Chapter 6
Optimal Allocation of Groundwater Resources: Managing Water Quantity and Quality

Qiuqiong Huang, Scott D. Rozelle, Richard E. Howitt and James E. Wilen

Abstract Despite the importance of groundwater in the economy of the Hai River Basin (HRB), falling water tables and ***salinization*** of aquifers are both occurring in the region. Hydrological and hydrogeological studies have shown that increases in the salinization of parts of the freshwater aquifers are closely related to the extraction of groundwater. This study uses a framework that considers the interaction between water quantity and quality to examine how the presence of the prehistoric saline water layer affects groundwater management. Simulation results show that in a region where there is a salinization problem like in the HRB, it is optimal to pump at high rates in the early stage of extraction when the quality of groundwater is high. It is then optimal to reduce the pumping rate rapidly as the quality of groundwater deteriorates. Given this characteristic of the optimal pumping path, the heavy extraction currently observed in the HRB does not necessarily indicate that groundwater resources are being overused. However, unregulated extraction by non-cooperative users would eventually cause both the depletion of the water resource and the deterioration of water quality. Hence, joint quantity–quality management is required in the HRB. The study also shows that benefits to groundwater management are higher and costs are lower in regions with salinization problems.

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1 Introduction

Despite the importance of groundwater resources in stimulating the rapid development of the Hai River Basin (HRB), one of the main economic and political centers of China, the resource base is diminishing. Between 1958 and 1998, the level of the groundwater in the HRB fell by up to 50 m in certain shallow aquifers and by more than 95 m for some deep aquifers (Ministry of Water Resource et al. 2001). During a field trip to Cang County, Hebei province (the province in which most of the HRB resides) in July 2004, the authors observed farmers extracting deep aquifer freshwater from tubewells that were sunk to a level of more than 400 m. Given the fact that the depth to the bottom of the deep aquifers is between 500 and 600 m in most places in the HRB and that the current level of extraction of deep aquifer water far exceeds the rate of recharge (Chen 1999; Hebei Bureau of Geology Reconnaissance 2003), many policymakers are worried that China is using its water resources too rapidly.

In addition to the declining water levels, another potential problem is arising in some places in the middle and eastern parts of the HRB, namely the increased salinization of some aquifers. The salinity level of freshwater in certain parts of the deep aquifers, measured by the level of total dissolved solutes (TDS), is known to have increased by 14.3 mg/L annually in Hengshui County (Song and He 1996). In some places in Cang County, the TDS level increased from less than 2000 mg/L to more than 5000 mg/L, a level above which water is considered to be saline (Hebei Bureau of Geology Reconnaissance 2003).

Hydrological and hydrogeological studies have shown that increases in the salinization of parts of the freshwater aquifers are likely to be closely related to the extraction of groundwater (Mu and Zhang 2002; Zhu et al. 2002). Large layers of prehistoric saline water exist between the shallow and deep freshwater aquifers in the middle and eastern parts of the HRB. When groundwater is extracted and the stock of groundwater declines, the pressure difference between the freshwater and the overlying saline water layers increases. Although the saline water and freshwater are separated by a layer of clay (an aquitard), the pressure difference, according to hydrologists, can push saline water past the clay layer and into the freshwater layer—a process that can increase the salinity level of the freshwater.

Since the level of the groundwater and the degree of salinization are both related to the rate of extraction of groundwater, when determining how to use groundwater optimally, a framework that considers the interaction between water quantity and quality is required for complete and more efficient management of groundwater resources. When groundwater is pumped and the level of water changes in the HRB, water users incur two costs. First, pumping costs rise as groundwater levels fall—even when the quality of water remains the same. Second, when groundwater is extracted, the intrusion of saline water into the freshwater, as seen above, increases the salinity level of the groundwater. When this happens, the application of saline water in irrigation can cause salt accumulation in the soil. Agronomic studies have shown that crop productivity falls when there is an excessive level of salinity in the soil [e.g., Maas and Hoffman (1977)]. Because of these dual effects, it is likely that the way to optimally extract groundwater in the HRB is different from what it would be in an area without a salinization problem.

Given the importance of the North China Plain in the nation's economy, and given the changes that will continue in the future, one of the key issues facing policymakers is how to manage the quantity of water and maintain water quality. As an attempt to start addressing this issue, the overall goal of this contribution is to examine how the presence of the prehistoric saline water layer affects groundwater management. Specifically, we will address three questions: (1) How does the optimal allocation of groundwater resource (both the pumping path and the level of pumping lift and salinity level at the steady state) differ between the regions with salinization problems and regions without such problems? (2) In regions with salinization problems, how does the impact of pumping on groundwater quantity and quality differ when water users extract in a cooperative way as if they are managed by a social planner and when water users extract in a non-cooperative way? (3) What are the implications for policies that we can draw for managing groundwater in the HRB?

This study will make several contributions. It will be one of the first studies that take into account the interaction between the quantity and quality of groundwater to analyze the optimal use of groundwater resources in North China. Lessons learned from this study will also help tackle salinity problems that relate to groundwater extraction in other countries such as Australia, India, and Bangladesh. Few studies have utilized joint quantity–quality framework outside of China. Some of the exceptions are Roseta-Palma (2003, 2002). However, these papers only have a general model of quantity–quality problem. Scholars also have worked on salinity issues extensively (e.g., Dinar et al. 1993; Kan et al. 2002; Knapp 1992a, b, c). However, to our knowledge there have been none inside China, this study will contribute to the resource economics literature by providing an example of a resource problem where the interlinks between the quantity and quality are considered.

There are several subjects, however, that are not addressed in this study. First, the conjunctive use of groundwater and surface water is not considered. Water users in many places in the HRB depend solely on groundwater resources from deep confined aquifers. Unlike shallow aquifers that are recharged directly by surface water supply such as precipitation, deep aquifers in the HRB are recharged by a much slower horizontal flow from the mountain area, which is far less stochastic than surface water supply. Hence, a deterministic framework is used since the stochastic surface water is not included. Second, the irrigation salinity problem that occurs when irrigation water causes the water table to rise and brings salt to the surface is not addressed. In fact, this irrigation salinity problem ceased to exist after the heavy extraction of groundwater resources started in the 1970s in the HRB (Nickum 1988). This study only analyzes the increase in salinity due to the intrusion of saline water.

2 Analytical Framework and Propositions

We begin with the standard groundwater extraction problem, focusing on the decision of the extraction quantity that balances the current and future benefits from extraction versus the current and future costs of doing so. In the first instance, we ignore salinization (Model 1). Second, we then consider the case when there is a saline water layer and use a cooperative extraction model in which water users cooperate with each other and act as if their extraction is managed by a social planner (Model 2). Since the social planner is maximizing the total benefits of society, this solution will provide the optimal solution to groundwater extraction in the presence of a saline water layer in the aquifer. We use this model to establish a baseline against which we can compare the results when there is no social planner and there is inevitably a less optimal allocation of water. In the third step, we use a non-cooperative extraction model to more accurately reflect the real-world situation in the HRB where water users are not regulated (Model 3). Comparing the case with salinization when there is a social planner (Model 2) and without one (Model 3), we can show the differences that occur when there is no effective regulation of water use.

