Towards a Fuzzy Index of Skewness

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Abstract. The aim of this paper is to investigate the potential of fuzzy regression methods for computing a measure of skewness for the market. A quadratic version of the Ishibuchi and Nii hybrid fuzzy regression method is used to estimate the third order moment. The obtained fuzzy estimates are compared with the one provided by standard market practice. The proposed approach allows us to cope with the limited availability of data and to use all the information that is present in the market.

In the Italian market, the results suggest that the fuzzy-regression based skewness measure is closer to the subsequently realized measure of skewness than the one provided by the standard methodology. In particular, the upper bound of the Ishibuchi and Nii method provides the best forecast. The results are important for investors and policy makers who can rely on fuzzy regression methods to get a more reliable forecast of skewness.

Keywords: Fuzzy regression · Skewness · Forecasting · Italian market

1 Introduction

Moments of a distribution are of paramount importance in finance for portfolio allocation, risk management, trading strategies. Volatility of financial assets has attracted the interest of researchers and practitioners for decades. Only later, researchers have moved their interest towards higher-order moments of the distribution. The increasing importance of higher-order moments is supported by the introduction of the CBOE SKEW index for the S&P500 stock market, which measures the third order moment of the S&P500 risk-neutral distribution [6]. In the CBOE SKEW index, skewness is obtained from option prices by means of the Bakshi et al. formula [1] and reflects the investors' expectation of the realized third moment in the next thirty days.

The Bakshi et al. formula [1] is based on the strong assumption that a continuum of option prices with strike price ranging from zero to infinity is available. As in the market only a limited number of option prices is traded, it is standard market practice to

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generate the missing ones by means of an interpolation-extrapolation procedure of the quoted option prices. Moreover, standard statistical techniques are not able to deal with conflicting information. Therefore, when two options yield different implied volatility, standard market practice retains only out-of-the-money[1] ones and averages the two at-the-money implied volatilities, producing both a considerable loss of information and an element of arbitrariness in the estimation.

A few authors explore the potential of fuzzy techniques to estimate volatility from a limited and conflicting number of option prices (for a literature review see e.g. [15]). References [4, 8] explore fuzzy volatility in the Black-Scholes model [3]. Reference [5] extends previous contributions on the elicitation of the fuzzy volatility membership function in option pricing models by exploiting the Cox-Ross-Rubinstein framework for option pricing developed in [19].

In a model-free setting, [16, 17] combine the Bakshi et al. formula [1] with quadratic fuzzy regression methods (introduced in [18]) to obtain more informative volatility measures. Their methodology presents several advantages. First, it embeds in the estimation of the implied volatility smile function all the information coming from both call and put prices and avoids the a priori choice of discarding some option prices as in standard market practice. Second, the use of fuzzy regression methods ensures the convexity of the volatility smile, and, as a consequence, the absence of arbitrage opportunities. Third, empirical results suggest that the volatility estimates obtained through fuzzy regression methods perform better in forecasting future realized volatility than the volatility measures obtained using the standard procedure.

Given the increasing importance of measuring skewness of the return distribution for both investors and policy makers, and the unsolved problems in the implementation with market data of the Bakshi et al. formula [1], this paper represents the first attempt of computing a skewness index in a fuzzy setting. We complement the existing literature by investigating the potential of fuzzy regression methods to compute a fuzzy measure of skewness for the Italian market. The use of fuzzy regression methods is particularly suitable for this type of data (see [14]). Specifically, fuzzy regression methods allow us to cope with the limited availability of data, given that for the Italian market only a little number of pairs of strike prices and implied volatilities are available to be interpolated. Moreover, it allows us to embed the conflicting information coming from both call and put prices. In fact, for at-the-money strike prices, we have both a call and a put option with different implied volatilities, and standard regression techniques are not able to cope with interval values for the inputs.

