






TLS Survey and FE Modelling of the Vasari's Cupola of the Basilica dell'Umiltà (Italy). An Interdisciplinary Approach for Preservation of CH

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Abstract. This paper presents an interdisciplinary approach for identification and assessment of historic buildings that combines Terrestrial Laser Scanning (TLS) survey and Finite Element (FE) numerical modeling. The structural analysis of an historic building requires the development of an interconnected series of operations aimed at obtaining a satisfactory knowledge of the building, where usually in-situ investigations are performed together with advanced computational analyses. In this process, the geometric and topographic survey plays a pivotal role and therefore the possibility to rapidly acquire large amounts of spatial data (and to geo-reference any kind of information) allows to provide effective geometric and monitoring data that can be subsequently employed for structural analyses. In this respect, the interchange between Geo-informatics and Engineering sciences can be considered a challenging issue in the field of conservation/preservation of cultural heritage (CH). On the one hand, in fact, the accuracy of measured data directly affects decision-making and analysis process. On the other hand, the merging of digital documentation technologies with innovative computational techniques supports the creation of an inter/trans-disciplinary cooperation model towards sustainable preservation of CH. These issues are herein addressed through the discussion of an emblematic case study: the Cupola of the Basilica dell'Umiltà in Pistoia (Italy) designed and realized by Giorgio Vasari in the middle of the sixteenth century.

Keywords: 3D digitization of cultural heritage · Metric survey · Numerical modeling and structural analysis · Dome

1 Introduction

The capacity to rapidly acquire large quantities of spatial data, to geo-reference several type of information, to obtain detailed models that allow even more accurate analyses and simulations, puts Geo-informatics at the center of the attention of many research areas. Among these, particularly interesting is the use of these techniques for the analysis of the cultural heritage (CH). Assess the current stability of a building, identifying its degradation conditions, monitor the evolution over time of a failure,

preventing the potential causes of damage, simulate or reconstruct the behavior of a building under seismic actions, are some of the ways in which the geometric properties of a structure, acquired with the most up-to-date automated surveying systems, are employed to support structural integrity analyses.

From the point of view of the restoration science, a scientific approach that interprets the text of a construction must be based on a systematic observation and measurement of the building. The complexity of the relations between its elements, determined by their position in space, can be systematically approached only by a virtual model that takes into account the reciprocal spatial relations of the collected data. For this reason, well before that this analytic and descriptive approach was rigorously defined, the survey - and its subsequent architectural representation in two and three dimensions - has constituted the main instrument for the study of the CH [1, 2].

Nevertheless, even if the survey boasts a long and solid tradition in the field of restoration, its role has been truly appreciated and understood mainly in recent times. In fact, an examination of the so-called *Carte del Restauro*, reveals that references to the survey are both superficial and fragmentary. The *Carta di Atene* (1931) recommends “precise surveys” only for archeological excavations at sites that will be reburied. The *Carta di Venezia* (1964) discusses “precise documentation in the form of analytical and critical reports, illustrated with drawings and photographs”, without specifying the metrological value of the drawings. The *Carta del Restauro* (1972), finally, indicates the content of the restoration project, stating that “it should be based on a complete graphic and photographic survey to be interpreted metrologically.” It was not until the *Carta del Rilievo Architettonico* of 1999–2000 [3] that was provided a study explicitly dedicated to the survey. This document recommends that the regulatory criteria for a survey should incorporate a project as well as oversight of the operations and testing. The survey should additionally include a report indicating the implemented criteria, objectives and degree of precision, so as to make qualitative evaluation possible. The survey is further defined as “an open system of knowledge” that brings together all relevant data and whose creation involves multiple professional specializations. This document emphasizes the structural survey among the themes for detailed study, pointing to the dual objective of “illustrating the structural model in its overall configuration” and “documenting the geometric characteristics and the materials necessary for the engineer to conduct the required assessments and tests,” in consideration of the need for further experimentation to integrate the survey methods with non-destructive analyses of structures covered by plaster.