2.1 Cooperative Extraction Model

Model 1

In most studies in the literature, groundwater is pumped in an environment in which the pumping of groundwater does not have any impacts on its quality (e.g., Burt 1964). We begin with this assumption and treat the resource from the viewpoint of a social planner to analyze the optimal extraction of groundwater. Under such a circumstance, each water user extracts cooperatively (or under the guidance of a social planner) in order to maximize the total benefit from utilizing groundwater (Model 1). Thus, he is called a *cooperative user*. Cooperative users are solving the following *quantity-only* problem:

$$\begin{aligned} &\operatorname*{Max}_{\{w_t\}} M \sum_{t=0}^{\infty} \beta^t f(w_t, h_t) \\ &\text{s.t.} \quad h_{t+1} = h_t + \phi_1 M w_t - \phi_2 R \end{aligned} \tag{1}$$

In Model 1, M is the number of water users. We assume water users are identical so they have the same net benefit function $f(\cdot)$. The net benefit is a function of the pumping rate w_t. It is also a function of the pumping lift h_t, since the pumping cost is directly associated with the pumping lift.[1] The change in h_t is a function of the difference between the recharge to the aquifer R and the net aggregate withdrawal

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[1] Pumping lift is defined as the depth from the ground surface to groundwater.

of all M users Mw_t. The parameters ϕ_1 and ϕ_2 convert changes in the volume of groundwater stock into changes in the lift of pumping. The net benefit function satisfies the following properties: $f_w > 0$, $f_{ww} < 0$, $f_h < 0$, $f_{hh} \leq 0$ and $f_{wh} < 0$. In Eq. (1), we did not include the return flow of irrigation water since the return flow will mostly stay in the shallow aquifer. In this way, we are modeling the actual water consumption.[2]

Model 2

In the HRB, when there is a saline water layer present in the aquifer, the increase in the pumping lift will induce the intrusion of saline water and then leads to an increase in the salinity level of the water resource, denoted as E_t in this contribution. Net benefits are now a function of three variables: w_t, h_t, and E_t. Since the marginal productivity of a given quantity of water decreases with its salinity level ($f_{wE} < 0$), the net benefit decreases in E_t ($f_E < 0, f_{EE} < 0$ and $f_{Eh} = 0$).[3] In regions with salinization problems, water users need to solve a more complicated problem that we henceforth call the *quantity–quality* problem:

$$\begin{aligned} &\underset{\{w_t\}}{Max}\ M\sum_{t=0}^{\infty}\beta^t f(w_t, E_t, h_t) \\ &s.t. \quad h_{t+1} = h_t + \phi_1 M w_t - \phi_2 R \\ &\qquad E_{t+1} = E_t + \delta(h_{t+1} - h_t) \end{aligned} \tag{2}$$

where δ measures the impact of changes in the depth to groundwater on the changes in its level of quality. One unit increase in the pumping lift leads to a δ unit increase in the salinity level of groundwater. The derivation of the equation of motion for E_t is in Appendix 1.

After solving problem (2), cooperative users will follow the optimal allocation rule (Appendix 2):

$$f_{wt} = -M\sum_{l=1}^{\infty}\beta^{\ell}\phi_1 f_{h_{t+\ell}} - M\sum_{l=1}^{\infty}\beta^{\ell}\phi_1\delta f_{E_{t+\ell}} \tag{3}$$

Equation (3) says that along the optimal pumping path, the marginal net benefit of groundwater f_{w_t} is equated with the marginal cost of pumping, which is made of costs that occur in the future. Since pumping one unit groundwater leads to both

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[2] In practice, what determines the level of the water table is the actual water consumption, not pumping rates. In most uses of groundwater, some of the water pumped is returned to the groundwater system. The only water that does not return to the aquifer is what evapotranspires from crops and soils. The part of evapotranspiration is the actual water consumption. Pumping rates may be irrelevant to the level at which a water table stabilizes. For example, Kendy (2003) shows that pumping decreases in some counties in Hebei province by more than 50%, yet the water table declines at the same rate over years. The modeling in this study also reflects this fact.

[3] The benefit and cost function are separate in the net benefit function. Since E_t only enters the benefit function and h_t only enters the cost function, the cross-derivative, f_{Eh}, is zero.

a ϕ_1 unit increase in the pumping lift and a $\phi_1\delta$ unit increase in the salinity level (both of these changes are in the future), the marginal cost has two components. The first term $-M\sum_{\ell=1}^{\infty}\beta^{\ell}\phi_1 f_{h_{t+\ell}}$ reflects an increase in the pumping cost of all M water users due to a larger pumping lift in all the future periods. The second term $-M\sum_{\ell=1}^{\infty}\beta^{\ell}\phi_1\delta f_{E_{t+\ell}}$ reflects the decrease in the benefit for all M water users due to a higher salinity level in all the future periods. The term on the right-hand side discounts future costs into current ones.

Comparing the decision rules in the quantity-only problem (Model 1) and that in the quantity–quality problem (Model 2), it can be seen that the decisions made by the social planner in seeking the optimal extraction of groundwater resources are different. One fundamental difference is that the optimal steady-state water level differs. In a region in which the pumping of groundwater does not affect the salinity level (i.e., in the case of the quantity-only problem), the second term on the RHS of Eq. (3) vanishes. Therefore, the marginal cost of extraction is higher in a region with a salinization problem. Higher marginal costs results in lower pumping rates, which in turn leads to more water left in the ground at the steady state. Based on this set of ideas, we develop the first proposition:

Proposition 1: *The socially optimal pumping lift is smaller in a region with a salinization problem, compared to that in a region without such a problem (Proof in Appendix 3).*

A second difference is that the value of the groundwater resources also differs in the two cases. Specifically, when δ is higher (i.e., when the change in the quantity of groundwater by pumping has a greater impact on the change in quality), the aquifer becomes more saline given the same volume of pumping. Since higher salinity levels reduce benefits from using groundwater in the future, the value of groundwater is lower. Following this logic, we establish a second proposition:

Proposition 2: *In a region with a salinization problem, the value of groundwater (the present value of net benefits from using groundwater in all future periods) decreases in the magnitude of impact that groundwater extraction has on the salinity level of groundwater (Proof in Appendix 4).*

2.2 Non-cooperative Extraction Model

Model 3

Unlike the assumption of the social planner model, water users in China are not regulated when withdrawing water from a common aquifer. Without any regulations, water users are not likely to cooperate among themselves. Each individual water user extracts groundwater in order to maximize his own profit independent of that of others and thus is called a *non-cooperative user*. Mathematically, a non-cooperative user i is solving the following *non-cooperative extraction* problem (Model 3):

$$\underset{\{w_{it}\}}{\text{Max}} \sum_{t=0}^{\infty} \beta^t f(w_{it}, E_t, h_t)$$

$$\text{s.t.} \quad h_{t+1} = h_t + \phi_1[w_{it} + \sum_{j \neq i}^{M} w_j^*] - \phi_2 R$$

$$E_{t+1} = E_t + \delta(h_{t+1} - h_t) \tag{4}$$

Here, user i is involved in a non-cooperative difference game with other water users. When user i makes his decision, he takes the pumping rates of other users, w_j^* as given. The solution of this model is a feedback Nash equilibrium. In our work, since we will only solve the non-cooperative extraction problem in the case in which there is a saline water layer in the aquifer, we identify Model 3 as the non-cooperative extraction problem (although its complete name would be the non-cooperative extraction problem in the presence of salinization). After solving problem (4), the non-cooperative user i will follow the decision rule that can be expressed as (Appendix 5):

$$f_{w_{it}} = -\sum_{\ell=1}^{s} \beta^\ell \phi_1 f_{h_{t+\ell}} - \sum_{\ell=1}^{s} \beta^\ell \phi_1 \delta f_{E_{t+\ell}} + \sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \frac{\partial w_j^*}{\partial h_{t+\ell}} + \sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \delta \frac{\partial w_j^*}{\partial E_{t+\ell}} \tag{5}$$

