Schema-Based Instruction: Supporting Children with Learning Difficulties and Intellectual Disabilities

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Children who have learning difficulties or intellectual disabilities have similar challenges when solving mathematical word problems, including creating an internal representation of the problem structure and organizing the information to generate a solution strategy. Students with learning difficulties in mathematics and those with intellectual disabilities benefit from mathematics instruction that incorporates visual aids and repetition, and promotes strategy flexibility to help develop conceptual understanding. With regard to mathematical word problem solving, one approach has shown promise for individuals with learning difficulties and typically developing youth. Schema-Based Instruction (SBI) uses visual representations to teach students the mathematical structure of word problems. In this paper, we draw on existing literature to outline some of the cognitive deficits that have been observed in children with learning difficulties in mathematics and in those with intellectual disabilities and describe the ways in which those deficits can manifest themselves in the context of mathematical problem solving. We then describe the data we collected from our own delivery of SBI to a group of students with intellectual disabilities and compared their performance to students with and without learning difficulties. We focus on instances of meaningful problem solving after the intervention, with a focus on how the students may have circumvented or compensated for specific cognitive deficiencies. We conclude the chapter with a discussion about the elements of the instruction that may account for the students' performance after the intervention.

Classrooms are made up of many different types of learners, ranging from students who excel academically to those who struggle to learn the material being taught. Some students may have learning difficulties, intellectual disabilities, or other challenges, such as autism spectrum disorders. Indeed, it is estimated that in Canada, 57.6% of children aged 5–14 with intellectual disabilities are within the mild to moderate

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range, indicating that more than half are likely placed in inclusive school settings (Human Resources and Skills Development Canada (HRSDC), 2011).

Existing educational policies in North America stipulate that children with learning difficulties or intellectual disabilities should have access to equal opportunities for high-quality education that meet their needs (e.g., Education Act of Ontario, Ontario Ministry of Education, 1990; No Child Left Behind Act (NCLB), 2001; Ministère de l'éducation et de l'enseignement supérieur, 1999). Further, policies advocate for access to general education in the regular classroom to help these children meet the developmental goals established for all students (Individuals with Disabilities Education Improvement Act (IDEA), 2004).

Typical school mathematics curricula and pedagogies are rarely tailored to children with special needs (in our case, children with learning difficulties or intellectual disabilities), despite the fact that they are often integrated with other students in the classroom (Rose & Rose, 2007). Given the potential number of students who need support in mainstream classrooms, it is of utmost importance that appropriate programs are put into place to help support all children's mathematical development.

Although research on effective mathematics instruction is often focused on the typically developing student population, there is less research on effective practices specifically for children with learning difficulties. Even less research attention has been paid to mathematics instruction for children with intellectual disabilities and the pedagogies that can support their learning. It would appear that, for the most part, the mathematics instruction provided to children with intellectual disabilities has by and large emphasized rote, procedural instruction, with little focus on the development of conceptual understanding (Baroody, 1999; Cawley, Parmar, Yan, & Miller, 1998). More recently, Powell, Fuchs, and Fuchs (2013) argued that for students with mathematics difficulties, instruction should increase its focus on fluency with basic arithmetic skills, which are needed to solve problems that require conceptual knowledge. Traditional views of children with mathematics difficulties and intellectual disabilities are that they are "passive learners," capable of learning lower-level skills, but unable to devise or learn new strategies or transfer the skills previously learned. Research has shown, however, that children with intellectual disabilities are indeed capable of learning a number of mathematical skills and concepts with proper instruction (Baroody, 1999; Fletcher, Lyon, Fuchs, & Barnes, 2006; Ginsburg, 1997).

In this chapter, we describe an instructional intervention that we delivered to a small group of students with intellectual disabilities and compare their performance to that of children with and without learning difficulties. For purposes of clarity, we use the term "learning difficulty" to describe students who are either (a) diagnosed with a specific mathematics disability according the researcher's criteria, or (b) students who are underperforming in mathematics relative to their peers in the classroom, but who have no known diagnoses. When we use the term "mathematics learning disability," we are referring only to those students with specific diagnoses reported by the researchers. The intervention we delivered was a modified version of schema-based instruction (SBI; Jitendra & Star, 2011), an empirically validated teaching approach designed to support students' understanding of the mathematical structure of word problems.

