given in a plane; (2) to construct the center of a sphere four points of which are given in space. He defined the notion of orthogonal projection and the two planes of projection, then he explained that, before solving problems, we have to examine the representations of a point, of a straight line, and of a plane in their most "remarkable positions" (Vallée 1819, p. 10) and he gave them in a set of figures (Fig. 2.1). The first problems only concerned points, straight lines, and planes. In the same manner, he began to study the projections of curves before those of surfaces. He showed the usefulness of the notion of tangent to a curve, since the projection

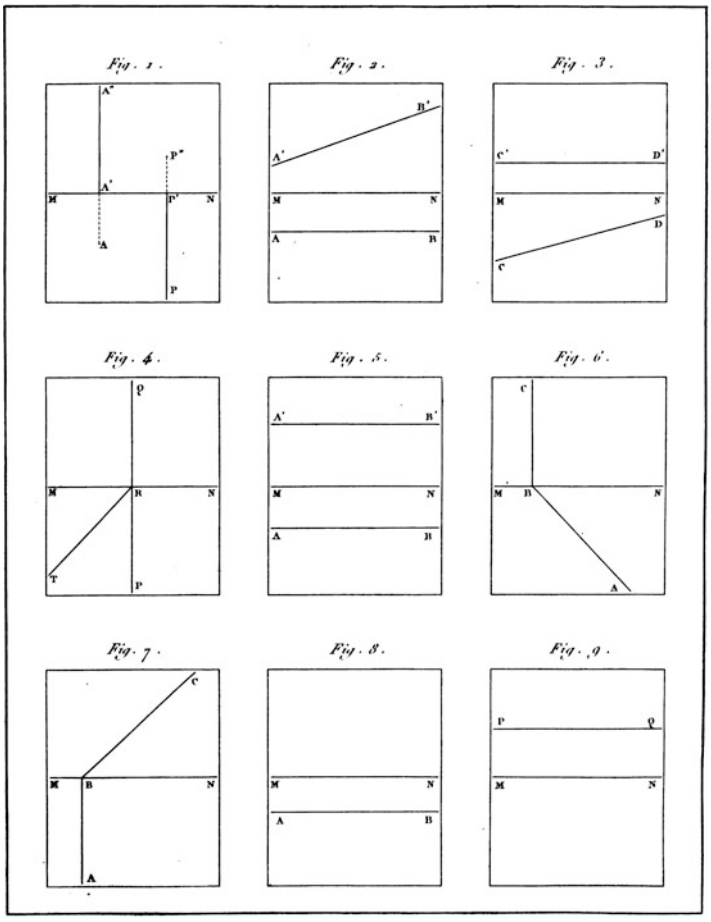

Fig. 2.1 Table of "traces" in Vallée's *Traité de géometrie descriptive* (n. p.)

of the tangent is tangent to the projection of the curve. Thus, Vallée introduced two orders, different from Monge's one, which will often be used by the successors: (1) a decomposition of the projections according to the simplicity of the figures, those of a point, a straight line, and a plane, with their different "traces" on the planes of projection and their problems; (2) a study of the projection of curves, with their "traces" and problems on tangents. As in Monge's *Leçons*, the part on surfaces began with their general conception, as generated by the motion of a curve, but it continued with cylindrical and conical surfaces, surfaces of revolution, warped and envelope surfaces, tangent planes, and then intersections of surfaces. Vallée used the word "rabattement" (rebatment), introduced by Charles Potier in 1817 (Barbin 2015b), to name the result of the motion of the vertical plane to the horizontal position, but here the notion was also introduced to solve problems on angles by turning a plane around a straight line (on Potier, see Gouzevitch, Chap. 13, this volume).

In 1821 Vallée wrote his *Traité de la science du dessin* devoted to artists, in which he introduced descriptive geometry, and not only perspective, contrary to other authors, like the teacher of drawings Thénot (1834). For Vallée, "descriptive geometry is an indispensable help to deeply penetrate all that concerns the mechanism of the eye. [...] As it furnishes the means to rigorously represent points, lines and surfaces defined with exactness; we conceive that it has to serve as a basis of the science of drawing" (Vallée 1821, pp. viii–ix). To represent an object and its shadow, he introduced two planes of projection, one horizontal (its plane) and one vertical (its cutting), and he deleted the ground line, for instance, for a niche (Fig. 2.2). The second edition of the book contained the enthusiastic support of Joseph Fourier, Gaspard de Prony, and François Arago in the name of the Royal academy of sciences.

