Chapter 36
Nonclassical Bending Behavior of Thin Strips of Photochromic Liquid Crystal Elastomers Under Light Illuminations

Yang Zhang & Yongzhong Huo

Abstract Photochromic liquid crystal elastomers (LCEs) bend when irradiated by light of suitable wavelength. However, due to the rotation of the liquid crystal director, rather large shear strains are inevitably produced and some basic assumptions of the classical simple beam theory of Euler-Bernoulli fail to be satisfied. In this work, we use the first-order shear deformation beam theory of Timoshenko to model the unusual quasi-soft bending behavior of soft LCEs under light illuminations. The results show that in addition to the large shear strain, the effect of initial effective length ratio makes a great difference to the deflections due to the rotation of director. This represents the first direct verification that Euler-Bernoulli beam theory fails to deal with such nonclassical bending of soft LCEs, while Timoshenko beam model can work sufficiently well, which also gives a possible method to measure the effective opto bending moment experimentally.

Keywords: Liquid crystal elastomer · Photochromic materials · Beam theory

36.1 Introduction

Photochromic LCEs doped with rod-like groups, such as azobenzenes, which undergo trans-cis isomerization on absorption of UV photons, are found to contract when irradiated at suitable wavelengths since the local order is disrupted by the kinked dopant groups (Warner and Terentjev, 2007; Finkelmann et al, 2001). Since light is absorbed by the material (Corbett and Warner, 2006, 2008), the reduction in intensity through the thickness of a cantilever gives a gradient of response and then

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non-uniform strains lead to the bending of a cantilever of the active material as in Ikeda et al (2003); Camacholopez et al (2004); Yu et al (2003); Jin et al (2006).

The light induced bending behavior of LCEs with a beam model based on simple bending assumptions has been studied by several authors (Jin et al, 2006; Warner and Mahadevan, 2004; van Oosten et al, 2007; Jin et al, 2010a,b, 2011; Dunn, 2007; Dunn and Maute, 2009; He, 2007; Modes et al, 2011; Warner et al, 2010; Warner and Corbett, 2010; Zeng et al, 2010). Besides, due to the unusual soft or semi-soft behavior of LCEs, the rotation of the LC director can have a strong effect on the mechanical response of the materials as shown in Warner and Terentjev (2007); Jin et al (2010b, 2011). Lin et al (2012) proposed the constitutive equation including the effect of the photo isomerization, and found that the opto-mechanical behaviors are also affected by the soft behavior . Large shear strains occur in the quasi-soft bending due to the anisotropy and its very special mechanical properties (soft elasticity) of LCEs. However, on this occasion, straight lines normal to the mid-plane of LCE beams before deformation won't remain normal to the mid-plane after deformation, which finally leads the classical Euler-Bernoulli beam assumption to fail as discussed in Lin et al (2012).

The first-order shear deformation beam theory of Timoshenko allows for the effect of transverse shear deformation which is neglected in the Euler-Bernoulli beam theory. In the first-order shear deformation theory, the transverse shear strain is assumed to be constant with respect to the thickness coordinate, so shear correction factors are introduced to correct for the discrepancy between the actual transverse shear force distributions and those computed using the relations of the TBT in Timoshenko (1921); Reddy et al (1997).

In this paper, the Timoshenko beam model for quasi-soft bending of photochromic LCEs under light illuminations is presented. Based on the assumption of the form of the displacement and the stress field, the governing equations and the general solutions of rotations and deflections of beams are obtained. The finite element results are compare with the theoretical results of TBT model for various external loads. A numerical method is used to evaluate shear correction factor introduced in TBT.

36.2 TBT Model for Optical-mechanical Bending of Beam Shaped Specimens

36.2.1 Optical-mechanical Constitutive Relations

As shown in Fig. 36.1, we consider a uniform LCE beam with length L, thickness h and width w. The director of the sample is parallel with x direction, i.e. $\mathbf{n}_0 = (1,0,0)^{\mathrm{T}}$ and it is illuminated upward by unpolarized light along the y direction from the bottom. Here a linearized opto-mechanical constitutive relation of soft LCEs for infinitesimal deformations obtained by one of the authors, see Lin et al (2012), is applied to study the light induced bending behavior of photochromic LCEs. As $\mathbf{n}_0 = (1, 0, 0)^{\mathrm{T}}$, the components of Cauchy stress take the form

$$
\begin{aligned}
\sigma_{xx} &= \frac{E}{1+\nu}\left(\varepsilon_{xx} - \varepsilon_{xx}^{r}\right) - p, \\
\sigma_{yy} &= \frac{E'}{1+\nu'}\left[\left(\varepsilon_{yy} - \varepsilon_{yy}^{r}\right) + \frac{\nu - \nu'}{(1+\nu)(1-\nu')}\left(\varepsilon_{xx} - \varepsilon_{xx}^{r}\right)\right] - p, \\
\sigma_{zz} &= \frac{E'}{1+\nu'}\left[\left(\varepsilon_{zz} - \varepsilon_{zz}^{r}\right) + \frac{\nu - \nu'}{(1+\nu)(1-\nu')}\left(\varepsilon_{xx} - \varepsilon_{xx}^{r}\right)\right] - p, \\
\sigma_{xy} &= 2G'\left(\varepsilon_{xy} - \frac{r_0 - 1}{r_0 + 1}\omega_{xy}\right), \\
\sigma_{xz} &= 2G'\left(\varepsilon_{xz} - \frac{r_0 - 1}{r_0 + 1}\omega_{xz}\right), \\
\sigma_{yz} &= 2G\varepsilon_{yz},
\end{aligned}
\tag{36.1}
$$

where ε_{ij} are the Cauchy strains, ω_{ij} are the antisymmetric parts of the displacement gradient, ε_{ij}^{r} are the light-induced strains and p is the lagrangian multiplier introduced due to the incompressibility. The elastic constants and the light-induced strains are given by