We begin the chapter by referring to the existing literature to outline some of the cognitive deficits that have been observed in children with learning difficulties and intellectual disabilities and the ways in which those deficits can manifest themselves in the context of mathematical problem solving. We will then turn our attention to the student data we collected to describe particular instances of meaningful problem solving after the intervention, with an eye toward how the students may have circumvented or compensated for specific cognitive deficiencies. We conclude the chapter with a discussion about the elements of the instruction that may account for the students' performance after the intervention. We note that the conclusions that we can draw are limited because of the small number of participants in our study. Because of this limitation, we do not claim that our data generated any robust effects, causal, or other. Because of the dearth of research on the problem solving of children with intellectual disabilities, however, our findings nevertheless make an important contribution to this literature.

Mental Representation in Problem Solving

De Corte, Verschaffel, and De Win (1985) proposed a theoretical model of the processes involved in solving mathematical word problems. The model consists of five stages. In the first stage, the child processes the verbal text and creates a mental representation of the word problem structure. In the second stage, the child selects the appropriate arithmetic operation for finding the unknown. This selection is in large part dependent on the mental representation constructed in the first stage. In the third stage of the problem-solving model, the child executes the operation he or she has chosen. During the fourth stage, the child reactivates the mental representation, inserting the answer that was calculated. In the last stage, the child verifies if the answer is correct.

A study by Boonen, de Koning, Jolles, and van de Schoot (2016) nicely illustrates how successful problem solving is contingent on a correct mental representation of the problem structure. The authors found that children tended to write numbers sentences with operations that were consistent with the relational terms (i.e., "more than," "less than") used in the problem. In other words, the children were more likely to write number sentences with "+" when "more than" was used in the word problems than when "less than" was used, even when subtraction may have been a legitimate operation for the problem. This indicates that for children to be successful on inconsistent problems (when the relational terms do not match the required operation), they need to rely on a mental representation of the problem structure and cannot get by with superficial aspects of the problem text (e.g., the words "more" or "less").

Students with learning difficulties are often challenged when creating mental representations of problems and identifying the relevant information for solving them (Xin, Jitendra, and Deatline-Buchman, 2005). This has been shown to negatively impact problem solving in a number of ways, including reduced accuracy, difficulty generating number sentences, and applying inappropriate strategies for solving the problems (Hutchinson, 1993; Montague & Applegate, 1993). Some of the difficulties they have in creating mental representations of mathematics problems can be explained by executive functioning deficits that have been identified in the literature for children with intellectual disabilities (Oznoff & Schetter, 2007). Children with learning difficulties are challenged when they solve problems that require visualization and working memory capacity (Stein & Krishnan, 2007).

SBI specifically targets the first stage of the model—students' internalization of an appropriate problem structure. Without an appropriate mental representation, students will be hindered in choosing an appropriate operation, which in turn, will affect subsequent computations. Appropriate mental representations of word problem structures help students see the relationships among the quantities in the problem, which then supports the identification of suitable strategies for solving it (Lucangeli, Tressoldi, and Cendron, 1998).

Domain General Predictors of Word Problem Solving

Daroczy, Wolska, Meurers, and Nuerk (2015) proposed a model of the cognitive factors that are predictive of successful word problem solving. Their model includes domain general abilities as well as linguistic and numerical capabilities, and the authors describe the ways in which these factors account for student performance. In this section, we focus specifically on domain general abilities, such as executive functioning skills (e.g., working memory, shifting, and inhibition). The research has identified two specific executive functions, namely working memory and cognitive flexibility, as being especially important in mathematical problem solving (e.g., Geary, 2004; Geary, Hoard, Byrd-Craven, Nugent, and Numtee, 2007). Below, we present a brief overview of the literature describing the impact of working memory and flexibility on specific aspects of word problem solving.

Working memory. Working memory is the ability to hold a mental representation of information in one's mind while simultaneously using other mental processes to complete a task (Geary et al., 2007). Working memory plays a major role in predicting problem-solving accuracy (Andersson, 2007; Swanson & Beebe-Frankenberger, 2004; Zheng, Swanson, and Marcoulides, 2011); it is involved in all aspects of the problem-solving process because the students need to keep a number of pieces of information in mind during text comprehension, all while selecting an appropriate operation, executing the operation, and verifying the response.

Several researchers have demonstrated that students with learning difficulties and those with intellectual disabilities have significant working memory deficits, which in part explain their difficulty solving mathematics problems (Geary, 2004; Henry, Messer, and Poloczek, 2018). These deficits are manifested in various ways when children solve problems. For example, children with mathematics learning difficulties and with intellectual disabilities have trouble executing the required operation, and as such will use less mature strategies and make more errors than children without difficulties. For instance, children with learning difficulties often resort to finger-counting because it reduces the demands on working memory (Geary, 2004).

