Chapter 16
Descriptive Geometry in Czech Technical Universities Before 1939

Vlasta Moravcová

Abstract All branches of industry were developing rapidly in the Czech Lands in the first half of the nineteenth century. This caused an increasing demand for specialists with technical education. Descriptive geometry (as an important part of this education) first appeared as a subject in polytechnic schools, but soon also in secondary schools, especially in real-schools.[1] The greatest boom in Czech descriptive geometry came in the second half of the nineteenth century and it was still reverberating at the beginning of the twentieth century. Secondary schools, technical universities and other schools with the Czech teaching language were established and Czech textbooks and original scientific works were published. In this chapter, we give fundamental information about descriptive geometry education in Czech technical universities and about the most significant results, which were published by geometers who lectured at these universities.

Keywords Czech language · Descriptive geometry · Jan Sobotka · Karel Pelz · Polytechnic schools · Real-schools · Rudolf Skuherský · Technical universities

[1]The real-school (from German: *die Realschule*, in Czech: *reálka or reálná/reální škola*) was a special kind of secondary school with emphasis on mathematics and natural sciences as opposed to grammar school (gymnasium), where more lessons of Greek, Latin, etc. were provided. These real-schools were instituted in Austria in 1849 by the Exner-Bonitz reform.

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1 Beginnings of Descriptive Geometry Teaching in the Czech Lands

The first polytechnic school in the Czech Lands,[2] *Königlich-böhmische ständische Lehranstalt zu Prag* (royal Czech educational institution of the estates in Prague), was founded in Prague in 1806 in the manner of the *École polytechnique* in Paris. It originated from the *Ständische Ingenieurschule in Prag* (engineering school of the estates in Prague), which was established in 1707. At first, this polytechnic school[3] was a part of the *Karl-Ferdinand Universität in Prag* (Charles-Ferdinand university in Prague),[4] but it became an independent school in 1815. Its first principal[5] was František Josef Gerstner (1756–1832).

The main aim of the polytechnic school was the preparation of students for practice. Therefore the character of education was directed not only at theory, but particularly at applications of technical and natural sciences.

Descriptive geometry was introduced by Karel Wiesenfeld (1802–1870) as a part of civil engineering lectures at the polytechnic school in Prague in the 1830s. Students did not know this subject from secondary school, therefore Wiesenfeld at least provided the students with bases of Monge's projection and perspective. Moreover, orthogonal projection and construction of shadows were included in the syllabus for machine engineering students, thanks to professor Karel Wersin (1803–1880) in 1840. The education was provided according to César Nicolas Louis Leblanc's textbook *Choix de modéles appliqués à l'enseignement du dessin des machines, avec un texte descriptif* (selected models used for teaching of machine drawing with a descriptive text) (Leblanc 1830).

Inclusion of descriptive geometry in the lectures proved very useful and as a result, the department of descriptive geometry was established at the polytechnic school in Prague in 1850. The course *Beschreibende* (descriptive) *Geometrie* became obligatory for all first-year students, except for students of chemical engineering. At first, the lectures were provided by the professor of civil engineering Karel Wiesenfeld. The first professor of descriptive geometry, Rudolf (Rudolph) Skuherský, was appointed 2 years later. Rudolf Skuherský (1828–1863) studied at the polytechnic school in Prague and later in Vienna. There he published two

[2]The Czech Lands (also known as the Lands of the Bohemian Crown) were a constituent part of the Habsburg Monarchy, which was formally unified as the Austrian Empire (1804–1867) and later as the Austro-Hungarian Empire (1867–1918). The independent state Czechoslovak Republic (consisting of Bohemia, Moravia, Czech Silesia, Slovakia and Sub-Carpathian Ruthenia) was declared in 1918 and dissolved as a result of a German invasion in 1939 (more precisely, as a result of the Munich Agreement in 1938).

[3]This school had various official names throughout history (see Moravcová 2015, p. 159).

[4]The university in Prague, known as the Charles university in Prague, was named after Kaiser Ferdinand III between the years 1654–1918.

[5]This school did not gain a university statute immediately, it was approved in 1863. There was similar situation in other technical schools in the Austrian Empire.

original papers on descriptive geometry *Die orthographische Parallelperspective* (Skuherský 1851) and *Die Theorie der Theilungspunkte als Beitrag zur Lehre von der freien Perspektive* (theory of dividing points as a contribution to free perspective) (Skuherský 1851), that ensured him the post of a descriptive geometry assistant of professor Johann Hönig (1810–1886) in the school year 1851/1852 (see Velflík 1906, 1909, pp. 389–396).

Skuherský gave 15 h of lectures in descriptive geometry a week (including seminars) according to his own treatises, Johann Hönig's textbook *Anleitung zum Studium der darstellenden Geometrie* (introduction to the study of descriptive geometry) (Hönig 1845) and Charles François Antoine Leroy's work *Traité de Géometrie descriptive* (Leroy 1834). The lectures contained various methods of projections including Skuherský's own method (see Sect. 7), a projection of polyhedra with regard to crystallography, a theory of curves and surfaces, spherical geometry, illumination and anamorphosis. Moreover, Skuherský organized supplementary lectures on perspective and applications such as stereotomy or gnomonic projection.