$$
\begin{aligned}
\beta = \frac{r}{r_0},\ E = \mu\beta^{\frac{1}{3}}\frac{2+\beta}{\beta},\ E' = 2\mu\beta^{\frac{1}{3}}\frac{2+\beta}{1+\beta},\ \nu' = \frac{1}{1+\beta},\ \nu = \frac{1}{2}, \\
G = \frac{E'}{2(1+\nu')},\ G' = \frac{1}{2}\frac{r_0+1}{r_0-1}\left(\frac{1}{\beta} - 1\right), \\
\varepsilon_{xx}^{r} = -\frac{1-\beta}{2+\beta},\ \varepsilon_{yy}^{r} = \varepsilon_{zz}^{r} = -\nu\varepsilon_{xx}^{r},
\end{aligned}
\tag{36.2}
$$

where μ is the effective shear moduli, r and r_0 denote the anisotropy of the shape distribution of nematic network in the current configuration and in the reference configuration, respectively. Under light illuminations, the anisotropy denoted by r

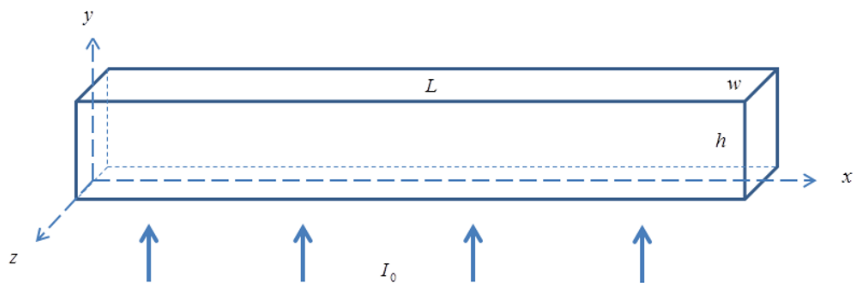

Fig. 36.1 The schematic of the beam shaped specimen under upward unpolarized light.

decreases due to the photo isomerization and decays with the penetration depth of light. Here we assume that the change is very small, that is, $(r_0 - r)/r_0 \ll 1$. Thus β is less than but approximately equal to 1. The parameters used for calculations in the following part are the same as those in Lin et al (2012). Note that for this unusual constitutive relation in Eq. (36.1), the first terms of three normal stresses represent the deviatoric part of stress tensor and p is the spherical part, which can be determined by using the incompressibility constraint. More details on the derivation of Eq. (36.1) has been given in Lin et al (2012).

If there is no light illumination, which means $r = r_0$, the light induced strain ε^r_{xx} is zero and the soft material behaves as an isotropic and incompressible Hookean material, except for its vanished in-plane shear moduli G', which is often referred as the soft behavior as predicted by the neo-classical elastic energy. However, the material behavior is rather different under light illumination. Due to $\beta \neq 1$, two elastic moduli and two Poisson's ratio arise in constitutive relation of Eq. (36.1), and single domain LCEs become transverse isotropy in the plane perpendicular to the director. In addition, the light induced change of the effective length ratio $r_0 - r$ will produce nonzero shear moduli G' and light induced strain ε_r.

Besides, the Young's moduli and light induced strain depend on the effective length ratio , which are affected by the light illumination conditions (incident light intensity i_0 and light decay distance d). Since light is absorbed by the material, the reduction in light intensity through the thickness of a cantilever gives a gradient of response and non-uniform strains lead to the bending of a cantilever of the active material. Therefore, the LCE material under light illumination becomes a functional gradient material. Moreover, the light induced decrease of the effective length ratio r implies the light induced anisotropy.

As observed in experiments and discussed in several theoretical works, the elastic moduli of single domain LCEs are anisotropic and depend strongly on the temperature. However, it is necessary to take into consideration that the stress induces a biaxiality of the liquid crystal molecules in order to obtain this anisotropy. In the present paper, the biaxiality is neglected for simplicity. Thus, the elastic moduli are taken as isotropic under mechanical loading and the anisotropy is induced by the light illumination.

36.2.2 Timoshenko Beam Model

Beam theories are developed by assuming the form of the displacement or stress field as a linear combination of unknown functions and the thickness coordinate. In Timoshenko beam theory (TBT), for stress components, we have basic assumption

$$\sigma_{yy} = \sigma_{zz} = 0. \tag{36.3}$$

Combining (36.3) and the incompressibility $tr(\varepsilon) = 0$, we get

$$p = \frac{E'}{2(1+\nu')}\varepsilon_{yy} = \frac{E'}{2(1+\nu')}\varepsilon_{zz} = -\frac{E'}{(1+\nu')}\varepsilon_{xx}. \tag{36.4}$$

Hence, we can obtain the following constitutive equations for the bending of LCE beams

$$\sigma_{xx} = E\left(\varepsilon_{xx} - \varepsilon_{xx}^{r}\right),\ \sigma_{xy} = 2G'\left(\varepsilon_{xy} - \frac{r_0 - 1}{r_0 + 1}\omega_{xy}\right). \tag{36.5}$$

The Timoshenko beam theory (TBT) is based on the in-plane displacement field at $z = 0$

$$u(x,y) = u_0(x) + (y - \bar{y})\,\phi(x),\ v(x,y) = v_0(x) + \tilde{v}(y), \tag{36.6}$$

where $u_0(x)$ and $v_0(x)$ is the displacements of the point $(x, \bar{y})$ on plane $z = 0$, $\phi(x)$ denotes the rotation of straight lines normal to the mid-plane about z axes and $\tilde{v}(y)$ denotes the difference of displacements between the two points (x, y) and $(x, \bar{y})$. In view of the displacement field given in Eq. (36.6), the in-plane strains and rotation components are given by

