Barrier Coverage Problem in 2D

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Abstract. This paper deals with the NP-hard problem of covering a line segment by n initially arbitrarily arranged circles on the plane by moving their centers to the segment in such a way that the sum of the Euclidean distances between the initial and final positions of the centers of the disks would be minimal. In the case of identical circles, a dynamic programming algorithm is known, which constructs a $\sqrt{2}$–approximate solution to the problem with $O(n^4)$–time complexity. In this paper, we propose a new algorithm that has the same accuracy, but the complexity of which is reduced by n^2 times to $O(n^2)$.

Keywords: Sensor networks · Mobile sensors · Barrier coverage

1 Introduction

The sensor network consists of devices, each of which collects data within a proximity, which is called a *coverage area*. On the plane, a coverage area most often is a circle (disk) with a sensor in its center [5,7,19,24]. Though both an ellipse [16] and a sector [17] can be a coverage area of the sensor. In the wireless sensor networks energy of the sensors is often irreplaceable because the recharge or change of the battery is either impossible or impractical. The energy of the sensors defines network's lifetime. Rational use of energy prolongs the lifetime of the sensor network [5,7]. For energy efficient operation of the sensor network, it is necessary to solve several optimization problems. One of the problems is optimal placement of sensors and determination of the values of their parameters. As sensing energy consumption is proportional to the coverage area, this problem is reduced to the classical min-density covering problem [5–7,13,24].

In barrier monitoring, it is necessary to detect an unauthorized crossing of a barrier separating the two territories. In some cases, the barrier is considered as a line [1,4,10–14,20,21], in others as a strip [17,23]. The barrier can be covered by stationary sensors [2,9,10,15,17,20,22,23,25], and by mobile sensors [1,4,11–14,18]. A coverage area is often considers as a circle [4,11–14,20], but sometimes (in the case of directed devices) it is a sector [17,22,25]. In [20] a notion of weak coverage is introduced and the critical conditions for the existence of weak barrier coverage in a randomly deployed sensor network is proposed. Later, in [9] the

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algorithm of guaranteed detection and localization of intruders, the trajectory of which is located in the area of sensors placement, is proposed. How to assess and ensure the quality of the barrier coverage is examined in [10].

If a sensor is mobile, the movement energy consumption is proportional to the distance traveled by a sensor. The important optimization problem for mobile sensor networks is minimization of the total distance traveled by sensors [1,4, 11–14,18]. It is necessary to move the sensors in such a way that each point of the barrier (the line segment) belongs to the coverage area of at least one sensor, and the total length of the relocations would be minimal. In [13] the NP-hardness of the problem is proved. The efficiency of the different placement strategies of sensors for a barrier coverage is studied in [21]. There was also studied the question how to improve a barrier coverage after the placement using the mobility of the sensors. The authors of [21] presented an algorithm for efficient improvement of barrier coverage with a wide range of parameters of placement the sensors. Circular barriers in the plane were studied in [4,11]. Paper [4] presents a $O(n^2)$–time algorithm for the special case of the circular barrier covering problem (when the sensors are placed along the boundary of the region uniformly) with approximation ratio $1 + \pi$, where n is the number of sensors. Later, in [1], this result was improved by presenting an algorithm with the same running time and approximation ratio 3. A PTAS was also proposed for this problem in [4], which was improved in [8]. In [3] for the case, when arbitrary disks are lying on the line containing the segment, and the disks in the cover do not intersect, an FPTAS is proposed.

In the literature on barrier monitoring, as a rule, the problem of covering a *line segment* with *identical* circles when the centers of the circles move *to the segment*, is considered. In case of the Euclidean metric it is nothing known about the complexity of this problem, however, there is a polynomial $\sqrt{2}$–approximation algorithm [12]. A line segment coverage problem in the special case when equal disks initially lie on the segment is considered in [1] and a $O(n \log_2 n)$–time algorithm is proposed to solve this problem.