Joseph Adhémar was a private teacher of mathematics and a prolific author of textbooks on descriptive geometry and its applications, intended for beginners, craftsmen, and civil engineers (Barbin 2015b). His textbooks were published by Carillan-Gœury, the bookseller of the *Corps des Ponts et Chaussées* and the *Corps des Mines*. In his *Cours de géométrie descriptive* of 1823, like Vallée, he decomposed the projections of a point, a straight line, and a plane. In 1832, he edited a collection of textbooks named *Cours de Mathématiques à l'usage de l'ingénieur civil*, and the first one is devoted to descriptive geometry. Like Vallée, he introduced a study of projections of curves before coming to surfaces. In another collection, named Cours de *Mathématiques à l'usage des architectes, ingénieurs civils, etc.*, he treated the applications of descriptive geometry: cutting of stones (1834), shadows (1840), frames (1849), and bridges (1853).

One novelty introduced by Adhémar in 1832 concerned the choice of the planes of projection. He stressed: "always, in the applications, we will have to choose the system of coordinate planes or auxiliary planes on which the projections will be the simplest. And as long as we change nothing in the data and their related position, the generality of the question will remain complete". He added: "the choice of the planes of projection is one of the most essential parts of the solution of problems" (Adhémar 1832, p. 158). Later, in his *Traité de géométrie descriptive* edited in 1841,

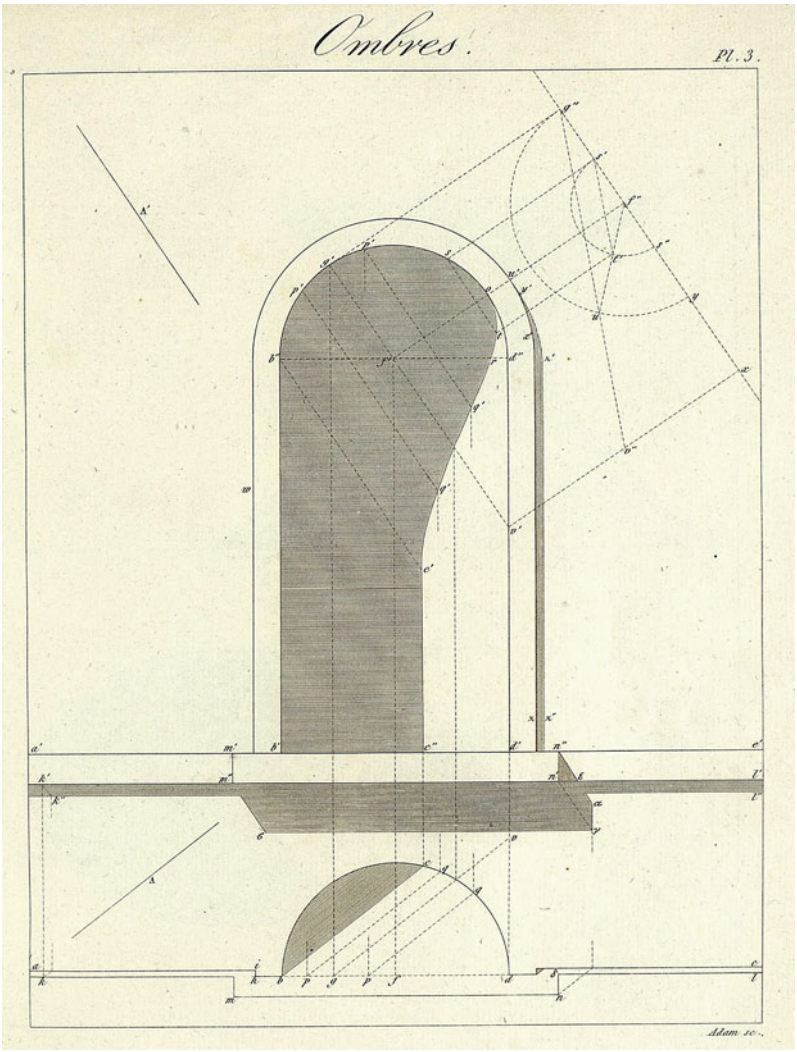

Fig. 2.2 Drawing of a niche in Vallée's *Traité de la science du dessin* (plate 3)

he stressed the advantages of using "auxiliary planes" for making a construction easier and he introduced a special chapter of 12 pages entitled "Rebatments". For him, this operation corresponded to the "transformation of coordinates" in algebraic analysis. He used rebatments to construct several problems and he used the properties of the rotations without making them explicit. He wrote:

> To have the true magnitude of a portion of a straight line, which joins two points, we have to turn this line until it becomes parallel to one of the planes of projection. This operation, named *rebatment*, served in the solution of several previous questions. The importance that

> the *rebatments* have to acquire in the further applications of descriptive geometry has to engage us to present some general considerations on this kind of operation right now. [...] When we make a *rebatment*, each point describes a circle in space whose center is on the axis of rotation, and whose radius is the distance of this axis to the turning point (Adhémar 1832, pp. 44–46).

