Minimum of Constrained OWA Aggregation Problem with a Single Constraint

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Abstract. In a recent paper we found an analytical formula for the constrained ordered weighted aggregation problem (OWA) when we need to maximize the objective function. In this note we prove that the method works in the case when we need to minimize the objective function. If in the case of the maximization problem we need to rearrange the coefficients in the constrained in nondecreasing order, for the nontrivial minimization problem, it suffice to rearrange them in nonincreasing order.

Keywords: OWA operators · Constrained optimization · Constrained OWA aggregation

1 Introduction

The OWA operators (ordered weighted average operators) were introduced by Yagger in paper [8]. Since then, OWA operators were successfully used in research fields that belong in broad sense to decision making. One interesting problem is to optimize the OWA operator. This type of investigation started with paper [7] and since then it became a challenging problem for researchers. The issue is that we lack an analytical formula for the solution function. In order to avoid repetition, we refer to our recent paper [2] where the problem is discussed in detail. Then, we refer to the surveys [3] and [4] where the reader can find about many optimization problems related to the OWA operators. Our interest in this topic is to find those types of optimization problems where we can find an analytical expression for the solution function. In paper [7] the idea was to transform the problem into a mixed integer linear problem. As the number of variables increases significantly an some of them are restricted to be integers, it seems hard to find an analytical expression for the solution function in general. The first such concrete result can be found in paper [1] in the special case when we have a single constraint and all coefficients are equal to one. This result was

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generalized recently in paper [2] where the coefficients are arbitrary this time. The method used in this paper to obtain the analytical expression of the solution function in the case when we maximize the objective function can be adapted in order to find the analytical solution function when we need to minimize the objective function. This is what we will do in this note. A common feature in solving all these problems is that the constrained OWA aggregation problems are transformed into linear programs and the analytical expression of the solution function is obtained using the dual of these linear programs. It is important to mention that there are other works too where one uses the dual of linear programs in order to obtain the solution of certain type of constrained OWA aggregation problems (see papers [5,6]).

The paper is organized as follows. In Sect. 2 we recall the basic theory on the constrained OWA aggregation problem and we also recall our result from the recent paper [2] where we found the analytical expression of the solution function when we have a single constraint with arbitrary coefficients and the objective function needs to be maximized. In Sect. 3, this time we will need to minimize the objective function. Again, we will have a single constraint with arbitrary coefficients. If in the case of the maximization problem we need to rearrange the coefficients in nondecreasing order, for the minimization problem it suffice to rearrange them in nonincreasing order. This similar approach is a consequence of an inequality (often referred as Chebyshev inequality) on finite sequences of reals. There are some differences considering the two types of problems but the cases when the coefficients are positive give a similar type of solution function. It is important to note that in the case of the minimization problem it is not indicated to transform the problem into a maximization problem by considering the opposite of the objective function. In this case, we lose the positiveness of the weights and the solving becomes more complicated. What is more, we cannot use the formulae obtained in paper [2] because there the positiveness of the weights is essential. Indeed, as we said, the solution of this problem is obtained by using the solution of the dual of a linear program. But this solution needs to have positive components and this does not hold if instead of positive weights we consider they opposite values. Section 4 presents an example where both problems, maximization and minimization, are solved according to the expressions of the solution function. The paper ends with conclusions where the main results are discussed and further research on the topic is addressed.

2 Optimization of OWA Operators

Suppose we have the nonnegative weights $w_1, ..., w_n$ such that $w_1 + ... + w_n = 1$ and define a mapping $F : \mathbb{R}^n \to [0, 1]$,

$$F(x_1, ..., x_n) = \sum_{i=1}^{n} w_i y_i,$$

where y_i is the i-th largest element of the sample $x_1, ..., x_n$. This is called an OWA operator associated to the weights $w_1, ..., w_n$ (see [8]). Then consider a matrix A of type (m, n) with real entries and a vector $b \in \mathbb{R}^m$. A constrained OWA aggregation problem corresponding to the above data, is the problem

$$\max F(x_1, ..., x_n)$$

subject to

$$Ax \leq b,\ x \geq 0.$$

This problem was proposed by Yagger in [7]. A difficult task is to find an exact analytical solution to this problem. Yagger used a method based on mixed integer linear programming problem which employes the use of auxiliary integer variables and therefore, this method is not always effective. In the special case where we have the single constraint $x_1 + ... + x_n = 1$, the first analytical solution for the constrained OWA aggregation problem is given in paper [1]. This result has been generalized recently in paper [2] where the coefficients in the constraint are arbitrary. This problem can be formulated as