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \phi'(x)(y - \bar{y}) = \varepsilon_{xx}^{0} + \phi'(x)(y - \bar{y}),\\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} = \frac{\partial \tilde{v}}{\partial y},\\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\phi(x) + v_0'(x)\right),\\
\omega_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\phi(x) - v_0'(x)\right).
\end{aligned} \tag{36.7}$$

Insert expressions of Eq. (36.7) into constitutive equations Eq. (36.5) and we can express the bending moment M_{xx} and shear force Q_x in terms of $v_0(x)$ and $\phi(x)$

$$\begin{aligned}
M_{xx} &= \iint_A \sigma_{xx}(y - \bar{y})\,\mathrm{d}A = \iint_A E\left(\varepsilon_{xx}^{0} + \phi'(x)(y - \bar{y}) - \varepsilon_{xx}^{r}\right)(y - \bar{y})\,\mathrm{d}A\\
&= D\phi'(x) + M_{\mathrm{eff}},\\
Q_x &= \iint_A \sigma_{xy}\mathrm{d}A = \iint_A \frac{2G_1}{1 + r_0}\left(\phi + r_0 v_0'\right)\mathrm{d}A = K_{\mathrm{s}} A_{xy}\left(\phi + r_0 v_0'\right),
\end{aligned} \tag{36.8}$$

where

$$\begin{aligned}
D &= \iint_A E(y - \bar{y})^2\mathrm{d}A,\\
M_{\mathrm{eff}} &= -\iint_A E\varepsilon_{xx}^{r}(y - \bar{y})\,\mathrm{d}A,\\
A_{xy} &= \iint_A \frac{2G'}{1 + r_0}\mathrm{d}A.
\end{aligned} \tag{36.9}$$

and K_s is the shear correction factor that has been introduced to compensate for the error caused by assuming a constant shear stress distribution through the beam depth. The shear correction factor depends not only on the material and geometric parameters but also on the loading and boundary conditions. Here,

$$\bar{y} = \frac{\iint_A Ey\mathrm{d}A}{\iint_A E\mathrm{d}A} \approx \frac{h}{2}. \tag{36.10}$$

From Eq. (36.8), we can write the relations of generalized displacement field and externally applied loads as

$$\phi'(x) = \frac{M_{xx} - M_{\mathrm{eff}}}{D},\ v_0'(x) = -\frac{1}{r_0}\phi(x) + \frac{Q_x}{r_0 K_s A_{xy}}. \tag{36.11}$$

Then institute Eq. (36.11) into the balance equations of moments and forces

$$\frac{\mathrm{d}M_{xx}}{\mathrm{d}x} = Q_x,\ -\frac{\mathrm{d}Q_x}{\mathrm{d}x} = q, \tag{36.12}$$

we can get the governing equation of deflections

$$v_0''(x) = \frac{M_{\mathrm{eff}} - M_{xx}}{r_0 D} + \frac{q}{K_s A_{xy} r_0}. \tag{36.13}$$

Note that without considering the last term in Eq. (36.13), the solutions of Eq. (36.13) are reduced to Euler-Bernoulli beam theory when $r_0 = 1$. In other words, there exist large discrepancies between the two theories EBT and TBT in the bending of soft LCE beams.

Furthermore, the solutions for the Timoshenko beam under the light actuations and external distributing loads q may be readily obtained by integrating the fourth-order differential equation and using two boundary conditions from at each end of the beam to evaluate the integration constants. By integrating Eqs. (36.12) and (36.11) with respect to x field, we can express general solutions of the bending moments, shear forces, rotations and deflections of beams as

$$\begin{aligned}
M_{xx} &= M_{xx}^L - Q_x^L(L-x) - \int_x^L \int_\xi^L q\mathrm{d}\eta\mathrm{d}\xi, \\
Q_x &= Q_x^L + \int_x^L q\mathrm{d}\xi, \\
\phi(x) &= \phi^0 + \int_0^x \frac{1}{D}(M_{xx} - M_{\mathrm{eff}})\mathrm{d}\xi, \\
v_0(x) &= v_0^0 - \frac{\phi^0}{r_0}x + \int_0^x \frac{1}{r_0}\int_0^\xi \frac{1}{D}(M_{\mathrm{eff}} - M_{xx})\mathrm{d}\eta\mathrm{d}\xi + \int_0^x \frac{Q_x}{r_0 K_s A_{xy}}\mathrm{d}\xi.
\end{aligned} \tag{36.14}$$

where v_0^0, ϕ^0, M_{xx}^L, Q_x^L are constants of integration. These constants are to be determined by using the boundary conditions of the particular beam.

For free (F), simply supported (S) and clamped (C) ends, boundary conditions are given by

$$F:\ Q_x = M_{xx} = 0,\ S:\ v_0 = M_{xx} = 0,\ C:\ \phi = v_0 = 0. \tag{36.15}$$

36.3 Examples of Cantilever Beams and Numerical Results

The most important class of problems involves cantilever beams, which are usually tested experimentally in mechanics. Here, we take cantilever beams for examples and use three simple cases to demonstrate our theoretical model. The first case with $q = 0$ shows only the effect of light illuminations, and the other two with a point load $q = f\delta(x - L)$ and uniformly distributed loads $q = q_0$ represent the coupled effect of optical and mechanical loads.

In all the three cases, the cantilever beams is clamped at $x = 0$ and is free at $x = L$. Thus according to Eqs. (36.15), the boundary conditions are set by $v_0^0 = \phi^0 = 0$ and $M_{xx}^L = Q_x^L = 0$. Substitue the conditions into Eqs. (36.14) and we obtain the solutions to cantilever beams as

$$\begin{aligned}\phi(x) &= \int_0^x \frac{1}{D}(M_{xx} - M_{\mathrm{eff}})\mathrm{d}\xi,\\ v_0(x) &= \int_0^x \frac{1}{r_0}\int_0^\xi \frac{1}{D}(M_{\mathrm{eff}} - M_{xx})\mathrm{d}\eta\mathrm{d}\xi + \int_0^x \frac{Q_x}{r_0 K_{\mathrm{s}} A_{xy}}\mathrm{d}\xi.\end{aligned} \tag{36.16}$$

For brevity, we assume that beams have the uniform anisotropy at the initial state, i.e. r_0 is a constant, and we only consider uniform cantilever beams under uniform light illuminations, so D and M_{eff} are independent of x coordinate with the form $D = D_0$ and $M_{\mathrm{eff}} = M_{\mathrm{eff}}^0$. Besides, notice that the shear correction factor should be taken into account if nonzero shear forces are present.