Another novelty was introduced in 1841, with the role given to cylindrical surfaces. Adhémar did not begin with the general conception of surfaces and neither with ruled surfaces. He started with the cylindrical surfaces, not as the simplest surfaces but as "the first and most essential curved surfaces" (Adhémar 1841, p. 134). For him, cylindrical, conical, and spherical surfaces are "the essential basis of almost all the combinations of the industry" (Adhémar 1841, p. 264). He defined a cylindrical surface as generated by a straight line that moves in parallel to itself, following a curve named the generative curve of the cylinder. He used this surface to show that "to construct a tangent to a given curve at a given point, it is sufficient to construct the two tangents of the projections of the curve at the projections of the given point. These lines are the projections of the tangent to the curve" (Adhémar 1841, p. 143). In this way, he stressed a general study of the projection on one plane only.

Notions élémentaires de géométrie descriptive, edited by M. F. Amadieu in 1838, was the first textbook for the candidates to the *École polytechnique* that took into account the novelties of Adhémar. The author was a former student of the *École militaire de Saint-Cyr* and he was a teacher at the *Lycée* of Versailles (near Paris). The textbook was small, with only 110 pages: it contained "preliminary notions" with theorems, two pages on drawings and a list of "problems to solve". It followed Vallée's textbook with the drawings of the nine projections of a straight line depending of its different positions with regard to the planes of projection, etc. In the part called "Resolution of problems by the method of rebatments", the properties of the motion of rotation are used implicitly (Amadieu 1838, pp. 19–20). In the part named "Changing the vertical plane", the problems of the change of planes are treated systematically: given the two projections of a point to find the projection on a new vertical plane, then the same problem for a straight line and for a plane.

4 Descriptive Geometry in the *École centrale des arts et manufactures* (1829–1853)

The *École centrale des arts et manufactures* was created in 1829 to train civil engineers and managers for industries and to develop the applications of the new sciences. It was an initiative of an industrialist man, Alphone Lavallée, with the chemist Jean-Baptiste Dumas, the physicist Eugène Péclet, and the mathematician Théodore Olivier. Lavallée was a shareholder of the journal Le Globe, created in 1824 to spread the saint-simonian doctrine, which granted a major role to the engineers in society (Comberousse 1879). Olivier was a former student of the

École polytechnique (1811) and of the *École d'artillerie* of Mézières. He was a student of Monge and Hachette and he remained friendly with Hachette. In 1851, he wrote, about the creation of the *École centrale des arts et métiers*, that, from 1816, the students of the *École polytechnique* received the same teaching as those of the *École normale*, so many of them preferred to become teachers than engineers, which means "philosophers" than "workers" (Olivier 1851, pp. xiii–xxiii). For him, many textbooks were edited for the entrance examination to the *École polytechnique* because of a "thirst of lucre" but without making any progress in science. He concluded that the industry needed civil engineers, who were not trained anymore in this school. At this time, courses for technicians were given in the *Conservatoire National des Arts et Métiers*, created in 1794, and in some towns, like in Metz by Poncelet (Fox 1992), while *Écoles centrales* in Châlons, Angers and Saint-Étienne prepared supervisors. For Olivier, the purpose of the creation of the *École centrale des arts et manufactures* was to "recreate the ancient *école centrale des travaux publics* like Monge had conceived it" to train engineers and not "scholars" (Olivier 1851, p. xx). Four sciences had to be taught: geometry, mechanics, physics, and chemistry. The teaching of mechanics contained a part on analytical geometry and analysis. The teaching of geometry was reduced to descriptive geometry and occupied a great part of the timetable, with 2 h every Tuesday and Saturday morning in first year, only on Tuesday in second year (Comberousse 1879, p. 46). During the 3 years of studies, many hours were devoted to drawings.

