

# The Signature of Organic Urban Growth



## Degree Distribution Patterns of the City's Street Network Structure

Leonard Nilsson and Jorge Gil

**Abstract** Cities are complex, perhaps one of the most complex kinds of structure created by humans. Some cities have been planned to a large extent while others have grown organically. The outcome of planning and growth processes is the diverse morphological built form and street patterns observed in cities today. So far, in urban planning and history research, cities are classified as planned or organic, largely based on the visual assessment of their urban morphology. To understand cities and their characteristics, it is necessary to develop methods that quantitatively describe these characteristics and enable objective comparisons between cities. Using graph representations of the street network of cities, it is possible to calculate measures that seem to exhibit patterns with scale-free properties (Jiang in *Phys A Stat Mech Appl* 384(2):647–655, 2007). And thanks to progress in the field of complexity studies, it is also possible to test if and to what extent the distribution of a measure in a collection of elements fits a power law distribution (Clauset et al. in *SIAM Rev* 51:43, 2009). In this chapter, we show that the degree distribution of a city's street network seems to fit a power law distribution for cities that are considered to have mostly grown organically. On the other hand, cities that are planned to a large extent do not exhibit such a good fit. This result is relevant since a power law distribution is a signature of multiplicative growth processes. Furthermore, the result of the quantitative classification method correlates well with the results of earlier qualitative morphological classifications of cities.

**Keywords** Street networks · Spatial morphology · Growth patterns · Power law · Degree distribution

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# 1 Introduction

Cities have been a very important part of human civilization since agriculture-based society emerged. They have been built independently from each other in different parts of the world and by different cultures. In order to better understand and study cities, researchers in sociology, economy, physics and geography have for long resorted to quantitative methods based on functional characteristics (Harris 1943; Nelson 1955; Beaverstock et al. 1999) and population size (Clark 1951; Bettencourt et al. 2010), to describe, rank and produce classifications of cities. From an urban morphology studies perspective, a basic way of classifying cities has been by their growth process and the resulting morphological pattern, using a simple dichotomy: organic cities that have grown gradually, versus planned cities that have emerged from larger planning interventions. Another way of approaching this dichotomy has been using the concepts of bottom-up growth (organic) versus top-down development (planned) (Batty 2013).

Some cities are entirely planned in one sweep, while others have mainly grown organically in a piecemeal process governed by local decisions. But often these processes have alternated back and forth over time (Kostof and Tobias 1991). Hence, growth does not manifest itself in an absolutely distinct dichotomy, but it is rather a question of different degrees of planning resulting in different morphological patterns that combine these two main forces. Nevertheless, it is still a dominant morphological classification of cities and urban areas.

A peculiar aspect is that these organic patterns have been for the past decades regarded as desirable in cities, and therefore efforts to mimic them have often been made in planning (Kostof and Tobias 1991). Despite research on the topic of their growth, design and generation, there are still few consistent ways of classifying and describing these morphologic patterns. The classification approach used is largely based on the visual judgement of the perceived patterns. It is, therefore, necessary to develop methods that quantitatively measure their characteristics and enable the classification of cities and parts thereof, towards their objective comparison and the understanding of city growth.

The street network of a city is one of its most durable structures, with a crucial influence on its land use and building development over time. It is therefore important to understand this complex network. Using graph representations of the street network it is possible to calculate topological measures that describe different aspects of its configuration (Hillier 1996), and efforts to find properties of entire cities based on such measures show that these seem to exhibit patterns with scale-free properties. Nevertheless, the results are diverging and sometimes contradictory. In some studies, the various network centrality measures like degree, betweenness and others, reveal a scale-free behaviour in many cities (Porta et al. 2006a, b; Jiang 2007; Volchenkov and Blanchard 2008; Shpuza 2014). Other studies refute this and have shown that there are differences and exceptions from these patterns (Jiang and Claramunt 2004; Crucitti et al. 2006a, b; Porta et al. 2006a, b).

By studying the statistical distribution characteristics of some individual centrality measure, the emerging distribution can tell us something about the processes shaping this measure. One of the most common distributions is the normal distribution (hence the name ‘normal’). This distribution emerges as a result of numerous random factors added together. Exponential distributions, on the other hand, are the result of time-dependent processes. A distribution that has caught the interest of many scientists in the last 20 years is the power law distribution, which has scale-free properties and is the result of a multiplicative growth process. This is often called a complex or self-organizing process.

In this study, we measure the degree centrality distribution of the street network’s topology in a number of cities with different morphological patterns and test if they fit the power law distribution, to uncover whether there are universal mathematical patterns of cities as found in earlier studies, or if these vary between cities as other studies suggest. The difference in morphological structure and growth could explain the contradictory results found in those latter studies. The hypothesis is that these mathematical patterns are indeed not universal, but rather depend on the extent to which the cities are organic or planned. If that holds true, then these measures provide a signature of organic urban growth for the objective categorization of cities, which can extend our understanding of cities in general.

In the next section, we provide an overview of relevant background theories and related work, followed by a description of the two case studies—a longitudinal study of the growth of Paris and a cross-sectional study of several world cities—and the methodology used, leading to the presentation of the studies’ results, and finally wrapping up with a summary of findings and pointers to further work.

## 2 Of Street Networks and Degree Centrality Distributions

Urban morphology has been studied as an academic discipline for a long time, but is has often been done in qualitative studies based on perceived patterns of urban morphology (Kostof and Tobias 1991). Those patterns are usually derived from the geometric properties of urban elements, such as buildings, parcels and streets, and their spatial arrangements. Since the 1990s, with increasing computing power and availability of spatial analysis software such as Geographic Information Systems (GIS), we have witnessed the exploration of new approaches to identify and quantify urban morphological patterns. Webster (1995) proposes the analysis of urban satellite images to extract a set of urban texture measures of streets and blocks, thus identifying the fingerprint and profile of different neighbourhoods. Gil et al. (2012) pursue a similar goal by looking at individual elements of the city, i.e. urban streets and blocks, and classifying them according to a set of geometric and topological measures to create summary profiles of urban areas. Louf and Barthélemy (2014) also use geometric attributes of urban elements, but instead develop indices based on the ratios of the distribution of said attributes in order to classify different cities across the world.

