Chapter 12
Mathematical Modeling and Numerical Simulation of Fixtures for Fork-Type Parts Manufacturing

Check for updates

Vitalii Ivanov, Ivan Pavlenko, Ivan Kuric, and Mykyta Kosov

12.1 Introduction

The main tasks of fixtures are the precise locating and reliable clamping of workpieces during the machining on the machine tools. Fixtures are an integral part of the closed-loop technological system "machine tool–fixture–cutting tool–workpiece". Fixtures considerably effect on the quality and the cost of final products, as well as productivity and flexibility of production [1, 2]. The statistical data [3, 4] confirmed the importance of fixtures for products manufacturing. Fixture design is complicated and time-consuming process [5, 6]. It is important to find the optimal solution between flexibility and productivity, as well as to ensure the required accuracy [7]. Therefore, fixture requirements were identified and should be considered during design process [8–17]. The problem of fixture design is very urgent and relates to the multidisciplinary tasks on ensuring the stable locating of workpiece in the fixture during the machining process.

12.2 Literature Review

For many years fixture design has been the focus of academic research with significant progress in both theoretical and practical studies.

V. Ivanov (✉) · I. Pavlenko · M. Kosov
Sumy State University, Sumy, Ukraine
e-mail: ivanov@tmvi.sumdu.edu.ua

I. Kuric
University of Zilina, Zilina, Slovak Republic

Research paper [18] presents the methodology of modeling of the system "fixture–workpiece". In addition, the influence of the previous load from the impact of clamping and cutting forces on the error of the work surface was determined [18]. Friction between elements of the mechanical system "fixture–workpiece" has been researched and have been determined the deformations, which appear in the contact points [19]. The methodology of analysis of stability of the system "fixture–workpiece" has been developed and calculation of the minimum clamping force, required for machining process, has been presented; also, the influence of sequence of workpiece clamping has been researched [20]. The influence of modes of cutting and fixture compliance on the workpiece stability was analytically calculated and researched [21]. The simplified analytical model of the contact interaction between workpiece and clamping elements was developed [22]. The finite element model for determination of the stability of workpiece locating in fixture and the methodology of optimization of the previous loading were developed [23]. The mathematical model of fixture interaction with workpiece and analysis of deterministic positioning of fixture was developed [24]. Many researches in fixture accuracy and analysis of the fixture stability were realized, based on the consideration of 2D problem and introduction of "operative factor," which considers the friction forces [25]. The methodology on determination of points and clamping forces for ensuring the stable locating of workpiece in fixture has been developed [26]. The methodology of control of clamping force considering the contact interaction between workpiece and clamping elements by means of the methods of nonlinear programming was presented [27]. The problem of the temporariness of the stability of fixture locating considering the restriction of the force and direction of its action in the system "fixture–workpiece" has been researched [28]. The influence of material removal on the dynamic state of the system "fixture–workpiece" was considered as opposed to quasistatic approach of predecessors [29]. Moreover, scientific and methodological approach for identification of mathematical models of the mechanical system "fixture–workpiece" using artificial intelligent systems is proposed in paper [30]. Paper [31] focuses on the investigation of the affect that an automated flexible fixture has on the vibrational characteristics of parts in a reconfigurable manufacturing environment, through modeling the modal characteristics of test parts, of differing geometries, for various fixturing setups.

The goal of the proposed work is the mathematical modeling and numerical simulation of the system "fixture–workpiece" based on manufacturing features of the process of locating and clamping of the workpiece in the fixture considering dynamic analysis of the workpiece in fixture under the effect of spatial system of cutting and clamping forces.

12.3   Research Methodology

12.3.1   Mathematical Model

The fixture for multiaxis machining of parts is developed based on the analysis of requirements for fixture designing and fork-type parts manufacturing (Fig. 12.1). The main advantage of the proposed fixture configuration is the possibility of readjusting its functional elements when changing the workpiece dimensions.

The conservative mechanical system including a workpiece and clamped by supports is considered in a local orthogonal coordinate system xyz with the origin in the mass center C. The relative coordinates of the current point B_j, forces F_x^j, F_y^j, F_z^j and moments M_x^j, M_y^j, M_z^j for modeling space movement of a workpiece are presented on the related design scheme (Fig. 12.2).

The displacements x_i, y_i, z_i of the workpiece supporting surfaces in a fixture are determined as static displacement. They can be expressed by geometric dependencies with respect to six independent parameters (degrees of freedom of the workpiece): x, y, z, φ, ψ, θ [32].