As a mathematician, Olivier was author of works on geometry, descriptive geometry, and the mechanics of gears. His thesis in 1834 concerned the geometrical study of curves and surfaces of second order and the applications of gears. He had been professor of descriptive geometry in *École centrale des arts et manufactures* from 1829 until his death in 1853, and also in the *Conservatoire National des Arts et Métiers*. In this latter school, in order to help students to understand ruled surfaces, he designed concrete "models", which had been manufactured by Fabre de Lagrange in 1872. Made out of threads, some of them were static (like those imagined by Monge) or some were articulated. For instance, a cylinder could be transformed in a hyperboloid, and then to a cone (Sakarovitch 1994, pp. 332–333). Olivier wrote many textbooks on descriptive geometry, like his *Cours de géométrie descriptive* in two volumes (1843), *Développements de géométrie descriptive* (1843), *Compléments de géométrie descriptive* (1845), *Applications de la géométrie descriptive* (1846).

In his *Cours de géométrie descriptive*, Olivier began with an inaugural problem, not easily solved by ordinary geometry, which is to fix the direction of the perpendicular of a plane passing through a given point. Then, he transformed two ideas of Vallée and Adhémar into two methods: the "method of point, straight line and plane" and the "method of changes". He wrote about the first method: "as soon as we will know how to represent a point, a right line and a plane by the method of projections [...], we will know descriptive geometry" (Olivier 1843, p. vi). The different possible "traces" of a point or a line are called "alphabet". He justified the "method of changes", meaning changes of planes of projection or rotations of

figures around an axis, by remarking that a figure drawn on the planes of projection can be very complicated, but difficulties "will disappear by a suitable choice of planes of projection; we can also keep the same planes and change the position of the figure, this last operation is always made by turning the figure around on an axis" (Olivier 1843, p. 2). He introduced the operation of rebatment: "if we make a plane turn around its intersection with another plane, until it meets with this one, we say that we rebat ('rabattre') the first plane on the second. This operation is frequently used in descriptive geometry" (Olivier 1843, pp. 18–19). For him, this operation is identical to Euler's formula employed in analysis to find an equation of the section of a solid. The properties of what he called "rotation" are clearly expressed, and are used systematically. Like Monge, Olivier defined the surfaces in general and insisted on problem solving. He treated developable surfaces, tangent planes to conical and cylindrical surfaces, envelope surfaces and surfaces of revolution.

It is interesting to compare the contents of Olivier's textbooks with manuscripts of his students, preserved in the archives of the school. Every student had two notebooks, one for lessons, where the figures were made freehand, and another one for exact drawings. This last notebook was constituted by a list of problems, and many of them concerned motions. For instance, in 1839–1840, problem 12 asked to carry a given cone parallel to itself in such a manner that its summit reaches a given point (Fig. 2.3). In a notebook of 1845–1846, many problems concerning rotations

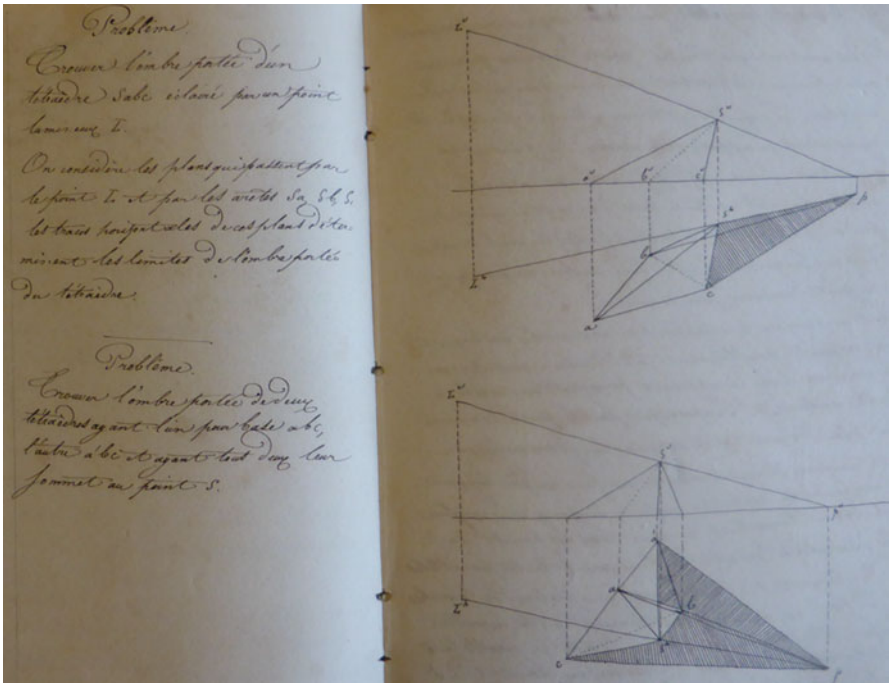

Fig. 2.3 From a notebook of drawings (OLI1 1839–1840)

were studied for themselves. Olivier's interest in rotations came from descriptive geometry but also from his study of gears. The notebook contained problems of application with drawings of carpentry, stones, and stairs.

