Chapter 5
The (In)Vulnerability of 20 Voting Procedures to the No-Show Paradox in a Restricted Domain

Abstract The No-Show paradox occurs whenever a group of identically-minded voters is better off abstaining than by voting according to its preferences. Moulin's (Journal of Economic Theory 45:53–64, 1988) result states that if one wants to exclude the possibility of the No-Show paradox, one has to resort to procedures that do not necessarily elect the Condorcet winner when one exists. This paper examines 10 Condorcet-consistent and 10 Condorcet-non-consistent procedures in a restricted domain, viz., one where there exists a Condorcet winner who is elected in the original profile and the profile is subsequently modified by removing a group of voters with identical preferences. The question asked is whether the No-Show paradox can occur in these settings. It is found that only 2 of the 10 Condorcet-consistent procedures investigated (Minimax and Schwartz's procedure) are invulnerable to the No-Show paradox, whereas only 3 of the 10 non-Condorcet-consistent ranked procedures investigated (Coombs's, the Negative Plurality Elimination Rule, and the Majority Judgment procedures) are vulnerable to this paradox in the restricted domain. In other words, for a No-Show paradox to occur when using Condorcet-consistent procedures it is not, in general, necessary that a top Condorcet cycle exists in the original profile, while for this paradox to occur when using (ranked) non-Condorcet-consistent procedures it is, almost always, necessary that the original profile has a top cycle.

Keywords Condorcet-consistency · Domain restrictions · No-Show paradox · Voting paradoxes · Voting procedures

5.1 Introduction

The theory of voting is known for its many apparently negative results that amount to demonstrating the impossibility of satisfying several social choice desiderata. Arrow's (1963) impossibility theorem is the best-known result of this kind, but it is by no means the only one. In fact, the incompatibility of two or more desirable

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properties is one standard method of expressing findings that are related to the study of voting rules; if one wishes that one's rules always behave in a plausible way in all circumstances, then one has to be prepared for the possibility that the behavior is not plausible in another sense.

One important result on voting rules was achieved by Moulin (1988). It states that if the number of alternatives under consideration is at least 4 and the number of voters is at least 25, then no Condorcet-consistent rule is compatible with the property known as Participation.[1] A rule satisfies Participation if any group of voters with the same preference ranking over the alternatives is under no circumstances better off abstaining than voting according to its preferences. Following Fishburn and Brams (1983), situations where Participation is not satisfied are called *No-Show paradoxes*. Condorcet-consistent rules share the defining property that in all circumstances they always result in the Condorcet winner being the sole alternative chosen, whenever there is a Condorcet winner in the profile under investigation. Condorcet winner, in turn, is an alternative that—according to the voters' preferences—would be preferred to any other alternative by a majority of voters.

Moulin's result has subsequently been refined and augmented (see e.g., Brandt, Geist, & Peters, 2017; Brandt, Hofbauer, & Strobel, 2018; Felsenthal & Nurmi, 2017; Pérez, 1995, 2001), but the basic incompatibility between the two social choice desiderata, Condorcet-consistency and Participation, remains intact. Our main interest here is to find out what this incompatibility would mean in terms of the design of voting institutions. In other words, under what kinds of circumstances can we expect the incompatibility to materialize? More specifically, if one adopts a Condorcet-consistent voting system, how likely is it that an instance of the No-Show paradox will be encountered?

A straight-forward way to address these questions is to construct a probability model of the process generating the preference profiles and to determine those giving rise to the No-Show paradox. The relevance of the models depends on the degree in which they mimic the process that underlies the emergence of the profiles of the decision making body under examination.[2] Our goal is more modest. We aim at determining the effect of one important profile characteristic, viz., the presence or absence of a Condorcet winner, on the possibility of the No-Show paradox. Our main problem is to determine whether various Condorcet-consistent, as well as various ranked Condorcet non-consistent procedures, are vulnerable to the No-Show paradox in the restricted domain characterized by the presence—and the election—of the Condorcet winner in the initial profile. Probability and simulation results suggest that the probability of a Condorcet winner existing in randomly generated preference profiles is in general higher than the probability of majority cycles (see, e.g., Gehrlein, 1983;

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[1]Brandt et al. (2017) corrected Moulin's result by showing that there exists no Condorcet-consistent rule which satisfies Participation when there are at least 12 (rather than 25) voters and 4 alternatives.