Comparison of the RHSs of Eqs. (3) and (5) shows that, given the same pumping lift and the same level of salinity, non-cooperative users (Model 3) extract more than cooperative users (Model 2, Appendix 6). Similar to the quantity–quality case, the term $-\sum_{\ell=1}^{s} \beta^\ell \phi_1 f_{h_{t+\ell}} - \sum_{\ell=1}^{s} \beta^\ell \phi_1 \delta f_{E_{t+\ell}}$ reflects the marginal cost of pumping due to higher pumping lifts and higher salinity levels in the future. However, when water users are pumping groundwater in an environment characterized by non-cooperative extraction, no single individual accounts for the social cost of his pumping, which is the increased future pumping costs of other users that will accrue due to the drawing down of the water level. Hence, non-cooperative users underestimate the marginal cost by $(M-1)\sum_{\ell=1}^{s} \beta^\ell \phi_1 f_{h_{t+\ell}} + (M-1)\sum_{\ell=1}^{s} \beta^\ell \phi_1 \delta f_{E_{t+\ell}}$. In addition, water users also react to a lower pumping lift or a salinity level by increasing their pumping rates ($\frac{\partial w_j^*}{\partial h_t} < 0$ and $\frac{\partial w_j^*}{\partial E_t} < 0$, Appendix 6). This strategic behavior of water users (a water user may pump more than what he would had there been no other users to discourage the extraction of others) is discussed in detail in Negri (1989) and Provencher and Burt (1993). Knowing this, user i places a lower value on the marginal cost of pumping (by a degree of $\sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \frac{\partial w_j^*}{\partial h_{t+\ell}} + \sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \delta \frac{\partial w_j^*}{\partial E_{t+\ell}}$). This lower valuation occurs because any water he conserves for future use (as would occur in the case of the social planner in the quantity–quality problem) will be pumped out by other users. As a result, given the same pumping lift and the same salinity level, non-cooperative users will extract more than what they would extract had water users cooperate in pumping.

Although the over-extraction of non-cooperative users does not necessarily lead to both higher pumping lifts and higher salinity levels at the steady state, it does in this case. The linear relationship between changes in salinity level and changes in pumping lifts makes it that higher pumping lifts are always accompanied by higher salinity levels. Following this logic, we form

Proposition 3 *In regions with salinization problems, compared to cooperative extraction, non-cooperative extraction leads to both a higher pumping lift and a higher salinity level at the steady state* (*Proof in Appendix* 6).

3 Empirical Specification and Parameterization of the Model

Analyses of the theoretical models in the previous models have provided a basic understanding of the way to optimally use groundwater in the specific environment of the HRB where extraction affects both the quantity and quality of groundwater. The next step is to empirically examine the propositions developed from the theoretical models. In this section, before presenting the results of the empirical analysis, we will first specify the functional form of the benefit function and introduce the data sources and information that will be drawn on to parameterize the models.

The specified net benefit function is as follows:

$$f(w_t, h_t, E_t) = \mathrm{e}^{\alpha E_t - \theta (E_t)^2}(aw_t - 0.5bw_t^2) - ch_t w_t \tag{6}$$

Following several economics studies that analyze a quantity-only problem (Feinerman and Knapp, 1983; Rubio and Casino, 2001), we use a quadratic benefit function, $aw_t - 0.5bw_t^2$, where a and b are the intercept and slope of the demand function for irrigation water, respectively. Pumping cost is a function of the volume pumped, w_t, and the pumping lift, h_t. The parameter c is the marginal cost of lifting one unit of groundwater by one unit of pumping lift. Unlike the studies that analyze a quantity-only problem, in the net benefit function there is an exponential function $\mathrm{e}^{\alpha E_t - \theta (E_t)^2}$ that measures the magnitude of the reduction in the crop yield in response to higher salinity level of the irrigation water. This exponential function has been used in several agronomic studies, and the parameters α and θ in the exponent are also estimated in these studies [e.g., Van Genuchten and Hoffman, 1985].

3.1 Parameterization of the Model

Parameter values are obtained from different sources. Values of parameters in the quadratic benefit function and the pumping cost are estimated using the China Water Institutions and Management (CWIM) survey data that we collected in 2004.[4] Net benefit from water use is calculated as revenue from agricultural production minus cost of variable inputs other than water. These inputs include labor, fertilizer, pesticide, herbicide seed, plastic sheeting, and machinery (for most rural households in China, this means the cost of renting machine). The linear parameter a and quadratic parameter b are estimated using a random-coefficient model with net benefit as the dependent variable and water use in linear and quadratic terms as explanatory variables. In the 2004 CWIM, we collected information on the depth of water in the village, characteristics of pumps (size, water per hour, lift, etc.), electricity price, level of water use, and the amount farmers pay for water. Using this set of information, we calculate the average pumping cost to be used in the model.

Rarely will farmers know the level of salinity of the irrigation water they use, so we are unable to estimate parameters in the exponential function $e^{\alpha E_t - \theta (E_t)^2}$ using our survey data. These parameters are estimated using the experimental data on the levels of crop yields and different salinity levels of irrigation water that are reported in Shao et al. (2003).

Even less is known about the exact relationship between changes in salinity level and changes in the depth of water. Hengshui County is among the areas that have the highest degree of salinization problem in Hebei province. The salinity level measured by EC increases by 0.81 dS/m ($\approx$0.5427 g/L) between 1975 and 1995; the water levels dropped by 43.53 m during the same period (Fig. 1).

The value of δ, the parameter that measures the relationship between the changes in the salinity level and the water level, is around 0.012 gL/ m for Hengshui County. Hence, a value of 0.02 can be considered as large. Since the value of δ will differ across places, in the simulations, we choose a range of values of δ. These values range from very small to large, 0.0001, 0.04, 0.1, and 0.2.

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[4] In the 2004 China Water Institutions and Management Survey, the enumeration team collected data in 24 communities in Hebei province. In order to guarantee an adequate sample of communities in each of several water usage situations, the communities were chosen randomly from three randomly selected counties according to location, which in the Hai River Basin often is correlated with water scarcity levels. Xian County is located along the coastal belt (the most water scarce area of China); Tang County is located along the inland belt (an area with relatively abundant water resources that are next to the hills and mountains that rise in the eastern part of Hebei province); and Ci County is located in the region between the coastal and inland belts. The survey was conducted by interviewing three different types of respondents in each community (or village): the community leader; well manager (typically three randomly selected well managers per community); and households (four randomly selected households). We use separate questionnaires for each type of respondents. Although most of the data in the analysis come from the household questionnaire, we also use some data from the community leader and well manager questionnaires. Two major blocks of data are used from the household survey: data on household production activities and data on household water use.

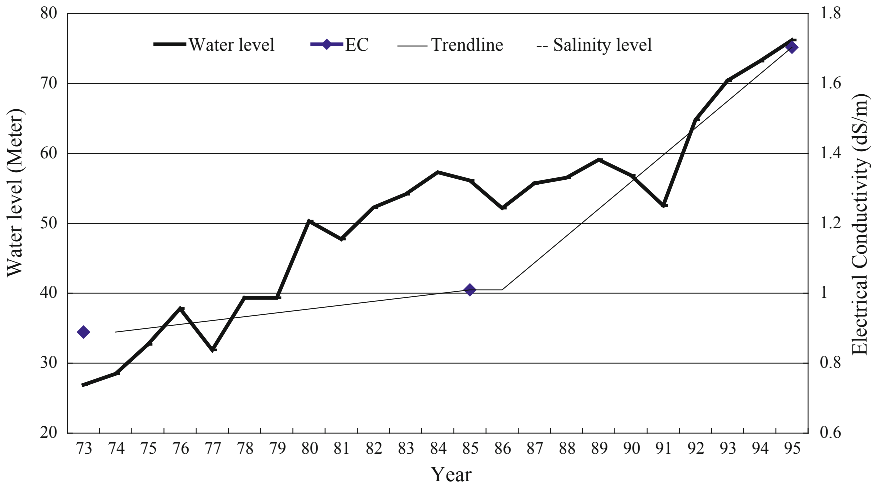


Fig. 1 Changes in the water level and the salinity level 1975–1995, Hengshui County, Hebei Province. *Source* Hebei Bureau of Geology Reconnaissance (2003)

Table 1 Values of parameters in the model

Parameter	Description	Value
β	Discount factor	0.9434
a	Intercept of marginal net benefit	\$0.39/m^3
b	Slope of marginal net benefit	0.007
EC_0	Initial salinity level	0.582 g/L
h_0	Initial depth-to-water	60 m
S	Specific yield	0.00157
c	Marginal pumping cost	\$0.000128/m3/m lift
α	Coefficient on the linear term in salinity-yield function	0.0025627
θ	Coefficient on the quadratic term in salinity-yield function	0.0111101

A discount rate of 6% is used. The initial water depth is set at 60 m. The value of a specific yield is taken from Chen et al. [P167, 1999]. The level of recharge is set at 5 cm expressed in terms of water depth, which is the value used by Shen et al. (2000). In an analysis in Table 2, we also increase it to 1 m. The rest of the parameter values are listed in Table 1.