5 From the Preparatory Grades to Secondary Schools (1847–1869)

Olivier's "method of changes" was criticized by Émile Martelet in the 4th edition of Leroy's *Traité*. For him, the method led to complicated constructions, but it could be a good subject for exercises (Leroy 1855, p. 394). In the 2nd edition of the *Traité élémentaire de géométrie descriptive* of Eugène Catalan and Henri Charette de Lafrémoire, Lafrémoire wrote: "during these recent years, the auxiliary projections were excessively recommended. It was believed that we had to recourse to their use in every circumstance, and in all the problems of descriptive geometry [...]. A new thing needed a new name: the Method of Planes of Projection was invented!" (Lafrémoire de et al. 1852, p. 120). The controversy continued in the *Nouvelles Annales de mathématiques* from 1851 to 1856 (Barbin 2015b).

Jules de La Gournerie also criticized Olivier's method. He was a former student of the *École polytechnique* and of the *École des ponts et chaussées*, and he succeeded to Leroy as professor of descriptive geometry in the first school. In the preface of his *Traité de géométrie descriptive* of 1860, he defined descriptive geometry as the "abstract science of the drawing line" (La Gournerie 1860, p. v) and he wrote four pages against Olivier's method, considering that it was not suitable for applications and was not new—Abraham Bosse used it in 1643 and he was not approved, while Monge did not use it in his drawings (La Gournerie 1860, p. viii). He quoted Hachette, Leroy, Vallée, and Fourcy. Like Vallée and contrary to Monge, he began treating the simplest elements, straight lines, and planes, and the simplest surfaces, cylinders, and cones. The first volume contained chapters on the "quoted geometry", used in topography, and on the axonometric perspective. The second volume (1862) and the third volume (1864) gave a complete study on surfaces and their curvatures, using analysis, presented recent results and gave applications on shadows. La Gournerie also gave lessons to the *Conservatoire des arts et métiers*, to promote descriptive geometry among workers and artists.

Despite this negative advice, Olivier's method had been adopted in many textbooks for candidates to the *École polytechnique* and to other schools of government. As soon as 1847, Bertaux-Levillain, a former student of the *École polytechnique* and a teacher, wrote in his *Éléments de géométrie descriptive* that he "followed the movement shown by a learned professor, M. Olivier": "it seemed to me that it was useful to put into the hands of students a textbook where [...]they could find the notions of change of planes of projection, of rebatment of a plane on another in the beginning, notions that one rejects in the middle of the course wrongly, because they are indispensable for properly understanding the solution of almost all the

problems" (Bertaux-Levillain 1847, p. xiv). In 1852, Henri Édouard Tresca, a former student of the *École polytechnique* and a teacher of mechanics in the *Conservatoire des arts et métiers*, edited a *Traité élémentaire de géométrie descriptive*, "written in accordance with the textbooks and the lessons of Th. Olivier". The textbook began with the method of the projections of "point, straight line, plane", then it gave the representations of the projections of curves, of cylindrical and conical surfaces and surfaces of revolution. It presented two methods required to solve problems: "the general method of changes of planes" and "the general method of motions of rotations", which replaced the method of rebatment (Tresca 1852, p. 95). Here, the idea of rotation introduced by Olivier to define the changes of planes was enlarged to become a notion, that was the basis of a general method. Tresca gave what he called "the rules" to execute motions of rotations of one or several points.

In 1853, Antoine Amiot edited his *Leçons nouvelles de géométrie descriptive*, intended for the students of the preparatory grades to the *École polytechnique*, but also for the *École normale supérieure*, which trained future teachers. He was a former student of this latter school, a teacher at the *Lycée Saint-Louis* and the *École des beaux-arts*. The textbook was a small textbook of 190 pages only (without figures), which belonged to the collection of textbooks written by Amiot. It did not begin with an inaugural problem. In some chapters, there were theorems followed by a list of problems given as exercises, like in many textbooks of geometry of this period. Other chapters were composed of problems, like in Monge's. It began with the decomposition of the projections of point, straight line, and plane. Then Amiot introduced what he called "transformations of projections", which are the changes of plane of Olivier, and the rotations, which are systematically used to solve problems. He wrote: "it is M. Th. Olivier who gave a scientific character to the ideas expounded in this chapter under a particular title, and making it a basis of a method of resolution of questions in space geometry" (Amiot 1853, pp. 25–26). Like Monge, he defined general surfaces, as generated by the motion of a line, which meets one or many fixed guiding lines called "directrices". He continued with tangent planes to cylindrical surfaces and their problems, solved with the theorem on the projection of a tangent: "the tangent at a point M of any curve generally has, for orthogonal or oblique projection on any plane, a tangent to the projection of the curve, in the projection m of the point M" (Amiot 1853, p. 150).