[2]Brandt et al. (2018) examined the incidence of the No-Show paradox displayed by three Condorcet-consistent procedures (Black's, Minimax and Tideman's rule) using Ehrhart theory and extensive computer simulations. They found that for a small number of alternatives (4) the probability that these procedures display the No-Show paradox is negligible and as the number of alternatives increases (up to 30) the No-Show paradox becomes much more likely.

Gehrlein & Lepelley, 2011, 2017). So, should the absence of a Condorcet winner be a necessary prerequisite of the No-Show paradox, this would significantly diminish the practical importance of Moulin's theorem. In what follows it will, however, be seen that the No-Show paradoxes are possible in the presence of a Condorcet winner for nearly all those 10 Condorcet-consistent systems that we will focus upon, whereas the No-Show paradox cannot occur under 4 of the 6 ranked non-Condorcet-consistent procedures investigated when a Condorcet winner is present and elected in the original profile.

5.2 Assumptions and Definitions

We shall focus on rules that aggregate individual opinions into collective ones in the following sense. Each individual is assumed to be endowed with a complete (or connected) and transitive preference relation (ranking) over the decision alternatives (candidates, policies, etc.). We denote the set of individuals (voters) by N and assume that it consists of n voters. The rules that specify the aggregation process are set-valued social choice correspondences so that for each n-tuple of individual preference rankings (called the *preference profile*), the rule indicates the set of chosen alternatives, the winners. The set of alternatives is denoted by A and it contains k elements. We assume that the rules are anonymous so that the number, not the identity, of the voters having each preference ranking determines the decision outcome when the rule is applied.

Our basic tool in the analysis of the preference profiles is the *pairwise comparison matrix* which contains k rows and k columns so that the element in cell (i, j) indicates the number of individuals strictly preferring alternative i to alternative j. The k entries along the main diagonal are left blank. By completeness of the individual preference relations we can assume that each non-diagonal entry is non-empty. In all our examples the individual preferences are not only complete and transitive, but also strict, meaning that if x is preferred to y by an individual, this implies that y is not preferred to x by that same individual. From this it follows that the pairwise comparison matrix is reciprocal, i.e., the sum of entries (i, j) and (j, i) is always n.

5.3 Examples Demonstrating the Possibility of No-Show Paradox Under Eight Condorcet-Consistent and Three Non-Condorcet-Consistent Procedures When a Condorcet Winner Exists in the Initial Profile

In the following examples we use notation such as '3 voters $a > b > c$' to denote three voters having (transitive) preference ordering among alternatives a, b, c such that they prefer alternative a to b, b to c, and hence also a to c. The descriptions of all the voting procedures listed in this as well as in subsequent sections appear in Chap. 2.

5.3.1 Black's Procedure

Suppose an initial profile with 23 voters whose preference orderings are as follows:

3 voters: $a > c > b > d$
3 voters: $a > c > d > b$
6 voters: $b > d > a > c$
6 voters: $c > a > d > b$
5 voters: $c > d > b > a$

In this initial profile a is the Condorcet winner. Now, if *ceteris paribus*, the six $c > a > d > b$ voters abstain then the social preference ordering becomes cyclical ($a > c > b > d > a$)[3] in which case the winner is determined according to Borda's procedure. This winner is c (with a Borda score of 27) which is a preferable result for the six abstainers than the election of a. Therefore Black's procedure is vulnerable to the No-Show paradox even when the initial profile contains a Condorcet winner.

5.3.2 Kemeny's Procedure

The example used in Sect. 5.3.1 to demonstrate the vulnerability of Black's procedure to the No-Show paradox under our restricted domain assumption applies to Kemeny's procedure too. In the initial profile a is the Condorcet winner and is elected under Kemeny's procedure. If the six $c > a > d > b$ voters abstain the social preference ordering becomes cyclical in which case the transitive social preference ordering according to Kemeny's procedure is $c > b > d > a$ (with the highest sum of 58 pairwise voter agreements with this possible social preference ordering), i.e., c is elected according to Kemeny's procedure which is preferable for the six abstainers to the election of a. It is therefore concluded that Kemeny's procedure too is vulnerable to the No-Show paradox even if a Condorcet winner exists (and is elected) in the initial profile.