An empirical analysis performed in the Italian market (see [18]) concludes that the best estimation method for the volatility smile function is the Ishibuchi and Nii regression method [10], with the preferred h-cut at $h = 0.8$. Therefore, we adopt the quadratic extension of the Ishibuchi and Nii fuzzy regression method to estimate the skewness of the Italian market. In order to assess whether the proposed fuzzy regression method outperform the standard market practice in estimating skewness, we adopt a two-step methodology. First, we evaluate the proposed skewness measure with

[1] An option is said to be at-the-money, in-the-money, or out-of-the-money if it generates a zero, positive, or negative payoff, respectively, if exercised immediately.

respect to its forecasting power on future realized skewness using the mean squared error (MSE) indicator, which provides robust results in the presence of noise in the proxy of skewness. Second, we perform the Model Confidence Set test (see [9]) on the MSE loss function to find the best forecast for future realized skewness. Third, we adopt a defuzzification procedure in order to condense all the information content of fuzzy skewness estimates (which provides investors with an interval of possible values and a most possible value within the interval) in a unique value.

The results of this paper suggest that the skewness indices obtained using fuzzy regression methods are closer to the subsequently realized measure of skewness than the one provided by the standard methodology. This result is in line with previous findings in [16, 17] for volatility, indicating that the use of fuzzy regression methods in computing skewness of the option implied distribution enhances its predictive power on future realized skewness. In particular, the best estimate of subsequently realized skewness is the one that combines the Bakshi et al. formula [1] with the upper bound of the Ishibuchi and Nii fuzzy regression method [10].

The paper proceeds as follows. In Sect. 2, we discuss the financial problem. In Sect. 3, we describe the procedure adopted to embed all the information coming from both call and put prices in the estimation of skewness. In Sect. 4, we present the results of the empirical application on the Italian market. In Sect. 5 we present the defuzzification procedure. In Sect. 6 we evaluate the goodness of the measures by assessing their forecasting power on future realized skewness. The last section concludes.

2 Skewness Obtained from Option Prices: From the Smile Function to Skewness Estimation

The standard market formula used to extract volatility and higher order moments from a cross-section of option prices is the model-free formula proposed in [1]. This formula is called model-free since it does not rely on any option pricing model, being consistent with many asset price dynamics. According to [1] model-free skewness can be obtained from the following equations:

$$Skewness(\mathrm{t},\tau) \equiv \frac{e^{r\tau}W(\mathrm{t},\tau) - 3e^{r\tau}\mu(\mathrm{t},\tau)V(\mathrm{t},\tau) + 2\mu(\mathrm{t},\tau)^3}{\left[e^{r\tau}V(\mathrm{t},\tau) - \mu(\mathrm{t},\tau)^2\right]^{3/2}} \tag{1}$$

where $\mu(\mathrm{t},\tau)$, $V(\mathrm{t},\tau)$ and $W(\mathrm{t},\tau)$ are based on the first, second and third moments of the distribution, respectively, and are obtained from call and put prices as follows:

$$\mu(t,\tau) \equiv E^q \ln[S(t+\tau)/S(t)] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau) \tag{2}$$

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1-\ln[K/S(t)])}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{2(1+\ln[S(t)/K])}{K^2} P(t,\tau;K)dK \tag{3}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\ln[K/S(t)] - 3ln[K/S(t)]^2}{K^2} C(t,\tau;K)dK - \int_{0}^{S(t)} \frac{6\ln[S(t)/K] + 3ln[S(t)/K]^2}{K^2} P(t,\tau;K)dK \quad (4)$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\ln[K/S(t)]^2 - 4ln[K/S(t)]^3}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{12\ln[S(t)/K]^2 + 4ln[S(t)/K]^3}{K^2} P(t,\tau;K)dK \quad (5)$$

$C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of a call and a put option at time t with maturity τ and strike K, respectively, and $S(t)$ is the underlying asset price at time t.

In order to compute the integrals in Eqs. (2)–(5), a continuum of option prices with strike price ranging from zero to infinity is required. However, this hypothesis is not fulfilled in the reality of financial markets. In particular, for European peripheral countries, such as the Italian one, only a small number of strike prices is available (around 15 per day) and the strike prices are spaced by a fixed range of basis points (e.g. for the Italian market, 250–500 basis points depending on the maturity). As a consequence, truncation and discretization errors may occur if a finite range of strike prices and a discrete summation are used to approximate the integrals in Eqs. (2)–(5).