In 1865, descriptive geometry became an autonomous part of secondary school teaching at the "elementary mathematics" level (students aged 17–18 years), which prepared the students for the *Baccalauréat*, the military school of Saint-Cyr, and the naval school of Brest. The program was close to Amiot's textbook, with projections of point, line, and plane, method of rebatments, projections of prisms, pyramids, cylinders, cones, plane intersections of polyhedrons (Belhoste 1995, p. 404). A typical textbook was the *Éléments de géométrie descriptive*, edited in 1869 by Charles Briot and Charles Vacquant, two former students of the *École normale supérieure* and teachers at the *Lycée Saint-Louis* and at the *Lycée Henri IV*. In the beginning of the textbook, the authors explained that an ordinary drawing is not sufficient to have exact lengths, but there exists the method of projections. They gave

the figures of the "traces" of the projections of a point, considered as "an alphabet" and they proposed problems on point, straight line, and plane. The chapter entitled "Methods in descriptive geometry" contains three methods, namely rebatments, rotations, changes of planes. The following chapter began with the projection of a curve and with the theorem on the projection of the tangent to a curve (Briot and Vacquant 1869, p. 74). The last two chapters treated the projections of polyhedrons and their intersections. Consequently, the projection and its properties appeared as the major subject of the problems.

6 Descriptive Geometry and Modern Geometry at the Turn of the Century

The teachings of geometry and descriptive geometry became closer at the end of the nineteenth century. It was the result of two movements. On one hand, from the 1860s, the teaching of descriptive geometry widened to allow other ways of representing space objects and took the study of projections as a preliminary. On the other hand, from the 1870s, authors proposed to enlarge the teaching of geometry to "modern geometry" where the projections play an important role, as well as methods coming from descriptive geometry, like rotations. Indeed, the accent on the introduction of methods to solve problems of descriptive geometry and the tendency to understand these methods as subjects for teaching led to abstract notions, which can be fruitful for solving any kind of geometrical problems (Moussard 2012, 2015; Chevalarias 2014).

An interesting actor and witness of the connection between these two movements is Eugène Rouché, who taught descriptive geometry in the *École centrale des arts et manufactures*, from 1867 to 1888, and edited *Éléments de géométrie descriptive* for "special secondary school teaching" in 1875. In 1872, 30 years after the first edition, he reedited Olivier's *Cours de géométrie descriptive*. As he was a student of Jules de La Gournerie in the *École polytechnique*, we can interpret that as a choice between two teaching methods. He is also an author with Charles de Comberousse of an important *Traité de géométrie élémentaire* in 1866, which had been a book of initiation to modern geometry among teachers in France and had an impact for the teaching of mathematics (Barbin 2012). On that date, the two authors were teachers of preparatory grades in the famous *Lycée Charlemagne and Collège Chaptal* in Paris. Charles de Comberousse was a former student of the *École centrale* and he taught kinematics and applied mechanics in this school from 1862 on.

Rouché did not edit his lessons of descriptive geometry given in the *École centrale* but the archives of this school contain many series of students' notebooks. In 1871–1872, the first lesson began with definitions of projections of points, straight lines, then continued with the notions of sheaf of rays and anharmonic ratio, in the spirit of Poncelet's geometry. The problem of the section of a cone was treated with projections on three planes (Fig. 2.4). Thus, the conception of descriptive geometry

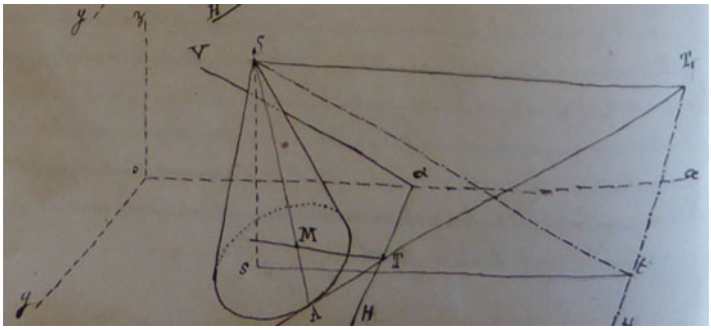

Fig. 2.4 A section of a cone in a notebook (ROU14 1871–1872, n. p.)