Parallel to this work, other research groups started in the 1990s to analyse cities by means of growth simulations, often using cellular automata (Batty and Longley 1994), measuring characteristics of the boundary and general shape of the city. In this field, cities are often said to be fractal, which means that certain measures have scale-free properties. This property is claimed to be the result of a bottom-up growth process, and by analogy, organic cities might also be called fractal and possess scale-free properties.

## ***2.1 The Topology of Street Networks***

Around the 1980s a new method for representation and quantitative analysis of spatial structures was developed called space syntax (Hillier and Hanson 1984). As the name indicates, it addresses the configuration of spaces: architectural, urban streets or open spaces. Spatial configuration, is best defined as how spaces, and in the case of cities usually streets, are connected and related to each other (Hillier 1996; Marshall 2015). A basic concept of the original space syntax theory is that it analyses the topology of the street network, and leaves out metric information (Hillier 1999). The geometry of space only plays a role in defining its representation as discrete elements and in constraining their connections.

Since the late 1990s, parallel to the growth of space syntax as a research field, another important field of research that has been rapidly evolving is that of network science (Watts 2004). Topological analysis of graphs is also the fundamental model for this scientific field, where many kinds of networks are studied, including city street networks (Barthélemy 2011). As computers have become more powerful, methods appeared for calculating properties of large networks based on existing theories, such as small-world network properties and scale-free properties (Watts and Strogatz 1998). This offered the possibility to study cities from an entire new perspective (Batty 2013).

Based on this progress in complex network analysis in general, new methods for analysing road and street network patterns have been developed, evolving the research field pioneered by space syntax (Jiang et al. 1999; Porta et al. 2006a; Jiang 2007). These methods use the results from topological network analysis and thereafter analyse various properties, both individual measures and statistical distributions (Jiang 2007; Volchenkov and Blanchard 2008; Shpuza 2017). These elaborations of the space syntax research field, combined with the general network science field, might enable the quantification of cities in a richer way.

## ***2.2 Street Network Representation***

When the space syntax research field started in the 1980s, the main element of analysis of street configurations was the axial line (Hillier and Hanson 1984). An axial line is an unobstructed line of sight, and in order to analyse an area or a city, an

axial map is created made of the fewest and longest unobstructed lines of sight. The axial map is converted into an unweighted undirected graph, and analysed based on its topology (Hillier 1996). This model has been dominating the research field, but in the early 2000s efforts were made to integrate space syntax in ordinary GIS research and make it possible to use the more common road centreline maps for larger scale studies (Jiang et al. 1999; Dalton et al. 2003).

One approach widely used today is to convert the road centreline maps to a model of street segments and analyse its network centrality. This can be achieved using a so-called ‘primal’ approach (Porta et al. 2006b) with geographic weight on the links between street junctions that are nodes in the graph; or using a so-called ‘dual’ approach (Porta et al. 2006a), where the nodes are the street segments and the links have angular (fractional) weight based on the degree of direction change between street segments, instead of topological (discrete) steps (Dalton 2001; Turner 2007). The latter model is similar to the segment model derived from an axial map and equivalent in terms of analysis results (Krenz 2017; Kolovou et al. 2017). However, both street network models based on street segments lose topological structural properties of the network as a whole.

Another approach for using road centreline maps and replace axial lines has been to aggregate street segments based on the concept of ‘Natural roads’ (Jiang et al. 2008), which is similar to that of ‘Strokes’ (Thomson 2003) and of ‘Continuity lines’ (Figueiredo and Amorim 2005). These concepts are based on the Gestalt principle of good continuation (Jiang 2007), and applying improved street segment aggregation algorithms results in a network model that reveals the structure of the city (Jiang and Liu 2009). Its correlation with pedestrian and vehicular movement is on par with the other methods from the space syntax research field, such as angular segment analysis and axial integration (Jiang 2009; Jiang and Liu 2009), which supports its validity for representing urban structure.

The different approaches of representing streets as entities continuing across intersections open up a particular view of urban street networks. Since streets are often planned and built in their entire length and seldom subject to changes over time, a method that reconstructs the street as an entity makes it possible to represent and understand the city as a collection of streets added together as the city grows. In this way, space syntax and similar topological approaches to street network representation can be used to understand urban growth and development.

### ***2.3 Degree Distribution and Power Law Fitting***

The most basic measure of centrality obtained from network analysis is degree centrality, also called connectivity in space syntax studies, i.e. the number, per node, of connections with other nodes. When using the street network models based on street segments described in the previous section, degree centrality only classifies street junctions by type, which is considered uninteresting for studying urban configuration (Porta et al. 2006a). However, when using a street network model based

on ‘Natural roads’ or axial lines degree centrality provides an interesting measure of street hierarchy.

When analysing the distribution of degree centrality values of street networks, a certain pattern often occurs, namely that there are far more low connected elements than highly connected ones. This pattern seems to follow a power law distribution, which has scale-free properties (Porta et al. 2006a; Jiang 2007) and is a signature of a natural process governing the creation of the distribution. In other fields of network science, such as social networks, flight connections or power grid structures, the same pattern occurs, and many studies rely on degree centrality as the single measure of distribution to test for power law fitting (Clauset et al. 2009; Barabasi and Albert 1999; Volchenkov and Blanchard 2008). In urban studies, this aspect has not been extensively researched, but seems to offer the possibility of new and interesting discoveries.

## 2.4 *Related Work*

Some efforts to find properties of cities as a whole using street network models have been made (Crucitti et al. 2006a, b; Porta et al. 2006a, b; Jiang 2007; Volchenkov and Blanchard 2008; Shpuza 2014), although many of these studies were carried out on a small sample area of a city or on a small number of cities, which makes it harder to extract general conclusions from the results. The results are also diverging and sometimes incompatible. It has been shown in some studies that various network measures like degree, betweenness and other centrality measures, reveal a scale-free behaviour in many cities (Porta et al. 2006a; Jiang 2007; Volchenkov and Blanchard 2008). While other studies refute this observation, and have shown that there are differences and exceptions from these patterns (Jiang and Claramunt 2004; Crucitti et al. 2006a, b; Porta et al. 2006b). To further support or refute these claims, a more extensive study is needed. The results of these earlier works can be discussed in light of the results from the present study, even though the preconditions and methods are not exactly identical.