4 Results of the Empirical Analysis

We solve the dynamic optimization problems numerically (the quantity-only problem, the quantity–quality problem, and the non-cooperative extraction problem) using the general algebraic modeling system (GAMS). When using the dynamic programming technique with the value-iteration algorithm to solve the problems numerically, we use the "collocation" method described in Judd (1998) and Miranda and Fackler (2002). In particular, the Chebychev polynomial is used to approximate the infinite horizon value function of water users. The solutions will provide the pumping path, the level of value function, and the level of the pumping lift and salinity level at the steady states in different problems. Using these results, we will compare the dynamic properties of the pumping path and test the propositions developed in the previous section.

4.1 Quantity-Only Problem Versus Quantity–Quality Problem

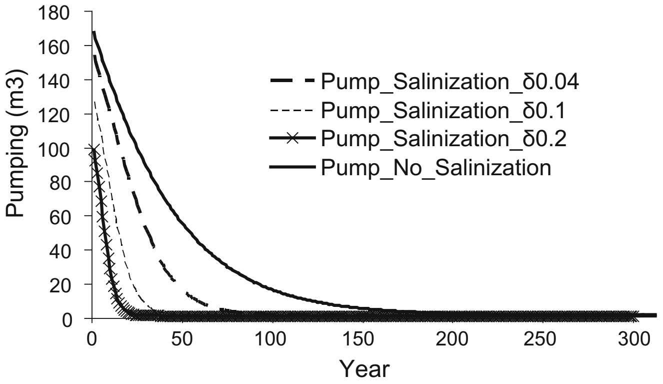
In the first part of the empirical analysis, we vary the value of the parameter, δ which measures the impact of groundwater extraction on changes in the salinity level. When δ is zero, there is no salinization problem and users are solving a quantity-only problem (Model 1). When δ takes on nonzero values, there is a salinization problem and users are solving a quantity–quality problem (Model 2). Comparison of the solutions to the two problems will enable us to examine Propositions 1 and 2 in order to answer the question "How does the optimal allocation of groundwater resource differ between the regions with salinization problems and regions without such problems?"

Comparison of the solution to a quantity-only problem and the solution to a quantity–quality problem shows that the way to optimally use groundwater is different between a region with a salinization problem and a region without a salinization problem. The pumping paths differ (Fig. 2a).

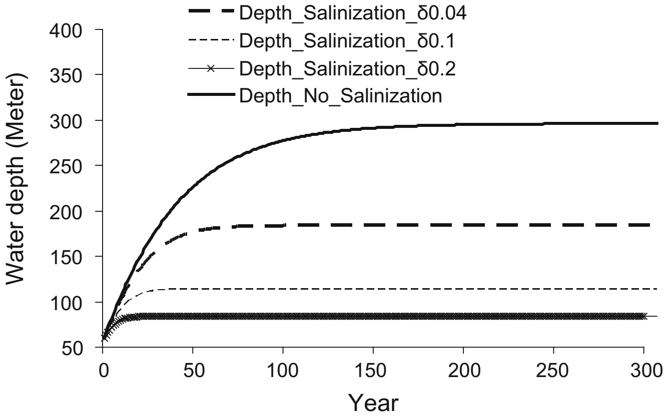
When the value of δ is between 0.04 and 0.2, compared to a water user in the region without a salinization problem, in a region with a salinization problem a water user pumps less at all times. The pumping path is more complicated when the value of is 0.0001 (Fig. 3a).

Compared to a water user in the region without a salinization problem and in a region with a salinization problem, a water user pumps more at the beginning of extraction. The rate of fall in his pumping rate is higher. After a certain period of time, his pumping rate drops below that of a water user in a region without a salinization problem. Intuitively, in the region with a salinization problem, pumping will lead to an increase in the salinity level of groundwater and thus reduces the benefit of future groundwater use. Under such a circumstance, if the impact of changes in water depth on the salinity level is small, it is optimal to pump more at the beginning when the

(a) *Comparison of the pumping path*



(b) *Comparison of the pumping lift*



(c) *Comparison of the salinity level*

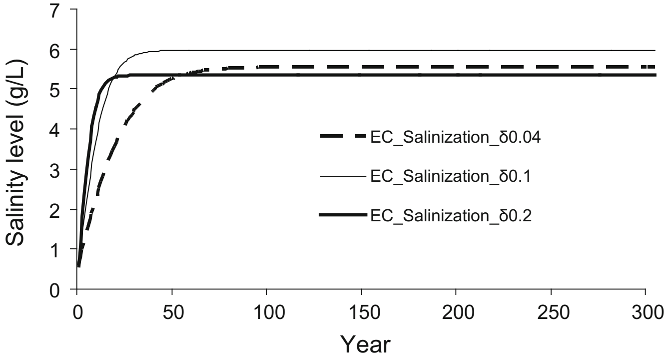
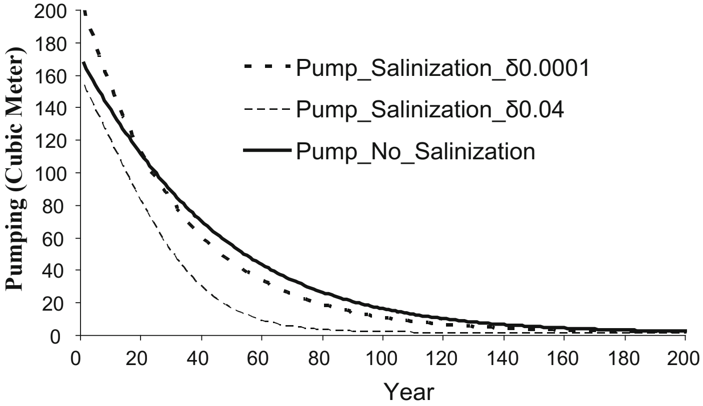
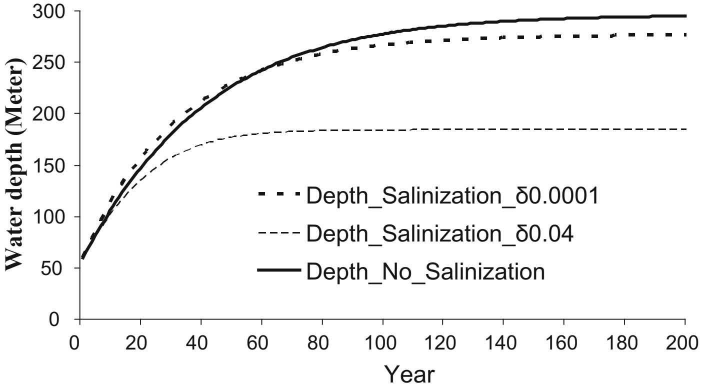


Fig. 2 Comparison of cooperative extraction with and without salinization problem ($\delta = 0.04; 0.1; 0.2$). **a** Comparison of the pumping path. **b** Comparison of the pumping lift. **c** Comparison of the salinity level