A commonly used solution is the one proposed in [11], who suggest to mitigate both truncation and discretization errors by exploiting an interpolation-extrapolation method. Given that standard statistical techniques are not able to cope with conflicting information, standard market practice uses only a subset of available option prices (it retains only at-the-money and out-of-the-money option prices, therefore put options for strikes below and call options for strikes above the current asset price). Moreover, it averages the two at-the-money implied volatilities (when the strike price equals the current asset price) in a single estimate. It is obvious that this technique produces both a considerable loss of information and introduces an element of arbitrariness in volatility and skewness estimation.

3 The Smile Function Obtained Through Fuzzy Regression Methods

In this section we present the approach adopted in order to include all the available information in the market in the smile estimation procedure to obtain more informative skewness estimates. This methodology represent an appealing solution to deal with a framework characterized by conflicting information that needs to be aggregated (e.g. interval values for the inputs).

Following [16–18], we propose to exploit fuzzy regression methods in order to incorporate all the uncertainty embedded in the data in the smile estimation procedure, without losing the information in the original data. Starting from the initial grid of strike prices (x_p) and implied volatilities (y_p), we compute the minimum and the maximum volatility for each strike price x_p, $p = 1,\ldots,n$ as:

$$\sigma_{min}(x_p) = \min(\sigma_C(x_p), \sigma_P(x_p)) \tag{6}$$

$$\sigma_{max}(x_p) = \max(\sigma_C(x_p), \sigma_P(x_p)) \tag{7}$$

where $\sigma_C(x_p)$ and $\sigma_P(x_p)$ are the volatility of the call and the volatility of the put option associated to the strike price x_p. In this way, for a given strike price x_p, we have a range of possible values for volatility y_p given by $y_p = [\sigma_{\min}(x_p), \sigma_{max}(x_p)]$. In order to include all the observations in the smile estimation, we resort to fuzzy regression methods, which are capable to deal with interval values for the inputs. Given that the relationship among strike prices and implied volatilities takes the form of a smile, the so-called volatility smile, we adopt a quadratic fuzzy regression model, in order to achieve the best fit to the data.

The quadratic fuzzy regression model takes the following form:

$$\sigma(x) = A_0 + A_1 x + A_2 x^2 \tag{8}$$

where $\sigma(x)$ is the fuzzy output (i.e., the implied volatility associated to each strike price), x is a non-fuzzy input vector of strike prices and A_i, i = *0,..,2,* are the fuzzy coefficients of the second order polynomial. Since we deal with strictly positive variables, the lower bound $(\sigma^L(x))$, the upper bound $(\sigma^U(x))$, and the central value $(\sigma^C(x))$ of the fuzzy regression model can be rewritten as:

$$\sigma^L(x) = a_0^L + a_1^L x + a_2^L x^2$$
$$\sigma^U(x) = a_0^U + a_1^U x + a_2^U x^2$$
$$\sigma^C(x) = a_0^C + a_1^C x + a_2^C x^2$$

Relying on a previous empirical analysis performed on the Italian market, we adopt the quadratic extension of Ishibuchi and Nii fuzzy regression method proposed in [18] to estimate the volatility smile function. This approach is based a two-step methodology. In the first step, the coefficients a_0^C, a_1^C, a_2^C of the central regression $\sigma^C(x) = a_0^C + a_1^C x + a_2^C x^2$ are derived using the ordinary least squares:

$$\min z = \sum_{p=1}^{m} \left[y_p - (a_0^C + a_1^C x_p + a_2^C x_p^2) \right]^2 \tag{9}$$

where $y_p = (\sigma_{\min}(x_p) + \sigma_{\max}(x_p))/2$ is the average of the two implied volatilities which is adopted here to facilitate the use of the least squares estimation for the calculation of the central equation.

In the second step, the lower $\sigma^L(x)$ and the upper $\sigma^U(x)$ bounds are derived by means of the following optimization problem:

$$\min z = \sum_{p=1}^{m} \sigma^U(x_p) - \sigma^L(x_p) \tag{10}$$

where

$$\sigma^U(x) = a_0^U + a_1^U x + a_2^U x^2$$
$$\sigma^L(x) = a_0^L + a_1^L x + a_2^L x^2,$$

subject to:

$$h\,\sigma^C(x_p) + (1-h)\sigma^L(x_p) \le y_p = \sigma_{\min}(\mathrm{x}_p)\,,\ \ p = 1, \ldots, m$$

$$h\,\sigma^C(x_p) + (1-h)\sigma^U(x_p) \ge y_p = \sigma_{\max}(\mathrm{x}_p)\,,\ \ p = 1, \ldots, m$$

$$a_i^L \le a_i^C \le a_i^U, \quad i = 0,\ 1,\ 2$$

where a^C is pre-determined in the first step.