## 3 **Datasets and Methods**

In this work we carry out two experiments, one longitudinal and one cross-sectional, in which we analyse the degree centrality distribution pattern of cities and test to what extent they fit a power law distribution, to test the hypothesis that these patterns are not universal, but rather depend on the extent to which the cities are organic or planned. In the following sections we explain in detail the ‘what’ and the ‘how’ of this work.

### ***3.1 Objects of Study***

The first experiment is a longitudinal study of Paris, France, testing how the signature of degree centrality, and thus the scale-free nature of cities, changes over time as the city grows. Growing from a medieval walled core like many European cities, the city of Paris has undergone periods of profound transformation to its urban structure and character, namely: through the building of new bridges and public spaces in the seventeenth century; the famous Haussman plan in the nineteenth century that introduced a network of avenues and boulevards; and the urban expansion of its periphery in the twentieth century with projects like La Défense.

The second experiment is a cross-sectional study of a selection of world cities, testing how the signature of degree centrality relates to the planning process, and changes across geographic and cultural contexts, to try to confirm or refute the existence of universal patterns. There are no standardized or consistent ways of classifying cities, therefore the selection of cities is mainly based on earlier classifications focused on urban patterns resulting from different development processes (Kostof and Tobias 1991; Batty and Longley 1994; Hillier 1996). In addition, other criteria for selection have been chosen in order to have variation in the sample of cities.

The first and foremost criterion used has been the degree of planning or self-organization, aiming to have a somewhat balanced collection of organic and planned cities. The second criterion has been the dominant culture that has shaped the city morphology, since different cultures may play a large role in the cities' street network structure (Hillier 1996). This is probably because of the different social structure, organization and interaction found in different cultures (Kostof and Tobias 1991). The third criterion has been geographic location, since the preconditions for urban development differ considerably in different parts of the world, namely the geology, climate, building materials supply and cost. Therefore, the cities selected for this study are chosen from different continents or continental areas. Finally, the age of the city can also play a role, in that organic urban transformation is a slow process that takes long time to manifest itself, whereas building a planned city can be done quite fast in comparison.

Based on these criteria, ten cities have been chosen for this study (Table 1). Including a larger number of cities in the study could bring benefits, but since the aim is not to reach a conclusive answer, but rather to explore the path for new theories, the number of cities chosen (10) can be considered sufficient. This selection should be seen as a starting point for further and deeper studies.

### ***3.2 Sources of Street Network Data***

The street network of the selected cities is the main object of study, therefore it is essential to use a reliable data source. For the first experiment, a data set of the historical evolution of the Paris street network was obtained from the GeoHistorical-

**Table 1** Summary of properties for the cities in the cross-sectional analysis

City	Degree of planning	Shaped by culture and religion	Location	Age	Geographical characteristics
Rio de Janeiro	Many self-organized favelas	Catholic South American	Brazil	Modern (60s)	Coastal city
Venice	Self-organized villages grown together	Catholic European	Europe	Middle age	Several islands
Damascus	Old roman city organically, reshaped when under Arab government	Originally roman, later Arabic/Islamic	Middle east	Roman age	Desert/plain
London	Low, self-organizing	Anglo-Saxon, Lutheran	Europe	Middle age	River city
Tokyo	Low, self-organizing	Japanese all the time, although became capital later	Eastern Asia	Middle age	Coastal city
Beijing	High, since it became capital	Chinese all the time, although it became capital later	Eastern Asia	Antiquity	Topographical constraints
Moscow	High, as an expression of power	Russian Orthodox, capital all the time	Russia	Middle ages	River city
New York	High, the original grid is virtually unchanged	American all the time	North America	About 1600	Several islands
Brasilia	Very high, entire city planned at once	Very new, built as capital	Brazil	60s	Exploited jungle
Paris	High, since both the Baroque planning and Haussmann's boulevards	Catholic European	Europe	Middle age	River city



Data project (<http://www.geohistoricaldata.org>), with 10 maps spanning from 1300 to the present day. Figure 1 shows a temporal sequence of maps of the street network of Paris. Historical maps are relatively scarce and are produced at irregular time intervals. Therefore, it is troublesome to use some kind of strict selection criterion, because the resulting selection would be too sparse. That is the reason for using many maps although some of them are quite close in time.

For the second experiment, in order to make a worldwide comparison, it is essential to obtain a common data source with relatively consistent data classification and collection. Publication restrictions and copyright issues make the usage of official land surveys or proprietary navigational street data (e.g. TomTom, Navteq, Google) hard. Instead, Open Street Map (OSM) was used as the source for the street network data. OSM is one of the most successful Volunteered Geographic Information (VGI) initiatives that aim to create global public domain map data. It is open for anyone to view, use, edit and create new features; and it has a worldwide coverage, although the level of detail and classification criteria might differ between different areas.