Besides, the finite element method proposed by Lin et al (2012) is used to model the deformation of the specimen under light illuminations and finite element results are used to compare with the theoretical results of TBT model. To investigate whether straight lines normal to the mid-plane of LCE beams will remain straight after deformation or not, the displacements in the axial direction vs. y coordinates are plotted in Fig. 36.2. It shows the displacement u changes linearly with the y coordinate and the plane cross-section assumption is still valid.

36.3.1 First Case: no Load

In this case, beams are only driven by optical loads, i.e. $q = Q_x = M_{xx} = 0$ and thus solutions read

$$\phi(x) = -\frac{M_{\mathrm{eff}}^0}{D_0}x,\ v_0 = \frac{M_{\mathrm{eff}}^0}{2r_0 D_0}x^2. \tag{36.17}$$

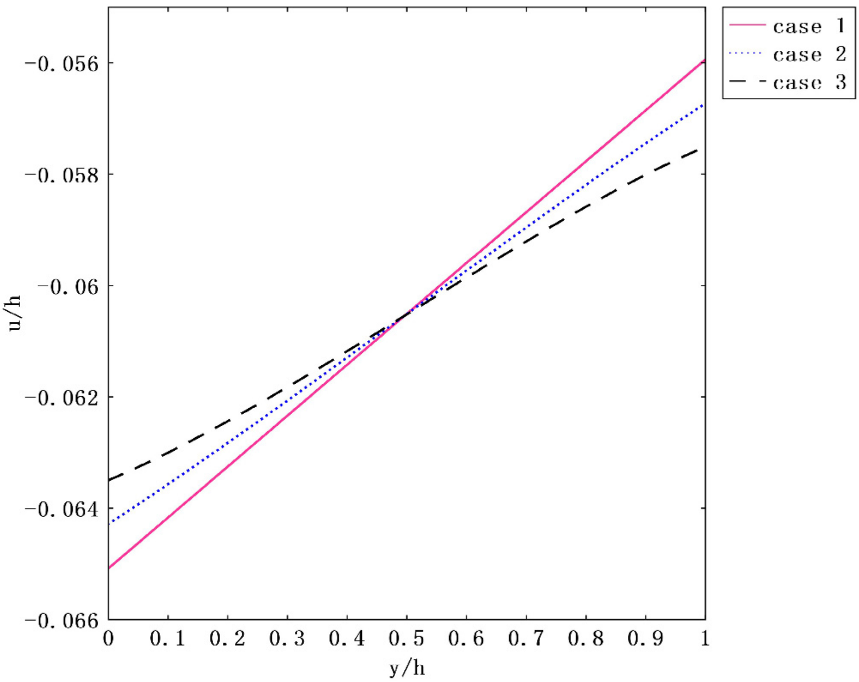

Fig. 36.2 Displacements $u$of cross section along y axis for three cases. $(r_0 = 3,\ i_0 = 2,\ d/h = 1)$

Obviously, the maximum deflection occurs at $x = L$

$$v_0^{\max} = \frac{M_{\text{eff}}^0}{2r_0 D_0} L^2. \tag{36.18}$$

In an experiment, we can obtain the effective optical bending moment through measuring the maximum deflection of cantilever beams.

The following expression can well describe the relation of the solution of EBT and the solution of TBT in this case

$$v_0^L = \frac{1}{r_0} v_0^E, \tag{36.19}$$

where the superscript "T" and the superscript "E" respectively denotes the quantity in TBT and the quantity in EBT. It's obvious that the solutions of TBT is reduced to Euler-Bernoulli beam solutions if $r_0 = 1$. For anisotropy LCEs, it holds $r_0 > 1$, which implies that the effect of r_0 finally leads the classical EBT to fail. Figures 36.3 and 36.4 indicate that theoretical results of TBT model agree well with the finite element results for different initial anisotropy r_0 and different dimensionless incident light intensites i_0.