5.3.3 Nanson's and the BER (Baldwin's) Procedures

Let the initial profile be the same 23 voters and 4 candidates as under Black's procedure. In this initial profile a is the Condorcet winner and is therefore elected under

[3]Read: the majority of voters prefer a to c, c to b, b to d, and d to a.

both Nanson's and Baldwin's procedures. Now, if *ceteris paribus*, the six $c > a > d > b$ voters abstain then the Borda scores of a, b, c, d in the first count are 24, 26, 27, and 25, respectively, with an average score of 25.5—so according to Nanson's procedure both a and d are deleted and thereafter c beats b (11:6) and becomes the Nanson winner—which is preferable for the six abstainers. Similarly under the BER procedure: after the first counting round a (whose Borda score is 24) is deleted, in the second counting round d (whose Borda score is 14) is deleted, and in the third counting round c beats b (11:6). Thus both Nanson's and the BER procedures are vulnerable to the No-Show paradox even under our restricted domain assumption.

5.3.4 Successive Elimination Procedure

Suppose the initial profile has seven voters with the following preference orderings (cf., Felsenthal & Nurmi, 2017, pp. 64–65). Suppose further that under this procedure the elimination is conducted as follows: first candidate a competes against b and the loser is eliminated; thereafter the winner competes against c.

2 voters: $a > b > c$
2 voters: $b > c > a$
1 voter: $c > a > b$
2 voters: $c > b > a$

Accordingly, in this initial profile b is the Condorcet winner. Now suppose that, *ceteris paribus*, the two $c > b > a$ voters abstain. In this case a would be elected in the first stage, and thereafter c will beat a and will become the final winner—which is preferable for the abstainers to the election of b. This demonstrates the vulnerability of the Successive Elimination procedure to the No-Show paradox even in (initial) profiles containing a Condorcet winner.

5.3.5 Young's Procedure

Let the initial profile be one with 49 voters whose preference orderings are as follows (Felsenthal, 2012, pp. 87–88; Nurmi, 2012, pp. 266–267):

11 voters: $b > a > d > e > c$
10 voters: $e > c > b > d > a$
10 voters: $e > d > a > b > c$
10 voters: $a > c > d > b > e$
2 voters: $e > c > d > b > a$
2 voters: $e > d > c > b > a$
2 voters: $c > b > a > d > e$
1 voter: $d > c > b > a > e$

1 voter: $a > b > d > e > c$

Here d is the Condorcet winner. Now suppose that, *ceteris paribus*, the 10 $e > d > a > b > c$ voters abstain. As a result in the diminished profile candidate e needs the removal of only 12 voters in order to become the Condorcet winner (the 10 $a > c > b > d > e$ voters and the 2 voters whose top preference is c), whereas each of the other candidates needs more removals in order to become a Condorcet winner. So according to Young's procedure e becomes the winner in the reduced profile—which is preferable for the abstainers to the election of d. Thus Young's procedure can display the No-Show paradox even under the restricted domain where a Condorcet winner exists in the initial profile.

5.3.6 Copeland's Procedure

Consider the initial profile with 21 voters whose preference orderings are as follows:

3 voters: $a > c > b > d$
3 voters: $a > c > d > b$
5 voters: $b > d > a > c$
5 voters: $c > a > d > b$
5 voters: $c > d > b > a$

Here a is the Condorcet winner. Now suppose that the five $c > a > d > b$ voters decide to abstain. As a result c becomes the Copeland winner which is preferable for the five abstainers to the election of a. Thus, the No-Show paradox can occur when using Copeland's procedure even in profiles containing a Condorcet winner.

5.3.7 Dodgson's Procedure

We start with the initial profile where there are 19 voters whose preference orderings are as follows:

5 voters: $d > b > c > a$
4 voters: $d > a > b > c$
4 voters: $b > c > a > d$
3 voters: $a > d > c > b$
3 voters: $a > d > b > c$

Here a is the Condorcet winner. Now suppose that, *ceteris paribus*, the four $d > a > b > c$ voters decide to abstain. As a result, in the reduced profile it would take for d only three preference switches to become a Condorcet winner (if three $b > c > a > d$ voters change their preference ordering to $b > c > d > a$) whereas each of the other candidates needs more than three preference switches in order to become a Condorcet winner. So according to Dodgson's procedure *d* becomes the winner in the reduced profile which is a preferable outcome for the four abstainers.