Sometimes the problem of barrier coverage is considered in 3D space [22]. We within this paper consider a problem on a plane. Let barrier be a line segment on abscissa axis, and let us number the circles according to the nondecreasing abscissas of their centers. A solution in which after moving the sensors the order preserves is called an *order-preserving covering* (OPC). In the general case may not exist an optimal OPC [12]. In [12] the authors presented a $O(n^4)$–time $\sqrt{2}$–approximation algorithm.

In this paper, we propose a dynamic programming $\sqrt{2}$–approximation algorithm that solves the problem with $O(n^2)$–time complexity. Compared with the known algorithm [12], the degree of the time complexity polynomial is halved.

This paper is organized as follows. Section 2 presents a mathematical formulation of the problem. Section 3 gives the description of new dynamic programming $\sqrt{2}$–approximation algorithm $\mathcal{A}$. In Sect. 4 it is proved that the time complexity of the algorithm $\mathcal{A}$ is $O(n^2)$. The Conclusion section contains summary and further directions of the research. In the Appendix A we describe in detail the solution of one example of a covering of a given line segment by three identical circles.

2 Problem Formulation

Let barrier is a L-length line segment on the plane. It is required to cover it by mobile sensors with circle coverage areas. We introduce a coordinate system in such a way that the barrier is a segment between the points $(0, 0)$ and $(L, 0)$. Let S be a set of disks (corresponding to the coverage areas of the sensors), $|S| = n$, each of which is given by initial coordinates of its center $p_i = (x_i, y_i)$ and radius $r_i > 0$, $i \in S$. We assume that the sensors are numbered from left to right according to the values x_i, $i = 1, 2, \ldots, n$.

Definition 1. *The function $\hat{p}\colon S \to R^2$ is called a covering assignment if the segment is completely covered when the final positions of the sensors are $\hat{p}_i = (\hat{x}_i, \hat{y}_i)$, $i \in S$.*

Let $d(p_i, \hat{p}_i)$ be a distance between the points p_i and $\hat{p}_i$. The problem of barrier coverage by mobile sensors is to find a covering assignment $\hat{p}^*$ of minimum cost, which is the solution of the problem

$$cost(\hat{p}^*) = \min_{\hat{p}} cost(\hat{p}) = \min_{\hat{p}} \sum_{i \in S} d(p_i, \hat{p}_i). \tag{1}$$

In the general case, the covering of a segment can be obtained not necessarily by moving the sensors *to a segment*. However, in [12–14] the special case of the problem (1) when the sensors move *on the barrier* is considered. In this paper, we also consider this case, though we can modify our algorithm in such a way that it builds the solution in a general case. However, within the framework of this paper, we do not set ourselves the goal of describing the general case.

In the case when disks have different radii, the problem (1) is known to be NP-hard even to approximate up to a constant factor [13,14]. However, if the circles are identical it is unknown whether this problem is NP-hard or it is polynomially solvable [12,14]. Paper [12] presents a dynamic programming algorithm for finding $\hat{p}$ that determines an optimal OPC under L_1 metric with $O(n^4)$–time complexity. Meanwhile, the optimal solution of the problem (1) under metric L_1 is a $\sqrt{2}$–approximate solution under the Euclidean metric [12].

3 Algorithm $\mathcal{A}$

In the following, as earlier, we shall identify the centers of the circles (disks) with the sensors. Let the circles be numbered in the nondecreasing order of the abscissas of their centers. We start with the known definitions and simple observations.

Definition 2. *A covering assignment $\hat{p}$ is order-preserving if for every $i, j \in S$ we have $\hat{x}_i < \hat{x}_j$ iff $i < j$.*

Lemma 1 [12]. *If the circles are identical, then there is an optimal order-preserving covering assignment under L_1 metric.*

Lemma 2 [12]. *If the circles are identical, then any optimal order-preserving covering under L_1 metric is a $\sqrt{2}$-approximate solution to the problem (1) under Euclidean metric.*

Further, in this section, we present a new dynamic programming algorithm $\mathcal{A}$ that constructs an OPC, which, in the case of identical circles, is an optimal solution to the problem (1) under the metric L_1. The algorithm consists of one forward recursion and one backward recursion.