The second polytechnic school in the Czech Lands was founded in Brno in 1849. Descriptive geometry was initially taught there by an assistant, Anton Mayssl (1826–1899), according to the second edition Georg Schaffnit's textbook *Geometrische Constructionlehre, oder darstellende Geometrie* (geometrical constructions or descriptive geometry) (Schaffnit 1837). Two years later Georg Beskiba was appointed professor. Georg Beskiba (1819–1882) studied at the polytechnic school in Vienna. He became professor of civil engineering at the polytechnic school in Lviv in 1846. There he also lectured in descriptive geometry (see Šišma 2002, pp. 36–38).

Descriptive geometry lectures at the polytechnic school in Brno were obligatory for students of engineering in the first year of studies and their number fluctuated between 3 and 13 h a week (including seminars) in the 1850s.

2 Origins of Czech Technical Universities

Although the Czech language was allowed by the law at the Charles-Ferdinand University in Prague since the revolutionary year 1848, the polytechnic school in Prague only provided German lectures. Skuherský was the first one who offered Czech lectures (in parallel with German lectures) in 1861. Owing to the great interest in these lectures they continued to be provided. This was one of the steps that resulted in the division of the technical university in Prague into the separated Czech technical university, *Český polytechnický ústav království českého* (Czech polytechnic school of the Kingdom of Bohemia) and the separated German technical university, *Deutsches polytechnisches Institut des Königreiches Böhmens* (German

polytechnic school of the Kingdom of Bohemia), in 1869.[6] Both of these schools used in their official names the term technical university from 1879.

The polytechnic school in Brno was finally established as a German school and it was reorganized as a technical university in 1873. However, the *Česká vysoká škola technická v Brně* (Czech technical university in Brno) was established in 1899. Jan Sobotka was appointed professor of descriptive geometry in the same year. Jan Sobotka (1862–1931) studied at the Czech technical university in Prague. He then worked as an assistant in descriptive geometry there between the years 1886–1891. Later he studied in Zürich and Wrocław, taught at the real-school in Vienna (from 1894) and at the technical university in Vienna (from 1896). Sobotka influenced teaching of geometry in Brno from 1899 to 1904, then he was appointed professor of mathematics at the Charles-Ferdinand university in Prague. He is known for his textbook *Deskriptivní geometrie promítání parallelního* (descriptive geometry of parallel projection) (Sobotka 1906) and for the works on axonometry (especially oblique axonometry, see Sect. 9) and differential geometry (see Kašparová and Nádeník 2010). Sobotka created a syllabus[7] of descriptive geometry, organized a mathematical library and arranged a collection of geometrical models in Brno (Unknown Author 1911).

3 Descriptive Geometry in Czech Secondary Schools

Descriptive geometry also appeared in secondary schools, particularly real-schools, in the 1850s in connection with the development of technical studies.

Real-schools were instituted as a 6-year secondary school in 1849 by *Entwurf der Organisation der Gymnasien und Realschulen in Oesterreich* [outline of the organization of grammar schools and real-schools in Austria] (Unknown Author 1849). The number of classes increased to seven in about 1870. Czech and German real-schools were opened in the Czech Lands during the second half of the nineteenth century. At first, the education was provided mainly in the German language in Czech real-schools, but from the 1860s the Czech language began to be used more frequently.

Descriptive geometry was being integrated into education gradually and without a given curriculum in the 1850s and 1860s. The curriculum of descriptive geometry for Czech real-schools was first determined in 1874 and modified in 1898, 1909 and 1933 (see Moravcová 2015, pp. 29–38). Initially it contained orthogonal projection onto two planes (Monge's projection) and central projection (especially linear

[6]The period 1864–1869 (also called the utraquist period), when the school provided all lectures in German and Czech in parallel, preceded the division of the technical university in Prague (for history of the technical university in Prague see Lomič and Horská 1978).

[7]The syllabus contained Monge's orthogonal projection, oblique projection, central projection, axonometry and technical curves and surfaces.

perspective). The other projection methods like orthogonal axonometry and oblique projection were introduced in the first half of the twentieth century, but all parts were reduced and simplified as well.

The first graduation exams in some real-schools took place in 1869 (in connection with an extension to seven classes); from 1872 the exams were defined by law (before that the exams could be taken only in grammar schools). The descriptive geometry exam was one of the obligatory parts of graduation exams in real-schools, it had a written form and lasted 5 h. The exam was very difficult in comparison to current requirements in secondary education.

The advanced level of descriptive geometry in real-schools before World War II can be supported with several extant materials found in libraries or archives like graduation exam exercises, students' drawings, school notes, etc. (see Moravcová 2015).

The development of the Czech descriptive geometry curriculum can also be observed through the study of Czech textbooks for real-schools. The first one *Zobrazující měřictví* (descriptive geometry) (Ryšavý 1862, 1863) was written by Dominik Ryšavý (1830–1890). Even though it contained many errors, it was crucial for Czech descriptive geometry as it started the formation of the Czech terminology.[8] Problems with the terminology were worked out by Vincenc Jarolímek (1846–1921) in his textbook *Deskriptivní geometrie pro vyšší školy reálné* (descriptive geometry for high real-schools) (Jarolímek 1875, 1876, 1877). This one was supplemented with a German–French–Czech glossary of all the terms used. Moreover, the topics were organized very clearly and logically and the signage was similar to the contemporary one. The textbook was exceptional and there was a high demand for it abroad. It was translated into Bulgarian and published in Plovdiv[9] in 1895. Furthermore, Jarolímek published the first Czech collection of descriptive geometry exercises (Jarolímek 1873). It contained more than one thousand exercises on Monge's projection which Jarolímek created from his own experience. Before this collection (first published in 1873 and reprinted in 1880 and 1904), there had been no similar German collection in use in Czech schools.