The fuzzy regression output is used to generate call and put prices to plug into Eqs. (2)–(5). In order to have a benchmark for the proposed fuzzy-regression-based measures of skewness, we also compute a skewness measure by applying the standard cubic spline methodology. Moreover, given the importance of having a constant 30-day measure for skewness (most of the risk measures for financial markets are calculated for a reference time horizon equal to 30 days), a linear interpolation procedure is adopted:

$$Skewness_{30} = w\ Skewness_{near} + (1-w)Skewness_{next} \tag{11}$$

with $w = (T_{next} - 30)/(T_{next} - T_{near})$, and T_{near} (T_{next}) the time to expiration of the near (next) term options, $Skewness_{near}$ (resp. $Skewness_{next}$) is the estimate which refers to the near (resp. next) term options. In general, a first option series with a maturity of less than 30 days (near) and a second series with time to maturity greater than 30 days (next) are used.

4 Fuzzy Skewness for the Italian Market

In this section, we present the results for the skewness measures of the Italian market based either on the standard interpolation-extrapolation methodology or the fuzzy regression method. The data set consists of daily closing prices on FTSE MIB-index options (MIBO), recorded from 1 January 2010 to 28 November 2014. The data set for the MIBO is kindly provided by Borsa Italiana S.p.A, while the time series of the FTSE MIB index, the dividend yield and the Euribor rates are obtained from Datastream. Several filters to the option data set are used in order to eliminate arbitrage opportunities and other irregularities in the prices (for a detailed discussion see e.g. [12, 13]).

We perform the procedures described in Sects. 2 and 3 on the option prices that meet the filter constraints and we obtain 1233 daily observations for each of the 10 estimates of skewness (we choose to use the upper and lower bounds of the h-cuts, with h = 0, 0.25, 0.5, 0.75, 1 and the standard method). We also compute the subsequently realized measure of skewness (obtained from historical series) using daily FTSE MIB log-returns and a rolling window of 30 calendar days. In this way the physical measure refers to the same time period covered by the measures computed using option prices, which represent the investors' expectation (under the risk-neutral measure) of the former. In Table 1 we report the average value of realized skewness (first column) and the estimates of skewness computed from option prices (columns 2–7). Specifically, the estimate obtained using the standard procedure is reported in column 2. On the other hand, the upper bound and the lower bound for each h-cut, is reported in columns 3–7.

Several observations are noteworthy. First, it is straightforward to note that all the skewness measures obtained from option prices are on average lower than zero, pointing to a negative risk-neutral skewness (i.e. the risk-neutral distribution is skewed to the left). On the other hand, the subsequently realized distribution is almost symmetrical, the measure of skewness estimated from the historical series of the underlying asset being equal to −0.012 on average. Second, the skewness estimate obtained by setting h equal to one is lower than the one obtained using the standard interpolation-extrapolation methodology. Third, the skewness estimate that is the closest to the subsequently realized measure of skewness is the one provided by the upper bound of the Ishibuchi and Nii (h = 0) fuzzy regression method.

5 The Defuzzification Procedure

In Sect. 3 we presented the advantages of skewness estimates obtained using fuzzy regression method. In particular, the proposed skewness measures allow to extrapolate further information with respect to the standard methodology since they provide not only a most possible value for skewness, but also an interval of possible values around the most possible one.

However, investors may prefer to condense all the information content of the skewness estimates obtained using the fuzzy regression method into a unique value (crisp output). This objective can be achieved by exploiting a defuzzification procedure. An appealing solution in order to synthesize all the information embedded in the skewness estimates is the one proposed in [19], who suggest that that the best defuzzifier is the scalar that is "closest" to the triangular fuzzy number:

$$x = \frac{a^L + 2a^C + a^U}{4} \tag{12}$$

where a^L, a^C and a^U are the lower, the central and the upper bound of the triangular fuzzy number.

