

# Chapter 28

## Validation of Automatic Modal Parameter Estimator on a Car Body-in-White



N. Gioia, Pieter-Jan Daems, and J. Helsen

**Abstract** Noise, vibration and harshness (NVH) problems are critical issues to be tackled by automotive industry to ensure comfort of people. The key role of this problem is played by the susceptibility of the structure to vibrations. Therefore, design values such as the modal parameters (i.e. eigenfrequencies, damping ratios, mode shapes and modal scaling factors) are essential and their experimental validation is of high interest. At this purpose, both Experimental and Operational Modal analysis (EMA and OMA) represent a powerful approach. Previous works have investigated and implemented a completely automated OMA technique for continuously tracking the modes of machines under normal operating conditions. In this work the automatic modal parameter estimator is used to perform automated experimental modal analysis on data acquired from a car body-in-white excited by means of multiple shakers. Fully automated modal analysis is performed with special focus on damping value and mode shape validation. The results obtained with the manual and automatic modal parameter estimators are compared in order to show the validity and performance of the implemented method. Modal parameters estimation is based on the state-of-the-art pLSCF algorithm. To make it suitable for continuous analysis, the algorithm is improved by eliminating all the required human interactions.

**Keywords** Automatic operational modal analysis · Car body-in-white · Automatic experimental modal analysis · Mode shapes validation · Damping values validation

### 28.1 Introduction

In machine industry, noise, vibration and harshness (NVH) problems are critical design issues. The key role of this problem is played by the susceptibility of the structure to vibrations. Therefore, design values such as the modal parameters (i.e. eigenfrequencies, damping ratios, mode shapes and modal scaling factors) are fundamental. In initial design stages, the vibration response of the structure is simulated using simplified models. The further the design process evolves, the more complex these models become, e.g. finite element and multibody simulations. Although industry utilize sophisticated modeling tools that already allow an optimized configuration at this stage, the experimental verification of the design values is fundamental for model validation and for guaranteeing safety and reliability of the structure[1]. Since damping and boundary conditions depend on the vibrations amplitude and the modal parameters depend on the (rotating) speed of the structure or the parts, it is important to experimentally verify the design values in normal operating conditions, i.e. around the operating point. The needed validation can be performed both in laboratory conditions or real operating conditions. Experimental and Operational Modal Analysis (EMA and OMA) represent powerful approaches to identify systems with few inputs and hundreds of outputs. EMA is used in case of laboratory tests, when both the vibration response of the structure and the excitation source are measurable. On the other hand OMA is used for identifying a structure excited by (unmeasured) natural and operational forces. The interest in having an automatic operational modal analysis algorithm is due to the possibility of extracting modal parameters from operating machines. The advantage of considering operating machines is the fact that the system is loaded in a representative way. Preferably long-term measurements are used that encompass the different loading conditions the system is subjected to during its lifetime. This requires the use of automatic algorithms able to process the data continuously and autonomously. Not only for these operational modal analysis cases the use of automated modal analysis tools is interesting. Also for experimental modal analysis automation has the advantages of increasing repetitiveness of the analysis. In previous researches [2, 3], we implemented a completely automatic modal parameter estimator coupled with autonomous pre- and post-processing methodologies (signal validation/classification, harmonics removal and modal parameter tracking). The

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complete chain has been used to process data acquired on a wind turbine drivetrain. In this work the automatic modal parameter estimator is used to perform automated experimental modal analysis on data acquired from a car body-in-white excited by means of multiple shakers. The choice of the scope of the analysis is linked to several reasons. First of all, experimental modal analysis is of high interest in automotive industry to solve NVH problems and ensure safe and comfortable cars. The availability of an automatic tool to process experimental data, represents a significant advantage to increase the repeatability of the tests. Secondly, the processed data, have been already analyzed with a commercial manual algorithm (*TestLab*). The knowledge of the modal parameters of the system, allows to focus on damping values and mode shapes validation. In this research indeed, the results obtained with the manual and automatic modal parameter estimators are compared in order to show the validity and performance of the implemented automatic modal analysis method. For this research, the so called *polyreference least-squares complex frequency-domain* estimator has been used as starting algorithm [4].