5.3.8 Coombs's and the Negative Plurality Elimination Rule (NPER) Procedures

Consider the initial profile with 19 voters whose preference orderings are as follows (Felsenthal, 2012, p. 78; Nurmi, 2012, pp. 266–267):

5 voters: $d > b > c > a$
4 voters: $d > a > b > c$
4 voters: $b > c > a > d$
3 voters: $a > d > c > b$
3 voters: $a > d > b > c$

In the initial profile *a* is the Condorcet winner and although Coombs's and the NPER procedures do not necessarily elect a Condorcet winner when one exists, *a* is nevertheless elected under Coombs's and the NPER procedures in this example. (As in the initial profile there is no candidate who is ranked first by the majority of voters, candidate *c*—who is ranked last by the plurality of voters—is eliminated from all ballots according to Coombs's and the NPER procedures, thereafter candidate *b* is eliminated from all ballots, and thus finally candidate *a*, the Condorcet winner, becomes the candidate listed first in the ballots of the majority of voters and is therefore declared the winner according to Coombs's and the NPER procedures).

Now suppose that, *ceteris paribus*, the four $d > a > b > c$ voters abstain. As in the reduced 15-voter profile there is no candidate who is ranked first by the majority of voters, candidate *a* (who is ranked last by the plurality of voters) is eliminated from all ballots according to Coombs's and the NPER procedures. Thereafter candidate *d* is ranked first by a majority of voters and hence is elected according to Coombs's procedure—which is a preferable outcome for the four $d > a > b > c$ abstainers to the election of *a* (the Condorcet winner in the original profile). After the elimination of *a* in the first counting round, candidates *c* and *b* are eliminated in the second and third counting rounds, respectively, according to the NPER procedure, so *d* becomes the winner according to this procedure too.

5.3.9 The Majority Judgment Procedure

The MJ procedure is vulnerable to No-Show paradox also under the restricted domain assumption. To see this consider the following example.[4]

[4]This example refutes the statement made by Felsenthal and Nurmi (2018) that the MJ procedure is invulnerable to the No-Show paradox under the restricted domain assumption.

Suppose there are seven voters, V_1–V_7 who rank originally two candidates, *x* and *y*, on a scale from A (lowest) to F (highest), as follows:

Alternative/voter	V_1	V_2	V_3	V_4	V_5	V_6	V_7	Median
x	A	A	C	F	B	F	C	C
y	B	B	B	E	D	D	A	B

Here *x* is the Condorcet winner and is elected according to the MJ procedure because its median rank is higher than *y*'s.

Now suppose that, *ceteris paribus*, voters V_1 and V_2, who prefer the election of *y*, decide to abstain. As a result one obtains the following distribution of grades among the two candidates:

Alternative/voter	V_3	V_4	V_5	V_6	V_7	Median
x	C	F	B	F	C	C
y	B	E	D	D	A	D

As a result of this abstention *y* wins—which would be a preferable outcome for the two abstainers than the original election of *x*.

5.4 Proofs Regarding the Impossibility of the No-Show Paradox Under Two Condorcet-Consistent and Three Non-Condorcet-Consistent Procedures When a Condorcet Winner Exists and Is Elected in the Initial Profile

5.4.1 Minimax Procedure

The Minimax procedure is one of two Condorcet-consistent procedures investigated which is invulnerable to the No-Show paradox when a Condorcet winner exists in the initial profile (the second procedure is Schwartz's as explained in 5.4.2 below). This can be seen from the following argument. Denote by $n(x, y)$ the number of voters strictly preferring *x* to *y* in profile P and denote by $n'(x, y)$ the number of voters strictly preferring *x* to *y* in profile P′. These profiles are now defined. Let *c* be the Condorcet winner in the original profile P, and let a group G consisting of *g* voters with identical preferences and strictly preferring another alternative *x* to *c* leave P, so that the remaining electorate constitutes profile P′. With *g* identically minded voters now abstaining, *x*'s support in all pairwise comparisons involving alternatives that G ranks lower than *x* (including, *inter alia*, *c*) diminishes by *g* votes, while all other *x*'s pairwise comparisons remain the same. Since G ranks x higher than c, for all those alternatives z that differ from c and x, if $n(c, z) - n'(c, z) = g$ then also $n(x, z) - n'(x, z) = g$, but not conversely, i.e., there is at least one alternative, w, such that $n(x, w) - n'(x, w) = g$, but $n(c, w) - n'(c, w) = 0$. It then follows that if min $n(c, z) >$ min $n(x, z)$, so must be min $n'(c, z) >$ min $n'(x, z)$. Thus the Minimax procedure cannot lead to a No-Show paradox when the initial profile contains a Condorcet winner.