The ICOMOS Charter - ISCARSAH Principles (2003) - devotes a section to the structural surveys, beginning with the mapping of visible damage and cracks and of the materials and their state of decay, and moving on to a consideration of geometric and structural irregularities and the relations with the environmental context. Although it highlight the importance of the survey in the restoration project, these documents lack precise specifications, regarding, for example, scale, level of detail, the instruments needed to carry out and verify results. Such shortcomings are probably due to the great variability of situations involved in creating a survey of an architectural structure. Typically, those most familiar with a monument were surveyors armed with tape measures and plumb lines who gauged spaces and noted shapes and dimensions in accordance with their training in the history of architecture; yet the badges of land

surveyor or “measurer” have not generally been happily worn by architects or engineers, who consider themselves to be in possession of a different set of skills. Measurements were essentially limited to distances, related to planes whose spatial positioning was not simple to realize. The traditional survey, therefore, was often unsatisfactory above all for measuring buildings with complex floor plans or spaces of great height [4].

For these reasons, researchers have for decades been experimenting with the use of technical instruments which have already been tested for land surveying and cartographical production, such as topography and photogrammetry. In the last decades, with the advances in computer science and the transformation to digital platforms, it have been seen a dramatic change in documentation systems. Terrestrial Laser Scanning (TLD) and Structure from Motion (SfM) digital photogrammetry have for several years now allowed to survey complex geometries at lower costs. If the geospatial market as applied to constructions is considered, attention is no longer directed toward the acquisition itself but rather toward integration and automation of the various technologies:

- Autonomous Technologies - collision avoidance, data visualization, and real-time navigation support are driving commercial adoption of autonomous vehicles in a wide variety of field operations;
- Portable Scanners - handheld scanners, indoor mapping carts, and backpack systems are meeting the needs to extract specific objects and generate 3D models from tight, busy spaces;
- Immersive Data Visualization - moving beyond abstract data sets is enabling professionals to fully manipulate and interact with data in three dimensions;
- All-digital Environments - be it Building Information Modelling or Virtual Design and Construction, AEC firms are rapidly moving to an all-digital design environment to save significant time and money, improve design quality, and increase worker safety;
- Data Processing Automation - organizations are breaking through the bottleneck of overwhelming digital datasets by automating data processing and 3D feature extraction.

All of these techniques aim to define the position of high-density points in space. Their results can be integrated as long as they are expressed in a single reference system defined by a reference network created with a total station or with GPS. Moreover, the permanent materialization of the network and the preservation of all data and metadata allow us to verify the accuracy of the result and its successive integration as well as to monitor the evolution of conditions over time [5–7].

The most interesting aspect of the availability of new digital instruments for reality acquisition - from the point of view of Engineering science - is that they enable to obtain virtual models that are to be understood not only in the architectural or computer-graphic sense as 2- or 3-dimensional images of a building, but from the broader perspective of a conceptual representation that is faithful and objective (apart from measurement uncertainty and inaccuracies in following the adopted procedures, which are in any case verifiable through quality statistical parameters), and thus useful for describing specific phenomena [8–11]. As in the past, proximity to the object, both

during acquisition and data processing, places the surveyor in a better position to identify aspects that are less easily detected from a more comprehensive perspective. Three-dimensional acquisition can also be employed as a diagnostic instrument to identify anomalies and irregularities and investigate the deviation between the real geometry and ideal surfaces used as reference points, to monitor the development of instability or of other phenomena, or to automatically characterize decay [12–14].

In summary, surveyors before the digital era, following a principle of economy, acquired data that was sufficient to produce the two-dimensional representations necessary for defining a conceptual model of three-dimensional reality, a model so abstract as to be fully comprehensible only by specialists. Today, on the other hand, digital transformation, driven by technological innovation, allows surveyors not only to acquire 3D geometric information (which can be reduced to 2D data for operational requirements) but also to create 4D models that also take the temporal dimension into account with continuous or repeated measurements. It is therefore evident that the idea of the survey, understood as a synthesis of an open system of knowledge which is always organized and articulated in a spatial form, today finds application in a 3D digital model that is considered the framework for the geolocation of every type of data.

2 Surveyed Structure: The Basilica dell'Umiltà

What has been reported above, delineates a path of organic information acquisition that has been traced with regard to its principle aspects. It must now be shown that in practice its stages are still often applied in a discontinuous way, in accordance with the respective needs of individual projects. It may therefore prove useful to investigate for which aims and in which ways the techniques of geomatics have been used in projects for structural evaluations of significant historical buildings.