In addition, although little is known about the role of working memory in the creation of a mental representation, it has been shown that working memory is implicated in the construction of the mental number line (Geary, Hoard, Nugent, and Byrd-Craven, 2008). It is therefore reasonable to attribute children's struggles in constructing a useful mental model for a word problem to their working memory challenges. Furthermore, students with intellectual disabilities and learning difficulties likely struggle to keep a mental representation in mind throughout the problem-solving process (Lee, Ng, & Ng, 2009; Swanson & Sachse-Lee, 2001).

Judd and Bilsky (1989) attempted to alleviate cognitive load by providing a visual aid (i.e., dots that represented the quantities in the word problem) to students with and without intellectual disabilities while they were solving addition and subtraction word problems. The authors observed that students in both groups who were provided the visual representations were better able to retain the relevant information in the problem. They also showed that of all the students who were provided the visual aids, those who employed counting strategies were more likely to be successful (i.e., accurate) relative to those who were not provided the visual aids. Finally, Judd and Bilsky found that the errors made by children with intellectual disabilities were often characterized by overcounting or undercounting during the execution of the solution strategy.

Strategy flexibility. Flexibility in strategy use is defined as adapting one's strategies to the characteristics of the task at hand (Van der Heijden, 1993, cited in Verschaffel, Torbeyns, De Smedt, Luvwel, and Van Dooren, 2007). Ostad (1997) examined the addition strategies of children with mathematics learning difficulties and found that the children in the primary and upper elementary grades relied more on "back-up" strategies than retrieval strategies relative to their typically developing peers, which appears to be related to working memory deficits (Geary, Hoard, Byrd-Craven, and DeSoto, 2004). Back-up strategies are overt strategies that are visible or audible, such as counting on one's fingers. Retrieval strategies are those in which the answers are retrieved from long-term memory and can support performance on complex tasks because they require fewer demands on working memory (Powell et al., 2013; Raghubar, Barnes, and Hecht, 2010). In addition, Ostad found that children with mathematics learning difficulties exhibited considerably less flexibility in their strategy use compared to those students without difficulties. In particular, they tended to use one strategy repeatedly, as opposed to their typically developing peers, who used a range of strategies when solving problems. Ostad used the term "strategy rigidity" to describe those children who repeatedly use a smaller number of primitive back-up strategies when solving word problems.

Children with intellectual disabilities also struggle with strategy flexibility. Children with autism spectrum disorders, for example, have significant limitations with regard to cognitive flexibility and planning (Oznoff & Schetter, 2007), specifically at the conceptual level, making it difficult for these individuals to shift from one concept to the next. This is especially problematic when they engage in mathematics activities that require frequent shifting between various strategies or operations. It also appears that there is a relationship between the construction of an accurate representation of a problem's structure and flexibility in strategy use. Indeed, children who lack conceptual understanding of the quantitative relationships in a problem—that is, who lack an accurate mental representation of the problem structure—tend to be rigid in terms of the solution strategies they generate (Baroody, 1999; Lee et al., 2009).

Word Problem-Solving Instruction and SBI

There is evidence that children with learning difficulties and children with intellectual disabilities can acquire the same mathematical knowledge as their typically developing peers if additional and appropriate instruction is provided (e.g., Clements & Sarama, 2009; Fletcher et al., 2006). For example, young children with intellectual disabilities can learn oral counting, one-to-one correspondence, and cardinality (Baroody, 1999). In addition, they can learn basic numeracy skills (e.g., counting and subitizing) to the same level as their typically developing peers, as long as instruction is explicit and provides opportunities for practice (Bird & Buckley, 2001).

Three reviews of the literature focused on the effects of mathematics interventions on the learning of children with intellectual disabilities (i.e., Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakemanxya, 2008; Butler, Miller, Lee, and Pierce, 2001; Mastropieri, Bakken, and Scruggs, 1991) and of children with autism spectrum disorders (Browder et al., 2008). Together, these reviews showed that interventions targeting word problem solving with children and teenagers presenting with intellectual disabilities and autism were effective when they focused on the training of cognitive self-control strategies (e.g., checklists), the analysis of problem statements, and the use of concrete objects during the execution of solution strategies.