3.1 Forward Recursion

Let $S_k(l)$ be a minimum sum of the distances $d(p_i, \hat{p}_i) = |x_i - \hat{x}_i| + |y_i - \hat{y}_i|$, $i = 1, \ldots, k$, for the first k, $k = 1, \ldots, n$, sensors that form an OPC of the segment $[0, l]$, $0 \le l \le L$. Without loss of generality, we suppose that $y_i \ge 0$, $i \in S$. Then we can calculate the cost

$$S_1(l) = \begin{cases} d(p_1, \hat{p_1}(l)), & 2r_1 \ge l \\ +\infty, & 2r_1 < l. \end{cases}$$

Here the point $\hat{p_1}(l)$ is lying on the segment, it is the nearest point to the point p_1, and the segment $[0, l]$ is covered by disk 1. The value $d(p_1, \hat{p_1}(l))$ is defined analytically depending on the initial position of the sensor 1. We assume that the center of the disk 1 moves to the point $(x, 0)$ on the segment (see Fig. 1). Then

$$d(p_1, \hat{p_1}(l)) = \begin{cases} -x_1 + y_1, & x_1 \le 0, \quad l \le r_1, \quad x = 0 \\ l - r_1 - x_1 + y_1, & \max\{x_1 + r_1, r_1\} \le l, \quad x = l - r_1 \\ y_1, & 0 \le l - r_1 \le x_1 \le r_1, \quad x = x_1 \\ x_1 - r_1 + y_1, & x_1 > r_1, \quad r_1 \le L, \quad x = r_1 \\ x_1 - L + y_1, & x_1 > r_1 > L, \quad x = L. \end{cases}$$

Let now there are two disks 1 and 2 with radii r_1 and r_2. If $l \le 2\min\{r_1, r_2\}$, then either two sensors, or one of the sensors can be used in the cover. Let's consider the following possible cases.

1. The sensor 2 is not used in the cover.
2. The sensor 1 is not used in the cover.
3. Both sensors 1 and 2 are used in the cover.

In the first case, we have $S_2(l) = S_1(l)$. In the second case, we let $S_1^2(l)$ be the minimum distance of movement of a sensor 2 for covering $[0, l]$ (suppose that $S_2(l) = +\infty$, if $2r_2 < l$). Let now both sensors are used in the cover of the

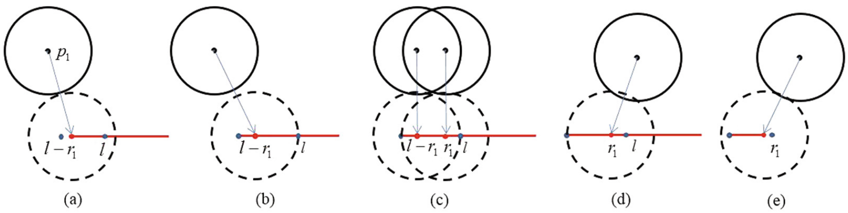

Fig. 1. Movement of disk 1 depending on r_1, p_1 and l. The original location (p_1) and the final location ($\hat{p_1}$) are connected by arrow. (a) If $l - r_1 \le 0$, $x_1 \le 0$, then $x = 0$. (b) If $x_1 \le l - r_1$, $l - r_1 \ge 0$, then $x = l - r_1$. (c) If $0 \le l - r_1 \le x_1 \le r_1$, then $x = x_1$. (d) If $x_1 \ge r_1$, $r_1 \le L$, then $x = r_1$. (e) If $x_1 > r_1$, $r_1 > L$, then $x = L$.