An important example of the great potential residing in the adoption of the techniques of geomatics concerns the survey of the great domes [15, 16]. This has always been a challenging work because it involves broad and tall structures which are difficult to measure with precision by direct means. It is further difficult to measure their thicknesses because the extrados is only visible in cramped crawl spaces: if no direct openings are available, creating a topographic network to connect the internal and external surfaces becomes a daunting task. The Vasari's Cupola of the Basilica of the Madonna dell'Umiltà in Pistoia (Italy) (Fig. 1), in this respect, is a broader case for the application of the techniques of geomatics for the study of domes.

2.1 History

The construction of the Basilica della Madonna dell'Umiltà in Pistoia (Fig. 1) started at end of the fourteenth century, following the original design of Giuliano da Sangallo (1445–1516), and required more than seventy years to be completed. After the death of Giuliano da Sangallo the construction was first continued by the architect Ventura Vitoni from Pistoia (1442–1522), and subsequently by Giorgio Vasari (1511–1574) that worked on the Basilica from 1561 to 1567. Vasari designed the last level of the octagonal tambour, together with the Cupola.



Fig. 1. The Basilica della Madonna dell'Umiltà.

The design of the cupola was clearly inspired to the Brunelleschi's cupola of Santa Maria del Fiore in Florence; in his design Giorgio Vasari reproduced the main key features of the Brunelleschi's dome such as the double masonry shell system, the ribs between the webs and several materials and outer decorative details. Nevertheless, despite such similarities, Vasari did not adopt any of the technical innovation proposed by Brunelleschi in his cupola. Main technical differences can be summarized as follows: (i) the shape of the Cupola of the Basilica della Madonna dell'Umiltà shifted from the gothic to the hemispherical one; (ii) the brick pattern used to build the Cupola was different from that employed by Brunelleschi in the Florentine dome: Vasari laid the brick courses along the parallels disregarding the double curved herringbone courses of the Brunelleschi's cupola; (iii) the connection between the inner and the outer shell was realized with weak joints, disregarding the strong ring and the rib system that held together the inner and outer shell of the Brunelleschi's dome [17, 18].

Due to the three above mentioned constructive drawbacks, and others not discussed here such as the lateral force-carrying capacity of the tambour, a complex and wide-spread crack pattern formed soon after the completion of the dome (even before the installation of the heavy lantern) that evolved during centuries [19]. Vasari himself carried out the first retrofitting works, through the insertion of steel chains inside and outside the Cupola. Additional chains have been inserted over the centuries due to the evolution of the cracking pattern.

Since 2008, the Cupola of the Basilica dell'Umiltà has been interested by a series of studies, experimental investigations and analyses - promoted by *Soprintendenza* (the Institution in charge of conservation of historical buildings) and financed by the *Cassa di Risparmio di Pistoia e Pescia* Foundation - aimed at the conservation of the structural complex. The project consisted of the restoration of the whole Basilica; the restoration of the Cupola started in 2012 and concluded in 2014 with the renovation of the lantern.

2.2 Geomatic Survey

Recent methodological developments in analyzing masonry structures have led to fresh interest in studying the basilica with the aid of geomatics technologies. The survey is not only the basis on which to create structural models, but especially in such sophisticated and complex cases represents the only tool which allows correlation of the overall geometry, the construction techniques, the characterization of the texture of the materials, the position of the tie-rods, the cracks detection and mapping, etc. Combining these data with historical and archival information enables researchers to gain access to the reasoning behind the initial project and to the modifications carried out over the centuries.

In this case as well, the greatest challenges in performing the survey (extended to the entire basilica) regarded:

- The difficulty in carrying out topographical measurements and laser scans both in the tight crawlspace between the two domes and in surveying the exterior, given that the building is quite high and located in a network of narrow roads;
- The need to conduct a detailed survey of small elements, such as decorative features and the position of cracks, in spite of these limitations.

The survey allowed researchers to more clearly understand the morphology of the work, in particular of elements which are not directly visible. Vasari himself, when the first cracks appeared, modified the system of vertical connections and the crawlspace structures of the dome, adding eight ribs in the crawlspace to sustain the weight of the lantern [20, 21].

The survey (Figs. 2 and 3) highlighted and made it possible to quantify:

- The number of ribs, their correct placing, form and dimension;
- The exact placing and dimension of the internal and external tie-rods;
- The actual thickness of the two calottes; the building materials and techniques;
- The consolidation of the interventions; the internal connections, both horizontal and vertical;
- The cracks present on both calottes, highlighting the most damaged awnings;
- The links among the geometry, structure and deformations shown over time.