The defuzzification procedure is used to convert, for each strike price, the different fuzzy regression results in the defuzzified volatility level. The obtained values for volatility are subsequently converted in terms of call prices and used as input in Eqs. (2)–(5) in order to obtain a unique skewness estimate.

The result for the defuzzified skewness estimate obtained with the Ishibuchi and Nii method is reported in Table 1 (last column). We can see that the defuzzified skewness estimate (−0.368) is close to the central estimate of the Ishibuchi and Nii fuzzy regression method ($h = 1$). This suggests that the skewness estimate obtained using the Ishibuchi and Nii fuzzy regression method do not show a pronounced asymmetry.

Table 1. Average value of the estimated skewness measures.

RSkew	Std. Meth.	Ishibuchi and Nii					*def*
		h = 1	h = 0.75	h = 0.50	h = 0.25	h = 0	
−0.012	−0.387	−0.368	−0.359	−0.352	−0.345	−0.334	−0.368
			−0.383	−0.397	−0.413	−0.434	

We report in the first and second column the average value for daily realized skewness (RSkew) and the skewness estimate obtained using the standard interpolation-extrapolation method (Std. Meth.), respectively. In columns 3–7 we report the average value for daily skewness measures obtained combining the Bakshi et al. skewness formula (Eq. 1) with the Ishibuchi and Nii fuzzy regression method [10]. The results are reported for different values of h. For each value of h we report the upper bound (first row) and the lower bound (second row) estimate of skewness. Finally in the last column we report the average value for the skewness estimate obtained using the defuzzification procedure.

6 The Assessment of the Best Skewness Forecast

We are interested in evaluating whether fuzzy regression methods to estimate skewness enhance the predictive power on future realized skewness. Given the large number of forecasts (11) for skewness proposed in Table 1, we resort to the model confidence set procedure (MCS) to identify the best model, or a smaller set of best models (see [9]). In order to evaluate the forecasting performance of the proposed models, in line with Patton (2011), we adopt the Mean Squared Error (MSE) error indicator, which provides robust results in the presence of noise in the proxy of skewness:

$$\mathrm{MSE} = \frac{1}{m}\sum_{k=1}^{m}(forecast_k - realized_k)^2 \tag{13}$$

where $forecast_k$ and $realized_k$ are the forecasted and realized measures of moments, respectively, and $forecast_k$ is proxied by the different skewness measures obtained using option prices. The average value of the MSE loss functions are reported in Table 2. We can see that the best forecast for future realized skewness is the one provided by the upper bound of the Ishibuchi and Nii (h = 0) fuzzy regression method.

Moreover, also the most possible value provided by the Ishibuchi and Nii (h = 1) fuzzy regression method yields a lower error than that of the standard methodology. We also evaluate the forecasting performance of the proposed defuzzified skewness measure on future realized skewness by computing the Mean Squared Error (MSE) error indicator (Eq. (13)). The result, reported in Table 2 (last column), indicates that the unique value of skewness obtained using the defuzzification procedure obtains a slightly worse performance with respect to the central estimate of skewness (h = *1*). However, the defuzzified skewness estimate is still better than the standard methodology in forecasting future realized skewness (MSE is equal to 0.165 and 0.188 for the defuzzified and the standard method, respectively) and the improvement is significant from a statistical point of view (this result is supported by a t-test, where errors are corrected by Newey West, t-stat = −3.47, p-value = 0.00).

Therefore, investors who prefer to have all the information content of the skewness estimates condensed into a unique value could refer to the estimate obtained by means of the defuzzification procedure to have a more reliable forecast of future realized skewness.

Table 2. Forecasting skewness: MSE error indicator.

	Std. Met.	Ishibuchi and Nii					*def*
		h = 1	h = 0.75	h = 0.50	h = 0.25	h = 0	
MSE	0.188	0.161	0.153	0.148	0.143	0.138	0.165
			0.174	0.189	0.209	0.238	

The table reports the results of the skewness forecasting exercise performed using the mean squared error (MSE) indicator defined as follows: $\mathrm{MSE} = \frac{1}{m}\sum_{k=1}^{m}(forecast_k - realized_k)^2$ where $forecast_k$ and $realized_k$ are the values of option based forecast of skewness and realized skewness, respectively. For a definition of the skewness measures, see Table 1, (upper bounds in the first row and lower bunds in the second row).