5.4.2 Schwartz's Procedure

Let c be the Condorcet winner in the original profile P. By abstaining, *ceteris paribus*, and thereby creating profile P′, a group G of g like-minded voters can, at most, bring about a change in the choice outcome either (i) by replacing c with another Condorcet winner, say x, or (ii) by creating a multi-member choice set. In case (i) the outcome cannot be better for G since in P it prefers c to x. In case (ii) the Schwartz set consists of at least one candidate, say d, that is regarded worse than c by G, for otherwise d that is not in the Schwartz set in P would not be in the Schwartz set in P′. Hence, abstaining cannot bring about a better outcome for the abstainers in the restricted domain.

5.4.3 The Plurality with Runoff Procedure

Suppose x is the Condorcet and Plurality with Runoff winner in profile P and a group G consisting of g voters with the same preference ordering decides to abstain. Obviously, if x is their first-ranked alternative, they cannot benefit from abstaining. So assume that x is their second- or lower-ranked alternative. Now, to make a difference, G's abstaining has to change one of the runoff contestants, while the first-round support of the others remains as it was in the original profile. Suppose the runoff contestants in the original profile were x and w, while in the reduced profile they are x and z (x will have to be one of the runoff contestants in the reduced profile, since G did not rank it first. Hence x's plurality count remains the same as in the original profile). Can z now be preferred to x by G and at the same time defeat x in the second round of the reduced profile election? Now, if z was preferred to x by G in P, then it must be those voters not in G (i.e., voters in N-G) that turned the pairwise victory of z over x into a victory of x over z since x was the elected Condorcet winner in P. These voters are not abstaining in P′. Hence they still guarantee the victory of x over z in P′. So z cannot be the runoff winner in the reduced profile. Thus, the only way G can make a difference by abstaining is to bring about an outcome that is worse than x for G. Therefore, the Plurality with Runoff procedure is invulnerable to the No-Show paradox if the initial profile contains a Condorcet winner which at the same time is also the Plurality Runoff winner. (The possibility that the winner in P′ is found already on the first round cannot be a result of a successful abstaining of G since by abstaining G only affects the absolute and relative plurality count of alternatives that are lower than top in G's ranking. Suppose that G prefers *v* to the original Condorcet and Plurality with Runoff winner, *x*. Then by abstaining it can at most make *x* the first-round winner in P′ because either *x*'s plurality count now exceeds 50% or make no difference at all).

5.4.4 The Alternative Vote Procedure

The Alternative Vote procedure is also invulnerable to the No-Show paradox when the initial profile contains a Condorcet winner which is the Alternative Vote winner as well. This is so for the following reasons.

Let *x* be the Alternative Vote and Condorcet winner in the initial profile P. Suppose a group G of *g* identically-minded voters who prefer some other candidate to *x* leaves P, *ceteris paribus*, and denote the remaining reduced profile by P′. Can the ensuing Alternative Vote winner in P′ be preferred to *x* by G? No, for the following reason. If *x* is elected only on the basis of the top-ranked alternatives in P (that would make *x* the Absolute Winner in P and in P′), then a removal of G maintains *x*'s winning position *a fortiori* since *x* is not ranked first by G and the threshold of the required majority is smaller in P′ than in P. If, on the other hand, *x* is elected under the Alternative Vote procedure in P after removing (sequentially) some other alternative(s) which is (are) ranked first by a smaller number of voters than *x*, then the removal of G may decrease (due to the decreased required majority threshold) the number of candidates that must be removed in P′ before some candidate is ranked first by a majority of the voters. Can this candidate be different than *x*, say *z*, who is preferred by G over *x*? No. The fact that *x*, the Condorcet winner, is elected in P implies that a majority of voters in P preferred *x* to *z*. These voters do not belong to G and none of them is inclined to abstain in P′. All those individuals ranking *x* higher than *z* in P have the same preference in P′, while strictly fewer (namely *g* fewer) individuals rank *z* higher than *x* in P′ than in P. At the same time, the relative positions of all alternatives (including *x* and *z*) remain precisely the same in P and P′ among those voters who are not members of G. Since the voters preferring *x* to *z* continue to constitute the majority in P′ too, it is not possible that *z* be elected in P′.