(a) *Comparison of the pumping path*



(b) *Comparison of the pumping lift*



(c) *Comparison of the salinity level*

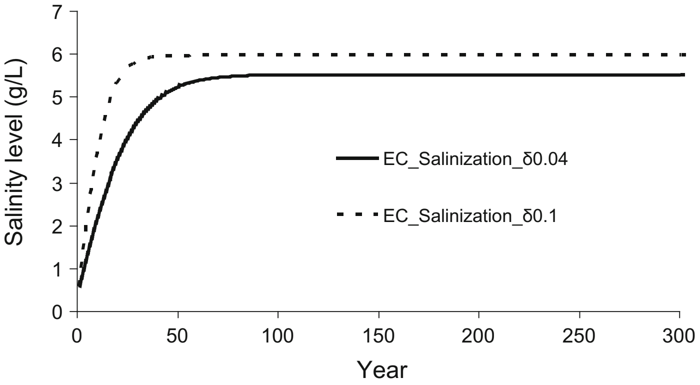


Fig. 3 Comparison of cooperative extraction with and without salinization problem ($\delta = 0.0001$; 0.04). **a** Comparison of the pumping path. **b** Comparison of the pumping lift. **c** Comparison of the salinity level

quality of groundwater is high. It is then optimal to reduce the pumping rate rapidly as the quality of groundwater deteriorates. If the impact of changes in water depth on salinity level is large, pumping will be penalized heavily in terms of benefit reduction even at the beginning, so it is optimal to pump less at all periods.

The comparison also provides evidence for Proposition 1 that the socially optimal pumping lift is smaller in regions with salinization problems (Figs. 3b and 4b). Even in the case of δ is 0.0001, although the heavy pumping of water users in the early stage leads to a more rapid increase in the pumping lift in the region with a salinization problem, the increase slows later when the pumping rate drops rapidly. At the steady state, the pumping lift is smaller when there is a salinization problem compared to when there is no salinization problem. Intuitively, since the pumping of groundwater leads to increases in the salinity level (Figs. 2c and 3c), the marginal net benefit of groundwater decreases and more water is left in the groundwater at the steady state.

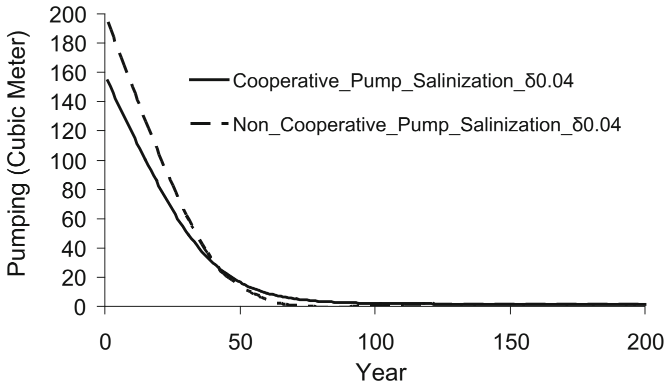
The result of comparison also supports Proposition 2 that in regions with salinization problems, the value of groundwater decreases in the magnitude of impact that groundwater extraction has on the salinity level of groundwater. When δ is 0.04 and assume the level of recharge is five centimeter expressed in terms of water depth, the present value of net benefits from using groundwater is reduced by more than 10% compared to the scenario when δ is 0 (Table 2, Column 3).

The present value of net benefits from using groundwater is reduced by almost half when δ is 0.2. It is also consistent with what we have observed in the field. In our pretest and formal interviews with farmers in the HRB, we asked the following question: "Suppose China's government starts a payment for water program. You will be paid to stop cultivation to conserve water, how much is your willingness to accept?" We interviewed farmers in two different counties. In Cang County, there is a serious salinization problem in most places, and in Luancheng County there is no salinization problem. In Cang County, the willingness of farmers to accept ($656/ha/year) is about $300 less than that in Luancheng County ($938/ha/year). In the region with a salinization problem, since the future benefits water users could obtain are lower due to the more saline water, water users value groundwater less.

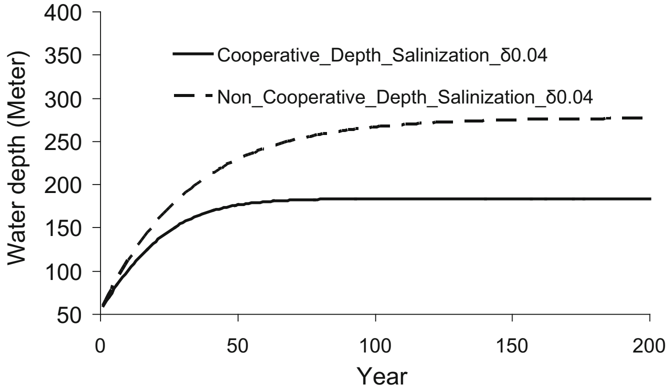
4.2 Cooperative Extraction Versus Non-cooperative Extraction

In the second part of the empirical analysis, we solve the quantity–quality problem numerically using the same nonzero value of δ as in the non-cooperative extraction problem (Model 3). A different game approach is used for the non-cooperative extraction problem. By comparing the solutions to the two problems, we are able to examine Proposition 3 in order to answer the question "In regions with salinization problems, how does the impact of pumping on groundwater quantity and quality differ when water users extract in a cooperative way and when water users extract in a non-cooperative way?"

(a) *Comparison of the pumping path*



(b) *Comparison of the pumping lift*



(c) *Comparison of the salinity level*

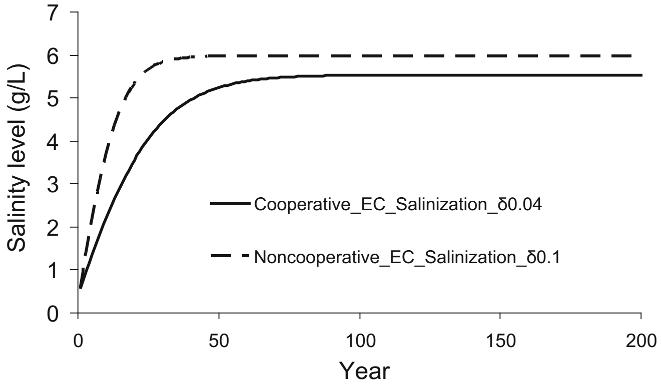


Fig. 4 Comparison of cooperative and non-cooperative extraction with salinization. **a** Comparison of the pumping path. **b** Comparison of the pumping lift. **c** Comparison of the salinity level

Table 2 Comparison of present value and gain from management

(1) Level of recharge	(2) δ	(3) Present value under optimal pumping	(3) Present value under non-optimal behavior		(4) Gain from management	(5) Ratio of gain from management under salinization to that under no salinization
5 cm	0^b	537,118	Purely myopic	518,135	18,983	
	0.04	479,839	Purely myopic	415,987	63,852	3.4
			Myopic with update	425,131	54,708	2.9
			Only salinity myopic	443,331	36,508	1.9
	0.2	293,390	Purely myopic	197,143	96,247	5.1
			Myopic with update	201,792	91,598	4.8
			Only salinity myopic	239,776	53,614	2.8
1 m	0	588,543	Purely myopic	564,063	24,480	
	0.04	550,350	Purely myopic	451,341	99,008	4
			Myopic with update	474,531	75,818	3.1
			Only salinity myopic	540,501	9848	0.4
	0.2	343,943	Purely myopic	186,802	157,140	6.4
			Myopic with update	232,826	111,116	4.5
			Only salinity myopic	321,709	22,233	0.9

Notes [a]Present value is the total value of benefits from water use of all households within a village in dollars

[b]$\delta = 0$ is the case when there is no aquifer salinization problem

Comparison of the solution to the quantity–quality problem and the non-cooperative extraction problem uncovers the difference in the pumping path of cooperative users and non-cooperative users (Fig. 4a). At the early stage of extraction, non-cooperative users pump more than cooperative users since non-cooperative users ignore the social cost of their pumping. As a result, the pumping lift that non-cooperative users face increases rapidly and they are forced to reduce their pumping rates sooner due to the higher pumping costs. In fact, after a period of time, non-cooperative users are pumping less than cooperative users.