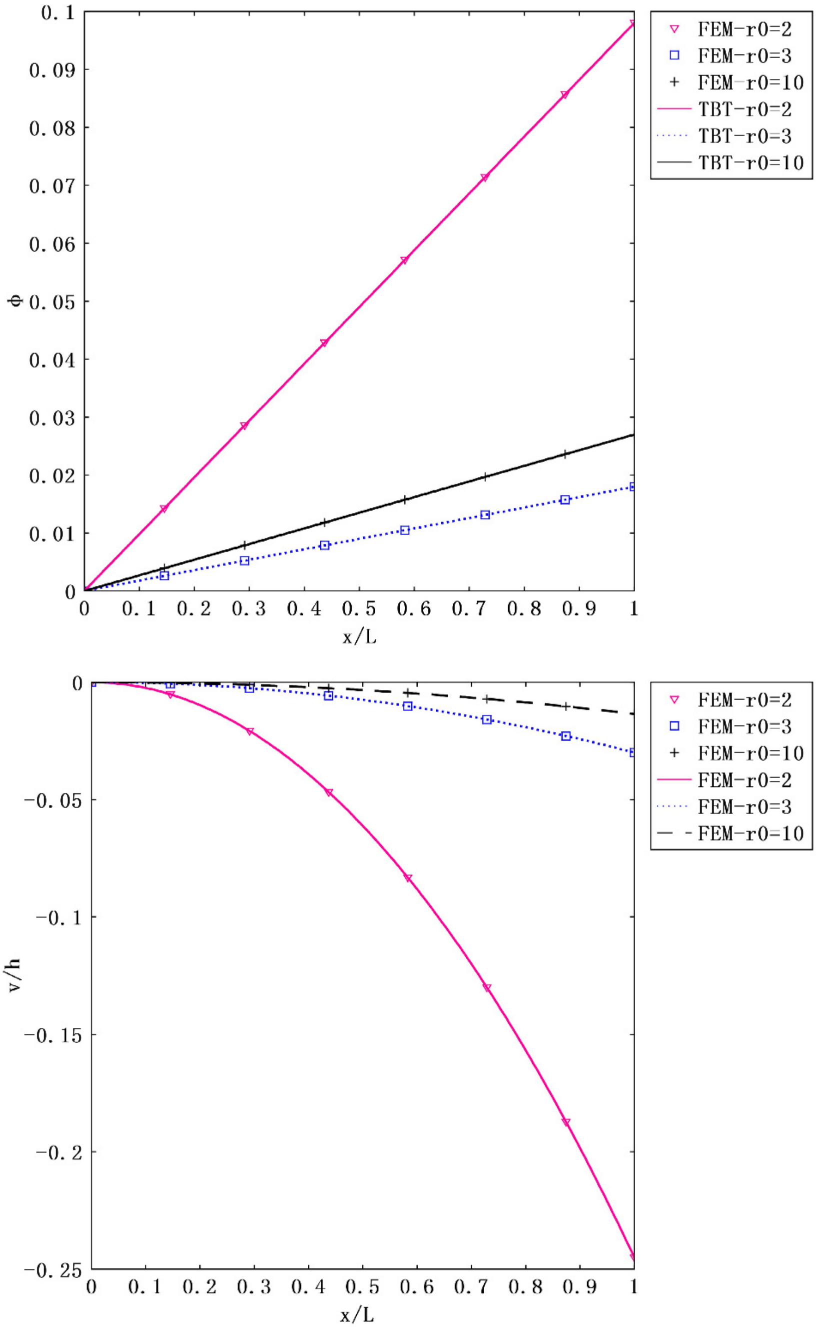

Fig. 36.3 (a) Rotation curves and (b) deflection curves for different r_0. ($i_0 = 2,\ d/h = 1$)

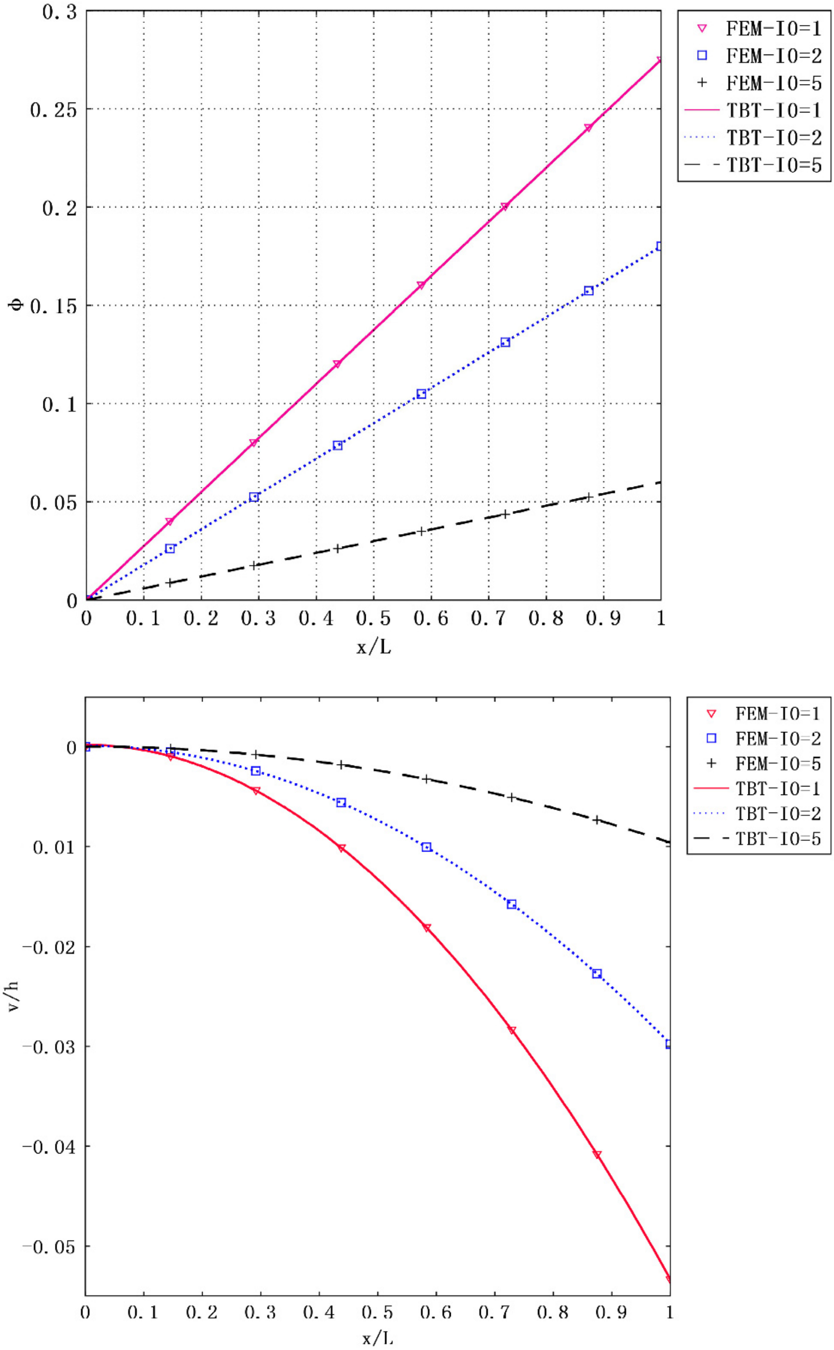

Fig. 36.4 Rotation curves and deflection curves for different i_0. $(i_0 = 3,\ d/h = 1)$