[8]The first Czech textbooks together with the first Czech lectures were fundamental acts for a creation of Czech terminology. The main personalities who were involved in it were Rudolf Skuherský (he started the Czech lectures on descriptive geometry at the Czech polytechnic school in 1861) and Dominik Ryšavý (who started to teach in the Czech language at the *První česká reálka v Praze* (first Czech real-school in Prague) in the same year).

[9]Czech mathematicians had great merit in the introduction of descriptive geometry in secondary education in Croatia and Bulgaria in the second half of the nineteenth century (see Bečvářová 2009).

Descriptive geometry was also taught in real-grammar schools[10] and some secondary industrial schools before World War II. However, descriptive geometry teaching in these schools was not as significant as in real-schools.

Real-schools ceased to exist during World War II (most of them were transformed into grammar schools) and since then the importance of descriptive geometry in secondary schools has been constantly decreasing.

4 Descriptive Geometry at the Czech Technical University in Prague

The Czech lectures in descriptive geometry at the Czech technical university in Prague were given by professor František Tilšer from 1864. Tilšer (1825–1913) was a student of law at first, later he studied at military schools in Olomouc and Vienna. This school was moved from Vienna to Louka u Znojma and Tilšer obtained a post of descriptive geometry professor there in 1854. He became professor at the technical university in Prague in 1864 and after its division (1869) he lectured at the Czech technical university. He simultaneously worked as a member of provincial assembly from 1870, therefore his lectures were often delivered by supply teachers. Tilšer elaborated a theory of illumination in the work *Die Lehre der geometrischen Beleuchtungs-Constructionen und deren Anwendung auf das technische Zeichnen* (essay on geometrical constructions of illumination and their application in technical drawing) (Tilscher 1862), in which he made efforts to generalize a construction of parallel illumination isophotes by using systems of tangential planes, which have a constant angle with the illumination direction, of a surface. Equally significant is Tilšer's two-volume work *System der technisch-malerischen Perspective* (system of technical-painting perspective) (Tilscher 1865, 1866) (see Velflík 1910, 1925, pp. 45–54).

Tilšer divided the lectures into 2 years of studies between the years 1870 and 1874, but their total number was not changed. At first, Tilšer taught like Skuherský, but he transformed the concept of descriptive education in 1875. He perceived descriptive geometry as the one means of human cognition. He distinguished two parts of descriptive geometry—*morphognosy* (a science on objects which are projected) and *iconography* (a science on projection of objects). Moreover, Tilšer created a special system of signage and he called the whole scientific discipline iconognosy (Mikulášek 1924). His philosophical approach was correct but too complicated.

[10]The real-grammar school (from German: *das Realgymnasium*, in Czech: *reálné gymnasium*) was a special kind of 8-year secondary school that was instituted in Austria in 1908 (schools of this kind had already been established in the Czech Lands after 1862) as a compromise between real-schools and classical grammar schools.

Tilšer wrote several works on descriptive geometry, some of them were written in the Czech language. He also published the first Czech descriptive geometry textbook for university students titled *Soustava deskriptivní geometrie* (system of descriptive geometry) (Tilšer 1870); however, he finished only the first volume which contained basics of projective geometry, polar coordinates and selected curve properties.

Tilšer's philosophical ideas were elaborated in his works *Grundlagen der Ikonognosie* (basics of iconognosy) (Tilscher 1878) and *Gasparda Monge-a Géometrie descriptive po stoletém vývoji čili u východiště z labyrintu* (Gaspard Monge's descriptive geometry after one-hundred-years development, or near the exit from the labyrinth) (Tilšer 1898).

The next professor of descriptive geometry at the Czech technical university in Prague, Karel Pelz, was chosen during a selection in 1896. Karel Pelz (1845–1908) studied at the technical university in Prague, then he worked as an assistant in descriptive geometry at the German technical university in Prague from 1870 to 1875. After that he taught at the real-schools in Těšín and Graz. He became professor of descriptive geometry at the technical university in Graz in 1878. Pelz was interested mainly in a theory of curves and surfaces, axonometry (see Sect. 9) and central projection. He wrote more than thirty original scientific works, many of them were positively appreciated abroad (see Sklenáriková and Pémová 2007).

At the technical university in Prague, Pelz simplified the syllabus and returned to standard conception of descriptive geometry teaching. Although he did not publish in the Czech language and his Czech was allegedly not excellent, he was appreciated by students as a great teacher. We can see the high quality of his lectures, thanks to extant lithographic notes.[11]

The number of students of the technical university in Prague increased, therefore parallel descriptive geometry lectures were introduced at the beginning of the twentieth century. As a result, the second department of descriptive geometry was created in 1907 and Vincenc Jarolímek was appointed the next professor. Vincenc (Čeněk) Jarolímek (1846–1921) studied at the technical university in Prague. He worked at the real-school in Písek since 1868, subsequently he became the head at the real-school in Hradec Králové (1891–1893), Karlín (1893–1895) and at the first Czech real-school in Prague (1895–1904). He was a provincial school inspector from 1904, simultaneously he lectured at the Czech technical university in Brno from 1905. He moved to the Czech technical university in Prague in 1906. Jarolímek continued Tilšer's research into isophotes in central illumination in the work *Centrálné osvětlení* (central illumination) (Jarolímek 1871). Other fields of his activities were theory of curves and surfaces, especially of the second order, and imaginary objects in geometry (see Sobotka 1916).