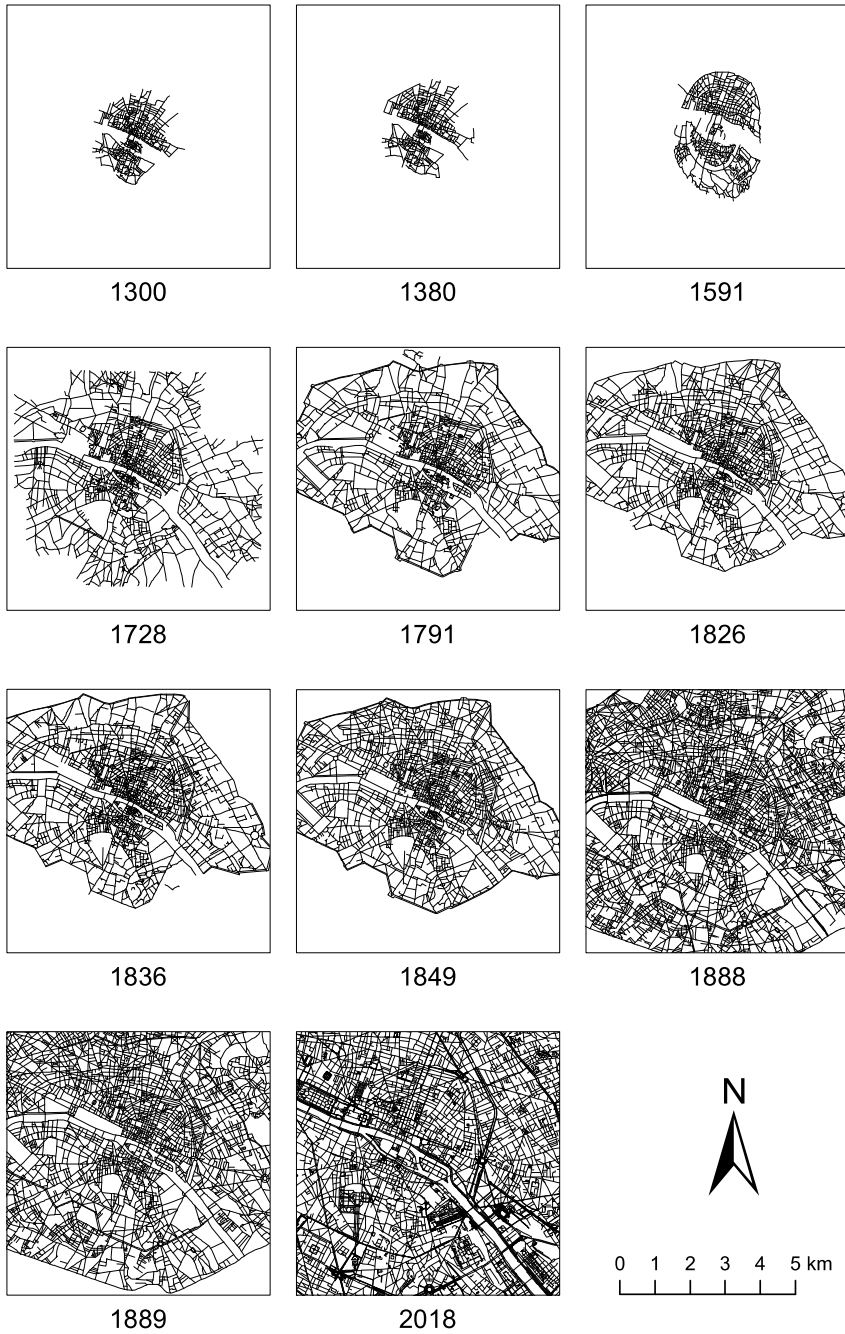
There has been some debate around the data quality of OSM, since anyone can modify it. However, the process is moderated, and previous research has suggested that the errors in this map data are on par with those in other data sets from authoritative sources (Haklay 2010; Dhanani et al. 2012; Krenz 2017). Therefore, the benefits of being able to use a common map source for all the cities in this study, without copyright or other legal restrictions, outweigh the possible data quality drawbacks of using OSM. The OSM data in shape file format was obtained from the download section of Geofabrik ([www.geofabrik.de](http://www.geofabrik.de)). Excerpts of the street network of the ten selected cities (listed in Table 1) are shown in Fig. 2.

### ***3.3 Street Network Model Considerations***

In this study, the street network data set is converted to a street network model, which is then translated to a graph to analyse its degree centrality. Other centrality measures can be calculated, and they offer possible directions of investigation on their distribution patterns, but to produce results that are easily comparable with other studies, also from other network research fields, only degree centrality will be used.

To obtain discrete and meaningful degree centrality results for each street, it is necessary to use an appropriate street network model. This rules out the street segment based models widely used today, presented in Sect. 2.2. As for aggregate street network model options, the axial line model would require the manual digitizing of the network, which is impractical for this extensive study. Since we have access to road centre line data of the selected cities, the ‘Natural roads’ street network model and analysis method, based on aggregating road centre line segments, is the most suitable.

A parameter that has to be taken into consideration is the extent of the studied areas in each city. In this work, the main goal is to capture the structure of the city