became larger than that of Monge. Eight years later, in 1879–1880, the presentation of descriptive geometry indicated: "it treats firstly, of geometrical notions, secondly, of various modes of graphical representations, thirdly, of stereotomy or cutting stones. The first part is disseminated in the middle of the others, so, as one goes along, the need for new theories appear" (ROU01, n. p.). The first lesson began with conical projections, and the first figure examined the case of the projection of a curve and its tangent, quite like many problems that considered one plane of projection only. Lessons included perspective and many applications, like carpentry or stereotomy, with lessons on vaults. Like Monge, Hachette, Vallée, Adhémar, Leroy, or Olivier, Rouché considered descriptive geometry for its applications: in 1893 with Charles Brisse, who was a professor in the *École centrale* and in the *École des Beaux-arts*, he edited a textbook entitled *Coupe des pierres* (cutting stones) (Rouché and Brisse 1893).

The *Éléments de géométrie descriptive* of Rouché, edited in 1875, were intended for the new "special secondary school teaching", which granted considerable importance to sciences and their applications, unlike "classical teaching". The program of 1866 introduced a part on descriptive geometry in the 3rd year (students aged 14–15 years), with the use of Olivier's models of the "Conservatoire des arts et métiers" (Belhoste 1995, p. 432). Rouché's textbook began with projective notions and their properties. Chapter VII, entitled "Rebatments and rotations", presented the two methods. The method of rotations was considered as a principle of solution, with the rotation of a point around a vertical axis, around a perpendicular axis to the vertical plane, then the rotations of a straight line, of a plane, etc. Here rotations were taught for themselves. As for his lessons in the *École centrale*, Rouché put forward general notions, which could be part of geometry teaching.

The *Traité de géométrie élémentaire* of Rouché and Comberousse was edited about 15 times from 1866 until 1935. The first edition was composed of 776 pages with two parts, plane geometry and space geometry. The historical introduction attached considerable value to Monge's descriptive geometry and Poncelet's theory of projections. The notion of rotation and its properties were used throughout the textbook, to study the similitude of polygons but also the homothetic figures

in space. The notion of rebatment also appeared, especially in the third edition where the ellipse was defined as the orthogonal projection of a circle (Rouché and Comebrousse 1873, pp. 320–330). In the following editions, rotation would be considered as a method for solving problems (Rouché and Comebrousse 1900, p. 264). In 1866, the notion of projection was introduced in the beginning of the part on geometry in space, it served, for instance, to introduce the notion of angle of a plane and a line and the shorter distance of two lines. It was proven that "the projection of a straight line AB on a plane is a straight line" (Rouché and Comebrousse 1866, p. 347). So, projection was considered as an abstract notion: properties were not considered as perfectly obvious but needed proofs. After having given the theorems on projection, the authors extended the notions of projection and perspective by considering many kinds of projections. The curves and usual surfaces were defined and studied in the part on space geometry. For instance, it was proven that the intersection of a circular cone by a plane is an ellipse, a hyperbola, or a parabola. Here, the notion of projection became central in a textbook of geometry (Barbin 2015a).

In 1864, Amédée Mannheim succeeded Jules de La Gournerie to teach descriptive geometry in the *École polytechnique*. He was a former student of this school and of the *École de Metz* before to become an officer. It is important to remark that his *Cours de géométrie descriptive de l'École polytechnique*, edited in 1881, began with various ways to represent bodies, like shadows, projections, conical perspective, axonometric perspective. It contained also a part on kinematics and its applications to descriptive geometry. Mannheim defended an important theoretical part with a quotation of Gabriel Lamé who considered that the principal utility of studies in the *École polytechnique* was "to exercise reasoning" (Mannheim 1880, p. ix). In two papers of 1882 and in the second edition of his textbook, Mannheim proposed to delete the ground line when it was not useful, to follow the habit of the engineers (Barbin 2015a). Ernest Lebon adopted this in the third edition of his *Traité de géométrie descriptive* intended for the level of "elementary mathematics" (Lebon 1901). He was a teacher at the *Lycée Charlemagne* and author of many papers on descriptive geometry. His textbook contained the three methods of descriptive geometry in this order: rotation, change of plane, rebatment. In 1891, the program of "modern secondary school teaching", which replaced "special secondary school teaching", introduced teaching of descriptive geometry for the "second and first levels" (students aged 15–17 years) (Belhoste 1995, pp. 543, 546). Lebon wrote a *Géométrie descriptive* intended to these students in 1891, where he presented the method of change only (Lebon 1891, pp. 57–64).