From 1907, lectures in descriptive geometry were provided in three specialized sections. The first one (also called the first department of descriptive geometry) was intended for students of civil engineering; the second one (the second department

[11] Lithographic (lithography is a method of copying) notes were written by students as a replacement of missing Czech textbooks.

Table 16.1 Overview of the descriptive geometry lecturers for civil engineering students at the Czech technical university in Prague between the years 1907–1939 (Moravcová 2015, p. 420)

Deskriptivní geometrie (descriptive geometry)		
1907–1908	K. Pelz	5/6, 4/6
1908–1913	V. Jarolímek	
1913–1915		5/5, 4/5
1915–1921	F. Kadeřávek/B. Procházka	
1921–1939	F. Kadeřávek	5/4, 4/4

Table 16.2 Overview of the descriptive geometry lecturers for machine engineering students at the Czech technical university in Prague between the years 1907–1939 (Moravcová 2015, p. 421)

Deskriptivní geometrie (descriptive geometry)		
1907–1908	V. Jarolímek	5/6, 4/6
1908–1921	B. Procházka/F. Kadeřávek	
1921–1925	B. Procházka	6/3, 0/5
1925–1927	J. Kounovský	
1927–1932		4/4, 2/4
1932–1933		5/3, 0/4
1933–1939		5/3, 0/3

Table 16.3 Overview of the descriptive geometry lecturers for the third section at the Czech technical university in Prague between the years 1907–1939 (Moravcová 2015, p. 421)

Deskriptivní geometrie (descriptive geometry)		
1907–1921	B. Chalupníček	6/6, 4/6
1921–1922		4/6, 4/3
1922–1930	F. Kadeřávek	
1930–1932		4/6, 4/4
1932–1939		4/4, 4/4

of descriptive geometry) was for students of machine engineering and the third one for ground building students (see Tables 16.1, 16.2, and 16.3).[12] A third department was not formally created.

All lectures contained the most common kinds of projections. Moreover, the lectures for the first section included spot height projection,[13] projective geometry and illumination. Lectures for the second one put emphasis on kinematic geometry and the third one included perspective. Furthermore, the fourth section for students of forest engineering, which contained spot height projection, nomography and basics of photogrammetry extra, was opened in 1919. Descriptive geometry lectures were provided only in the first semester of the first year of studies.

[12] Tables 16.1, 16.2, 16.3, and 16.4 provide information on lectures in descriptive geometry including the lectures' names and the numbers of lectures. A symbol x/y in the last column refers to the number of lectures/seminars a week. The lectures were intended for first-year students (unless otherwise stated).

[13] Spot height projection is an orthogonal projection onto a single plane. Projections of points are annotated with numbers (spots) which give information about the distance between the point and the plane of projection. Minus indicates the negative semi-space. This method is used mainly in cartography and civil engineering.

Bedřich Procházka, Josef Kounovský and František Kadeřávek were three other important descriptive geometry professors at the Czech technical university in Prague.

Bedřich Procházka (1855–1934) studied at the Czech technical university in Prague and from 1876 worked as an assistant in descriptive geometry there. He passed teachers' competence exams in mathematics and descriptive geometry in 1879. After that he taught at secondary schools in Prague, Chrudim, Pardubice and Karlín. He was awarded the position of senior lecturer in geometrical illumination in 1884 and subsequently in kinematic geometry in 1895. He worked as the head at the real-school in Náchod from 1897. He became professor of descriptive geometry at the Czech technical university in Brno in 1904 and in Prague in 1908 (see Bydžovský 1934).

Josef Kounovský (1878–1949) studied at the Czech technical university in Prague and worked as an assistant in descriptive geometry there from 1902. He passed teachers' competence exams in mathematics and descriptive geometry in the same year. He then taught at the real-schools in Prague and Hradec Králové. He was awarded the position of senior lecturer in geometry in 1912 and then he lectured at the Czech technical university in Prague, where he became professor in 1927 (see Kadeřávek 1950).

František Kadeřávek (1885–1961) studied at the Czech technical university in Prague and worked as an assistant in descriptive geometry there from 1906. He passed teachers' competence exams in mathematics and descriptive geometry in 1908. He was awarded the position of senior lecturer in synthetic geometry in 1912. He became professor at the Czech technical university in Prague in 1917 but he had already lectured in descriptive geometry there from 1915. Together with Josef Kounovský and Josef Klíma (see below) he wrote an important two-volume Czech descriptive geometry textbook for all university students *Deskriptivní geometrie I, II* (Kadeřávek et al. 1929, 1932). The book contains all substantial topics of descriptive geometry and is used even by contemporary students from time to time. Kadeřávek was interested in the history of descriptive geometry and wrote several popular science books on it (see Kepr 1955).

At the Czech technical university in Prague, Procházka and Kadeřávek often swapped the first and second section of descriptive geometry lectures and also gave the lectures for the third and fourth section.

Jarolímek and Procházka wrote the first Czech descriptive geometry textbook for students of technical universities titled *Deskriptivní geometrie pro vysoké školy technické* (descriptive geometry for technical universities) (Jarolímek and Procházka 1909). The book was distinguished by its logical structure and was in great demand.