36.3.2 Second Case: a Point Load

In this case, we consider a cantilever beam with a concentrated load f applied at the free end. The solutions are given by

$$\begin{aligned}\phi\left(x\right)&=-\frac{M_{\mathrm{eff}}^{0}}{D_0}x-\frac{f}{D_0}x\left(L-\frac{x}{2}\right),\\ v_0\left(x\right)&=\frac{M_{\mathrm{eff}}^{0}}{2r_0D_0}x^2+\frac{f}{2r_0D_0}x^2\left(L-\frac{x}{3}\right)+\frac{f}{r_0K_{\mathrm{s}}A_{xy}}x.\end{aligned}\tag{36.20}$$

Notice that shear correction factor K_{s} is introduced in the expression of deflections due to nonzero shear forces, which however, does not arise in the expression of rotations. Figure 36.5 indicates that the solutions of rotations in TBT model agree well with the finite element results in this case.

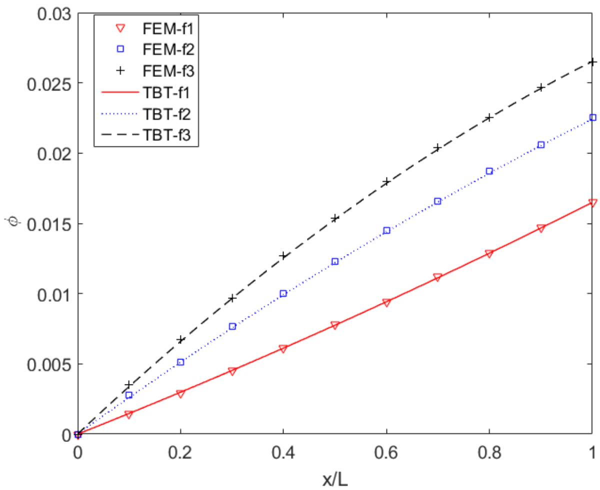

Fig. 36.5 Rotation curves for different concentrated loads. $(r_0=3,\ i_0=2,\ d/h=1)$

Here, a numerical method is used to evaluate the shear correction factor. For beams with different length, we can obtain the forces f, which satisfy $v_0\left(L\right)=0$. From the expression of Eq. (36.20), we have the following expression if $v_0\left(L\right)=0$ holds

$$\underbrace{-\frac{M_{\mathrm{eff}}^{0}}{2fL}}_{Y}=\frac{1}{3}+\frac{1}{K_{\mathrm{s}}}\underbrace{\left(\frac{D_0}{A_{xy}h^2}\right)\left(\frac{h}{L}\right)^2}_{X}.\tag{36.21}$$

So in FEM, we can find out a unique force f that can make the free end satisfy $v_0\left(L\right)=0$ and the obtained points for different length of beams (X,Y) are plotted in Figure 36.6. It shows that the shear correction factor is independent of length of

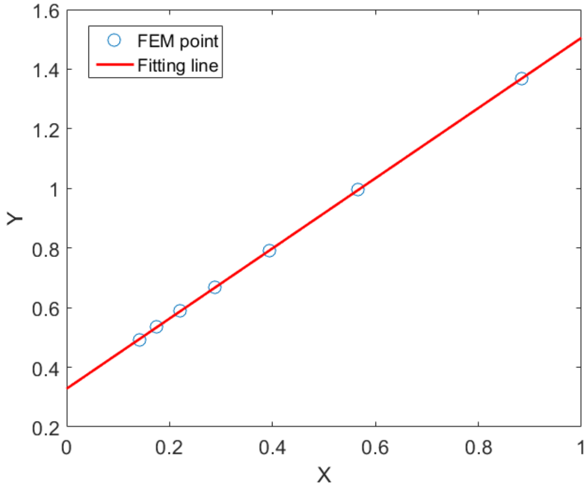

Fig. 36.6 Points of FE resutls are fitted by a line using the linear least square method. $(r_0 = 3,\ i_0 = 2,\ d/h = 1)$

beams and loads. And the points are fitted with the line $y = 0.328 + 1.177x$ by linear least square method, which implies that $K_\mathrm{s} = 1.177^{-1} = 0.850$. Figure 36.7 indicates that the theoretical results of deflections fit well with the finite element results in this case when $K_\mathrm{s} = 0.850$.

36.3.3 Third Case: Uniformly Distributed Load

The solutions of cantilever beams with uniformly distributed load $q = q_0$ are given by

$$\begin{aligned} \phi\left(x\right) =& -\frac{M^0_\mathrm{eff}}{D_0}x + \frac{q_0}{2D_0}\left(-L^2x + Lx^2 - \frac{x^3}{3}\right), \\ v_0\left(x\right) =& \frac{M^0_\mathrm{eff}}{2r_0D_0}x^2 + \frac{q_0}{2r_0D_0}x^2\left(\frac{1}{2}L^2 - \frac{1}{3}Lx + \frac{1}{12}x^2\right) \\ &+ \frac{q_0}{r_0K_\mathrm{s}A_{xy}}x\left(L - \frac{x}{2}\right). \end{aligned} \tag{36.22}$$

Figure 36.8 shows that numerical comparisons of both rotations and deflections between the theory and finite element results show good agreement when the shear correction factor $K_\mathrm{s} = 0.850$ has been taken into consideration.