It is also observed that at the steady state, both the pumping lift and the salinity level in the cooperative extraction problem are smaller than that in a non-cooperative problem (Figs. 4b and 5c). This result supports Proposition 3. Thus, over-extraction by non-cooperative users causes both the depletion of the water resource and the deterioration of water quality.

4.3 Cooperative Extraction Versus Different Types of Myopic Extraction

Since currently there is no effective management in the Hai River Basin, we are also interested in the extraction behaviors when there is no regulation. We look at three different types of water users that display different types of myopic behavior: the *purely myopic* water users; the *myopic with update* water users; the *only salinity myopic* water users. The purely myopic water users maximize their own net benefit from the current period. They completely ignore the dynamics of both water stock and water salinity. The myopic with update water users also only maximize one-period net benefit. However, they realize that somehow the benefit they obtain from the same amount of water is less than that from previous years, although they do not realize it is the result of the interaction between water stock and salinity level. So they will update their benefit function based upon observations from previous years.[5] The only salinity myopic water users maximize own net benefit over time, but they do not realize the interaction between water stock and salinity level. The difference between the cooperative water users and the only salinity myopic water users is the latter does not consider the dynamics of salinity when maximizing the present value of net benefit.

The results show that all types of myopic water users lead to higher water depth (Fig. 5b) and higher salinity level (Fig. 5c) at the steady state in comparison with the optimal pumping case. Among the myopic users, the purely myopic users are pumping more than other types of myopic users in all periods (Fig. 5a). As a result, they deplete the water stock and quality more severely than other types of myopic users. Between the only salinity myopic users and the myopic with update water users, the pumping behavior of the only salinity myopic users is closer to that of optimal users. This is because they are maximizing the net benefit over time. Hence,

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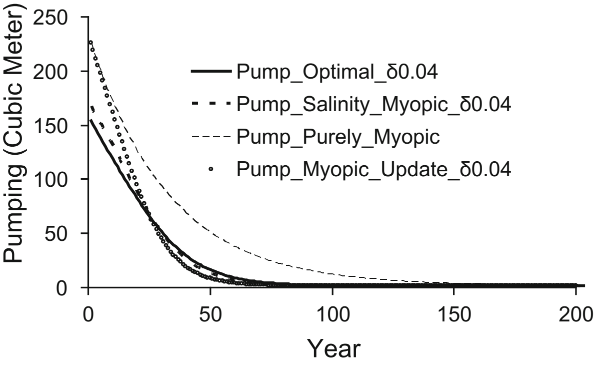
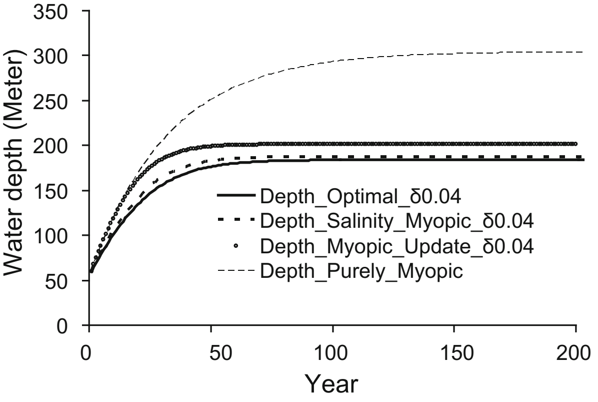
[5] In simulations, the benefit function is parameterized using values from the year before.

(**a**) *Comparison of the pumping path*



(**b**) *Comparison of the pumping lift*



(**c**) *Comparison of the salinity level*

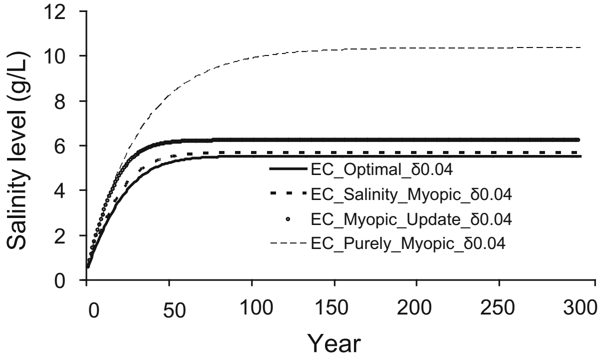


Fig. 5 Comparison of cooperative and myopic extraction with salinization. **a** Comparison of the pumping path. **b** Comparison of the pumping lift. **c** Comparison of the salinity level

depending on the characteristics of farmers and the village, pumping patterns are different when there is no regulation. In villages where the village leader puts efforts in playing the role of the social planner, farmers may be more like the only salinity myopic users since they are guided by the village leader to internalize their externality on other users. In those villages, the pumping of users is quite close to the optimal pumping case. In villages where farmers are more experienced or more motivated, farmers may behave like the myopic with update users, taking time to update their benefit function and revise their input uses. In these villages, farmers will also pump less than purely myopic users.

Our results also show that magnitudes of Gain From Management (GFM) are also different, depending on the degree of salinization types of behavior (Table 2, columns 5 and 6). In most cases, the GFM is larger when there is a salinization problem in comparison with when there is no salinization problem. The GFM can be double or more than five times higher when there is a salinization problem (column 6). The GFM is higher when water users are purely myopic than when water users are myopic with an update or only salinity myopic. However, when the recharge rate is high and farmers are only salinity myopic users, the GFM is much smaller than other cases. It is only 0.4 or 0.9 of the GFM when there is no salinization. Our findings indicate that the management decisions may differ across places. In villages where the aquifer receives high volume of recharge and farmers are only salinity myopic users, the cost of managing groundwater may be higher than the benefit, as pointed out in Gisser and Sanchez (1980). In other villages, however, the benefit of managing groundwater may overweigh the cost.

5 Conclusion

In this contribution, we have discussed the analytical framework for the optimal allocation of groundwater, both when there is no salinization problem and when there is a salinization problem. We also have compared the case when water users cooperate in pumping and the case when water users do not. Results from the numerical computation of the models are used to empirically examine propositions developed from the theoretical models. The lessons learned can help scholars and policymakers understand water use patterns in the HRB and provide some insights into how China should manage water (or whether they should manage it at all.

Results of the empirical analysis show that in a region where there is a salinization problem like in the HRB, the way to manage groundwater depends crucially on local conditions. For example, when extraction leads to small declines in the salinity level, it is actually optimal to pump at high rates in the early stage of extraction. Given this characteristic of the optimal pumping path, the heavy extraction we observe in some areas in the HRB does not necessarily indicate that groundwater resources are being overused. To judge whether we are overusing groundwater resources, we need to know which stage of extraction we are in. In fact, as already pointed out by Howitt and Nuckton (1981), even over-extraction is not necessarily bad during the earlier stages of extraction. If we are still in the early stage of extraction and it is still a long time before the steady state is reached, the pumping lift is still lower than the socially optimal level. Over-extraction is not harmful since it accelerates the convergence to the steady state by increasing the pumping lift rapidly.

Despite the potential danger of losing freshwater stock caused by both aquifer salinization and groundwater extraction, currently, the quality of groundwater and its quantity are managed in isolation of each other in the HRB. In most counties in the HRB, environmental protection bureaus are responsible for maintaining water quality and water resource bureaus are in charge of regulating groundwater extraction (Ministry of Water Resource et al. 2001). Under such a disjoint managing scheme, policies recommended by environmental protection bureaus may be inefficient. For example, massive investments in improving water quality may stimulate more extraction by farmers. Heavy extraction of water, by raising the salinity level of groundwater, makes these investments totally wasted. In fact, if the extraction of groundwater is not regulated, any measures that are intended to maintain the salinity level of freshwater will be in vain. In addition, the target level of optimal pumping lift set by the water resource bureaus will be incorrect. Hence, joint quantity–quality management is required in the HRB.