Bissonnette, Richard, Gauthier, and Bouchard (2010) conducted an overview of several reviews of the literature on mathematics interventions in students with learning difficulties. They demonstrated that explicit instruction is more effective than pedagogical methods based on constructivism for teaching word problem solving in children with learning difficulties. More recently, Jitendra, Nelson, Pulles, Kiss, and Houseworth (2016) reviewed the literature on instructional interventions centered specifically on teaching students the structure of mathematical problems with either visual representations (e.g., schematic drawings of part-whole and compare word problems) or with concrete representations, such as manipulatives. The 25 studies reviewed by the authors targeted students with learning difficulties and those at risk for mathematics learning disabilities. The findings showed that visual representations, whether on their own or in combination with concrete objects, positively impacted students' problem-solving performance.

SBI is an instructional approach that uses visual representations of problem structures to teach students how to solve a variety of word problems (Jitendra & Star, 2011). The findings from several studies have shown that SBI supports youth with learning difficulties, those at risk for mathematics learning disabilities, and typically developing students in their efforts to solve different types of word problems (Fuchs, Fuchs, Finelli, Courey, and Hamlett, 2004a; Fuchs et al., 2004b; Jitendra, DiPipi, and Perron-Jones, 2002; Jitendra & Star, 2011). A number of studies have also shown evidence of conceptual understanding following SBI as evidenced by performance on transfer problems (e.g., Fuchs et al., 2004a, 2004b; Jitendra et al., 2002), and Rockwell, Griffin, and Jones (2011) found maintenance effects for up to six weeks.

SBI's framework is based on schema theory. Schemata are knowledge structures that organize information in the learner's long-term memory (Bransford & Johnson, 1972; Griffin & Jitendra, 2009). In problem solving, schemata assist the learner in categorizing information, identifying the relationships between the quantities in a problem, and determining the best strategy for solving the problem (Chen, 1999). Chen (1999) found that when students are able to internalize what he termed "general schemata," defined as abstract representations of a problem's structure, their performance on transfer problems is enhanced. In addition, a general schema is one that is not linked to a specific procedure (Chen, 1999). When teachers provide students with a multitude of problems and diverse solution strategies, children can abstract a general schema which can then be used to solve novel problems, offering more flexibility across a range of contexts. The use of general schemata allows children to understand the semantic relations between the sets in the problem, which in turn supports a conceptual understanding about increases, decreases, and combinations involving sets (Cummins, 1991).

Xin and Jitendra (1999) argued that one of the reasons for the success of SBI is that it emphasizes conceptual understanding by creating representational links between the various aspects of word problems, thus enhancing students' ability to successfully solve them. SBI has been said to address the working memory and attention deficits of children with learning difficulties, and greatly differs from traditional mathematics teaching for children with intellectual disabilities, which tends to emphasize rote, procedural instruction (Cawley et al., 1998). Another possible reason for the success of SBI, particularly for children with learning difficulties, is that the creation of visual representations of the problem structure helps children solve problems by reducing cognitive load.

A Modified Version of SBI

Given that SBI has been found to be effective with students with a wide range of mathematical and cognitive abilities, we delivered a version of SBI to support the problem solving of three groups of first- and second-grade students: (a) a group of children with comorbid intellectual disabilities and autism spectrum disorders, (b) a group of children with learning difficulties (i.e., who were identified by their teacher as performing below the level of their peers), and (c) a group of children who were not struggling in mathematics in school.

Our delivery of SBI was a slightly modified version of the SBI protocol published by its designers (i.e., Fuchs et al., 2004a, 2004b; Jitendra & Star, 2011). We modified the typical SBI protocol by breaking the problem-solving process into smaller units for instruction. In addition, we also supplemented SBI with instructional features that were recommended by the National Mathematics Advisory Panel (2008), namely collaborative activity during problem solving and the sharing of solution strategies. We also encouraged students to solve the problems in whichever ways they found meaningful, which is also a departure from the typical SBI protocol. Finally, the instructor asked follow-up questions to encourage students to explain their strategies to their peers as clearly as possible, and also to encourage the students to identify any errors.

Participants

Our study had three groups of children, with three students in each group. All students were between 7 and 8 years old. The first group consisted of three students with comorbid intellectual disabilities and autism spectrum disorders who were finishing the first grade (ID group). The second group consisted of three second-grade students with learning difficulties, all from the same classroom (LD group). The final group consisted of three second-grade students, from the same class as the LD group, who exhibited average mathematics performance in school (AM group).