The reform of secondary school teaching in 1902–1905 strengthened the proximity of geometry and descriptive geometry teaching. Firstly, teaching of plane geometry was not separated from the one in space. Secondly, the teaching of geometry contained drawings, projections, and perspective, and thus many notions of descriptive geometry. Thirdly, the teaching of descriptive geometry began in the first grade (student aged 16–17 years), with projections, representations of point, line and plane on two planes of projection, rebatment of a plane on the horizontal plane, and the change of the vertical plane. The teaching in the last grade treated

rebatments, change of a plane of projection, and rotation around a perpendicular axis of a plane of projection. The teaching of descriptive geometry was teaching of methods with exercises and problems, but without concrete applications. This continued with the next reform of 1912, and a typical example is the *Cours de géométrie descriptive* of F. G.-M. (Frère Gabriel-Marie) in 1917. The "method of projection" occupied half the lessons for the first grade and the "theory of change of the horizontal plane", rebatment of a plane and rotations the third of those of the next grade (F. G.-M 1917). Many textbooks for preparatory grades were edited in this period, when, despite the criticisms of professors of the *Faculté des sciences* and of the *École polytechnique*, the teaching remained theoretical (Barbin 2015a). It is the case with the *Cours de géométrie descriptive* edited many times by Xavier Antomari for the candidates to the "great schools", which means *École polytechnique*, *École normale supérieure*, and *École centrale*. It was an impressive textbook of 641 pages, where the author quoted Rouché's method to determine the intersections between a surface and a straight line (Antomari 1910, pp. 486–491). As a result of the use of the methods, many problems of construction turned to examining the case of a projection on only one plane.

7 Conclusion: The Two Purposes of Monge and Their Historical Futures

The process of transformation of Monge's conceptions is linked with the emergence of two institutions. The first one is the preparatory grades for entrance into the *École polytechnique*, where students are prepared to answer exercises, far away from applications or drawings. The second one is the *École centrale des arts et manufactures*, created to train engineers for the industries. In some sense, these different teaching methods of descriptive geometry converge after the Cours of Olivier given in the *École centrale*, which was itself inherited from the textbooks for artists, craftsmen, and engineers written by Vallée and Adhémar. Indeed and maybe paradoxically, the introduction of "methods" by Olivier seemed equally suitable to train engineers for industries, "workers" who need applications, and to help students for examinations. In a long period, from 1815 until 1900, professors of the *École polytechnique* did not play a major role in the spreading of descriptive geometry. But, there existed a lineage of actors of changes, all linked with this school, who recognized their predecessor as a master: Monge, Hachette, Vallée, Olivier, and Rouché.

These five men promoted the applications of descriptive geometry. So, despite what the mathematician Carlo Bourlet wrote in 1906 (Sakarovitch 1998, p. 345), the first purpose of Monge was perennial in this period. Indeed, during the second part of the nineteenth century, descriptive geometry penetrated the world of craftsmen and technicians, as we can verify by comparing the first edition of 1844 of the *Nouveau manuel complet de la coupe des pierres* (cutting stones) written by the architect C.-J. Toussaint with its edition by F. Fromholt in 1902. The first book

quoted “the kind of method created by Desargues” and gave a short explanation on elementary descriptive geometry (Toussaint 1844, pp. 9–10). While, in his edition, Fromholt went further by introducing and making intelligible descriptive geometry to a simple worker, because this geometry “will teach him cutting of stones with considerable ease” (Toussaint 1902, p. 19).

The second purpose of Monge by teaching descriptive geometry at secondary level (Taton 1992) had been granted in 1865, even if it was far from his conceptions defended in the *École normale*. This teaching met projective geometry, inherited from his student Poncelet, at the end of the century. Thus, between 1902 and 1962, the teaching of descriptive geometry and geometry became closer at each reform. Finally, in 1962, descriptive geometry became only a part of the program of geometry in the last grade of the *Lycées*, from which it disappeared in 1966. It remained a subject for teaching in preparatory levels until it disappeared from the entrance examination to the “great schools” and schools of engineers in 1970.

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