An equally important aspect that requires consideration before leaders make policies to manage groundwater resources is the cost and benefit of management. Empirical results of our study show that without regulations, the total benefit of all non-cooperative users obtain from extracting groundwater is lower than that obtained by cooperative users who act as if their extraction is managed by a social planner. Hence, there is a gain from managing groundwater and it is measured by the increase in the total benefit from using groundwater. However, the cost of management may easily exceed the benefit due to the fragmented and small-scale nature of China's farmers in hundreds of villages in the HRB. For example, the cost of measuring and enforcing water use on tens of millions of small parcels throughout HRB and collecting fees on a farmer-by-farmer basis may exceed the benefits of volumetric pricing.

If the result of cost-benefit analysis favors the implementation of a certain policy, regions with lower cost of implementation should be given priority. One such policy could be the 'payment-for-water' program. Empirical results of our study indicate that the payment that farmers are willing to accept to retire land is lower in regions with salinization problem. Thus, if China's government is to implement payment-for-water program, regions with salinization problems should be given priority since the cost (the payment to retire land) is lower there.

Appendix 1. Derivative of the Equation of Motion for the Groundwater Salinity Level (E_t)

In this study, we simplify our analysis by only focusing on the case when the saline water moves into the deep aquifer. In Fig. 6, the extraction of deep aquifer water,

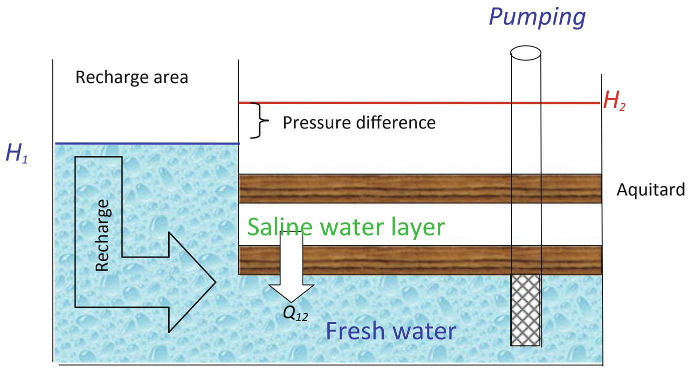


Fig. 6 The process of saline water intrusion due to groundwater extraction

Mw_t, leads to an increase in the depth-to-water in the deep aquifer, h_t. The hydraulic head in the deep aquifer, H_1, keeps declining as a result. When the pressure difference between the head in the saline water layer, H_2, and that in the deep aquifer is large enough, the saline water can move into the deep aquifer through the aquitard. The movement of saline water in response to the change in the head difference, Q_{12}, accounts for the phenomenon of increasing salinity level in the deep aquifer.

In the language of hydrology, Darcy's Law can be employed to formalize the movement of saline water.[6] Suppose the hydraulic head of the deep aquifer is linear in the depth-to-water: $H_{1t} = -c_1 h_t + d_1$ and $H_{2t} = c_2 \frac{Q_t}{As} + d_2$, where Q_t is the stock of the saline water, A is the area of saline water layer, and s is the specific yield, the volume of saline water that moves into the deep aquifer at time t can be expressed as

$$Q_t - Q_{t+1} = Q_{12} = -KA\frac{H_{1t} - H_{2t}}{b} = -\frac{KA}{b}\left[(-c_1 h_t + d_1) - \left(c_2 \frac{Q_t}{As} + d_2\right)\right] \tag{7}$$

where K is the hydraulic conductivity of the aquitard (unit: volume per unit of time), and b is the thickness of the aquitard. From (7), we have $Q_t - Q_{t+1} \propto h_{t+1} - h_t$.[7] We assume that the change in the level of salinity, $E_{t+1} - E_t$, is proportional to the total amount of intruded saline water at time t, Q_{12}. Hence, we have $E_{t+1} - E_t \propto h_{t+1} - h_t$. Suppose $E_{t+1} - E_t = \delta(h_{t+1} - h_t)$, we obtain the equation of motion for E_t:

$E_{t+1} = E_t + (h_{t+1} - h_t)$

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[6] Darcy's Law states that the volume discharge rate Q is directly proportional to the head drop $h_1 - h_2$ and to the cross-section area A, but it is inversely proportional to the length difference, L (Wang and Anderson 1995): $Q = -KA\frac{h_1 - h_2}{L}$ where K is the hydraulic conductivity of the medium (e.g., clay or sand). The negative sign signifies that groundwater flows in the direction of head loss.

[7] $Q_t - Q_{t+1} = \frac{KAc_1}{b}h_t + \frac{KAc_2}{Asb}Q_t - \frac{KA}{b}(d_1 - d_2) => Q_{t+1} = (1 - \frac{KAc_2}{Asb})Q_t - \frac{KAc_1}{b}h_t + \frac{KA}{b}(d_1 - d_2)$ $=> Q_{t+2} = (1 - \frac{KAc_2}{Asb})Q_{t+1} - \frac{KAc_1}{b}h_{t+1} + \frac{KA}{b}(d_1 - d_2) => Q_{t+1} - Q_{t+2} = (1 - \frac{KAc_2}{Asb})(Q_t - Q_{t+1}) + \frac{KAc_1}{b}(h_{t+1} - h_t)$.

Appendix 2. Derivation of the Euler Equation for the Cooperative Extraction Model

We rewrite (2) as

$$\begin{aligned}\underset{\{w[t]\}}{\text{Max}}\, L = M\sum_{t=0}^{\infty}\beta^{t} f(w_t, E_t, h_t)\\ -\sum_{t=0}^{\infty}\beta^{t+1}\lambda_{t+1}[h_{t+1}-(h_t+\phi_1 M w_t-\phi_2 R)]\\ -\sum_{t=0}^{\infty}\beta^{t+1}\mu_{t+1}[E_{t+1}-(E_t+\delta(\phi_1 M w_t-\phi_2 R)]\end{aligned}$$

The first-order condition for this problem gives:

$$\frac{\partial L}{\partial w_t} = f_{w_t}(w_t, E_t, h_t)+\beta\phi_1(\lambda_{t+1}+\delta\mu_{t+1}) = 0 \tag{8}$$

$$\frac{\partial L}{\partial h_t} = Mf_{h_t}(w_t, E_t, h_t)+\beta\lambda_{t+1}-\lambda_t = 0 \tag{9}$$

$$\frac{\partial L}{\partial E_t} = Mf_{E_t}(w_t, E_t, h_t)+\beta\mu_{t+1}-\mu_t = 0 \tag{10}$$

$$=> \lambda_{t+1}+\delta\mu_{t+1} = -f_{w_t}(w_t, E_t, h_t)/(\beta\phi_1) \tag{8}$$

Lagging (8) by one period gives:

$$\lambda_t+\delta\mu_t = -f_{w_{t-1}}(w_{t-1}, E_{t-1}, h_{t-1})/(\beta\phi_1) \tag{11}$$

(9) +δ^* (10)⇒

$$\beta(\lambda_{t+1}+\delta\mu_{t+1})-(\lambda_t+\delta\mu_t) = -M[f_{h_t}(w_t, E_t, h_t)+\delta f_{E_t}(w_t, E_t, h_t)] \tag{12}$$

Plugging (8) and (11) into (12) gives:

$$\begin{aligned}f_{w_{t-1}}(w_{t-1}, E_{t-1}, h_{t-1}) = \beta f_{w_t}(w_t, E_t, h_t)-\beta\phi_1 M[f_{h_t}(w_t, E_t, h_t)\\ +\delta f_{E_t}(w_t, E_t, h_t)]\end{aligned} \tag{11}$$