SBI and Student Data

All nine children received three instructional hours of SBI over four sessions (45 min each) that were delivered in small groups by the first author, a trained graduate student in educational research. During instruction, children were specifically taught "Action" problems (also known as join and separate problems; Carpenter, Fennema, Franke, Levi, and Empson, 2014; but also called "Group" or "Change" problems in the SBI literature; Jitendra & Star, 2011). Action problems describe an action where a given set is either increased or decreased, resulting in a different final quantity. In these problems, the unknown can be the initial set, the change set, or the end set. Consider the following word problem, "There are 9 apples in the bin. Five apples fall out of the bin. How many apples are left?" Described is a decreasing (or separating) action of apples falling from the bin, which changes the initial amount in the problem (9 apples). Two other problem types (part-whole and compare; Carpenter et al., 2014) were not used during our instruction, but were used as transfer problems on the assessments before and after the intervention.

Traditionally, SBI consists of two separate phases that we refer to here as the Problem Learning Phase and the Solution Generation Phase. In our study, each phase was completed in two instructional sessions spanning 45-min each. We used the same schematic representation, or schema,[1] for Action problems as those used in previous SBI studies (e.g., Fuchs et al., 2004a, 2004b), which can be found in Fig. 1. In the Problem Learning Phase, we provided explicit explanations of the different components of the schema, as is standard to SBI. Because the goal of this phase was for the children to learn the different components of the schema and where the numbers should be placed in it, we used story scenarios instead of word problems, which is also standard SBI practice. Story scenarios are word problems in which there is no unknown—that is, all of the numbers in the problem are provided. To illustrate using the word problem involving apples above, the corresponding story problem would be, "There are 9 apples in the bin. Five apples fall out of the bin. Now there are 4 apples left."

In the second phase, the Solution Generation Phase, the children were encouraged to generate their own strategies to solve a set of word problems, this time with unknowns. The students were provided with manipulatives (plastic chips) and paper and pencil to solve the problems in any way that was meaningful to them. The instructor also encouraged the students to use more than one strategy to solve a given problem and to share their strategies with the other students in their group. After the children had solved a given problem using a strategy of their choice, the instructor asked them if there was any other way the problem could be solved.

Before and after the modified SBI, we administered two tasks to the students to assess their mental representations of different problem types and their problem solving. Mental representations were assessed using the Problem Structure Test (PST), a multiple-choice test that we constructed specifically for this study. The PST contained six items, each of which required the student to read a word problem and choose one visual representation among three that best matched the structure of the problem (the fourth choice was "none of these"). A sample item from the PST is presented in Fig. 2. Four of the problems on the PST were action problems, and to assess transfer, the PST included one compare and one a part-whole problem. Correct answers were assigned 1 point and incorrect answers 0 points. The points were summed and converted to percent.

The students were also given word problems to solve before and after the intervention. Before the intervention, they solved six problems (i.e., four action, one compare, and one part-whole), and after the intervention, they solved eight problems, which

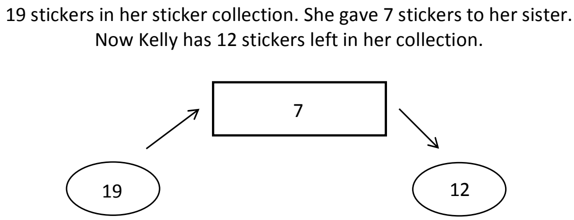

Fig. 1 Action schema illustrated on a workbook page used during the Problem Learning Phase

[1]For the remainder of the chapter, we use the term "schema" to refer to the schematic drawing that represents the structure of the word problem.

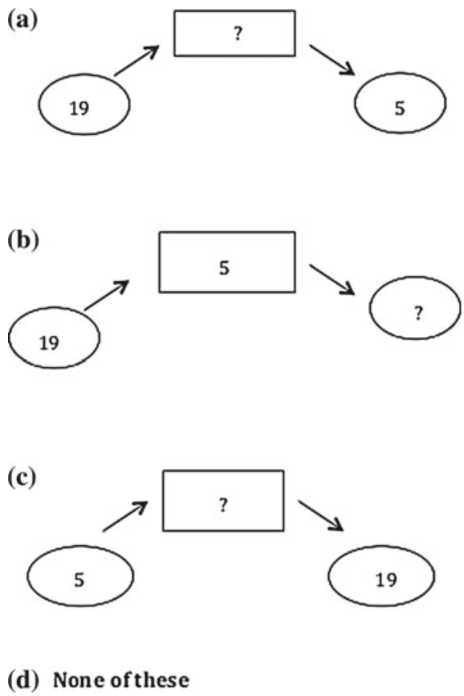

Fig. 2 Choices for the following problem on the Problem Structure Test: “Jesse’s mom made cookies for the bake sale. She sold 19 cookies and she has 5 left over. How many cookies did Jesse’s mom bake for the bake sale?”

consisted of isomorphic versions of the problems given before the intervention, and two additional action problems in which the quantities were not presented in the typical start–action–end order. The compare, part-whole, and atypical-sequence action problems were used to assess transfer.