5.4.5 Bucklin's Procedure

This voting procedure, too, is invulnerable to the No-Show paradox when the initial profile contains a Condorcet winner which is also the Bucklin winner. This is so for the following reasons.

Let *x* be the Bucklin and Condorcet winner in the initial profile P. Suppose a group G of *g* identically-minded voters who prefer some other candidate to *x* leaves P, *ceteris paribus*, and denote the remaining reduced profile by P′. Can the ensuing

Bucklin winner in P′ be preferred to *x* by G? No, for the following reason. If *x* is elected in P because it is ranked first by an absolute majority of the voters then this would make *x* also the Absolute Winner in P′. So a removal of G maintains *x*’s winning position *a fortiori* since *x* is not first ranked by G and the threshold of required majority is smaller in P′ than in P. If, on the other hand, *x* is elected under Bucklin in P after inclusion of lower than first ranked alternatives, then the removal of G may change the number of ranks that have to be taken into account in determining the Bucklin winner in P′. To wit, the majority threshold may be reached at an earlier stage by, say, *z*. Can *z* be preferable to *x* by G? No, it cannot since all alternatives ranked higher than *x* by G (including *z*) have equal or smaller first, second, etc., rank counts in P′ than in P. In other words, whatever advantage *z* gets in terms of shifting the number of ranks considered in order to find the winner in P′ is offset by the advantage accruing to *x* since the removal of G improves *x*’s relative standing *vis-à-vis z*.

5.5 Proofs Regarding the General Impossibility of the No-Show Paradox Under Four Non-Condorcet-Consistent Procedures

5.5.1 Plurality Voting

The Plurality Voting procedure is generally invulnerable to the No-Show paradox since the selected alternative, say *x*, which by definition is ranked first by the plurality of voters, can be changed to another winner, say *y*, only if some voters originally ranking *x* first, abstain. This is because the abstaining of any other voters only increases *x*’s plurality margin with respect to those candidates ranked first by the abstaining voters. Also those originally ranking *x* first cannot benefit from abstaining since thereby they decrease *x*’s plurality count, possibly even rendering *x* a non-winner. Thus, no voters can benefit from abstaining under the Plurality Voting procedure.

5.5.2 Approval Voting Procedure

The Approval Voting procedure is generally invulnerable to the No-Show paradox for the same reasons that the Plurality voting procedure is generally invulnerable to this paradox assuming that the improvement of a candidate’s position does not change its approvability status, i.e., the candidates approved initially will remain approved after improvement and the same holds for disapproved candidates.

5.5.3 Borda's Procedure

The Borda procedure is not susceptible to the No-Show paradox because the winning alternative under Borda's procedure is the alternative whose sum in the pairwise comparison matrix is largest. Therefore any single voter who abstains decreases by 1 the entries in the pairwise comparison matrix that fit his/her preference ordering. Thus, for example, if there are four alternatives, *a*, *b*, *c*, *d*, and, *ceteris paribus*, a voter whose preference ordering is $a > b > c > d$ abstains, then the entries in the cells *(a, b), (a, c), (a, d), (b, c), (b, d)* and *(c,d)* in the pairwise comparison matrix decrease by 1 each—consequently the sum in row *a* (the most preferred alternative) is decreased by 3, the sum in row *b* (the second most preferred alternative) is decreased by 2, the sum in row *c* (the third most preferred alternative) is decreased by 1, and the sum of row *d* (the least preferred alternative) is not changed. So a voter whose preference ordering is $a > b > c > d$ is not only unable to benefit by abstaining, but may even obtain a worse outcome by doing so as there is an increasing probability that the less preferable an alternative is, the more likely it may end up as the selected alternative because the decrease in the sum of its row becomes increasingly smaller.