5 Descriptive Geometry at the Czech Technical University in Brno

The lectures in descriptive geometry provided at the Czech technical university in Brno had a similar form as in Prague.

Bedřich Procházka, who became professor in 1904, introduced additional topics from projective and kinematic geometry in 1906, when the number of descriptive geometry lectures was increased (see Table 16.4).

Miloslav Pelíšek was appointed professor of descriptive geometry in 1909. Pelíšek (1855–1940) studied at the German technical university in Prague. From 1881, he worked as an assistant in descriptive geometry there, after that he taught at the secondary industrial schools in Plzeň and Prague. He lectured at the Czech technical university in Brno from 1908. Pelíšek focused on synthetic, kinematic and analytic geometry. His determination method of trajectories curvature centres is known as *Pelíšek's construction* in Czech geometry textbooks (see Hlavatý 1941).

At the Czech technical university in Brno, Pelíšek modified the syllabus of the descriptive geometry lectures, in which he added spot height projection and its applications. At the same time, he reduced kinematic and projective geometry in them, however, he offered optional lectures on these topics. As the number of the descriptive geometry lectures was decreasing in 1921, Pelíšek wrote a textbook *Deskriptivní geometrie* (Pelíšek 1922) that was tailored for students.

The number of students in Brno was smaller than in Prague, therefore parallel lectures were provided later, from 1928. These lectures were organized by Josef Klíma, who was appointed professor of descriptive geometry at the Czech technical university in Brno in 1927. The lectures were common for all students in the first semester. In the second semester, they were divided into two sections (one for civil engineering students and the other for mechanical and electrical engineering students). Josef Klíma (1887–1943) studied at the Czech technical university in

Table 16.4 Overview of the descriptive geometry lecturers at the Czech technical university in Brno (Moravcová 2015, p. 424)

Deskriptivní geometrie (descriptive geometry)		
1899–1904	J. Sobotka	6/6, 4/6
1904	V. Jarolímek	
1904–1905	B. Procházka	
1905–1906		6/6, 4/6
Deskriptivní geometrie spojená s geometrií polohy (descriptive geometry associated with projective geometry)		
1906–1908	B. Procházka	6/6, 6/6
1908–1914	M. Pelíšek	6/6, 6/6
Deskriptivní geometrie (descriptive geometry)		
1914–1921	M. Pelíšek	6/6, 6/6
1921–1926		5/5, 5/5
1926–1927	J. Klapka	4/4, 4/4
1927–1939	J. Klíma	

Prague and became an assistant in descriptive geometry at the same school in 1909. He taught at the real-schools in Vinohrady from 1917, later in Vršovice and Karlín. He was awarded the position of senior lecturer in descriptive geometry in 1924 (see Seifert 1946).

6 Summary of Descriptive Geometry Teaching in the Czech Technical Universities

Descriptive geometry was one of the main subjects for all the first-year students except for students of chemistry at technical universities in the Czech Lands. Lectures continued in the curriculum of real-school, i.e., Monge's projection was not included, but axonometry, central, oblique and spot height projections were taught from basic principles. The emphasis was put on theories of curves and surfaces, collineation, projective and kinematic geometry were lectured on as well. Moreover, illumination was a significant topic in the nineteenth century, later it was reduced.

The number of lectures and seminars was very high in the nineteenth century, but we can see its decrease in the tables above, after 1900. On the other hand, syllabi were expanded on new topics such as photogrammetry and nomography, so that they adapted to requirements of the narrow specialization of graduates.

Lectured topics on descriptive geometry were similar at both Czech technical universities in the twentieth century, the slight difference was in projective geometry teaching. Its basics were a part of the descriptive geometry lectures in most cases. Moreover, advanced problems of projective geometry were taught during extra lectures in Prague; these kinds of lectures were offered in Brno only by Antonín Sucharda (1854–1907) between the years 1901–1906 and by Miloslav Pelíšek between the years 1911–1918.

Finally, special lectures for students who were preparing for teachers' competence exams should be mentioned. The exams were introduced as special exams for students who meant to become secondary school teachers in 1850. They were supervised by committees which were set up within classical universities,[14] but lectures suitable for candidates taking the exam in descriptive geometry were only provided in technical universities in the Czech Lands in the nineteenth century.[15] These lectures were modified every year and contained various advanced topics, for example, central axonometry, theories of high-order surfaces, etc. Bedřich

[14] In the nineteenth century there was only one classical university in the Czech Lands—the above mentioned Charles-Ferdinand university in Prague, which was divided into Czech and German universities in 1882. The German university in Prague was closed down in 1945. The second Czech classical university (and the last one to be found before World War II) was established in Brno in 1919.

[15] Descriptive geometry lectures at classical universities in the Czech Lands were not provided systematically in the nineteenth century. Jan Sobotka started these lectures at the Czech technical university in Prague in 1910 for the purpose of preparing student for teachers' competence exams.

Procházka prepared a special six-volume textbook for these students titled *Vybrané statě z deskriptivní geometrie* (selected topics of descriptive geometry) (Procházka 1912–1918), which contained less known information about orthogonal and oblique axonometry and surfaces of the second order, central axonometry, central illumination, spatial curves, translational surfaces and kinematic geometry (see Moravcová 2015, pp. 247–248).