We examined two aspects of the students’ problem solving. First, regardless of counting or computation errors, we assessed whether the strategy used by the student reflected the structure of the problem. Those that were aligned with the problem structure were coded as “appropriate” and those that were not aligned were coded as “not appropriate.” In addition, we coded the type of strategies the children used on the same task. We used Carpenter et al.’s (2014) taxonomy of problem solving strategies as our coding scheme. Direct modeling strategies were characterized by physical representations of the objects and actions in the problem. Counting strategies were those where the child was able to first represent one quantity abstractly (i.e., without representing it physically) and used some tools, such as fingers or tallies, to keep track of counts to find the solution. Derived fact strategies were those where the student used a known fact (e.g., single-digit addition facts) to derive a solution. For example, for $6 + 7$, a child using a derived fact strategy may explain that the answer is 13 because she knows that 6 and 6 are 12, and then one more is 13. Strategies based on known facts only were coded as recall. Strategy type allowed us to examine whether students were able to use more than one strategy type after the intervention.

Aspects of Problem Solving Before and After the Intervention

Identification of Word Problem Structure

As reviewed earlier in this chapter, one of the biggest challenges experienced by children with learning difficulties and children with intellectual disabilities is to identify the underlying word problem structure. This is consistent with what we observed prior to SBI instruction. Students in the ID group and in the LD group were unable to identify the word problem structures before the intervention. Specifically, the mean PST percent score for the ID group was 5% (SD = 9.2) and for the LD group was 27% (SD = 9.8) before the intervention. This suggests that these students were either unable to extract an appropriate mental representation of the word problems on the test, or had difficulty interpreting the visual representations provided in the choices on each item. If they did identify the correct structure, it was almost always for the action problems. The AM group performed better than the other two groups before the intervention on the problem structure test with an average PST score of 44% (SD = 25.5) before the intervention. Although their performance left room for improvement, they identified the correct structure for action problems more frequently and were also better able to choose the appropriate structure for the part-whole problem on the test.

Following the intervention, the mean PST scores for the students in the ID and LD groups improved. Mean scores on the PST were at 45% (SD = 14.4) for the ID group and 62% (SD = 25) for the LD group, with the AM group's performance remaining unchanged. After the intervention, the students in both the ID and LD groups were more consistently able to identify the structure for action problems and most of the students (5 of 6) across both groups were also able to correctly identify the structure for part-whole problems. This was an interesting finding, as the students were not exposed to part-whole problems during the intervention. This, paired with the fact that students in the AM group were able to correctly identify the structure of part-whole problems before the intervention, suggests that there may be something intuitive about the representation we created for part-whole problems, which was a different representation of the one originally created for SBI. Further, it could be that learning the structure of action problems facilitates transfer to problems with different mathematical structures.

Strategy Use

The students in the ID and LD groups were observed to more often use appropriate problem-solving strategies after the intervention compared to before. It is possible that having a concrete way of organizing the information presented in the problem—that is, the schema—and to see the relevant information in the schema decreased working memory load, thereby allowing for the allocation of cognitive resources to finding an appropriate solution strategy. It is also possible that the availability of manipulatives used during problem solving alleviated cognitive load. They were able to physically represent the mathematical actions required to solve the problems (e.g., removing tokens or joining them). Because the children could represent the quantities in the problem using tokens, they did not have to keep the numbers in their minds, thus offloading valuable working memory resources.

The use of the schemas appeared to support the problem solving of the children in all three groups, but the ID and LD groups seemed to benefit the most from the visual representations. The majority of the students in these two groups combined (i.e., 5 of 6) continued to use the schema after the intervention to help organize the information in the problem. We observed one student in the LD group rely on the schema when she reached an impasse while attempting to solve the problems. When she was unsure of what solution strategy to use, she would draw the schema and insert the numbers into it. Organizing the information in this way seemed to help clarify critical aspects of the problem and she was then able to determine an appropriate strategy for solving it.