2.3 Damage Survey

The Cupola of the Basilica dell'Umiltà is affected by a widespread cracking pattern. The cracks initiated at the springing of the Cupola, and propagated towards the crown, cutting the entire thickness of the inner shell. To avoid the collapse of the cupola, the initial design was modified with the insertion of some tie-rods buried into the masonry during the construction of the dome. Vasari himself inserted the first two iron tie-rods at the base of the Cupola: the first in the corridor between the two shells and the second around the outer dome. In so doing, the construction of the dome was completed. Subsequently the lantern, which was designed by Vasari imitating the one of the Brunelleschi's dome, and particularly heavy, was installed. This increased the lower

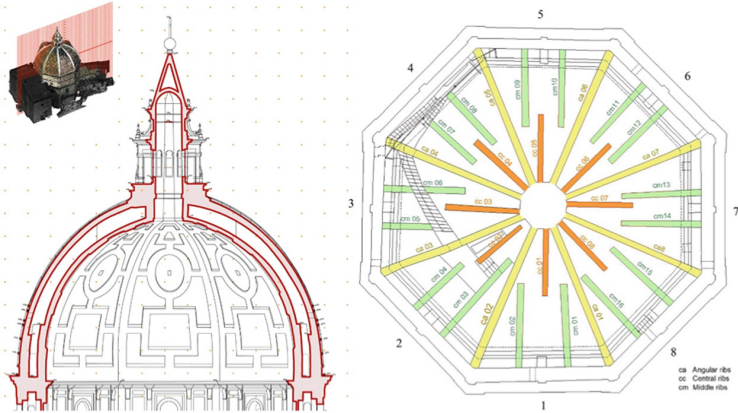


Fig. 2. Madonna dell'Umiltà in Pistoia: cross section of the dome (left) and the ribs between the two calottes (right).

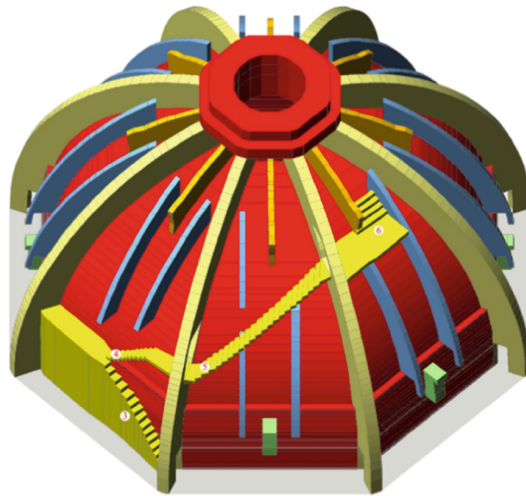


Fig. 3. Madonna dell'Umiltà in Pistoia: 3D model of the space between the two calottes.

thrust of the cupola, and contributed to the enhancement of the cracks on the structure [17]. A third tie-rod was inserted ten years after the completion of the works, around 1585, by Bartolomeo Ammannati (1511–1592) and two more tie-rods were inserted in 1592. Taking into account further interventions during the century, today, eight tie-rods are present on the Vasari's cupola, seven of which disposed at the extrados of the outer masonry shell.

3 Experimental Tests

The experimental campaign included single and double flat-jack tests, and was aimed at assessing the path of internal stresses on the tambour and on the cupola. Even if flat-jack tests are only slightly destructive (since after the test is completed, the flat-jack can usually be easily removed and the mortar layer restored to its original condition), position, number and distribution of tests were selected to minimize obtrusiveness and, at the same time, to obtain the effective structural information needed for the subsequent numerical modeling.

Both single and double flat-jack tests were executed in positions selected in accordance with the *Soprintendenza*. The location of the double flat-jack test takes advantage of previous single one (i.e., after the execution of the single flat-jack test a new flat-jack was inserted and the test was carried out) to reduce the obtrusiveness. Overall four levels (Lev. #1, #3, #4 and #5; Fig. 4) were investigated, alternating the walls in order to obtain a global picture of the state of stress. For the execution of the in situ tests, the following equipment was used: rectangular flat-jack (dimensions: $400 \times 200 \times 4.0$ mm); hydraulic circular saw ($\varnothing 350$ mm and thickness 3.5 mm); hydraulic hand pump [equipped with two manometers, with full scales of 25 and 100 bar (2.5 and 10 MPa)]; and mechanical displacement transducers (with a base length of 200 and 400 mm, with resolution equal to 0.01 mm).