7 Skuherský's Projection Method

The first attempts for a formation of graphic projection methods originated in England in the first half of the nineteenth century (see Lawrence, Chap. 18, this volume). Orthogonal axonometry was one of two approaches that were developed from these attempts. The other one was orthogonal parallel projection created by Rudolf Skuherský. Skuherský published his new theory that was based on transformations of an object in relation to the projection planes (Fig. 16.1) in the works *Die orthographische Parallelperspektive* (Skuherský 1850), *Die orthographische Parallel-Perspektive* (Skuherský 1858a) and *Die Methode der*

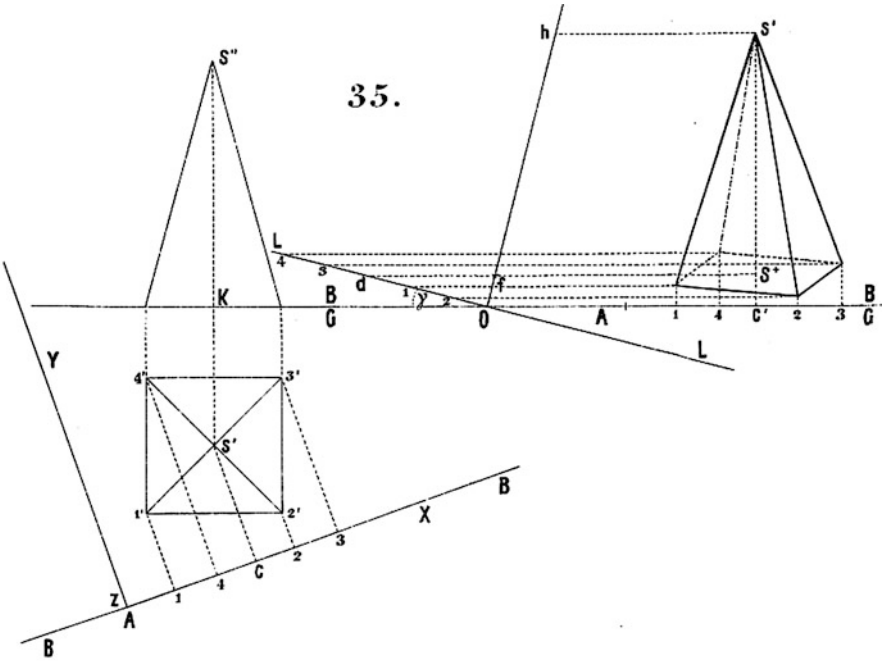

Fig. 16.1 Skuherský's projection of a pyramid from his work *Die Methode der orthogonalen Projekzion auf zwei Ebenen* (Skuherský 1858b, Tab. 2, Fig. 35)

orthogonalen Projekzion auf zwei Ebenen (method of orthogonal projection onto two planes) (Skuherský 1858b).

Skuherský's method can be explained easily through the projection of a cuboid $ABCDA'B'C'D'$, which is given in Monge's projection (Fig. 16.2a). We select