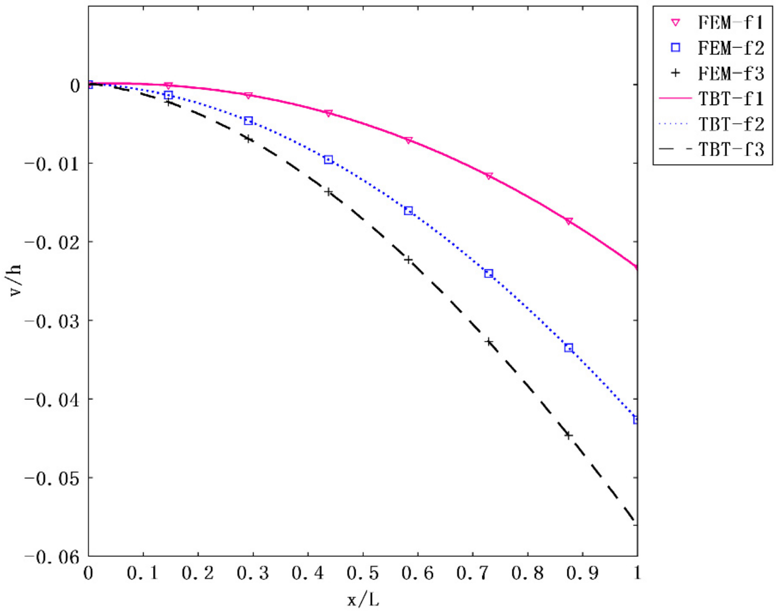

Fig. 36.7 Deflection curves for different concentrated loads. $(r_0 = 3,\ i_0 = 2,\ d/h = 1)$

36.4 Discussion About Shear Correction Factor

One of the main difficulties in using Timoshenko beam theory is the proper selection of the shear correction factor, since in TBT the shear correction factor is introduced to allow for the fact that the shear stress is not uniform over the cross section. In history, many authors have published definitions of the shear correction factor and have proposed various methods to calculate it. Most of these approaches fall into one of two categories. The first approach is to use the shear correction factor to match the frequencies of vibration of various beam constructions with exact solutions to the theory of elasticity. The second approach is to use the shear correction factor to account for the difference between the average shear or shear strain and the actual shear or shear strain using exact solutions to the theory of elasticity. At the present stage of theories and experiments, Timoshenko's expression in Timoshenko (1921) and Cowper's one in Cowper (1966) will be the most probable ones. Although not explicitly written in Timoshenko (1921), the shear correction factor obtained in the first manner for a rectangular beam is

$$K_s = \frac{(5+5\nu)}{(6+5\nu)}, \tag{36.23}$$

where ν is the Poisson's ratio.

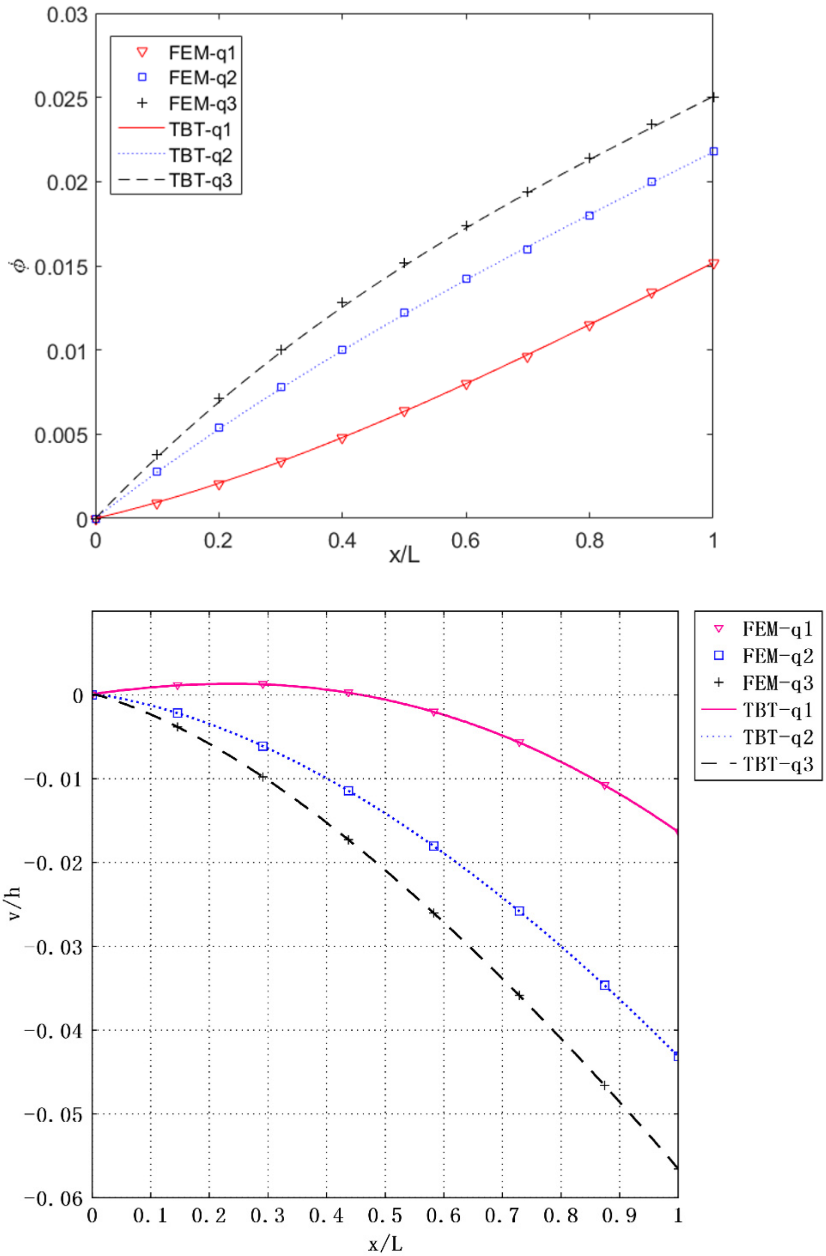

Fig. 36.8 (a) Rotation and (b) deflection curves for various uniformly distributed loads. $(r_0 = 3,\ i_0 = 2,\ d/h = 1)$

Cowper (1966) calculated the shear correction factor using an approach from the second category described above. For a rectangular isotropic homogeneous beam, Cowper found the following shear correction factor:

$$K_{\rm s} = \frac{10\,(1+\nu)}{12+11\nu}. \tag{36.24}$$

With regard to our incompressible materials of LCEs, the Poisson's ratio is close to 0.5, as indicated in Eq. (36.2). Hence, according to the Timoshenko's expression and Cowper's, the shear correction factor for the rectangle is respectively 0.882 and 0.857. Our numerical results indicate that of the shear correction factor is about 0.850 very close to Cowper's formula Eq. (36.24).