## 28.2 Theoretical Background

The Poly-reference Least-Square Complex Frequency-Domain (p-LSCF) method consists in a frequency-domain modal analysis method that requires as primary data the Frequency Response Functions (FRFs) and identifies a right matrix-fraction model:

$$[H(\omega)] = [B(\omega)][A(\omega)]^{-1} \quad (28.1)$$

where  $H(\omega) \in \mathbb{C}^{1 \times m}$  is the FRF matrix containing the FRFs between the  $m$  inputs and the  $l$  outputs;  $B(\omega) \in \mathbb{C}^{1 \times m}$  is the numerator matrix polynomial and  $A(\omega) \in \mathbb{C}^{m \times m}$  is the denominator matrix polynomial. Each row of the matrix-fraction model can be written as:

$$\forall o = 1, 2, \dots, l \quad \langle H_o(\omega) \rangle = \langle B_o(\omega) \rangle [A(\omega)]^{-1} \quad (28.2)$$

The numerator row-vector polynomial of output  $O$  and the denominator matrix polynomial are defined as:

$$\langle B_o(\omega) \rangle = \sum_{r=0}^p \Omega_r(\omega) \langle \beta_{or} \rangle \quad (28.3)$$

$$[A(\omega)] = \sum_{r=0}^p \Omega_r(\omega) [\alpha_r] \quad (28.4)$$

where  $\Omega_r(\omega)$  are the polynomial basis functions and  $p$  is the polynomial order. In the LSCF method, a z-domain model is used (i.e. a frequency domain model that is derived from a discrete-time model) and, by consequence, the basis functions are:

$$\Omega_r = e^{j\omega \Delta t r} \quad (28.5)$$

with  $\Delta t$  the sampling time.

The polynomial coefficients  $\beta_{or} \in \mathbb{R}^{1 \times m}$  and  $\alpha_{or} \in \mathbb{R}^{m \times m}$  can be grouped in matrices. The FRF model of Eq. (28.1) can be then written as a function of the coefficients  $H(\omega_k, \Theta)$  [4]. In order to find all the unknown model coefficients based on the measured FRFs a non-linear least-squares (NLS) equation error must be minimized. To do so, the equation errors of all the outputs and all frequency lines are combined in a scalar cost function that is minimized by putting its derivatives with respect to the unknown model coefficients equal to zero. Theoretically a complete right-matrix model can be obtained by computing the numerator coefficients solving all the equation deriving from the minimization of the cost function. However, a more practical approach is available. Generally, in modal analysis, one is not only interested in a good model as such, but more important in the accuracy of the estimated modal parameters. When trying to estimate from real data, is then a good approach to over-specify the model order, trying to fit high-order models that contain more modes that the one present in the data. Afterwards, the use of the stabilization diagram can help in separating the physical poles from the spurious ones.

On the stabilization diagram, the poles are shown for increasing model order, and their stability is labeled comparing the poles at one model order with one at a one-order-lower. The use of the stabilization diagram, solves the problem of the a-priori definition of the model order of the system that has to be identified. Once the stable poles and the corresponding

participation factors are selected on the stabilization diagram by the analyst, their value are used in the so-called pole-residue model (Eq. (28.6)) to find the mode shapes:

$$[H(\omega)] = \sum_{i=1}^n \frac{\{v_i\}\langle l_i^T \rangle}{j\omega - \lambda_i} + \frac{\{v_i^*\}\langle l_i^H \rangle}{j\omega - \lambda_i^*} - \frac{LR}{\omega^2} + UR \quad (28.6)$$

where  $n$  is the number of modes;  $\bullet^*$  is the complex conjugate of a matrix;  $v_i \in \mathbb{C}^l$  are the mode shapes;  $\langle l_i^T \rangle \in \mathbb{C}^m$  are the modal participation factors and  $\lambda_i$  are the poles, which occur in complex-conjugate pairs and are related to the eigenfrequencies  $\omega_i$  and damping ratios  $\xi_i$  as follows:

$$\lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2} \omega_i \quad (28.7)$$

$LR, UR \in \mathbb{R}^{l \times m}$ , introduced in Eq. (28.6) are respectively the lower residuals and upper residuals modelling the influence of the out-of-bands modes in the considered frequency band. In Eq. (28.6), the only unknown are then the mode shapes, the lower and upper residuals. They can be obtained by solving it in a linear least-square sense. This step is commonly called *least-squares frequency-domain* method.