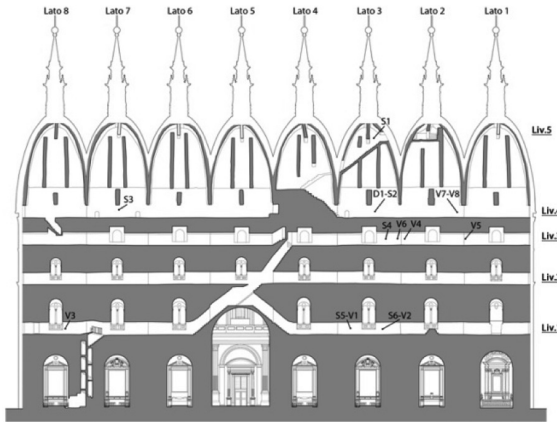


Fig. 4. Position of single (S) and double (D) flat-jack tests. Tests S1, D1s, S3, S4, and S5 were performed on the extrados of the inner tambour/cupola. Tests S2 and S6 were performed on the intrados of the outer tambour.

The measure of pressure applied by the flat-jack, taking into account the corrective factors (Table 1), approximately corresponds to the local pressure in masonry, and

Table 1. Estimated in-place stress and correction coefficients for the single flat-jack tests.

Test#	k_m	k_a	Stress (MPa)
S1	0.743	0.860	0.15
D1s	0.743	0.856	0.15
S2	0.743	0.860	0.18
S3	0.743	0.851	0.15
S4	0.743	0.842	0.28
S5	0.860	0.816	1.28
S6	0.860	0.825	1.28

therefore the average compressive stress in the masonry, σ_m , can be evaluated as follows:

$$\sigma_m = k_m \cdot k_a \cdot p \tag{1}$$

where k_a is the factor accounting for the ratio between the bearing area of the jack in contact with the masonry and the bearing area of the slot; k_m is the factor that accounts for the physical characteristic of the jack; and p is the pressure required to restore the original distance between the gauge points. Position of tests is reported in Fig. 4, while the results, expressed in terms of average compressive stress in the masonry, are summarized in Table 1. It is possible to observe that at the higher levels (Lev. # 3 and #4) the internal stresses are about the same and range between 0.15 MPa (Lev. #4) and 0.28 MPa (Lev. #3), while at Lev. #1 it increases up to 1.28 MPa. The Young's modulus, obtained with the double flat-jack test, was found to be about 800 N/mm².

4 Numerical Modelling

Numerical modeling has been proven to be an effective tool to help the comprehension of the structural behavior of ancient structures, and the inherent literature reports a large number of illustrative case studies. Lourenço et al. [22], through the discussion of the case study of the Monastery Jeronimos in Lisbon (Portugal), highlighted the role of advanced numerical simulations for the understanding of the structural behavior of CH. The finite-element technique was used by Taliercio and Binda [23] to analyze the Basilica of San Vitale in Ravenna (Italy), a building which suffers diffused cracking and excessive deformation. Chiorino et al. [24] analyzed the Dome of Vicoforte (Italy), the largest elliptical dome ever built, combining limit analysis and finite-element technique. The dome has been analyzed through models, which are able to provide reliable interpretations of its behavior and damage state. A careful use of numerical analyses dealing with practical engineering problems has been shown by Bartoli and Betti [25], who discussed the cracking pattern on the Prince's Chapel (the Medici's mausoleum) in the Basilica of San Lorenzo (Florence, Italy).

Based on this growing literature several numerical models of increasing complexity were developed to analyze the structural behavior of the Basilica dell'Umiltà. Unknown parameters were assumed by considering reasonable values for historic masonry. These preliminary elastic analyses, even if the hypothesis of linear behavior for masonry is to a large extent an unrealistic assumption, are exclusively aimed at describing the overall behavior of the entire structure, recognizing the role of its structural elements, and evaluating the entity of the principal stresses acting on its main elements from a qualitative point of view.