**Fig. 1** Sequence of street network growth and change in Paris, from 1300 to 2018

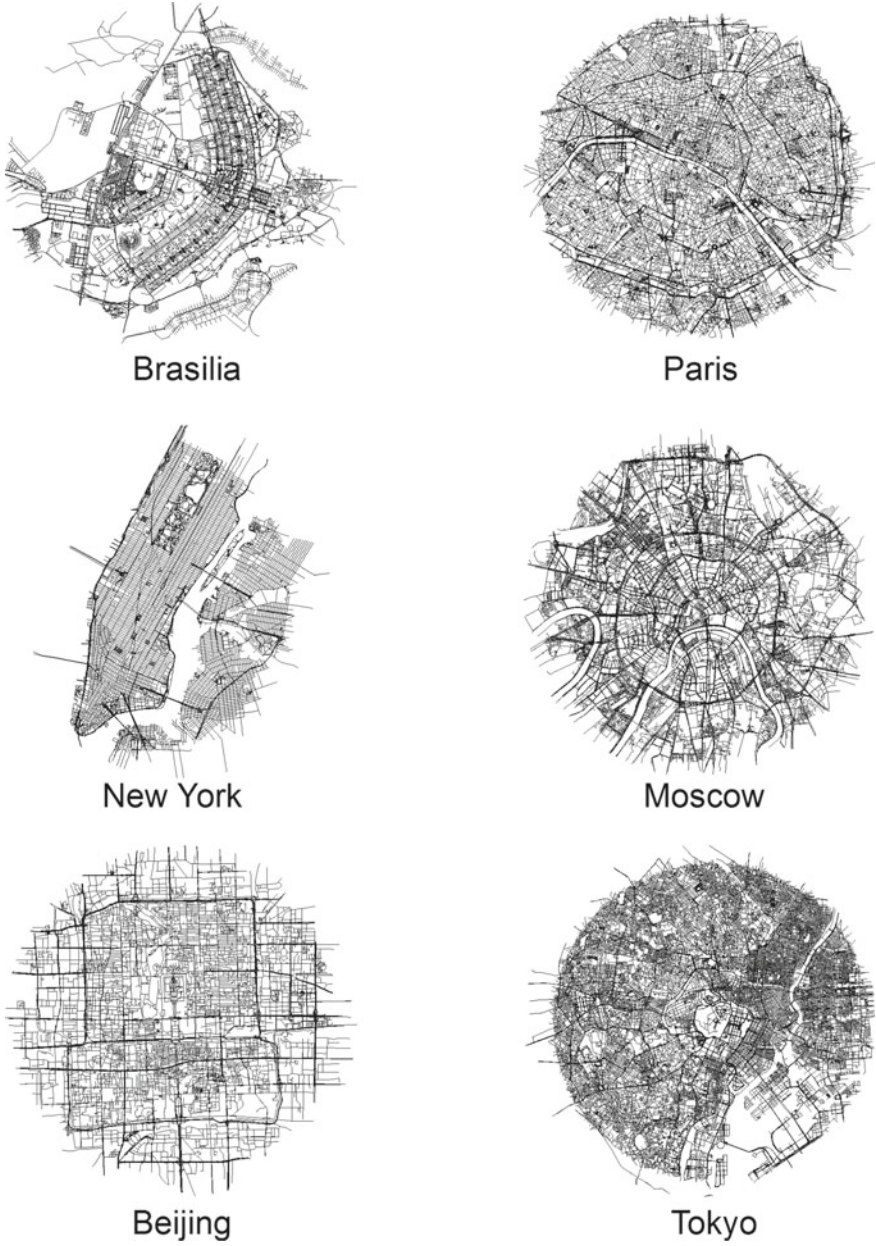
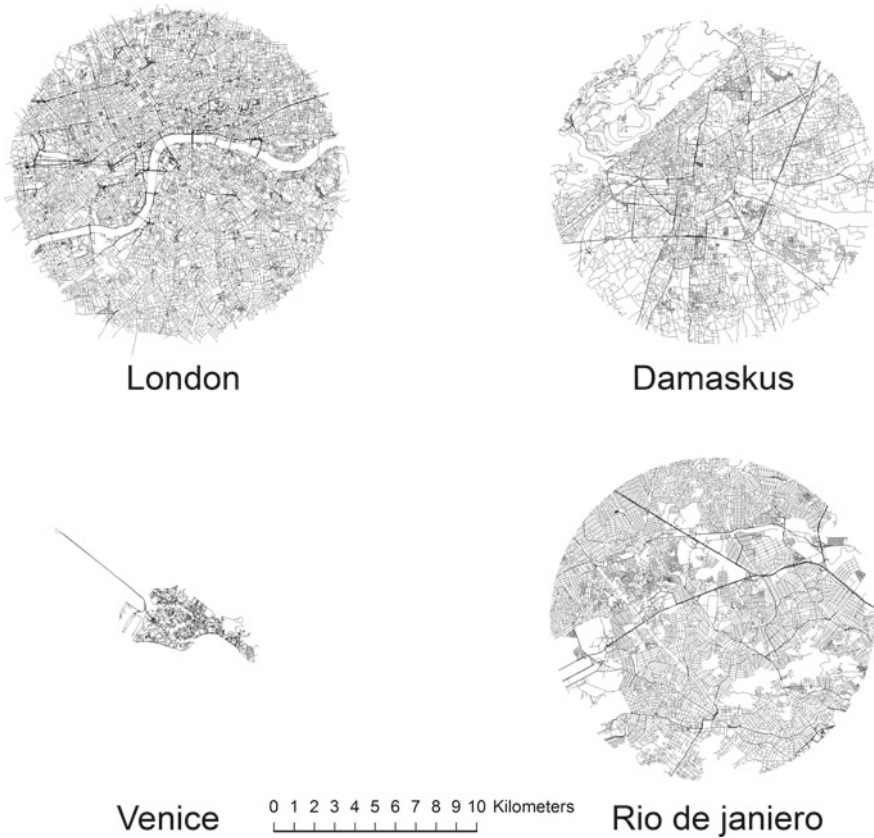


Fig. 2 The street networks of the ten cities in the cross-sectional analysis



**Fig. 2** (continued)

resulting from its historical growth to get an indication of whether the hypothesis holds. It can be argued that the larger the study area are better, in order to have a larger amount of data to analyse, and a certain number of cases is required to fit power law to the degree distributions. On the other hand, cities with organic patterns are often small, or have started small and possess small areas with organic pattern. To capture the organic parts of cities, and to make cities with different sizes more comparable, we use a circular sample area with a radius of 8 km. These areas are centred on the historical origin of the city (where applicable), like the Île de la Cité in Paris.

Finally, one aspect that must be considered is the boundary of the network model. In some cases, one can argue that this delimitation is arbitrary, while in other cases, like Venice, the city limits are very clear. Nevertheless, in every case we extract a significant section of the city. In addition, the degree centrality measure used is a local measure, meaning that edge effects of this boundary are much less pronounced (Gil 2017).

### 3.4 Methodological Process

Having defined the case studies, obtained the data and defined the street network model, the analytic methodology needs to be carried out.

The first step involves a series of data set extraction, cleaning and validation operations to enable the creation of the 'Natural roads' street network model. This includes clipping the data sets to extract the street segments in the 8 km study area; transforming the data to the WGS84 World Mercator projected coordinate system; breaking the road centre lines at every intersection, because by mistake some street segments continue across intersections, and this is a requirement for the natural streets interconnection algorithm to work; finally, identifying and deleting any isolated street segments or small 'islands' of streets, in order to obtain a network with a single connected component, again a requirement of the centrality algorithm.

The next step is to generate 'Natural roads', by joining the street segments at the junctions using the 'every best fit' algorithm with a threshold angle of  $45^\circ$  (Jiang et al. 2008). Once the 'Natural roads' have been created, we calculate degree centrality for each of the cities' street networks.