segment $[0, l]$ and x is a point where the center of the disk 2 moves. Thus we can calculate the cost

$$S_2(l) = \begin{cases} \min\{S_1(l), S_1^2(l), \overline{S}_2(l)\}, & 0 < l \le \min\{2(r_1 + r_2), L\} \\ +\infty, & 2(r_1 + r_2) < l, \end{cases}$$

where

$$\overline{S}_2(l) = \begin{cases} \min\limits_{x \in D_2(l)} \{|x_2 - x| + y_2 + S_1(x - r_2)\}, & l < x_2 + r_2 \\ l - r_2 - x_2 + y_2 + S_1(l - 2r_2), & l \ge x_2 + r_2, \end{cases}$$

and $D_2(l) = [\max\{r_2, l - r_2\}, \min\{2r_1 + r_2, l + r_2, L\}]$. Obviously, in the case 3, we have $x > r_2$ (see Fig. 2).

Fig. 2. Options of movement of the disk 2 in the case 3.

Let the values of all functions $S_i(l)$, $i = 1, \dots, k-1$, be counted, and let the segment $[0, l]$ is covering by disks $1, 2, \dots, k$. Then, the following recursions can be used to calculate $S_k(l)$, $k = 1, \dots, n$.

$$S_k(l) = \begin{cases} \min\left\{S_{k-1}(l), S_{k-1}^2(l), \overline{S}_k(l)\right\}, & 0 < l \le \min\{2\sum\limits_{i=1}^{k} r_i, L\} \\ +\infty, & 2\sum\limits_{i=1}^{k} r_i < l, \end{cases}$$

where

$$\overline{S}_k(l) = \begin{cases} \min\limits_{x \in D_k(l)} \{|x_k - x| + y_k + S_{k-1}(x - r_k)\}, & l < x_k + r_k \\ l - r_k - x_k + y_k + S_{k-1}(l - 2r_k), & l \geq x_k + r_k, \end{cases}$$

$S^2_{k-1}(l)$ is the cost function if only one disk k is covering the segment, and $D_k(l) = \left[\max\{r_k, l - r_k\}, \min\{2\sum\limits_{i=1}^{k-1} r_i + r_k, l + r_k, L\}\right]$.

After computing $S_n(L)$, the optimal position of the last disk n is found.

3.2 Backward Recursion

If the sensor n is used in the constructed cover then the position of its center $(\hat{x}_n, 0)$ is known, so the segment $[0, L - \hat{x}_n - r_n]$ is covered by the first $n-1$ disks. The formulas for calculating the value $S_{n-1}(l)$ is found for any l and hence for the argument $l = L - \hat{x}_n - r_n$ too. If the sensor n is not used in the optimal cover, then we consider the sensor $n-1$. If the sensor $n-1$ is used in the cover, then we know the position of its center $(\hat{x}_{n-1}, 0)$ and the segment $[0, L - \hat{x}_{n-1} - r_{n-1}]$ is covered by the first $n-2$ disks. Continuing the backward recursion, we find the covering of the whole segment $[0, L]$.

4 Time Complexity

In this section, we will prove that the proposed algorithm $\mathcal{A}$ can be implemented within the time complexity $O(n^2)$.

Definition 3. *We call $l_i \in [0, l]$ the* switching points *for the function $S_k(l)$ if in the segment $(l_i, l_{i+1}) \subseteq [0, l]$ the function is defined by one analytical expression $F_i(l)$ and $F_i(l_{i+1}) = F_{i+1}(l_{i+1})$, $F_i(l_i) = F_{i-1}(l_i)$, or if $S_k(l)$ is unlimited (equals $+\infty$).*

Lemma 3. *When adding the next disk k, the number of switching points for the function $S_k(l)$, $k = 1, \ldots, n$, increases by $O(1)$ with respect to the number of switching points for the function $S_{k-1}(l)$.*

Proof. Let first $k = 1$. If $2r_1 < l$ then $S_1(l) = +\infty$. Otherwise if $2r_1 \geq l$, then depending on p_1 and r_1, we have one of the three options for calculation $S_1(l)$.