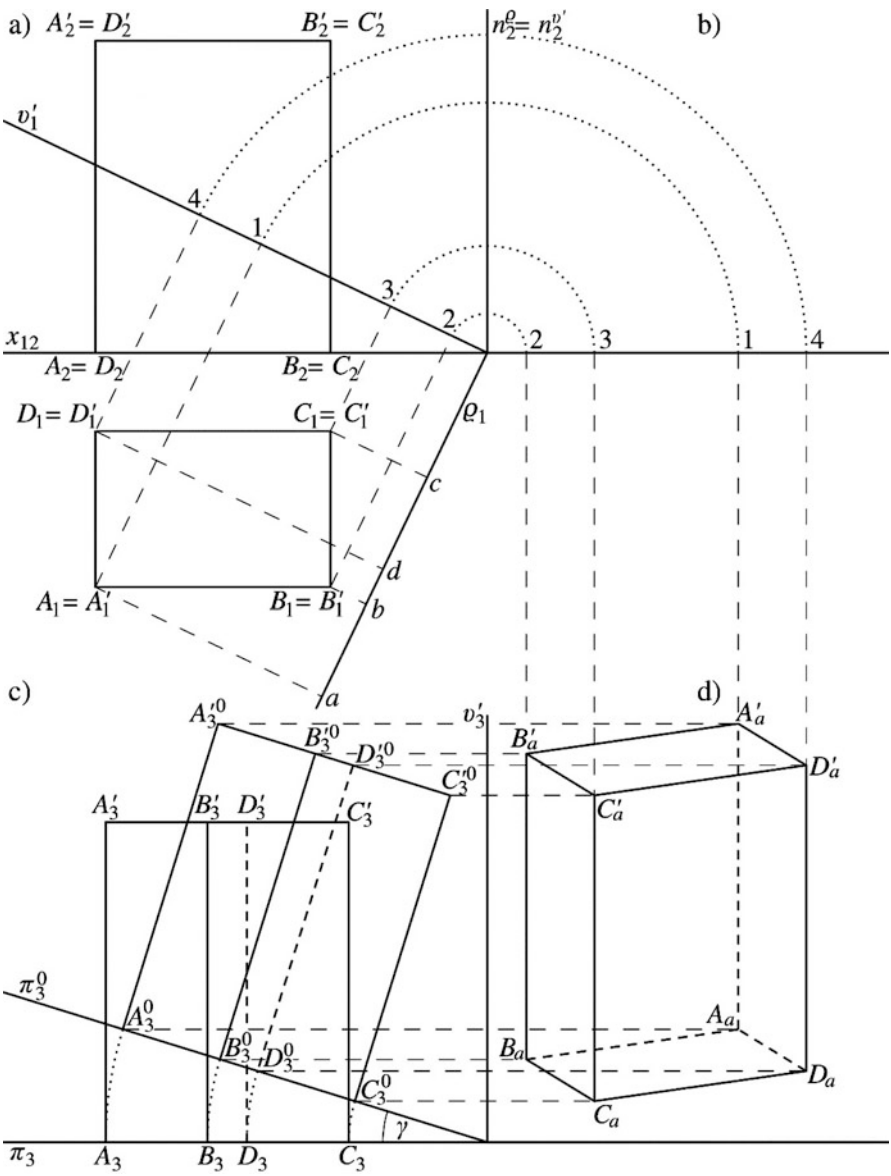

Fig. 16.2 Skuherský's projection of a cuboid (Moravcová 2015, p. 266)

a third projection plane ρ perpendicular to the first projection plane π. We then construct an orthogonal projection of the cuboid onto a plane ν' which is perpendicular to both π and ρ. We rotate ν_1' to the axis x_{12} (see points 1, 2, 3, 4 in Fig. 16.2b) and also rotate the third projection onto ρ at a given angle γ around the line of intersection of π and ν' (Fig. 16.2c). Finally, we can project this transformed cuboid onto ν' again (using the auxiliary points 1, 2, 3, 4 from Fig. 16.2b) and obtain the orthogonal projection of the cuboid in Skuherský's method (Fig. 16.2d).

Skuherský's orthogonal projection enables us not only to display any object but also to solve spatial problems in a plane. Although this method was replaced by smart orthogonal axonometry in the second half of the nineteenth century, we can find it or its elements in many textbooks, for example, Gustav Adolf Viktor Peschka's *Darstellende und projective Geometrie* (descriptive and projective geometry) (Peschka 1883) or Emil Müller's *Lehrbuch der darstellenden Geometrie für technische Hochschule, zweiter Band* (descriptive geometry textbook for technical universities, 2nd vol.) (Müller 1923).

8 Pelz's Contribution to Axonometry

Karel Pelz had great merit in the development of orthogonal axonometry. He focused on it during his stay in Graz and published four original papers in which he described many constructions in orthogonal axonometry: *Zur wissenschaftlichen Behandlung der orthogonalen Axonometrie I, II, III* (on scientific conception of orthogonal axonometry I, II, III) (Pelz 1880, 1881, 1884), and *Beiträge zur wissenschaftlichen Behandlung der orthogonalen Axonometrie* (supplement to the scientific conception of orthogonal axonometry) (Pelz 1885). In the first and second, Pelz introduced, inter alia, constructions of perpendicular lines and planes and projections of a circle in various planes. He was the first one who proved that an orthogonal axonometry is unequivocally defined by an axonometric triangle (Sklenáriková and Pémová 2007). In his subsequent papers, Pelz dealt with illumination of cylinders and cones (Fig. 16.3). In the last paper, he focused on a sphere and its parallel and central illumination (Sobotka 1910, p. 458).

Pelz did not write a textbook on descriptive geometry, but his discoveries were published by his assistant Rudolf Schüssler (1865–1942) in a book *Orthogonale Axonometrie, ein Lehrbuch zum Selbststudium* (orthogonal axonometry, a self-study book) (Schüssler 1905).

Pelz also dealt with the basic theorem of oblique axonometry known as *Pohlke's theorem* after its author Karl Pohlke (1810–1876). Pelz published a new original proof in a paper *Über einen neuen Beweis des Fundamentalsatzes von Pohlke* (on a new proof of Pohlke's theorem) in 1877 (Pelz 1877). According to Pohlke's assistant Hermann Schwarz (1843–1921), Pelz's proof was similar to Pohlke's own, but Pohlke did not publish it. Principles of Pelz's proof, which was based on a system of confocal conics, are elaborated in (Sklenáriková and Pémová 2007).

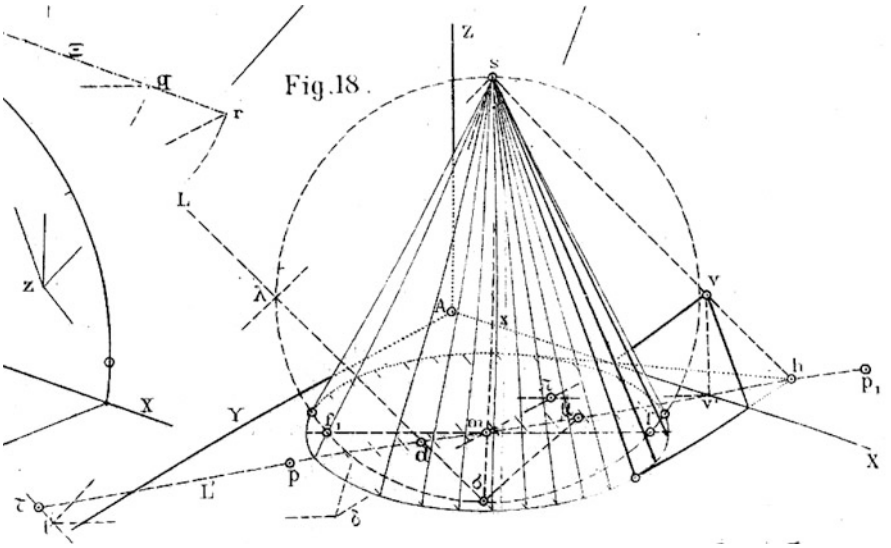

Fig. 16.3 Illumination of a cone in orthogonal axonometry from Pelz's paper *Zur wissenschaftlichen Behandlung der orthogonalen Axonometrie III* (Pelz 1884, Tab. 1, Fig. 18)