All the components of supports displacements are calculated by the following formulas:

$$
\begin{aligned}
x_i &= x_C - b_i\theta + h_i\phi; \\
y_i &= y_C + l_i\theta + h_i\psi; \\
z_i &= z_C + l_i\phi + b_i\psi,
\end{aligned}
\tag{12.1}
$$

where x_C, y_C, z_C are coordinates of mass center and l_i, b_i, h_i are local longitudinal, transverse, and vertical coordinates.
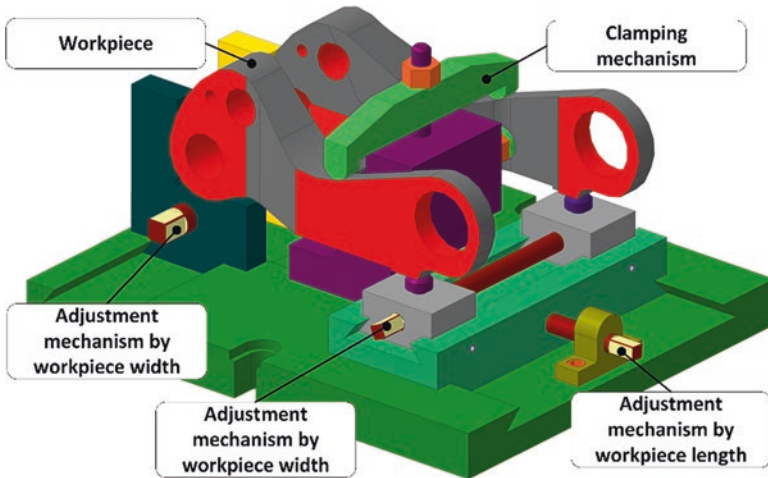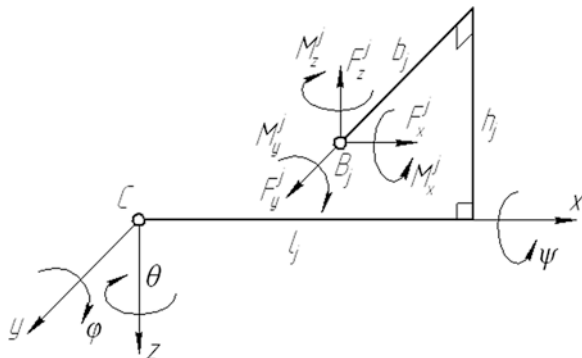

Fig. 12.1 The mechanical system "fixture–workpiece"

Fig. 12.2 The local
coordinate system for
modeling spatial
movement of the
workpiece

The spatial movement of the workpiece in a fixture can be determined by the center-of-mass theorem and the momentum theorem. The related mathematical model of dynamic analysis is realized by the system of 12th order differential equations with respect to the abovementioned independent parameters [33].

Local components of the related dynamic forces F_x, F_y, F_z and moments M_x, M_y, M_z have the following form:

$$
\begin{aligned}
F_x &= -\Sigma k_x^i \left(x_C - b_i \theta + h_i \phi \right); \\
F_y &= -\Sigma k_y^i \left(y_C + l_i \theta + h_i \psi \right); \\
F_z &= -\Sigma k_z^i \left(z_C + l_i \phi + b_i \psi \right); \\
M_x &= -\Sigma \left[k_y^i h_i \left(y_C + l_i \theta + h_i \psi \right) + k_z^i b_i \left(z_C + l_i \phi + b_i \psi \right) \right]; \\
M_y &= -\Sigma \left[k_x^i h_i \left(x_C - b_i \theta + h_i \phi \right) + k_z^i l_i \left(z_C + l_i \phi + b_i \psi \right) \right]; \\
M_z &= -\Sigma \left[-k_x^i b_i \left(x_C - b_i \theta + h_i \phi \right) + k_y^i l_i \left(y_C + l_i \theta + h_i \psi \right) \right],
\end{aligned}
\tag{12.2}
$$

where m is workpiece mass and J_{Cx}, J_{Cy}, J_{Cz} are central moments of inertia.

12.3.2 Modal Analysis

To prevent a resonance phenomenon during the process of workpiece machining, the oscillation frequency for fixture elements should not coincide with the main frequency of the cutting process. One of the ways to avoid this problem is to set another cutting mode. For this purpose, eigenfrequencies of the mechanical system "fixture–workpiece" are determined using the ANSYS software. As a result, the minimal value of the obtained frequencies is compared with the related frequency during the cutting process.