1. If $x_1 < 0$, then

$$S_1(l) = \begin{cases} -x_1 + y_1, & l \leq r_1, \quad x = 0 \\ l - r_1 - x_1 + y_1, & l > r_1, \quad x = l - r_1. \end{cases}$$

2. If $0 \leq x_1 \leq r_1$, then

$$S_1(l) = \begin{cases} y_1, & l \leq r_1 + x_1, \quad x = x_1 \\ l - r_1 - x_1 + y_1, & l > r_1 + x_1, \quad x = l - r_1. \end{cases}$$

3. If $x_1 > r_1$, then

$$S_1(l) = \begin{cases} x_1 - r_1 + y_1, & r_1 \leq L, \quad x = r_1 \\ x_1 - L + y_1, & r_1 > L, \quad x = L. \end{cases}$$

Thus, for any value of x_1 the number of switching points for function $S_1(l)$ is bounded by $O(1)$ (constant).

Let now the two first disks can be used in the covering. It is necessary to consider all cases for the calculation of $S_2(l)$. Due to the limitations of the space, we will consider in detail only two cases from the set of cases.

Let's consider, for example, the case when $0 < l < x_2 + r_2$. As both sensors are used in the cover of the segment $[0, l]$, then $x_2 > \max\{l - r_2, r_2\}$. Note that the sensor 1 does not cover the point $(0, l)$, and the sensor 2 does not cover the point $(0, 0)$. Therefore,

$$S_2(l) = \begin{cases} x_2 - x + y_2 + S_1(x - r_2), & x \in X_1 \\ x - x_2 + y_2 + S_1(x - r_2), & x \in X_2, \end{cases}$$

where $X_1 = [\max\{r_2, l - r_2\}, \min\{2r_1 + r_2, l + r_2, L, x_2\}]$,
and $X_2 = [\max\{r_2, l - r_2, x_2\}, \min\{2r_1 + r_2, l + r_2, L\}]$.

The function S_1 depends on the x_1, and it is computed as follows:

- if $x_1 < 0$, then

$$S_1(x - r_2) = \begin{cases} -x_1 + y_1, & x \leq r_1 + r_2 \\ x - r_1 - r_2 - x_1 + y_1, & x > r_1 + r_2; \end{cases}$$

- if $0 \leq x_1 \leq r_1$, then

$$S_1(x - r_2) = \begin{cases} y_1, & x \leq x_1 + r_1 + r_2 \\ x - r_1 - r_2 - x_1 + y_1, & x > x_1 + r_1 + r_2; \end{cases}$$

- if $x_1 > r_1$, then $S_1(x - r_2) = x_1 - r_1 + y_1$, $r_1 \leq L$.

Assume that $x_1 < 0$, $x \leq x_2$ and $x \leq r_1 + r_2$. Then we have the formula:

$$S_2(l) = x_2 - x + y_2 + S_1(x - r_2),$$

where $\max\{r_2, l - r_2\} \leq x \leq \min\{2r_1 + r_2, l + r_2, L, x_2\}$ and $S_1(x - r_2) = -x_1 + y_1$.

As a result, we have the following analytical expression

$$S_2(l) = x_2 - x + y_2 - x_1 + y_1,$$

where $\max\{r_2, l - r_2\} \leq x \leq \min\{2r_1 + r_2, l + r_2, L, x_2\}$
and $x = \min\{r_1 + r_2, x_2, l + r_2, L\}$.

Let's consider one more case when $l \geq x_2 + r_2$ and $x_1 < 0$. Then we have the formula $S_2(l) = l - r_2 - x_2 + y_2 + S_1(l - 2r_2)$, where

$$S_1(l-2r_2) = \begin{cases} -x_1 + y_1, & x_1 \leq 0,\, l - 2r_2 \leq r_1 \\ l - r_1 - 2r_2 - x_1 + y_1, & x_1 \leq 0,\, l - 2r_2 > r_1. \end{cases}$$

If $l - 2r_2 \leq r_1$, then in order to cover the segment $[0, l - 2r_2]$ the disk 1 moves to the point $(0, 0)$. Otherwise, if $l - 2r_2 > r_1$, the disk 1 moves to the point $(l - 2r_2 - r_1, 0)$.