$$\max F(x_1, ..., x_n) \tag{1}$$

subject to

$$\alpha_1 x_1 + ... + \alpha_n x_n \leq 1,$$
$$x \geq 0$$

Let us recall this result in the case when we can provide a nontrivial solution (these cases were solved in Propositions 1–2 in [2]). In what follows, S_n denotes the set of permutations of the set $\{1, ..., n\}$.

Theorem 1. *Consider problem (1). Then:*

(i) if there exists $i_0 \in \{1, ..., n\}$ such that $\alpha_{i_0} \leq 0$, then F is unbounded on the feasible set and its supremum over the feasible set is ∞;

(ii) if $\alpha_i > 0$, $i \in \{1, ..., n\}$, then taking (any) $\sigma \in S_n$ with the property that $\alpha_{\sigma_1} \leq \alpha_{\sigma_2} \leq ... \leq \alpha_{\sigma_n}$, and $k^* \in \{1, ..., n\}$, such that

$$\frac{w_1 + ... + w_{k^*}}{\alpha_{\sigma_1} + ... + \alpha_{\sigma_{k^*}}} = \max\left\{\frac{w_1 + ... + w_k}{\alpha_{\sigma_1} + ... + \alpha_{\sigma_k}} : k \in \{1, ..., n\}\right\},$$

then $(x_1^*, ..., x_n^*)$ is an optimal solution of problem (1), where

$$x_{\sigma_1}^* = ... = x_{\sigma_{k^*}}^* = \frac{1}{\alpha_{\sigma_1} + ... + \alpha_{\sigma_{k^*}}},$$
$$x_{\sigma_{k^*+1}}^* = ... = x_{\sigma_n}^* = 0.$$

In particular, if $0 < \alpha_1 \leq \alpha_2 \leq ... \leq \alpha_n$, and $k^* \in \{1, ..., n\}$ is such that

$$\frac{w_1 + ... + w_{k^*}}{\alpha_1 + ... + \alpha_{k^*}} = \max\left\{\frac{w_1 + ... + w_k}{\alpha_1 + ... + \alpha_k} : k \in \{1, ..., n\}\right\},$$

then $(x_1^*, ..., x_n^*)$ is a solution of (1), where

$$x_1^* = ... = x_{k^*}^* = \frac{1}{\alpha_1 + ... + \alpha_{k^*}},$$
$$x_{k^*+1}^* = ... = x_n^* = 0.$$

3 Minimizing the Objective Function

In this section we discuss the case when we search for the minimum in the objective function. It seems that we can apply a similar approach as in the case when the objective function is maximized. The general form is

$$\min F(x_1, ..., x_n) \tag{2}$$

subject to

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \leq \beta,$$
$$x \geq 0.$$

Again, we will consider one restriction but in general form. Obviously it suffices to consider only the following three problems (any other problem is reduced to one of them)

$$\min F(x_1, ..., x_n) \tag{3}$$

subject to

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \leq 0,$$
$$x \geq 0$$

$$\min F(x_1, ..., x_n) \tag{4}$$

subject to

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \leq 1,$$
$$x \geq 0$$

and

$$\min F(x_1, ..., x_n) \tag{5}$$

subject to

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \geq 1,$$
$$x \geq 0.$$

The solving of the first two problems is trivial. We observe that in both cases we have the unique solution $(0, 0, ..., 0)$, hence the minimum is 0 for both problems.

Let us discuss now the more interesting problem (5). The first result proves that in searching for the solution, in the case of positive weights it suffices to consider equality in the constraint.

Proposition 1. *Consider problem (5) If* $(x_1^*, ..., x_n^*)$ *is a solution of problem (5) and* $w_i > 0$, $i = 1, ..., n$, *then* $\alpha_1 x_1^* + ... + \alpha_n x_n^* = 1$.