### 28.3 Methodology

As stated in the previous section, the stabilization diagram is a powerful approach for avoiding the need of pre-defining the model order of the system. However, it requires the intervention of the analyst, that have to distinguish the stable poles from the spurious ones. To make the Poly-Reference Least-Square Frequency-Domain (p-LSFD) estimator automatic, the main algorithm-analyst interaction has been eliminated. Starting from the ideas investigated from other authors [5–7], the classical stabilization diagram [8] has been substituted with a clustering method that autonomously interprets the information collected in the stabilization diagram. In this work a clustering analysis has been used in order to group the poles showing stable characteristics (i.e. representative of the same mode). The physical modes have been then selected automatically exploiting statistical characteristics of the generated clusters. The advantage of the implemented methodology with respect to previous works [6–8] is the complete automation of the algorithm that selects the parameters required by clustering algorithm autonomously, based on some physical and statistical properties of the signal.

For the validation of the modal parameters estimator, FRFs generated during laboratory tests on a car body-in-white are used as input data. A shaker test was performed on the body-in-white with 2 fixed uniaxial excitation points and 52 triaxial measurement points. A schematic representation of the studied car body is shown in Fig. 28.1.

### 28.4 Results

The results obtained with *AutoMax*, are compared with the ones obtained with *TestLab*, in order to validate the implemented method. The analysis is performed in the frequency band of interest for the torsional mode of the car. Frequency and damping values have been normalized for confidentiality reasons. For both the frequency and damping values, a relative difference between the estimates obtained with *TestLab* and *AutoMax* is calculated as:

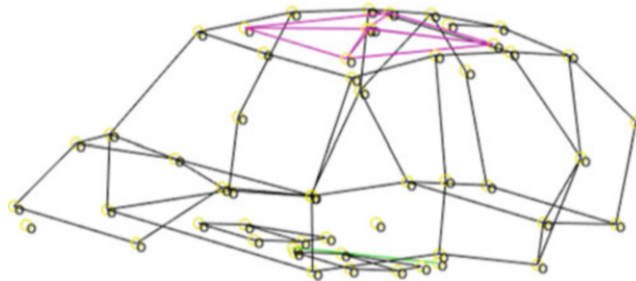
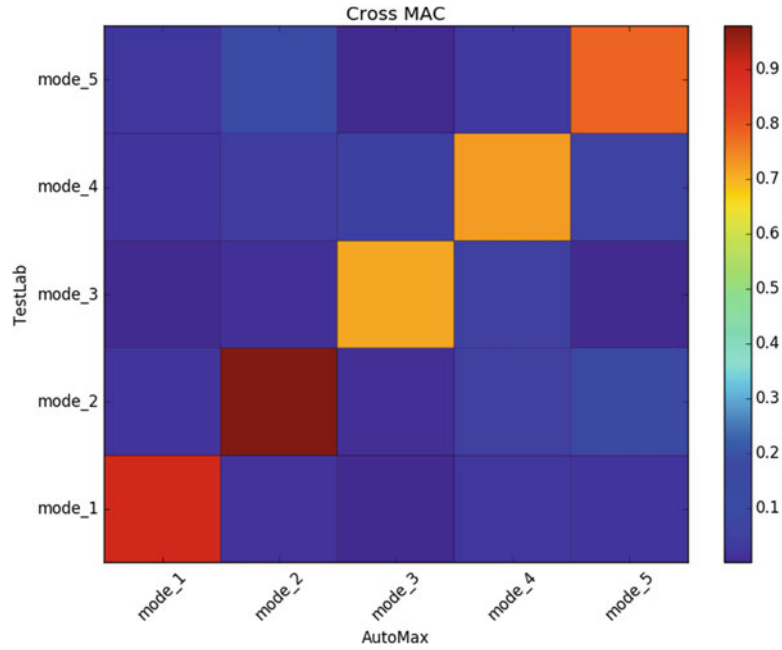