Assume that $l - 2r_2 \leq r_1$, then the function $S_2(l) = l - r_2 - x_2 + y_2 + y_1 - x_1$.

Other cases are considered similarly. Thereby the number of switching points for the function $S_2(l)$ is upper bounded by constant.

For an arbitrary number of sensors $k = 1, 2, \ldots, n$, we can calculate the cost as follows.

$$S_k(l) = \begin{cases} \min\left\{S_{k-1}(l), S^2_{k-1}(l), \overline{S}_k(l)\right\}, & 0 < l \leq \min\{2\sum\limits_{i=1}^{k} r_i, L\} \\ +\infty, & 2\sum\limits_{i=1}^{k} r_i < l, \end{cases}$$

where

$$\overline{S}_k(l) = \begin{cases} \min\limits_{x \in D_k(l)} \{|x_k - x| + y_k + S_{k-1}(x - r_k)\}, & l < x_k + r_k \\ l - r_k - x_k + y_k + S_{k-1}(l - 2r_k), & l \geq x_k + r_k, \end{cases}$$

$S^2_{k-1}(l)$ is the cost function if only one disk k is covering the segment, and $D_k(l) = \left[\max\{r_k, l - r_k\}, \min\{2\sum\limits_{i=1}^{k-1} r_i + r_k, l + r_k, L\}\right]$.

When calculating the value of $\overline{S}_k(l)$ it is considered two cases $l < x_k + r_k$ and $l \geq x_k + r_k$, and the number of switching points increases by constant. Hence, for calculation of the next value of $S_k(l)$, $k = 1, \ldots, n$, the constant number of switching points is added, that completes the proof.

Corollary 1. *When calculating the function $S_k(l)$ the optimal position of the center of the disk k, $k = 1, \ldots, n$ can be computed with time complexity equals $O(n)$.*

Remark 1. In the case of different disks may not exist an optimal order-preserving assignment under L_1 metric (see Fig. 3). Therefore, we can apply the proposed algorithm $\mathcal{A}$, but we cannot obtain a $\sqrt{2}$-approximate solution.

The main result of this paper is the

Theorem 1. *In the case of identical disks the algorithm $\mathcal{A}$ constructs a $\sqrt{2}$–approximate solution to the problem (1) with time complexity equals $O(n^2)$.*

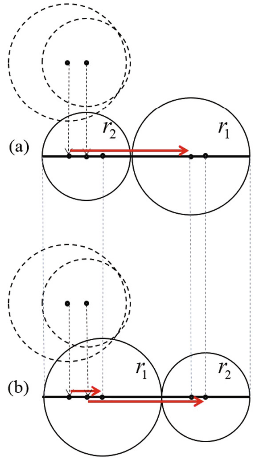

Fig. 3. (a) The optimal cover under L_1 metric. (b) The order-preserving cover which is worse by $2(r_1 - r_2)$ than the optimal cover.

Proof. It is known that in the considered case an optimal order-preserving covering under L_1 metric is a $\sqrt{2}$–approximate solution to the problem (1) under Euclidean metric [12]. Taking into account that the functions $S_k(l)$ are calculated n times and Corollary 1, we find that the complexity of the algorithm $\mathcal{A}$ is $O(n^2)$. The theorem is proved.

To illustrate the operation of the algorithm, in the Appendix A an example is given.

5 Conclusion

The paper deals with the problem of moving the centers of n circles located at arbitrary position on a plane on a given line segment of length L so that the line is completely covered by the circles while minimizing the cumulative Euclidean distance between the initial position of centers and their position on the segment. It is known that this problem is NP-hard in the case of a non-identical disks [13,14]. When the disks are identical the complexity of the problem is unknown, but there is a $O(n^4)$–time $\sqrt{2}$–approximation algorithm. In this paper, we propose a $O(n^2)$–time algorithm that is applicable in general case and constructs a $\sqrt{2}$-approximate solution to the problem in the case of n identical circles.