Strategy Flexibility

Most students demonstrated little change in the ability to use more than one type of strategy for the same problem after the intervention. This is consistent with the literature on strategy rigidity describing children with difficulties in mathematics using one type of strategy across problems (Ostad, 1997). We observed that when the researcher asked for an alternative strategy, most children gave a response in the same category as their first strategy, both before and after the SBI instruction. This was seen for most students in all three groups for all problems. For example, one child in the LD group used direct modeling (physically representing the quantities and actions in the problem) using tokens for his first strategy, but when asked if there was another way to solve the problem, he drew the objects in the problem on a piece of paper and acted on those representations by circling and crossing out objects. Although both strategies looked different on the surface, they were both coded as direct modeling, and as such, did not constitute strategy flexibility.

We also observed three students (two from the AM group and one from the LD group) change the operation for the second strategy, both before and after the intervention. That is, they used the standard written algorithm for their first strategy, and for the second, used the standard algorithm for a different operation. For example, on his first strategy, one child in the AM group performed the standard written algorithm for $198 - 116$ and correctly solved the problem. When asked for a second strategy, he used the standard algorithm for the inverse operation and computed $198 + 116$. Thus, we saw some evidence for strategy rigidity as most students would use the same type of strategy to solve a given problem.

Only one student in the LD group was able to successfully use more than one type of strategy on the problems after the intervention. On one problem, he used the standard written algorithm as his first problem-solving strategy for one of the end unknown problems. When asked for a different strategy, he explained what he would do to directly model the problem. He did not physically act it out by counting out tokens or drawing tallies, but clearly explained his direct modeling strategy verbally.

Pedagogical Implications

Despite the tentative nature of the conclusions that can be drawn from our small data set, the results are still promising with regard to intervention planning for inclusive classrooms. All students benefited from the instruction in one way or another. In the following section, we will describe some of our observations of the students' problem solving during the instruction that may account for their performance after the intervention.

Overall, the instruction appeared beneficial for children on most of the assessments we administered. Explicitly teaching the children the different components of the schema helped them prepare for the problem solving that followed. Indeed, we noticed that about half of the children continued to use the schemas during problem solving following the intervention, which presumably helped them to monitor their work and verify their responses. This may have supported the construction of appropriate mental representations of the mathematical structure of the problems, which could in part account for the greater use of appropriate strategies after the intervention.

A practice that we found to be especially helpful was when the instructor asked questions to encourage the children to reflect on their thinking so they could themselves correct any errors and find a more promising avenue for the solution. Instructor questions, together with the feedback of the other members of the group, encouraged the students to "talk out" the problem and change course during problem solving if necessary. This may have increased students' reflections about the relative appropriateness of a number of different solution strategies. By asking questions, the instructor was also provided with information about how the children were thinking about the problems. This, in turn, let the instructor modify her instruction to address the children's specific difficulties or misconceptions.

Another aspect that seemed to benefit the students was when the instructor focused on the structural and conceptual aspects of the problems when teaching the different problem types. For example, for action problems, the instructor would describe the action as something that would lead to a decrease or increase in the start number. When children understood this, their ability to monitor and correct their own work appeared to improve, thereby needing less prompting from the instructor. To illustrate, one child completed an action problem but made an error. To help the child see his error, the instructor asked questions about the structure of the problem. She reread the problem with the child. She asked him what would happen to the start number (the initial number of apples in the problem) if apples fell out of the bin. The child answered that the number would go down. The instructor asked him to look at the end number (the final number of apples after the action) and compare it to the start. In doing so, the child said, "Wait, that's not right. The number can't be bigger!"

When the children used the schemas during instruction, the visual representations appeared to help them organize the information provided in the problem. Prior to instruction, most children in the ID group struggled to make sense of how the numbers related to one another. At times, we observed some children in this group having difficulty using the correct numbers, despite having the information in front of them. The use of the schemas during and after instruction may have had a positive effect on students' ability to manipulate several pieces of information at once; the schemas presumably allowed the students to offload the numerical information in the problem so they could better focus on the problem structure and select an appropriate strategy. This is consistent with research showing that children with learning difficulties have working memory deficits and tend to rely on immature problem-solving strategies like finger counting as a result (e.g., Geary, 2004). The organizational support provided by the schemas appeared to provide easier access to key parts of the problem, thereby freeing up their working memory capacity.