The final step is to statistically analyse the degree centrality results of the cities to check if their distribution seems to fit a power law distribution. First, we do a power law fit, where the best-fit alpha and x-min values are calculated. Next, we test whether the data can fit a power law doing a Kolmogorov–Smirnov (K-S) test (Clauset et al. 2009). This test generates many samples based on an ideal power law distribution, using the x-min and alpha values derived from the best fit of the degree distribution being tested. Then, the samples are compared against the distribution by doing a regression analysis to derive a significance test. If the resulting  $p$ -value is above 0.1, the distribution has a probability of following a power law, but it is not certain. Even though a lot of empirical data looks like it follows a power law distribution, it is not always the case. The results of these tests (the  $p$ -value, x-min and alpha) are then used in the final assessment and judgement of the fit.

A parameter that deserves some attention is alpha, not only for the statistical tests but also as a distribution's characteristic. It tells us the scaling factor in the distribution, which is the relation between roads with low degree (minor roads) and roads with a high degree (main roads). A high alpha represents a steep slope of the best-fit line, showing that there are relatively fewer high degree roads, compared to the low degree roads. A low alpha represents the opposite. Having this information about the proportion of different road categories can be used in the interpretation and understanding of the results.

After a distribution passes the K-S test ( $p$ -value  $> 0.1$ ) it has to pass a final set of criteria in order to claim that the distribution is a power law. The distribution under test is compared to other possible distributions such as exponential and log-normal using a log-likelihood test. To meet the criteria the power law has to obtain a higher likelihood than these other distributions. This test is done because heavy-tailed distributions are often quite similar and can give (false) positive results on the

K-S test, especially in distinguishing between power law and log-normal. Based on these tests' results we are able to put all cities into various categories.

When all these measures are extracted, it is time to do a categorical comparison between the cities to see how the quantitative properties are distributed among the studied cities. A categorization is made based on the measures in order to investigate whether the measures might have some relation to their qualitative morphological characteristics.

### ***3.5 Software Used in the Process***

To carry out a largely automated and reproducible study, several pieces of software are required for data preparation, validation and analysis. First, the FME software is used for cleaning and validating the data sets. Then, the software Axwoman 6.3 (Jiang 2015) for ArcGIS is used to create 'Natural roads' and calculate degree centrality for each city. The degree distribution of the streets is imported, processed and analysed using the *powerlaw* package (Gillespie 2015) in R (R Core Team 2016), which is also based on the work by Clauset (2007). Finally, Microsoft Excel is used to collect and visualize the statistical results. The use of several pieces of software was necessary to reach the desired results, although it would have been simpler if a single application with all these capabilities combined existed.

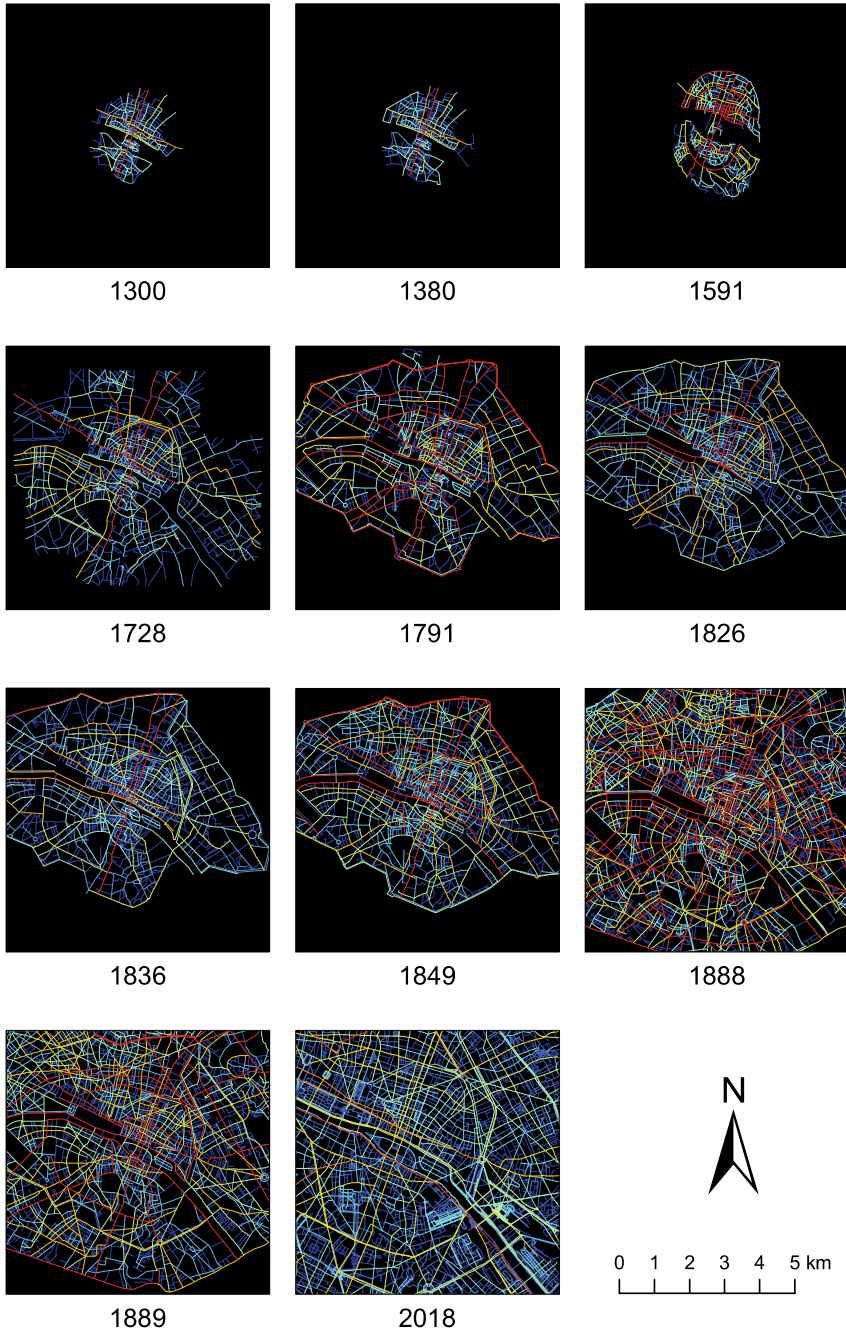
## **4 Results**

Following the geographical and topological analysis steps described in Sect. 3, applied to the objects of study, the result is a street network model of 'Natural roads' classified according to the measure of degree centrality. The result on the set of cities of the longitudinal study can be seen in Fig. 3 and of the cross-sectional study can be seen in Fig. 4. The results of the statistical analyses are presented in next sections, starting with a description of the different statistical parameters used. The number of street segment refers to the number before the natural roads' aggregation across intersections while the degree numbers refers to the natural roads (Tables 2 and 3).