In the case of zero value of forces and cutting moments, the mathematical model takes the following matrix form:

$$[M]\left\{\ddot{X}_C\right\}+[K]\{X\}=\{0\}, \tag{12.3}$$

where $\{0\}$ is zero column vector of external impact, $\{X\}$ is column vector of displacement, $[M]$ is matrix of inertia, $[K]$ is stiffness matrix, the components of which depend on the contact stiffness coefficients k_x^i, k_y^i, k_z^i of supporting surfaces:

$$
\begin{aligned}
&k_{xx}=\Sigma k_x^i;\ k_{yy}=\Sigma k_y^i;\ k_{zz}=\Sigma k_z^i;\ k_{\psi\psi}=\Sigma\left(k_y^i h_i^2+k_z^i b_i^2\right);\\
&k_{\phi\phi}=\Sigma\left(k_x^i h_i^2+k_z^i l_i^2\right);\ k_{\theta\theta}=\Sigma\left(k_x^i b_i^2+k_y^i l_i^2\right);\ k_{x\phi}=\Sigma k_x^i h_i;\\
&k_{x\theta}=\Sigma k_x^i b_i;\ k_{y\psi}=\Sigma k_y^i h_i;\ k_{y\theta}=\Sigma k_y^i l_i;\ k_{z\psi}=\Sigma k_z^i b_i;\\
&k_{z\phi}=\Sigma k_z^i l_i;\ k_{\psi\phi}=\Sigma k_z^i l_i b_i;\ k_{\psi\theta}=\Sigma k_y^i l_i h_i;\ k_{\phi\theta}=\Sigma k_x^i b_i h_i.
\end{aligned}
\tag{12.4}
$$

Six eigenfrequencies of the mechanical system "fixture–workpiece" are determined as natural values of the matrix of dynamic stiffness, which is equal to the roots of the following polynomial equation:

$$\left|[K]-\omega^2[M]\right|=0. \tag{12.5}$$

12.3.3   Harmonic Analysis

Spatial movement of the workpiece in a fixture as a 6-degrees-of-freedom system is determined analytically by the following matrix form:

$$[M]\left\{\ddot{X}_C\right\}+[K]\{X\}=\{F\}, \tag{12.6}$$

where $\{X\}$ is column vector of displacements and $\{F\}$ is column vector of external dynamic impact.

Monoharmonic external impact can be described by the expression $\{F\}=\{F_a\}\cdot\sin\omega t$, where $\{F_a\}$ is column vector of dynamic amplitudes; $\omega=\omega_0\cdot z$—frequency as a product of the spindle's rotation frequency and a number z of teeth of the operating cutting tool.

The general solution of the Eq. (12.6) can be presented as a column vector $\{X_a\}$ of displacement amplitudes of center of weight and rotation angles around the coordinate axes:

$$\{X_a\}=\left([K]-\omega^2[M]\right)^{-1}\{F_a\}. \tag{12.7}$$

The achieved dependence of $\{X_a\}$ components on the cutting frequency ω determines the amplitude-frequency response characteristics of the considered mechanical system "fixture–workpiece."

12.4 Results

12.4.1 Modal Analysis

The objects of the numerical simulation are free and forced oscillations of the mechanical system "fixture–workpiece". The fixture has the adjustment mechanisms for ensuring its readjustment when changing the object of machining. In this case, the workpiece needs to be machined from different sides.

The results of modal analysis for the first eigenfrequency are presented in Fig. 12.3 and allow detuning from a resonance. The related data is summarized in Table 12.1.

Additionally, the abovementioned data allows concluding that a fixture for the proposed manufacturing process is less compliant in comparison with the same fixture for the typical manufacturing process, since the first eigenfrequency increases in 1.64 times.
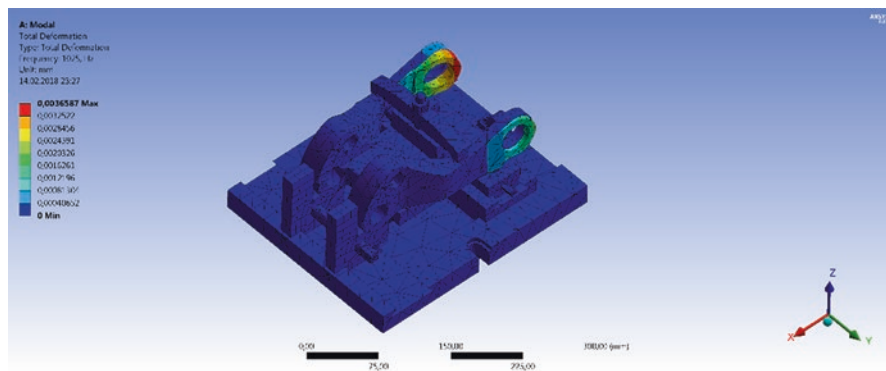