5.5.4 Range Voting Procedure

If *x* is the Range Voting winner in a profile, no voter ranking *x* first can improve the outcome by abstaining since by so doing s/he decreases the score of *x* thereby possibly making it a non-winner. The same applies to his/her second ranked candidate: by giving this candidate the second-largest number of points s/he might turn a non-winning candidate into a winner, and so on. So, whatever the distribution of points sums over candidates, the voter cannot benefit from abstaining when compared with voting according to his/her true preferences. Furthermore, if *x* is the Range Voting winner, no candidate *y* can become the winner in circumstances where a group of voters ranking *y* last joins the electorate. This is because *y* receives less value from the new entrants than any other alternative including *x*. Therefore, *y* cannot become the winner in the new profile. (This does not say that *x* remains the winner in the new profile, only that *y* isn't).

5.6 Concluding Remarks

Condorcet winners are usually considered to be relatively stable outcomes and hence the profile changes required to upset those outcomes are of considerable interest. Given Moulin's (1988) seminal result on the incompatibility, in general, of Condorcet-consistency and invulnerability to the No-Show paradox, we considered it worthwhile to examine whether this incompatibility is associated with only those profiles where a majority cycle—and hence a relatively unstable original setting—prevails. On the other hand, we were also interested in examining whether several ranked non-Condorcet-consistent procedures which are vulnerable, in general, to the No-Show paradox, would also exhibit this paradox when a Condorcet winner is present and elected in the initial profile. Our results reported above may seem a bit surprising: all Condorcet-consistent procedures examined except two (Minimax and Schwartz's procedure) are vulnerable to the No-Show paradox when a Condorcet winner exists in the initial profile, while all the non-Condorcet-consistent procedures examined except three (Coombs's, the NPER and the MJ procedures) are not vulnerable to this paradox in a restricted domain where a Condorcet winner exists and is elected in the initial profile. So it seems that under one type of voting procedures the existence (and election) of a Condorcet winner in the initial profile almost always guarantees a stable outcome, while most (seemingly more desirable) election procedures—which guarantee the election of a Condorcet winner if one exists in the initial profile—do not necessarily guarantee a stable outcome.

Exercises for Chapter 5

Problem 5.1
The strong No-Show paradox occurs in a profile where a group of identically-minded voters is not only better off abstaining, *ceteris paribus*, but gets its most preferred candidate elected, whereas by voting according to its preferences some less preferred candidate wins. Are there any instances of the strong No-Show paradox among the preceding examples?

Problem 5.2
A candidate, x, is said to be Pareto-dominated in a given a preference profile if *all* voters prefer some other candidate y to x. Show by way of an example that the Successive Elimination procedure may nevertheless lead to a Pareto dominated candidate being elected.

Problem 5.3
Given the example just constructed, would it be correct to state that (a) the elected candidate is necessarily a Condorcet loser and/or that (b) the elected candidate is never the first-ranked by any voter, and/or (c) every voter would have been better off had the voting agenda been different?

Problem 5.4
Show by way of an example that the Plurality with Runoff procedure is vulnerable to the No-Show paradox when there is no Condorcet winner in the original profile.

Problem 5.5
Show by way of an example that the Plurality with Runoff procedure is vulnerable to the No-Show paradox in the Condorcet domain when the Condorcet winner exists but is NOT elected in the initial profile.

Problem 5.6
How does the Alternative Vote procedure perform in the above two settings?

Answers to Exercises of Chapter 5

Problem 5.1
All profiles discussed in Sects. 5.3.1–5.3.8 exhibit the vulnerability of the respective procedures to the strong No-Show paradox.

Problem 5.2
Consider the following profile

1 voter: $a > b > d > c$
1 voter: $b > d > c > a$
1 voter: $d > c > a > b$

and the agenda of pairwise majority votes: (i) b versus d, (ii) the winner versus a, (iii) the winner versus c. Here c wins, but is Pareto-dominated by d.

Problem 5.3
(a) No, it is not; (b) yes, it is correct; (c) yes, it is correct.

Problem 5.4
Consider the following profile

6 voters: $a > b > c$
5 voters: $b > c > a$
4 voters: $c > a > b$

Here a wins. Now remove two $b > c > a$ voters, then c wins.

Problem 5.5
Consider the following profile

8 voters: $a > c > b$
5 voters: $b > c > a$
4 voters: $c > b > a$

Here c is the Condorcet winner, but b wins the runoff against a. Now, remove five $a > c > b$ voters and c becomes the winner.

Problem 5.6
In the same way as the Plurality with Runoff procedure, as is always the case in three-candidate contests.

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