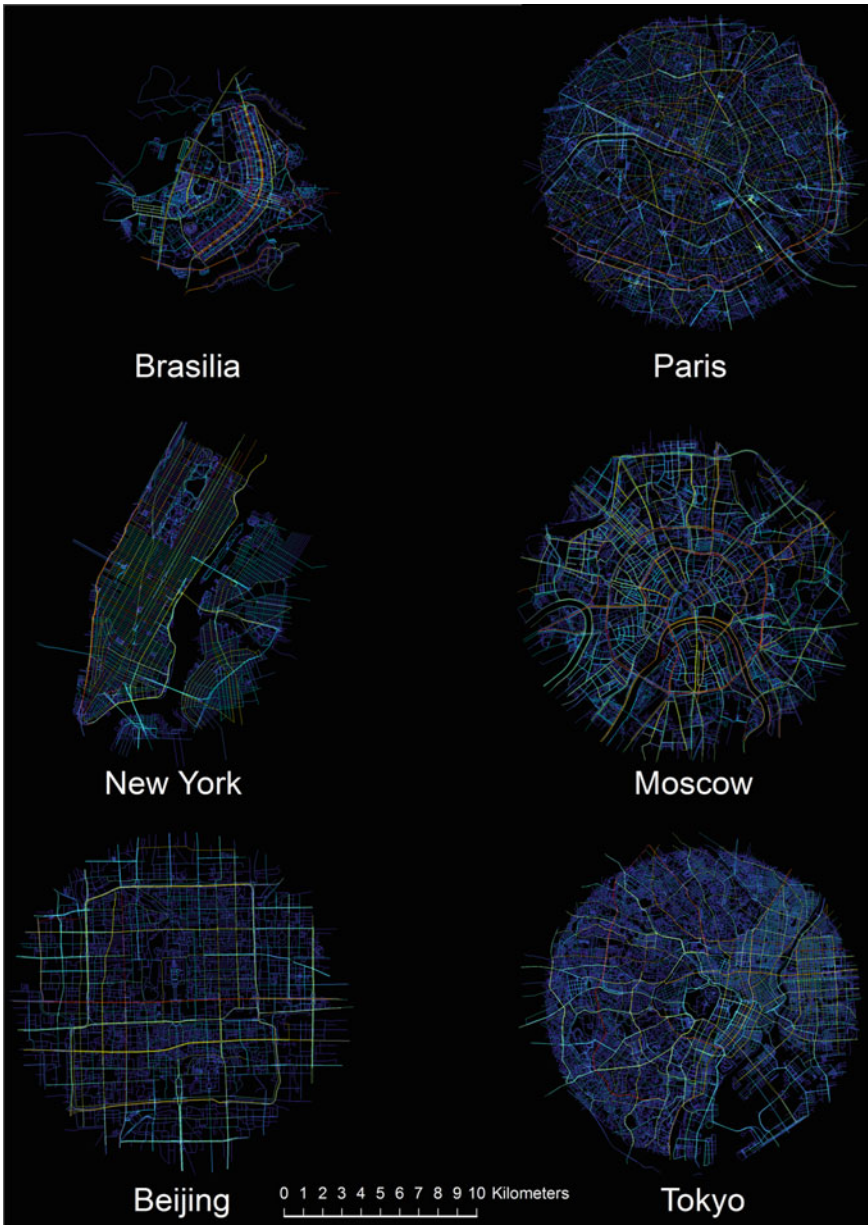
### ***4.1 Derived Parameters of Degree Distribution***

The results of the statistical analysis of degree centrality distributions is summarized in Table 4 for the longitudinal study of Paris and Table 5 for the cross-sectional study. Table 4 is sorted chronologically to understand the evolution and its expansion and restructuring. Table 5 is sorted on the  $p$ -value in ascending order, since the purpose is to understand the power law properties. The next parameter in these tables is Alpha,





**Fig. 3** The longitudinal analysis cities' dual graph network coloured according to degree centrality (red represents a high degree value, blue low)



**Fig. 4** The cross-sectional analysis cities' dual graph network coloured according to degree centrality (red represents a high degree value, blue low)



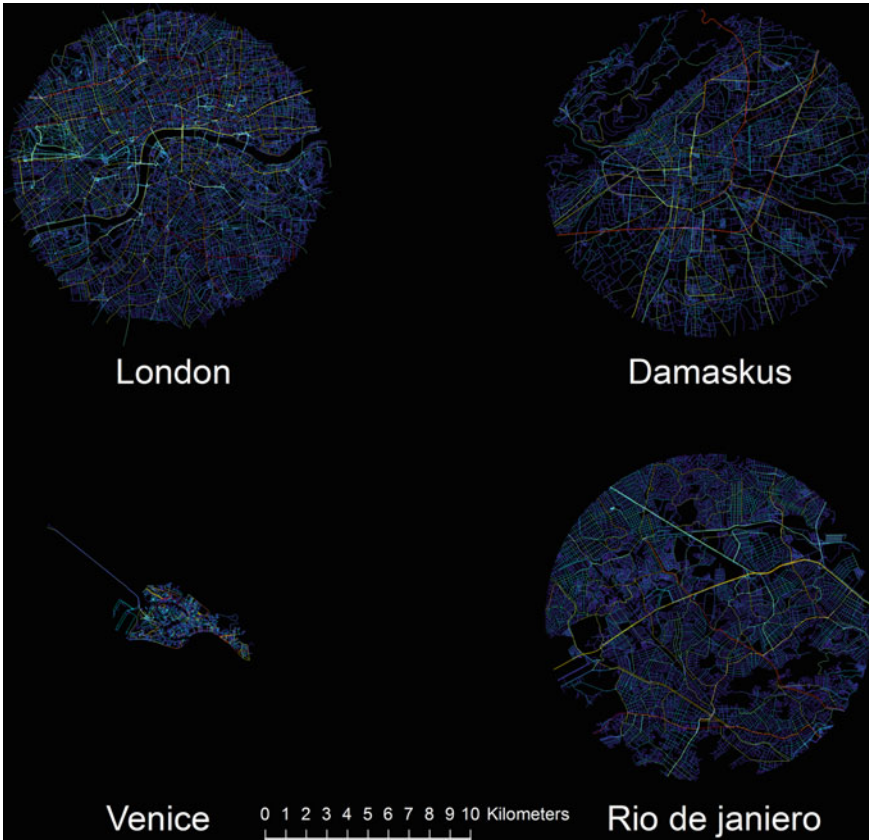


Fig. 4 (continued)