Fig. 28.1 Schematic view of the car and the sensors location

**Table 28.1** Comparison of the results obtained with the two different approaches in terms of frequency and damping estimates

Mode number	Normalized frequency			Normalized damping		
	TestLab	AutoMax	Difference [%]	TestLab	AutoMax	Difference [%]
1	0.63	0.63	0.006	0.64	0.62	3.125
2	0.80	0.80	0.024	0.98	1.00	2.04
3	0.85	0.86	0.064	0.45	0.44	2.22
4	0.98	0.98	0.03	0.55	0.52	5.45
5	0.99	1.0	0.09	0.57	0.54	5.26

The values of both the quantities are normalized



**Fig. 28.2** Cross MAC between the mode shapes obtained with TestLab and the one obtained with the new AutoMax. Data values are normalized to be between 0 and 1 for confidentiality reasons

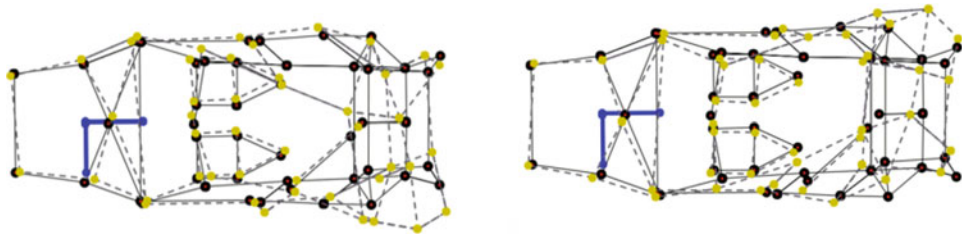
$$Diff_{rel}(x, x_{ref}) = \frac{|x - x_{ref}|}{x_{ref}} \quad (28.8)$$

where  $x$  is the frequency or damping estimates obtained with *AutoMax* and  $x_{ref}$  the frequency or damping estimates obtained with *TestLab*, used as reference values.

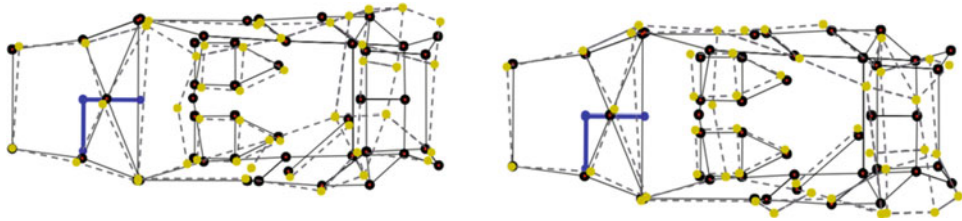
The comparison between frequency and damping (normalized) values is shown in Table 28.1. To validate also the mode shapes, the Cross Modal Assurance Criterion is used [9]:

$$MAC(\{\psi\}_r, \{\psi\}_s) = \frac{|\{\psi\}_r^H \{\psi\}_s|^2}{(\{\psi\}_r^H \{\psi\}_r)(\{\psi\}_s^H \{\psi\}_s)} \quad (28.9)$$

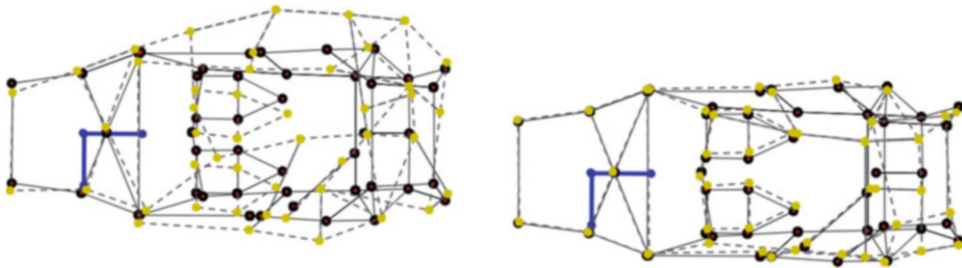
where  $\{\psi\}_r$  with  $r = 1, \dots, n_{modes_1}$  is the mode shape vector of the set of modes obtained with *AutoMax*,  $\{\psi\}_s$  with  $s = 1, \dots, n_{modes_2}$  is the mode shape vector of the set of modes obtained with *TestLab*,  $^H$  is the Hermitian transpose. The MAC matrix is based on the orthogonality conditions of the mode shapes, thus if  $\{\psi\}_r$  and  $\{\psi\}_r$  are estimates of the same physical mode shape, the modal assurance criterion should approach unity. If  $\{\psi\}_r$  and  $\{\psi\}_r$  are estimates of the different physical mode shape, the modal assurance criterion should be low or approaching zero. The results of the comparison of the mode shapes is shown in Fig. 28.2. The obtained mode shapes are shown in Figs. 28.3, 28.4, 28.5, 28.6, and 28.7.