The MSE reported in Table 2 are the inputs of the Model Confidence Set test, which is performed using the MCS package for R developed by [2]. The test allows to assess whether the difference in the forecasting power between the proposed models are significant from a statistical point of view (the statistic t_{ij} is used also in the well-known test for comparing two forecasts, see [7, 23]. The confidence level (1 − α) adopted in the test is equal to 0.95, the number of bootstrapped samples used to construct the statistic test is 1000 (B = 1000). The results for the Model Confidence Set test are reported in Table 3.

Table 3. Forecasting skewness: Model Confidence Set.

Superior Set Model created: (10 models are eliminated), indicator used: MSE						
	Rank	$T_{\text{max,M}}$	p-value	$T_{\text{R,M}}$	p-value	*Loss*
I&N f^{U} (0)	1	−6.896	0.000	−6.896	0.000	0.137

The table reports the Model Confidence Set for the skewness estimates obtained using either the standard methodology and the proposed fuzzy regression method. The input for the Model Confidence Set reported is represented by the MSE loss functions reported in Table 2. $T_{\max,M} = \max_{i \in M} t_i, T_{R,M} = \max_{i,j \in M} |t_{ij}|$ are the test statistics proposed in [9]; p-values for the tests are reported sideways in the p-value column, the corresponding rank is reported in the Rank column. The lower the value of T, the higher the rank. In the last column, we report the average loss of the model.

According to the Model Confidence Set test result reported in Table 3, the upper bound of the Ishibuchi and Nii fuzzy regression method (h = 0) is the best forecast for future realized skewness. All the other forecasts, included the one based on the standard procedure, are eliminated.

Given the relevance of correctly measuring skewness to assess the riskiness of asset return distribution, this result is very important for investors and regulators, who can rely on fuzzy regression methods in order to get a more reliable forecast for skewness.

7 Conclusions

In this paper we have proposed a method for estimating skewness from option prices by means of fuzzy regression methods. This approach offers several advantages. First, it is possible to incorporate conflicting information coming from both call and put prices, without having to make the a priori choice of discarding some option prices as in standard market practice. Second, fuzzy regression methods are particularly suited when a limited number of option prices is available. Last, fuzzy regression methods yield a more reliable estimate in the form of interval of possible values, containing the most possible one.

We offer an empirical application of the proposed method in the Italian market, during the 2010–2014 time-period. The measures of skewness are computed on a daily basis (closing values of option price are used) using five different level of h-cut: 0, 0.25, 0.50, 0.75, 1. The proposed skewness measures obtained through fuzzy regression are compared with the measure of skewness provided by the standard procedure, which are used as a benchmark. We also adopt a defuzzification procedure in order to condense all the information content of the fuzzy skewness estimate in a unique value.

We get several results. First, the skewness estimates obtained using the fuzzy regression method allow to extrapolate further information with respect to the standard least square regression, since the coefficients of the fuzzy regression model provide not only a most possible value for the coefficient, but also an interval of possible values around the most possible one. Second, the mean squared error (MSE) indicator suggests that the measures of skewness obtained through a fuzzy regression method are closer to the subsequently realized measures than the one obtained using the standard methodology. Third, the Model Confidence Set indicates that the improvement in the forecasting performance attained using fuzzy regression is significant also from a statistical point of view. Similar results are obtained for volatility estimates through fuzzy regression in [16, 17]. Specifically, the best forecast of subsequently realized skewness is the upper bound of the Ishibuchi and Nii fuzzy regression method ($h = 0$).

Since correctly measuring skewness is of paramount importance in finance in order to correctly assess the riskiness of asset return distribution, this result is very important for investors and regulators, who can rely on fuzzy regression methods to get a more reliable forecast of skewness. Future research should evaluate if the use of other fuzzy regression methods (such as Savic and Pedricz [21] and Tanaka et al. [22]) may improve the forecasting power of the fuzzy skewness estimates.

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