Rolling equation (11) forward one period gives:

$$f_{w_t} = \beta f_{w_{t+1}}-\beta\phi_1 M(f_{h_{t+1}}+\delta f_{E_{t+1}})$$

$$= \beta[\beta f_{w_{t+2}}-\beta\phi_1 M\big(f_{h_{t+1}}+\delta f_{E_{t+1}}\big)]-\beta\phi_1 M\big(f_{h_{t+2}}+\delta f_{E_{t+2}}\big)$$

$$= \cdots$$

$$= \beta^s f_{w_{t+s}} - \sum_{\ell=1}^{s} \beta^{\ell} \phi_1 M \left(f_{h_{t+\ell}} + \delta f_{E_{t+\ell}} \right) \tag{12}$$

Leading it forward into infinite future, we obtain

$$f_{w_t} = \lim_{s \to \infty} \left\{ \beta^s f_{w_{t+s}} - \sum_{\ell=1}^{s} \beta^{\ell} \phi_1 M (f_{h_{t+\ell}} + \delta f_{E_{t+\ell}}) \right\} \tag{13}$$

Since β is the discount factor that is well within the range of 0 and 1, (13) gives

$$f_{w_t} = -M \sum_{\ell=1}^{\infty} \beta^{\ell} \phi_1 f_{h_{t+\ell}} - M \sum_{\ell=1}^{\infty} \beta^{\ell} \phi_1 \delta f_{E_{t+\ell}} \tag{14}$$

Appendix 3. Derivation of Proposition 1

At the steady state, $w_{t\text{-}1} = w_t = w*$. We use w^*, E^*, and $h^*$to denote the value at the steady state. Equation (14) now becomes:

$$(1-\beta) f_w(w^*, E^*, h^*) + \beta \phi_1 f_h(w^*, E^*, h^*) + \beta \phi_1 \delta f_E(w^*, E^*, h^*) = 0$$

Using the implicit function theorem gives:

$$\frac{\partial h^*}{\partial \delta} = -\frac{\beta \phi_1 f_E(w^*, E^*, h^*)}{(1-\beta) f_{wh}(w^*, E^*, h^*) + \beta \phi_1 f_{hh}(w^*, E^*, h^*)}$$

We have $f_E < 0$, $1- > 0$, $f_{wh} < 0$ and $f_{hh} < 0$. Therefore,

$$\frac{\partial h^*}{\partial \delta} = -\frac{(-)}{(+)(-) + (-)} < 0$$

Appendix 4. Derivation of Proposition 2

Bellman equation for problem s(2) is:

$$\begin{aligned} V(h, E) &= \underset{w}{\text{Max}} \left\{ f(w, h, E) + \beta V(h', E') \right\} \\ \text{s.t.} \quad h' &= h + \phi_1 M w - \phi_2 R \\ E' &= E + \delta (h' - h) \end{aligned} \tag{15}$$

Using the Envelope Theorem gives:
$V_\delta = \beta V_{E'}(h',\ E') \cdot (h' - h) \Rightarrow V_\delta < 0$ since $V_{E'} < 0$ and $h' > h$.

Appendix 5. Derivation of the Euler Equation for Non-cooperative Extraction Model

we rewrite (4) as

$$\underset{\{w_{it}\}}{\text{Max}}\, L = \sum_{t=0}^{\infty} \beta^t f^i(w_{it}, E_t, h_t) - \sum_{t=0}^{\infty} \beta^{t+1} \lambda_{t+1}(h_{t+1} - h_t - \phi_1 w_{it} - \phi_1 \sum_{j \neq i}^{M} w_j^* + \phi_2 R)$$
$$- \sum_{t=0}^{\infty} \beta^{t+1} \mu_{t+1}(E_{t+1} - E_t - \phi_1 \delta w_{it}$$
$$- \phi_1 \delta \sum_{j \neq i}^{M} w_j^* + \phi_2 \delta R)$$

The first-order condition for this problem gives:

$$\frac{\partial L}{\partial w_t} = f^i_{w_t}(w_t, E_t, h_t) + \beta\phi(\lambda_{t+1} + \delta\mu_{t+1}) = 0 \tag{16}$$

$$\frac{\partial L}{\partial h_t} = f_{h_t}(w_t, E_t, h_t) + \beta\lambda_{t+1} - \lambda_t + \phi_1\beta(\lambda_{t+1} + \delta\mu_{t+1}) \sum_{j \neq i}^{M} \frac{\partial w_j^*}{\partial h_t} = 0 \tag{17}$$

$$\frac{\partial L}{\partial E_t} = f_{E_t}(w_t, E_t, h_t) + \beta\mu_{t+1} - \mu_t + \phi_1\beta(\lambda_{t+1} + \delta\mu_{t+1}) \sum_{j \neq i}^{M} \frac{\partial w_j^*}{\partial E_t} = 0 \tag{18}$$

Using the same manipulations as in the steps to obtain the Euler equation for the cooperative model, we obtain:

$$f_{w_{it-1}}(w_{it-1}, E_{t-1}, h_{t-1}) = \beta f_{w_{it}}(w_{it}, E_t, h_t) \left\{ 1 + \sum_{j \neq i}^{M} \phi_1 \frac{\partial w_j^*}{\partial h_t} + \sum_{j \neq i}^{M} \phi_1 \delta \frac{\partial w_j^*}{\partial E_t} \right\}$$
$$- \beta\phi_1 f_{h_t}(w_{it}, E_t, h_t) - \beta\phi_1\delta f_{E_t}(w_{it}, E_t, h_t) \tag{19}$$

Rolling equation (19) forward one period and continuing to substitute for terms in $t + 1$ gives:

$$f_{w_{it}} = \beta^s f_{w_{it}} + \sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \frac{\partial w_j^*}{\partial h_{t+\ell}} + \sum_{\ell=1}^{s} \beta^\ell \sum_{j \neq i}^{M} \phi_1 \delta \frac{\partial w_j^*}{\partial E_{t+\ell}}$$

$$-\sum_{\ell=1}^{s}\beta^{\ell}\phi_1 f_{h_{t+\ell}} - \sum_{\ell=1}^{s}\beta^{\ell}\phi_1\delta f_{E_{t+\ell}} \tag{20}$$

which as s goes to infinity becomes

$$f_{w_{it}} = \sum_{\ell=1}^{s}\beta^{\ell}\sum_{j\neq i}^{M}\phi_1\frac{\partial w_j^*}{\partial h_{t+\ell}} + \sum_{\ell=1}^{s}\beta^{\ell}\sum_{j\neq i}^{M}\phi_1\delta\frac{\partial w_j^*}{\partial E_{t+\ell}} - \sum_{\ell=1}^{s}\beta^{\ell}\phi_1 f_{h_{t+\ell}} - \sum_{\ell=1}^{s}\beta^{\ell}\phi_1\delta f_{E_{t+\ell}} \tag{21}$$

Appendix 6. Derivation of Proposition 2: Over-Extraction Under Non-cooperative Extraction

Applying the implicit function theorem to (16) gives

$\frac{\partial w_t^*}{\partial h_t} = -\frac{f_{wh}}{f_{ww}} = -\frac{(-)}{(-)} < 0$ and $\frac{\partial w_t^*}{\partial E_t} = -\frac{f_{wE}}{f_{ww}} = -\frac{(-)}{(-)} < 0$.
Therefore the term, $\phi_1 \sum_{j\neq i}^{M}\left(\frac{\partial w_j^*}{\partial h_t} + \frac{\partial w_j^*}{\partial E_t}\right)$, is negative.

Given the same depth-to-water and the same salinity level, the right-hand side of (5) is lower than that of (3). Consequently, $w_{it\text{-}1} > w_{t\text{-}1}$ since $f_{ww} < 0$.

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