Fig. 12.3 The results of modal analysis

Table 12.1 Comparative analysis of eigenfrequencies for the mechanical system "fixture–workpiece" in the case of fork-type parts machining, Hz

	Eigenfrequency			
Fixture design	1st	2nd	3rd	Cutting frequency
Typical manufacturing process	625	836	1550	1271
Multiaxis manufacturing process	1025	1055	1513	1271

12.4.2   Harmonic Analysis

Determining the oscillation amplitudes for the machined surfaces under the deviation of cutting forces and moments allows ensuring the sufficient rigidity of the mechanical system "fixture–workpiece". For this purpose, the following objectives are stated for the case of fork-type parts machining:

1. obtaining the amplitude-frequency response under the most loaded operating step,
2. evaluating the resonance frequency and comparing with the related cutting frequency.

Harmonic analysis is realized by using the ANSYS software. Amplitudes of the dynamic components of cutting forces are equal 20% from their nominal values. The proposed model takes into account the Coulomb friction between the contact surfaces [34], which have approximately the same roughness Ra = 1.6 μm with the friction coefficient 0.1. A range of oscillation frequencies is chosen for the reason of providing coverage of the first three eigenfrequencies obtained above. So, the maximum oscillating frequency is equal to 1550 Hz.

The amplitude-frequency response is obtained (Fig. 12.4), as well as pre- and under resonance modes are determined. For ensuring equal conditions, the comparison analysis is carried out for displacements under the resonance frequency. The maximum displacement for the proposed fixture design is equal to 25 μm. The result of harmonic analysis is presented in Fig. 12.4, and the related numerical simulation data is summarized in Table 12.2.
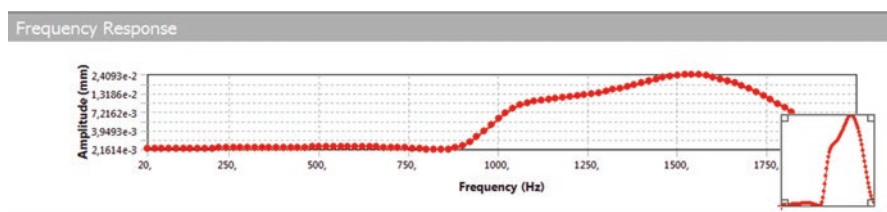

Fig. 12.4 The amplitude-frequency response for the fixture design while drilling holes in fork plugs

Table 12.2 The results of numerical simulation for the fixture design while drilling holes in fork plugs

Fixture design	Nominal cutting force, N	Amplitude of the dynamic force, N	Maximum displacement, μm
Typical manufacturing process	5398	1079	93
Multiaxis manufacturing process	5398	1079	25

12.5 Conclusions

In the research of recent challenges in fixture design for manufacturing complex parts, the existing approaches for designing CAx systems are systematized and generalized. As a result, a brand-new approach is developed based on a comprehensive analysis of free and forced oscillations of the mechanical system "fixture–workpiece" under the impact of clamping and cutting forces.

The mathematical model of free and forced oscillations for research of the dynamic states of the comprehensive mechanical system is proposed. It is proved on the example of fork-type parts considering contact stiffness of functional elements, required clamping forces, and dynamic components of cutting forces and moments. This model allows determining natural frequencies of free oscillations with the aim of detuning from a resonance. It should be noted that the first eigenfrequency 1025 Hz for the proposed manufacturing process is 1.64 times more than the same frequency for the typical manufacturing process. Consequently, the dynamic stiffness of the new design is significantly increased, and required deturning 20% from a resonance frequency 1271 Hz is ensured.

The research of forced oscillations of the proposed system under the impact of dynamic forces and cutting moments is presented. As a result, the amplitude-frequency response is obtained using the ANSYS software. It should be noted that the maximum displacement 25 μm does not exceed all the required tolerances and related manufacturing errors.

Further research will be aimed at developing the research methodology and related engineering approach for estimating parameters of the proposed mathematical model by the results of numerical simulation and experimental data using both of a linear regression procedure and a nonlinear regression analysis by means of artificial intelligent systems.