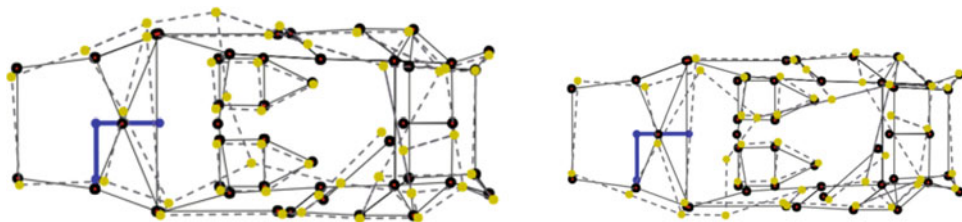
**Fig. 28.3** Mode 1: freq = 0.63, damp = 0.62



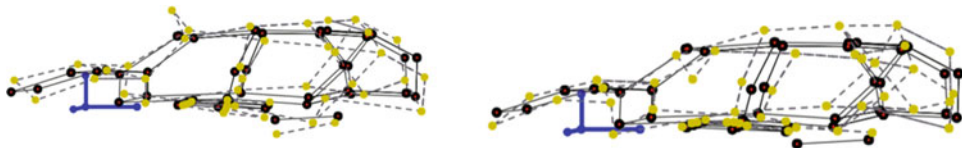
**Fig. 28.4** Mode 2: freq = 0.80, damp = 1.00



**Fig. 28.5** Mode 3: freq = 0.86, damp = 0.44



**Fig. 28.6** Mode 4: freq = 0.98, damp = 0.52



**Fig. 28.7** Mode 5: freq = 1.00, damp = 0.54

## 28.5 Conclusions

During this research the experimental validation of the modal parameters of a car body-in-white has been repeated with a new approach (*AutoMax*). The latter is an algorithm that performs experimental and operational modal analysis autonomously, i.e. without requiring the action of the analyst. The need of this tool is justified by several reasons, such as the elimination of the analyst expertise as aleatory variable, reduction of data processing time, increase of repeatability of the analysis and analysis of machines in real operating conditions. The automatic modal parameters estimator has already been presented in previous work, with the scope of automatically processing data coming from an operating wind turbine drivetrain.

During this work, the method implemented has been validated by means of a comparison of the estimates obtained with *AutoMax* and the ones obtained with *TestLab*, used as solid validated benchmark. The obtained results show the match of the estimates obtained with the two different modal analysis tool. In case of resonance frequencies, the maximum error is 0.09%, while in case of damping ratios the maximum error increases, reaching the 5.45%. A difference in the estimate values was expected, since the analysis performed is not exactly the same, due to some variable that has to be chosen, e.g. the maximum model order for the analysis, the model order at which the poles have been selected on the stabilization diagram from the analyst or algorithm. However, the error is limited, and it is not higher than the error that could have been obtained by manually repeating the analysis twice. Concerning the mode shapes, the cross MAC has been used to compare them in a quantitative way. Also in this case results show a good match of the estimates, having MAC values that are above 0.7 for the same modes and lower than 0.2 for different ones. The conclusions addressed for the damping estimates still hold true for the mode shapes. Moreover the visualization of the mode shapes obtained with *AutoMax* confirm that the mode shapes are the torsional modes that were expected.

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