Proof. If the conclusion were false, then we would have $\alpha_1 x_1^* + \cdots + \alpha_n x_n^* > 1$. Obviously, there exists at least one strictly greater than zero component in $(x_1^*, ..., x_n^*)$. Suppose these components are $x_{k_1}^*, ...x_{k_l}^*$. Then, there exists $\varepsilon > 0$ sufficiently small such that $\alpha_1 y_1^* + \cdots + \alpha_n y_n^* > 1$, where $y_{k_i}^* = x_{k_i}^* - \varepsilon > 0$, $i = 1, ..., l$, and all the other components are equal to 0 . Clearly this implies that $(y_1^*, ..., y_n^*)$ is feasible to our problem. What is more, we easily notice that $F(y_1^*, ..., y_n^*) < F(x_1^*, ..., x_n^*)$, which again, contradicts the minimality of $(x_1^*, ..., x_n^*)$.

Let us now discuss on the coefficients of the first constraint. If $\alpha_i \leq 0$ for all $i \in \{1, ..., n\}$ then we have no solution since the feasible set is empty. Next, suppose that there exists $i \in \{1, ..., n\}$ such that $\alpha_i \leq 0$. If $(x_1^*, ..., x_n^*)$ is a solution of problem (5) then it is sufficient to take $x_i^* = 0$ because otherwise, if $x_i^* > 0$, then it is really easy to prove that $(y_1^*, ..., y_n^*)$, where $x_j^* = y_j^*$ if $i \neq j$ and $y_i^* = 0$, belongs to the feasible set of problem (5) and $F(x_1^*, ..., x_n^*) \geq F(y_1^*, ..., y_n^*)$, hence $(y_1^*, ..., y_n^*)$ too, is a solution for (5). It means that if in problem (5) we have nonpositive coefficients in the restriction of problem (5), then we can reduce this problem to a problem where all coefficients are strictly greater than zero (we just eliminate the nonpositive coefficients and the weights from bottom, for example, if only $\alpha_1 \leq 0$ and $\alpha_2 \leq 0$, then in the new problem we eliminate these coefficients and the weights w_{n-1} and w_n) and a solution of the initial problem will be obtained by completing with zeros on the positions where the nonpositive coefficients were standing. For example, if only $\alpha_1 \leq 0$ and $\alpha_2 \leq 0$ then, if $(x_1^*, ..., x_{n-2}^*)$ is a solution of the problem where the coefficients α_1, α_2 and the last two weights are eliminated, then $(0, 0, x_1^*, ..., x_{n-2}^*)$ is a solution of the initial problem. It is important to mention that if the weights are positive and $\alpha_i \leq 0$ for some $i \in \{1, ..., n\}$, then it necessarily follows that $x_i^* = 0$. Indeed reasoning as above, this time we would get $F(x_1^*, ..., x_n^*) > F(y_1^*, ..., y_n^*)$, and this contradicts the minimality of $(x_1^*, ..., x_n^*)$.

Therefore, it will not be at all a limitation for the general case if in all that follows we assume that in problem (5) we have $\alpha_i > 0$, $i = 1, ..., n$. We start with the special case when $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$, and only after we shall discuss the general case. If $(x_1^*, ..., x_n^*)$ is a solution of the problem then let $\sigma \in S_n$ be any permutation such that $x_{\sigma_1}^* \geq x_{\sigma_2}^* \geq \cdots \geq x_{\sigma_n}^*$. It is well known that if $a_1 \geq a_2 \geq \cdots \geq a_n$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ then $\sum_{i=1}^{n} a_i b_i \geq \sum_{i=1}^{n} a_i b_{\tau_i}$ for any $\tau \in S_n$. This implies that $\sum_{i=1}^{n} \alpha_i x_{\sigma_i}^* \geq \sum_{i=1}^{n} \alpha_i x_i^*$ and hence $\sum_{i=1}^{n} \alpha_i x_{\sigma_i}^* \geq 1$. It means that $(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ is feasible and on the other hand, clearly we have $F(x_1^*, ..., x_n^*) = F(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$, which means that $(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ is a solution of problem (5) as well (in the case when the weights are positive, By Proposition 1 it also means that $\sum_{i=1}^{n} \alpha_i x_{\sigma_i}^* = 1$). But, this implies that $(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ is in addition a solution of the linear programming problem

$$\min \sum_{i=1}^{n} w_i x_i \tag{6}$$

subject to

$$\alpha_1 x_1 + ... + \alpha_n x_n \geq 1,$$
$$x_1 \geq x_2 ... \geq x_n \geq 0.$$

Indeed, it suffices to notice that the feasible set of this problem is included in the feasible set of problem (5) which combined with the fact that $F(x_1^*, ..., x_n^*) = F(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ and $(x_1^*, ..., x_n^*)$ solves (5) while $(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ is feasible to problem (6), all these imply that $(x_{\sigma_1}^*, ..., x_{\sigma_n}^*)$ is a solution of (6).