References

1. Bi, Z. M., & Zhang, W. J. (2001). Flexible fixture design and automation: Review, issues and future directions. *International Journal of Production Research, 39*, 2867–2894. https://doi.org/10.1080/00207540110054579.
2. Trojanowska, J., Kolinski, A., Galusik, D., et al. (2018). A methodology of improvement of manufac-turing productivity through increasing operational efficiency of the production process. In A. Hamrol, O. Ciszak, S. Legutko, & M. Jurczyk (Eds.), *Advances in Manufacturing* (pp. 23–32). New York: Springer. https://doi.org/10.1007/978-3-319-68619-6_3.

3. Gameros, A., Axinte, D., Siller, H. R., et al. (2017). Experimental and numerical study of a fixturing system for complex geometry and low stiffness components. *Journal of Manufacturing Science and Engineering, 139*(4), 045001-01–045001-12. https://doi.org/10.1115/1.4034623.

4. Gothwal, S., & Raj, T. (2017). Different aspects in design and development of flexible fixtures: Review and future directions. *International Journal of Services and Operations Management, 26*(3), 386–410. https://doi.org/10.1504/IJSOM.2017.081944.

5. Ansaloni, M., Bonazzi, E., Leali, F., et al. (2013). Design of fixture systems in automotive manufac-turing and assembly. *Advanced Materials Research, 712-715*, 2913–2916. https://doi.org/10.4028/www.scientific.net/AMR.712-715.2913.

6. Forstmann, R., Wagner, J., Kreiskother, K., et al. (2017). Design for automation: The rapid fixture approach. *Procedia Manufacturing, 11*, 633–640. https://doi.org/10.1016/j.promfg.2017.07.161.

7. Karpus, V. E., & Ivanov, V. A. (2012). Choice of the optimal configuration of modular reusable fixtures. *Russian Engineering Research, 32*(3), 213–219. https://doi.org/10.3103/S1068798X12030124.

8. Pehlivan, S., & Summers, J. (2008). A review of computer-aided fixture design with respect to infor-mation support requirements. *International Journal of Production Research, 46*(4), 929–947. https://doi.org/10.1080/00207540600865386.

9. Boyle, I., Rong, Y., & Brown, D. (2011). A review and analysis of current computer-aided fixture de-sign approaches. *International Journal of Robotics and Computer-Integrated Manufacturing, 27*(1), 1–12. https://doi.org/10.1016/j.rcim.2010.05.008.

10. Bakker, O. J., Papastathis, T. N., Ratchev, S. M., & Popov, A. A. (2013). Recent research on flexible fixtures for manufacturing processes. *Recent Patents on Mechanical Engineering, 6*(2), 107–121. https://doi.org/10.2174/2212797611306020003.

11. Tohidi, H., & AlGeddawy, T. (2016). Planning of modular fixtures in a robotic assembly system. *Pro-cedia CIRP, 41*, 252–257. https://doi.org/10.1016/j.procir.2015.12.090.

12. Hui, L., Weifang, C., & Shengjie, S. (2016). Design and application of flexible fixture. *Procedia CIRP, 56*, 528–532. https://doi.org/10.1016/j.procir.2016.10.104.

13. Posindu, B. A., Janaka, E. B., Kasun, D. T., et al. (2017). A novel fixturing system for complex shaped components. In IEEE (Ed.), *2017 Moratuwa Engineering Research Conference* (pp. 221–224). Moratuwa: IEEE. https://doi.org/10.1109/MERCon.2017.7980485.

14. Mohring, H.-C., Gessler, W., Konig, A., et al. (2017). Modular intelligent fixture system for flexible clamping of large parts. *Journal of Machine Engineering, 17*(4), 29–39. https://doi.org/10.5604/01.3001.0010.7003.

15. Erdem, I., Levandowski, C., Berlin, C., et al. (2017). A novel comparative design procedure for re-configurable assembly fixtures. *CIRP Journal of Manufacturing Science and Technology, 19*, 93–105. https://doi.org/10.1016/j.cirpj.2017.06.004.

16. Gothwal, S., & Raj, T. (2018). Conceptual design and development of pneumatically controlled flexible fixture and pallets. *International Journal of Services and Operations Management, 29*(2), 147–162. https://doi.org/10.1504/IJSOM.2018.089246.

17. Ivanov, V., & Zajac, J. (2018). Flexible fixtures for CNC machining centers in multiproduct manufacturing. *EAI Endorsed Transactions on Industrial Networks and Intelligent Systems, 4*(12), e4. https://doi.org/10.4108/eai.10-1-2018.153552.