36.5 Conclusions

Photochromic LCE is a currently developed smart material, which can contract and bend under suitable light illuminations. However, due to the unusual soft or semi-soft behavior of LCEs, the rotation of the LC director can have strong effect on the mechanical response of the materials. Large shear strains occur in the quasi-soft bending due to its very special mechanical properties (soft elasticity) of LCEs, which finally lead the classical Euler-Bernoulli beam assumption to a failure even for slender strips.

In this paper, the first-order shear deformation beam theory model of Timoshenko for quasi-soft bending of photochromic LCEs under light illuminations has been presented, which allows for the effect of transverse shear deformation. General solutions of the bending moments, shear forces, rotation and deflections of beams subjected to optical and mechanical loads are given, and the solutions show much difference between EBT and TBT. The effect of r_0 arises due to the free rotation of director of LCEs.

In TBT, the shear correction factor has to been taken into consideration due to the assumption of a constant shear stress distribution through the beam depth. The shear correction factor evaluated in the numerical method is 0.850, which shows good agreement with the value predicted by Cowper's formula. Numerical results indicate that TBT model we presented fits very well with finite element results for different geometric parameters and different loading and boundary conditions of beams.

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References

Camacholopez M, Finkelmann H, Palffymuhoray P, Shelley M (2004) Fast liquid-crystal elastomer swims into the dark. Nature Materials 3(5):307–10

Corbett D, Warner M (2006) Nonlinear photoresponse of disordered elastomers. Physical Review Letters 96(23):237,802

Corbett D, Warner M (2008) Polarization dependence of optically driven polydomain elastomer mechanics. Physical Review E Statistical Nonlinear & Soft Matter Physics 78(6 Pt 1):061,701

Cowper GR (1966) The shear coefficients in Timoshenko's beam theory. Journal of Applied Mechanics 33(2):335–340

Dunn ML (2007) Photomechanics of mono- and polydomain liquid crystal elastomer films. Journal of Applied Physics 102(1):307

Dunn ML, Maute K (2009) Photomechanics of blanket and patterned liquid crystal elastomer films. Mechanics of Materials 41(10):1083–1089

Finkelmann H, Nishikawa E, Pereira GG, Warner M (2001) A new opto-mechanical effect in solids. Physical Review Letters 87(1):015,501

He LH (2007) Surface deformation of nematic elastomers under striped illumination. Physical Review E Statistical Nonlinear & Soft Matter Physics 75(1):041,702

Ikeda T, Nakano M, Yu Y, Tsutsumi O, Kanazawa A (2003) Anisotropic bending and unbending behavior of azobenzene liquid-crystalline gels by light exposure. Advanced Materials 15(3):201–205

Jin L, Xin J, Huo Y (2006) Light-induced nonhomogeneity and gradient bending in photochromic liquid crystal elastomers. Science in China 49(5):553–563

Jin L, Yan Y, Huo Y (2010a) A gradient model of light-induced bending in photochromic liquid crystal elastomer and its nonlinear behaviors. International Journal of Non-Linear Mechanics 45(4):370–381

Jin L, Zeng Z, Huo Y (2010b) Thermomechanical modeling of the thermo-order–mechanical coupling behaviors in liquid crystal elastomers. Journal of the Mechanics & Physics of Solids 58(11):1907–1927

Jin L, Lin Y, Huo Y (2011) A large deflection light-induced bending model for liquid crystal elastomers under uniform or non-uniform illumination. International Journal of Solids & Structures 48(22):3232–3242

Lin Y, Jin L, Huo Y (2012) Quasi-soft opto-mechanical behavior of photochromic liquid crystal elastomer: Linearized stress–strain relations and finite element simulations. International Journal of Solids & Structures 49(18):2668–2680

Modes CD, Bhattacharya K, Warner M (2011) Gaussian curvature from flat elastica sheets. Proceedings Mathematical Physical & Engineering Sciences 467(2128):1121–1140

van Oosten CL, Harris KD, Bastiaansen CW, Broer DJ (2007) Glassy photomechanical liquid-crystal network actuators for microscale devices. Eur Phys J E Soft Matter 23(3):329–336

Reddy JN, Wang CM, Lee KH (1997) Relationships between bending solutions of classical and shear deformation beam theories. International Journal of Solids & Structures 34(26):3373–3384

Timoshenko SP (1921) On the correction for shear of the differential equation for transverse vibrations of prismatic bars. Philosophical Magazine 41(245):744–46

Warner M, Corbett D (2010) Suppression of curvature in nematic elastica. Proceedings of the Royal Society A 466(2122):3561–3578

Warner M, Mahadevan L (2004) Photoinduced deformations of beams, plates, and films. Physical Review Letters 92(13):134,302

Warner M, Terentjev EM (2007) Liquid Crystal Elastomers. Oxford University Press,

Warner M, Modes CD, Corbett D (2010) Curvature in nematic elastica responding to light and heat. Proceedings Mathematical Physical & Engineering Sciences 466(2122):2975–2989

Yu Y, Nakano M, Ikeda T (2003) Directed bending of a polymer film by light. Nature 425(6954):145

Zeng Z, Jin L, Huo Y (2010) Strongly anisotropic elastic moduli of nematic elastomers: Analytical expressions and nonlinear temperature dependence. Eur Phys J E Soft Matter 32(1):71–79