In view of the above discussion, we start by providing an analytical solution to problem (6). The reasoning is similar to those used in papers [1] and [2] in the cases when we have maximum instead of minimum in the objective function. It is convenient to write the dual of problem (6), which is

$$\max t_1 \tag{7}$$

subject to

$$\begin{aligned} \alpha_1 t_1 + t_2 &\leq w_1, \\ \alpha_2 t_1 - t_2 + t_3 &\leq w_2, \\ &. \\ &. \\ \alpha_{n-1} t_1 - t_{n-1} + t_n &\leq w_{n-1}, \\ \alpha_n t_1 - t_n &\leq w_n, \\ t &\geq 0. \end{aligned}$$

Summing up the first k inequalities from above, $k = \overline{1, n}$, we get

$$t_1 \leq \frac{w_1 + ... + w_k - t_{k+1}}{\alpha_1 + ... + \alpha_k}, \ k = \overline{1, n-1},$$
$$t_1 \leq \frac{w_1 + ... + w_n}{\alpha_1 + ... + \alpha_n}.$$

We easily notice that $t_1 \leq \frac{w_1 + ... + w_{k^*}}{\alpha_1 + ... + \alpha_{k^*}}$, where $k^* \in \{1, ..., n\}$ satisfies

$$\frac{w_1 + ... + w_{k^*}}{\alpha_1 + ... + \alpha_{k^*}} = \min \left\{ \frac{w_1 + ... + w_k}{\alpha_1 + ... + \alpha_k} : k \in \{1, ..., n\} \right\}.$$

It means that $(t_1^*, ..., t_n^*)$ is a solution of (7), where

$$t_1^* = \frac{w_1 + ... + w_{k^*}}{\alpha_1 + ... + \alpha_{k^*}},$$
$$t_{k+1}^* = \left(\frac{w_1 + ... + w_k}{\alpha_1 + ... + \alpha_k} - t_1^* \right) (\alpha_1 + ... + \alpha_k), k = \overline{1, n-1}.$$

From the duality theorem, if there exists $(x_1^*, ..., x_n^*)$ in the feasible set of problem (6), such that $\sum_{i=1}^{n} w_i x_i^* = t_1^*$, then $(x_1^*, ..., x_n^*)$ is a solution of problem (6). Obviously this solution exists since we can take

$$x_1^* = ... = x_{k^*}^* = \frac{1}{\alpha_1 + ... + \alpha_{k^*}},$$
$$x_{k^*+1}^* = ... = x_n^* = 0.$$

We are now in position to present an analytical solution for the general case of problem (5). We will just need to rearrange the order of the coefficients and variables in order to use the formula from above. We reiterate again the fact that it is not a limitation to assume that the coefficients are positive.

Theorem 2. *Consider problem (5). If $\alpha_i > 0$, $i \in \{1, ..., n\}$, then taking $\sigma \in S_n$ (it is possible to have multiple choices for σ) with the property that $\alpha_{\sigma_1} \geq \alpha_{\sigma_2} \geq \cdots \geq \alpha_{\sigma_n}$, and $k^* \in \{1, ..., n\}$, such that*

$$\frac{w_1 + \cdots + w_{k^*}}{\alpha_{\sigma_1} + \cdots + \alpha_{\sigma_{k^*}}} = \min \left\{ \frac{w_1 + \cdots + w_k}{\alpha_{\sigma_1} + \cdots + \alpha_{\sigma_k}} : k \in \{1, ..., n\} \right\},$$

then $(x_1^*, ..., x_n^*)$ is an optimal solution of problem (5), where

$$x_{\sigma_1}^* = ... = x_{\sigma_{k^*}}^* = \frac{1}{\alpha_{\sigma_1} + ... + \alpha_{\sigma_{k^*}}},$$
$$x_{\sigma_{k^*+1}}^* = ... = x_{\sigma_n}^* = 0.$$