18. Liao, Y., & Hu, S. (2001). An integrated model of a fixture – workpiece system for surface quality prediction. *International Journal of Advanced Manufacturing Technology, 17*(11), 810–818. https://doi.org/10.1007/s001700170108.

19. Kumbhar, N., et al. (2012). Finite element modelling and analysis of workpiece – fixture system. *International Journal of Applied Research in Mechanical Engineering, 2*(2), 60–65.

20. Kang, Y., et al. (2003). Computer-aided fixture design verification. Part 3. Stability analysis. *The International Journal of Advanced Manufacturing Technology, 21*(10), 842–849. https://doi.org/10.1007/s00170-002-1401-4.

21. Asante, J. N. (2010). Effect of fixture compliance and cutting conditions on workpiece stability. *The International Journal of Advanced Manufacturing Technology, 48*(1), 33–43. https://doi.org/10.1007/s00170-009-2284-4.

22. Cioata, V., & Kiss, I. (2009). The machining error due to contact deformation of workpiece—fixture system. *Acta Technica Corviniensis – Bulletin of Engineering, 2*(1), 33–36.

23. Zheng, Y. (2005). *Finite element analysis for fixture stiffness*. PhD Thesis. Worcester: Worcester Polytechnic Institute.

24. Asada, H., & By, A. (1985). Kinematic analysis of workpart fixturing for flexible assembly with automatically reconfigurable fixtures. *IEEE Journal on Robotics and Automation, 1*(2), 86–94. https://doi.org/10.1109/JRA.1985.1087007.

25. Rong, Y., & Bai, Y. (1997). Automated generation of fixture configuration design. *Journal of Manufacturing Science and Engineering, 119*(2), 208–219. https://doi.org/10.1115/1.2831097.

26. Chou, Y. C. (1993). Automated fixture design for concurrent manufacturing planning. *Concurrent Engineering, 1*(4), 219–229. https://doi.org/10.1177/1063293X9300100405.

27. Wu, Y., et al. (1997). Automated generation of dedicated fixture design. *International Journal Computer Application in Technologies, 10*(3–4), 213–235. https://doi.org/10.1504/IJCAT.1997.062249.

28. Trappey, A. J. C., et al. (1995). Computer-aided fixture analysis using finite element analysis and mathematical optimization modeling. *Journal of Manufacturing Science and Engineering, 2-1*, 777–787.

29. Deng, H. (2006). *Analysis and synthesis of fixturing dynamic stability in machining accounting for material removal effect*. Ph.D. Thesis. Atlanta: Georgia Institute of Technology.

30. Pavlenko, I., Trojanowska, J., Ivanov, V., & Liaposhchenko, O. (2019). Scientific and methodological approach for the identification of mathematical models of mechanical systems by using artificial neural networks. In: Machado J., Soares F., Veiga G. (eds.) Innovation, Engineering and Entrepreneurship. HELIX 2018. *Lecture Notes in Electrical Engineering*, 2019, *505*, 299–306. https://doi.org/10.1007/978-3-319-91334-6_41.

31. Slabbert, E., Walker, A., Bright, G. (2017). Modal analysis of machining processess on an automated flexible fixture for a reconfigurable manufacturing system. In: *24th International Conference on Mechatronics and Machine Vision in Practice (M2VIP)*. IEEE. doi: https://doi.org/10.1109/M2VIP.2017.8211498.

32. Ivanov, V., Pavlenko, I. (2018). Comprehensive approach for mathematical modeling of mechanical systems: Fixture design case study. In: Knapcikova L., Balog M. (eds.) *Proceedings of 2nd EAI Int. Conf. on Management of Manufacturing Systems, MMS-2017*, pp. 1–19. doi: https://doi.org/10.4108/eai.22-11-2017.2274154.

33. Ivanov, V., & Pavlenko, I. (2018). Fundamental approach for analysis of dynamic characteristics of fixtures. *EAI Endorsed Transactions on Industrial Networks and Intelligent Systems, 4*(13), e1. https://doi.org/10.4108/eai.20-3-2018.154366.

34. Pavlenko, I., Simonovskiy, V., Ivanov, V. et al. (2019). Application of artificial neural network for identification of bearing stiffness characteristics in rotor dynamics analysis. In: Ivanov V. et al. (eds.) Advances in Design, Simulation and Manufacturing. DSMIE-2018. *Lecture Notes in Mechanical Engineering,* 2019, pp. 325–335. doi: https://doi.org/10.1007/978-3-319-93587-4_34.