While our instruction brought forth positive change in students' problem solving, some modifications are required to further enhance performance in students with learning difficulties as well as those with intellectual disabilities. First, the Action problems presented during the instruction all described the start, action, and end sets in that order (e.g., Lisa had 324 pennies (start). She found some more pennies on the sidewalk (action). Now she has 434 pennies (end). How many pennies did Lisa find on the sidewalk?). This led to difficulties in successfully solving the atypical-sequence action transfer problems, especially for the ID group. These students began to enter the numbers into the schema in a rote manner. That is, they would automatically put the first number in the problem in the first part of the schema, the second number in the problem in the second component, and the third number in the last component, disregarding what each component meant within the structure of the problem. This was evident as most of the students in the ID and LD groups failed to correctly solve problems that presented information out of sequence at posttest. This is consistent with research on "psychological sets" (Duncker, 1945): Strategies can become automatic after repetitive use (i.e., in this case, putting the numbers into the schema in a certain order), so that it becomes rote. In these situations, the students did not appear to stop to think about the problem, but rather engaged in a behavior that had served them well in the past.

Another issue centered on the use of multiple strategies. Across the groups, both before and after the intervention, most students used only one strategy. Sometimes, they offered two different variations of the same strategy type category (e.g., direct modeling on fingers and direct modeling by drawing tallies on paper). There are several possible explanations for the lack of flexibility, which are important to consider in the design of future implementations of SBI with children with learning difficulties and children with intellectual disabilities. For one, the length of the intervention may have impacted the children's ability to demonstrate flexibility. As previously mentioned, all students received a total of three hours of instruction over a two-week period. It is possible that three hours are not enough time for children to become more flexible in their strategy use. Increased opportunities for practice are especially important for children who exhibit strategy rigidity (Baroody, 1996; Ostad, 1997). Perhaps with more practice and increased exposure to different types of strategies, children may have been more flexible following the intervention.

Although we believe more time would have been beneficial, we also suggest that flexibility gains were not observed because the strategies that were shared during the intervention may have all been of the same type. That is, as opposed to seeing a direct modeling strategy followed by a counting strategy, for example, children may have seen two different ways to directly model the same problem or two different ways to use counting strategies. In fact, evidence that this occurred was observed following instruction. Furthermore, previous mathematics instruction could also have played a role in children's choice to use the same strategy. It is possible that during the instruction they had received in school, they were not given the opportunity to explore different solution strategies.

Another possibility is that perhaps students with learning difficulties and children with intellectual disabilities need explicit instruction on how to use a variety of strategies to solve a given problem. In fact, there is a debate in the literature as to whether mathematics instructors should explicitly teach strategy flexibility to students who have learning difficulties. Specifically, the controversy centers on whether children should be taught to use a variety of strategies flexibly, or only a small handful of strategies for solving problems (Verschaffel et al., 2007). One argument is that children who struggle could benefit from using a small number of strategies repeatedly, which would alleviate pressures on working memory (Baxter, Woodward, and Olson, 2001). On the other hand, others have argued that flexibility should be explicitly targeted from the beginning (e.g., Butler et al., 2001; Verschaffel et al., 2007).

Conclusion

In this chapter, we described an instructional intervention based on SBI that we delivered to three small groups of children, one group with intellectual disabilities and autism spectrum disorders, a second group who were identified by their teacher because they were performing poorly relative to their peers, and a third group of students who were average performers. We described elements of improved problem solving after the intervention and speculated about how our instantiation of SBI may have supported students' performance. Our conclusions are necessarily tentative because our sample was small; nevertheless, the speculations we draw offer an existence proof that students with learning difficulties and intellectual disabilities can indeed learn and use key mathematical concepts in the context of appropriate instruction. Our approach deviates from existing work in the field in that we shifted attention away from students' deficits toward their strengths. In this way, our observations can inspire new lines of research on the mathematical potential of students with difficulties and intellectual disabilities.

Children with intellectual disabilities need to learn adaptive mathematical skills (such as problem solving) to help them become as autonomous as possible in their daily lives. In this chapter, we provided some evidence that students with intellectual disabilities and autism spectrum disorders, as well students presenting with mathematical learning difficulties, are able to develop both conceptual and procedural knowledge. Our observations are encouraging teachers and other practitioners who work with this population. Further, given the positive effects of instruction that was delivered in small-group settings, it appears that SBI could be a promising approach for teaching mathematical word problem solving in inclusive classrooms.

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