In particular, if $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$, and $k^* \in \{1, ..., n\}$ is such that

$$\frac{w_1 + \cdots + w_{k^*}}{\alpha_1 + \cdots + \alpha_{k^*}} = \min \left\{ \frac{w_1 + \cdots + w_k}{\alpha_1 + \cdots + \alpha_k} : k \in \{1, ..., n\} \right\},$$

then $(x_1^*, ..., x_n^*)$ is a solution of (5), where

$$x_1^* = ... = x_{k^*}^* = \frac{1}{\alpha_1 + \cdots + \alpha_{k^*}},$$
$$x_{k^*+1}^* = ... = x_n^* = 0.$$

As we said in the introduction, transforming problem (5) into a maximization problem, that is, considering $\max -F(x_1, ..., x_n)$ instead of $\min F(x_1, ..., x_n)$, will not lead to a simpler method to find the solution because we loose the positiveness of the weights which is essential in finding the solution of the dual problem that leads to the solution given in Theorem 1. There is another possibility to transform problem (5) into a maximization problem, but in this case too, we do not get an easier method. First, let us discuss the special case when all the coefficients in the constraint are equal to one, that is, we consider problem

$$\min F(x_1, ..., x_n)$$

subject to

$$x_1 + \cdots + x_n \geq 1,$$
$$x \geq 0.$$

As we know, without any loss of generality we may assume that the constraint is $x_1 + \cdots + x_n = 1$. Suppose that $(x_1^*, ..., x_n^*)$ is a solution of the problem from above. Denoting with $(y_1^*, ..., y_n^*)$ the vector that rearranges $(x_1^*, ..., x_n^*)$ in nondecreasing order, then using the substitutions $z_i^* = 1 - x_i^*$ and $y_i^* = 1 - t_i^*$, $i = \overline{1,n}$, we get

$$\begin{aligned} & F(x_1^*, ..., x_n^*) \\ &= w_1 y_1^* + \cdots + w_n y_n^* \\ &= w_1 (1 - t_1^*) + \cdots + w_n (1 - t_n^*) \\ &= \sum_{i=1}^n w_i - \sum_{i=1}^n w_i t_i^* \end{aligned}$$

and

$$\begin{aligned} & x_1^* + \cdots + x_n^* \\ &= n - \sum_{i=1}^n z_i^*. \end{aligned}$$

This easily implies that $(1 - x_1^*, ..., 1 - x_n^*)$ and any of its permutations is a feasible solution for the problem

$$\max \overline{F}(z_1, ..., z_n)$$

subject to

$$z_1 + \cdots + z_n = n - 1,$$
$$z \geq 0,$$

where

$$\begin{aligned} & \overline{F}(z_1, ..., z_n) \\ &= \overline{w}_1 t_1 + \cdots + \overline{w}_n t_n, \end{aligned}$$

$\overline{w}_i = w_{n-i}$ and t_i is the i-th largest element from the sequence z_1,...,z_n. Obviously, this later problem is a constrained OWA aggregation problem and the solution is immediate by applying Theorem 1, (ii). Unfortunately, $(1 - x_1^*, ..., 1 - x_n^*)$ it is not necessarily optimal since in general, the solution can have components strictly larger than 1. Actually, one can easily prove that if $(z_1^*, ..., z_n^*)$ is an optimal solution of problem

$$\max \overline{F}(z_1, ..., z_n)$$

subject to

$$z_1 + \cdots + z_n = n - 1,$$
$$z \geq 0,$$
$$z \leq 1$$

then $(1 - z_1^*, ..., 1 - z_n^*)$ and any of its permutations is an optimal solution of problem (5). Clearly, this problem in general is not of type (1). Now, considering the case of arbitrary coefficients we will arrive to a similar construction, that is, a more complex maximization problem having additional constraints.

Comparing Theorems 1 and 2, respectively, we can easily solve both problems (maximum and minimum) in the case of a single constraint. For the maximum problem we just need to rearrange the coefficients in nondecreasing order and in the case of the minimum problem, we need to rearrange them in nonincreasing order.