9 Sobotka's Constructions

The next Czech professor who claimed credit for the development of axonometry was Jan Sobotka. He devised three methods of transformation of oblique axonometry into orthogonal projection, which were published in the article *Axonometrische Darstellungen aus zwei Rissen und Koordinatentransformationen* (axonometric projection on the basis of two drawings and coordinate transformations) in 1901 (Sobotka 1901). These constructions are usually referred to as *Sobotka's constructions* in Czech literature.

The first construction transforms oblique axonometry into Monge's projection. It can be used in case the axonometry is defined by an axonometric triangle and a projection of the origin of coordinates. Oblique axonometry is first converted into orthogonal axonometry, then two of the auxiliary planes of projection are rotated to the axonometric plane and translated in the directions of the double-projection planes (see Fig. 16.4).

The second and third constructions are very similar to each other. They can be used if the axonometry is defined by oblique projections of the coordinates axes and the units of measurement on them. Sobotka excellently used the affine relation between the first and second auxiliary oblique projections and the first and second orthogonal projections (for detail see Moravcová 2015, pp. 273–275).

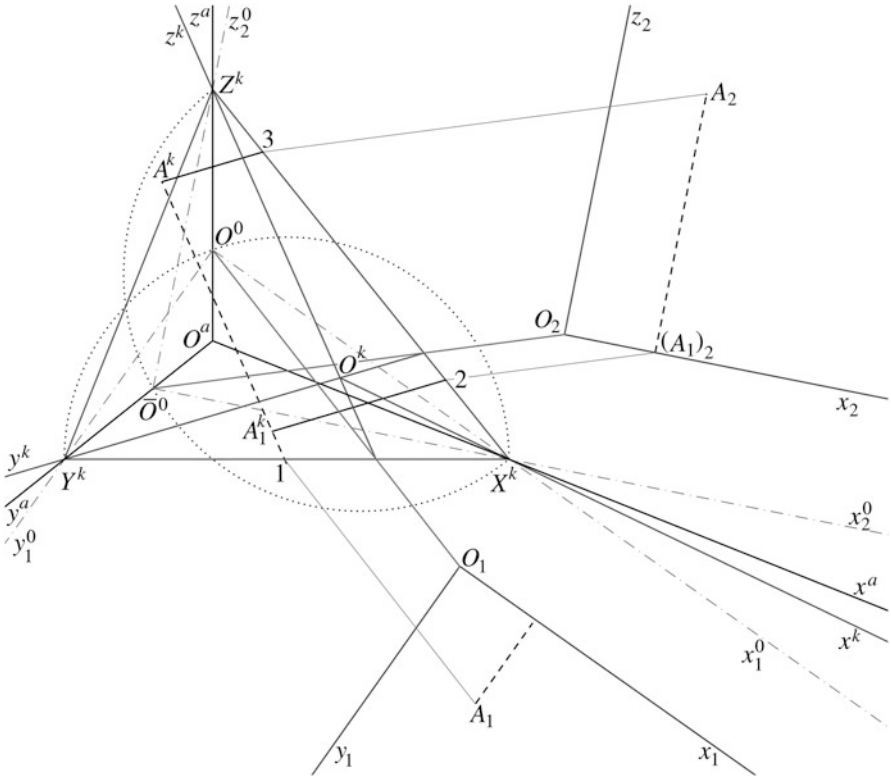

Fig. 16.4 Demonstration of Sobotka's first construction. A transformation of the oblique projection A^k of a point A into an orthogonal projection onto two perpendicular planes (Moravcová 2015, p. 272)

10 Conclusion

The most profound development of descriptive geometry in the Czech Lands was noticeable in the last third of the nineteenth century and its reverberations were still perceptible in the first half of the twentieth century. This science was integrated into the education of all engineers as an integral part of it. We have focused on Czech technical universities in this paper, but the situation and role of descriptive geometry at German technical universities in the Czech Lands was similar.

Origins of descriptive geometry in the Czech Lands were connected with its development in France, Germany and Austria; the greatest influence came from the polytechnic school in Vienna (see Stachel, Chap. 11 and Binder, Chap. 12, this volume). At first, German and French sources were studied until the 1860s. Nevertheless, in connection with Czech emancipation, we can observe the expansion of Czech science including the development of geometry (not only descriptive) as well. Many Czech geometers published works with new results on descriptive

geometry but also on projective or differential geometry (besides those mentioned, we can refer to Czech mathematicians such as Eduard Weyr, Emil Weyr, Josef Šolín and others). The term *Czech geometric school* is taken for these activities (see Folta 1982).

The interest in technical branches gradually decreased in the period between the World Wars and Czech education system and science were definitively repressed by World War II (Czech universities were forced to close in 1939). Czech education was restored after 1945, but descriptive geometry never gained such importance again.

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