In the further research, we plan to clarify the complexity of the problem in the case of identical disks and to give up a requirement of movement the sensors *on the segment*. Moreover, we are planning to design an FPTAS for the case when each barrier point is covered by exactly one disk.

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A Appendix

Example. Let it is required to cover the line segment $[0, 4.5]$ by three identical disks with radii equal to 1, which initial positions of the centers are $p_1 = (-0.5, 1)$, $p_2 = (2.5, 2)$ and $p_3 = (5.5, 0)$ (Fig. 4(a)).

Since $x_1 < 0$, then

$$S_1(l) = \begin{cases} -x_1 + y_1 = 1.5, & l \leq 1 \\ l - 1 - x_1 + y_1 = l + 0.5, & l > 1 \\ +\infty, & l > 2. \end{cases}$$

The disk 1 moves to the point $(0, 0)$, if $l \leq 1$ (Fig. 4(b)) and it moves to the point $(l - 1, 0)$, if $l > 1$ (Fig. 4(c)). Thus, we have the switching points $0, 1, 2$ and 4.5.

Let now two circles participate in the covering. If $l \leq 1$, then it is easy to see, that only disk 1 covers the segment [0,l] and $S_2(l) = 1.5$ (Fig. 4(d)).

If $1 < l \leq 2$, then the segment $[0, l]$ can be covered ether by one disk 1 or by one disk 2. We have that $d(p_1, \hat{p}_1) = l + 0.5 \leq d(p_2, \hat{p}_2)$. So, in this case only disk 1 covers the segment $[0, l]$. Suppose that both disks 1 and 2 participate in

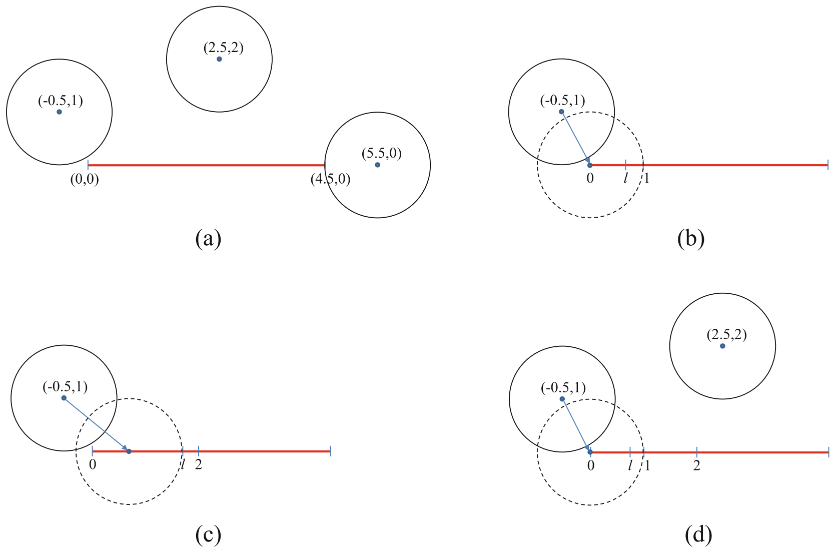

Fig. 4. (a) Initial position of the disks; (b) one disk in the case when $l \leq 1$; (c) one disk in the case when $1 < l \leq 2$; (d) two disks in the case when $l \leq 1$.