4 An Example for Both Maximization and Minimization Problems

Example 1. Suppose that $F(x_1, x_2, x_3, x_4) = \frac{1}{3}y_1 + \frac{1}{8}y_2 + \frac{1}{2}y_3 + \frac{1}{24}y_4$ and consider the constraint $x_1 + 4x_2 + 2x_3 + 3x_4 = 1$. Let us find the maximum point of F. Obviously, the minimum points are exactly the same if the constraint would be $x_1 + 4x_2 + 2x_3 + 3x_4 \leq 1$. Therefore, we can apply the conclusion of Theorem 1. We need a permutation of $\{1, ..., 4\}$ which would rearrange the coefficients in nondecreasing order. Such a permutation is

$$\sigma = \begin{pmatrix} 1\,2\,3\,4 \\ 1\,3\,4\,2 \end{pmatrix}$$

and by simple inspection, we get that

$$\max\left\{\frac{w_1 + ... + w_k}{\alpha_{\sigma_1} + ... + \alpha_{\sigma_k}} : k \in \{1, ..., 4\}\right\}$$

is achieved for $k^* = 1$. Applying the conclusion of Theorem 1, we get that $(x_1^*, ..., x_4^*)$, $x_1^* = 1$, $x_2^* = x_3^* = x_4^* = 0$, is a solution of our problem. We also notice that the maximum value is $F(x_1^*, ..., x_4^*) = \frac{1}{3}$.

Let us find now the minimum of F under the same constraint. Obviously, we have the same solutions if the constraint would be $x_1 + 4x_2 + 2x_3 + 3x_4 \geq 1$. It means that we can apply Theorem 2 for this problem. This time we need a permutation of $\{1, ..., 4\}$ which would rearrange the coefficients in nonincreasing order. Such a permutation is

$$\tau = \begin{pmatrix} 1\,2\,3\,4 \\ 2\,4\,3\,1 \end{pmatrix}$$

and by simple inspection, we get that

$$\min\left\{\frac{w_1+...+w_k}{\alpha_{\sigma_1}+...+\alpha_{\sigma_k}} : k\in\{1,...,4\}\right\}$$

is achieved for $k^*=2$. Applying the conclusion of Theorem 2, we get that $(\overline{x}_1,...,\overline{x}_4)$, $\overline{x}_1=\overline{x}_3=0$, $\overline{x}_2=\overline{x}_4=\frac{1}{7}$, is a solution of our problem. We also notice that the minimum value is $F(\overline{x}_1,...,\overline{x}_4)=\frac{11}{168}$.

5 Conclusions

In this note we completed the work in paper [2], as this time we found the analytical expression of the solution function in the case of minimization of OWA aggregation operators with single constraint. In the future, we are interested to extend the results in the case when we have more constraints. Although in general it seems that the method used in this research and in paper [2] cannot be generalized as we cannot find a single permutation to rearrange monotonically the coefficients in all constraints, some important special cases could be investigated. In the case of two constraints we have an ongoing research and results are promising. Another important problem would be to find the solution of the minimum problem from the solution of a derived maximum problem. This would ease on the computer implementation. This problem as well seems to be quite difficult since even in the simplest case when we have a single constraint with all coefficients equal to 1, we obtained a maximization problem that has additional constraints.

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References

1. Carlsson, C., Fullér, R., Majlender, P.: A note on constrained OWA aggregation. Fuzzy Sets Syst. **139**, 543–546 (2003)
2. Coroianu, L., Fullér, R.: On the constrained OWA aggregation problem with single constraint. Fuzzy Sets Syst. **332**, 37–43 (2018)
3. Emrouznejad, A., Marra, M.: Ordered weighted averaging operators 1988–2014: a citation-based literature survey. Int. J. Intell. Syst. **29**, 994–1014 (2014)
4. Nguyen, T.H.: Maximal entropy and minimal variability OWA operators weights: a short survey of recent developments. arXiv preprint arXiv:1804.06331 (2018). arxiv.org
5. Ogryczak, W., Śliwiński, T.: On efficient WOWA optimization for decision support under risk. Int. J. Approximate Reasoning **50**, 915–928 (2009)
6. Ogryczak, W., Śliwiński, T.: On solving linear programs with the ordered weighted averaging objective. Eur. J. Oper. Res. **148**, 80–91 (2003)
7. Yagger, R.R.: Constrained OWA aggregation. Fuzzy Sets Syst. **81**, 89–101 (1996)
8. Yagger, R.R.: On ordered weighted averaging aggregation operators in multicriteria decisionmaking. IEEE Trans. Syst. Man Cybern. **18**, 183–190 (1988)