Results of simplified model were then compared with those obtained from more sophisticated numerical analyses performed using the commercial code ANSYS and adopting solid hexahedral elements to model all the geometrical components [26].

The final three-dimensional solid model was developed based on the geomatic survey, encompassing the entire monument. In addition, taking into account the aim of the research, the modeling strategy used was the macro-modeling technique, which is, moreover, convenient for large-scale models. According to this technique, the masonry units and the mortar elements are assumed to be smeared, and an isotropic (or anisotropic) material represents the smeared units and mortars in the masonry. Other strategies, mainly suitable for small-size models, rely on micro-modeling approaches where units and mortar are modeled separately. The main results of simplified linear analyses are reported in Figs. 5 and 6. In Fig. 5 the experimental stress is compared with the one obtained with the numerical model. It is possible to observe a good agreement between the two values. The same can be observed for the remaining test, with the exception of test S6 (Fig. 6) where a difference of about 10% is observed.

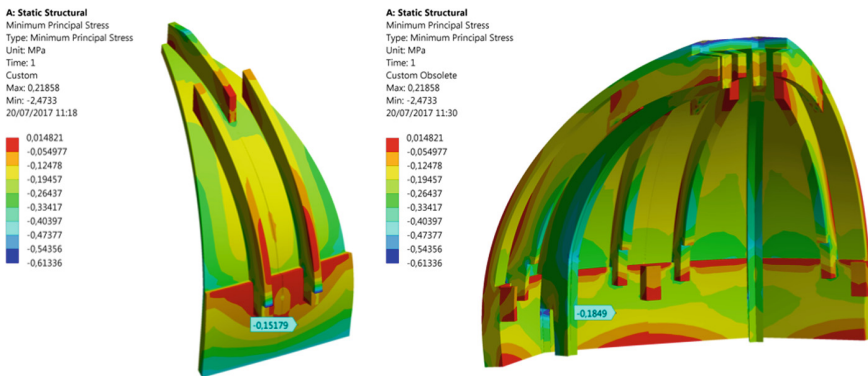


Fig. 5. Test D1s (left) - $\sigma_{m, exp} = 0.15$ MPa; Test S2 (right) - $\sigma_{m, exp} = 0.18$ MPa.

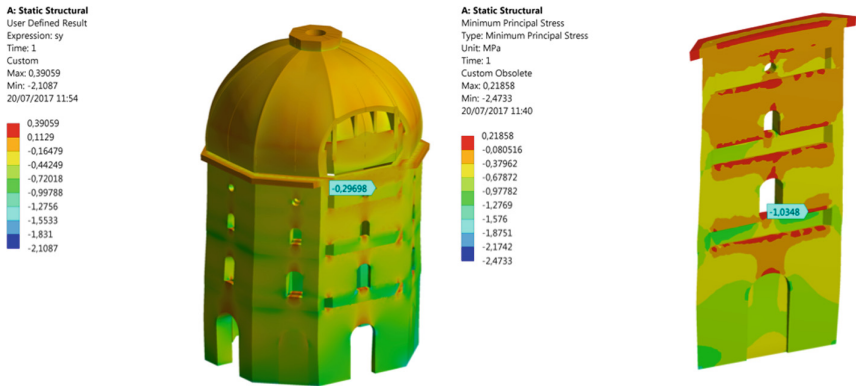


Fig. 6. Test S4 (left) - $\sigma_{m, \text{exp}} = 0.28 \text{ MPa}$; Test S6 (right) - $\sigma_{m, \text{exp}} = 1.18 \text{ MPa}$.

5 Conclusive Remarks

The paper showed, discussing the case study of the Basilica della Madonna dell'Umiltà in Pistoia, how the creation of an effective geometric model, through proper instruments of the geomatics, is a mandatory step in the interpretation of the architectural and constructive significance of the various elements that constitute the structure. It must be observed that, after all, this is the underlying approach of the Building Information Modelling that is steadily gaining ground in the design of new construction. The HBIM (Heritage-BIM), the specific application for cultural heritage buildings, is still only used sporadically and experimentally: nonetheless, it is evident that the possibility of gathering all the information about a structure in a single database, organized spatially and capable of progressive integration in light of later investigations, would be particular advantageous for those great complexes, about which knowledge is usually divided among many different specialized fields.

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