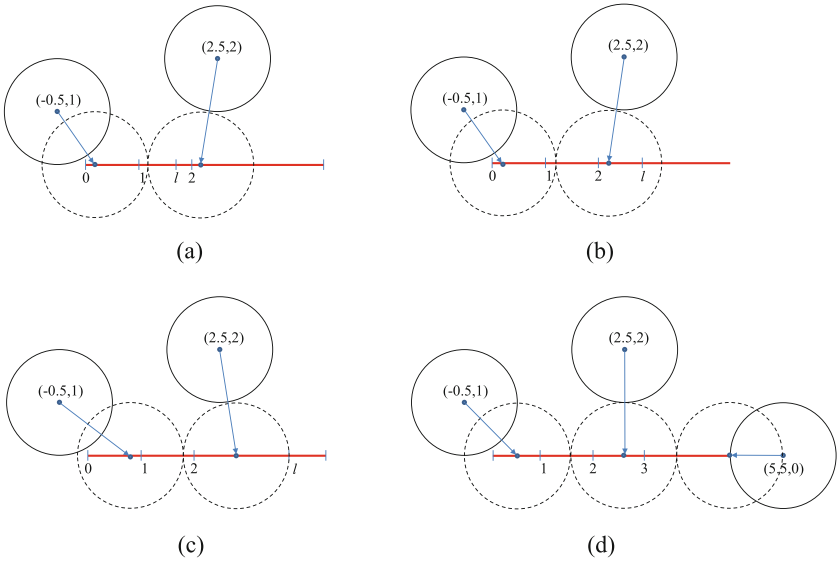

Fig. 5. (a) Two disks in the case when $1 < l \leq 2$; (b) two disks in the case when $2 < l \leq 3.5$; (c) two disks in the case when $3.5 < l \leq 4$; (d) the optimal OPC.

the covering of the segment $[0, l]$. Let us denote by $x \in (1, 3)$ the point at which the center of disk 2 moves. Then the segment $[0, x-1]$ must be covered by disk 1. If $x \leq 2.5$ then $S_2(l) = 2 + 2.5 - x + S_1(x-1) = 4$. If $2.5 < x \leq 3$ then $S_2(l) = \min\limits_{x \in [2.5,3]} \{2 + x - 2.5 + S_1(x-1)\} = \min\limits_{x \in [2.5,3]} \{2x - 1\}$. Therefore, in this case only the center of disk 1 moves to the point $(l-1, 0)$ (Fig. 5(a)).

If $2 < l \leq 3.5$, then the both disks 1 and 2 must participate in the covering of the segment $[0, l]$. If x is a point where the center of disk 2 moves, then the segment $[0, x-1]$ must be covered by disk 1. For any $x \in [l-1, 2.5]$, we get the same value of $S_2(l) = 4$ and set $x = 2.5$ (Fig. 5(b)).

If $3.5 < l \leq 4$, then both disks participate in the covering of the segment $[0, l]$. If $x \in [l-1, 3]$ is a point where the center of disk 2 moves, then the segment $[0, x-1]$ must be covered by disk 1. In this case $2.5 \leq x \leq 3$. Moreover, $x = l-1$ and $S_2(l) = l - 1 - 2.5 + 2 + 1 + l - 2 + 0.5 = 2l - 2$ (Fig. 5(c)).

Therefore, the following formula holds

$$S_2(l) = \begin{cases} 1.5, & 0 < l \leq 1 \\ l + 0.5, & 1 < l \leq 2 \\ 4, & 2 < l \leq 3.5 \\ 2l - 3, & 3.5 < l \leq 4 \\ +\infty, & l > 4, \end{cases}$$

where the switching points are $0, 1, 2, 3.5, 4, 4.5$.

Let now all three sensors participate in the covering. The center of disk 3 can move to the point $x \in [3.5, 4.5]$. Then the segment $[0, x-1]$ must be covered by disks 1 and 2 and

$$S_3(4.5) = \min_{x \in [3.5,4.5]} \{5.5 - x + S_2(x-1)\} = \min_{x \in [3.5,4.5]} \{9.5 - x\} = 5.$$

Then the center of disk 3 moves to the point $(4.5, 0)$.

The backward recursion allows us to restore the optimal coverage, which is shown in the Fig. 5(d).

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