Table 2 Descriptive statistics of the natural roads models of the longitudinal study

Year	Number of street segments	Number of natural roads	Mean degree	Max degree	Median degree	Length in kilometres
1300	1125	353	3.84	57	3	73.8
1380	1065	350	3.94	56	3	69.2
1591	1251	386	3.98	28	3	105.2
1728	3255	961	4.02	73	3	350.3
1791	3463	994	4.24	82	3	393.2
1826	3820	1111	4.35	61	3	407.6
1836	4368	1330	4.00	122	2	434.8
1849	4836	1276	4.79	93	3	497.5
1888	15,749	3511	5.03	88	3	1230.6
1889	9726	2413	5.43	109	3	1587.7

**Table 3** Descriptive statistics of the natural roads models of the cross-sectional study

Name	Number of street segments	Number of natural roads	Mean degree	Max degree	Median degree	Length in kilometres
Beijing	4862	4861	4.72	154	3	2575.8
Brasilia	7646	7645	3.57	189	2	2242.2
Damaskus	6337	6336	3.77	92	3	2189
London	16,786	16,785	3.83	217	2	3762.4
Moscow	14,471	14,470	4.35	210	3	4177.3
Newyork	3449	3448	6.59	173	3	2049
Paris	13,043	13,042	4.82	180	3	3985.8
Rio	8240	8239	4.02	118	3	2841.9
Tokyo	19,007	19,006	4.93	327	3	5212
Venice	2145	2144	3.42	50	2	319.3

which is the scaling factor of the hypothetical power law. Following that is  $x_{\min}$ , which is the starting value from where the distribution can fit a power law. Finally, the number of vertices and their mean degree value are in the next two columns for reference.

The comparison between possible distributions was made for two purposes. The first purpose is, as mentioned above, for those that pass the K-S test to exclude that other distributions could provide a better fit. The second purpose was to obtain an indication of which kind of distribution the non-power law objects could possibly fit. Those results can provide some clues for further research regarding the cities where power laws are ruled out. The results of these log-likelihood tests are commented in the last column. In some cases, the results of the test were not significant, which means that some other distributions cannot be ruled out.

One thing that needs to be mentioned about these tests is the finite-size effect in the street network. Due to the geographical constraints, city limits or sample area limit, the maximum possible degree for a road is limited. This maximum size limit leads to that all possible power laws will in reality be truncated power laws (power laws with an exponential tail). Almost all geographical networks with power law properties are in fact truncated power laws due to finite-size effects. Since this issue is something common to all the cities studied and the study is comparative, it is left out of the analysis. Another reason to leave it out is that the study is aimed at understanding the main part of the street network, not the small (exponential) tail part.

**Table 4** Results from the longitudinal study of Paris

Year	<i>p</i> -value	Alpha	x-min	Number of vertices	Mean degree	Comment	Plausible other distribution
1300	0.58	2.66	3	353	3.84	Medieval core	
1380	0.95	2.62	3	350	3.94	Medieval core	
1591	0.49	2.83	4	386	3.98	Medieval core	Log-normal cannot be ruled out
1728	0.77	2.83	5	961	4.02		
1791	0.95	2.69	4	994	4.24		
1826	0.31	2.70	4	1111	4.35		
1836	0.42	2.58	4	1330	4.00		
1849	0.72	2.61	5	1276	4.79		
1888	0.00	2.27	3	3511	5.03	Post-Hausmann	
1889	0.01	2.42	4	2413	5.43	Post-Hausmann	Possibly exponential
2018	0.00	2.44	4	13,043	4.82	Today	

### 4.2 The Evolution of Paris

The results from the evolution of Paris seem to indicate, as expected, that there are two distinct categories of city structures: Pre-Hausmann (1300–1849) and post-Hausmann (1888 until today). The main difference is observed in the *p*-value for power law probability. From medieval times until the mid-nineteenth century the results indicate that it is possible that the road network structure adheres to a power law, while three post-Hausmann maps fail the K-S test. Maybe the first map from 1889 could be a candidate since 0.1 is the test threshold, but when compared to all of the other maps’ *p*-value the result seems to be distinctly non-power law. Looking at the alpha values reveals that the four post-Hausmann maps have the lowest values while the other maps have a consistently higher alpha value. Altogether, these results show a clear distinction between the organically grown medieval city and the post-Hausmann Paris, subject to a very comprehensive restructuring.

**Table 5** Results of the calculations on the cities in the cross-sectional study

City	<i>p</i> -value	Alpha	x-min	Number of vertices	Mean degree	Growth process	Comparisons to other distributions
Brasilia	0.00	2.60	3	7646	3.566	Entirely planned from start	
Paris	0.00	2.44	4	13,043	4.822	Planned to a large extent	
New York	0.01	2.32	6	3449	6.590	Entirely planned grid	Log-normal seems to be a good fit
Moscow	0.03	2.52	3	14,471	4.354	Planned to express power	
Beijing	0.10	2.49	4	4862	4.736	Planned	
Tokyo	0.29	2.55	7	19,007	4.934	Organically grown	
London	0.39	2.73	6	16,786	3.832	Organically grown	
Damascus	0.40	3.09	8	6337	3.765	Organically grown on Roman grid	Log-normal cannot be ruled out
Venice	0.46	3.50	8	2145	3.425	Several cities organically grown together	Log-normal cannot be ruled out
Rio de Janeiro	0.88	3.13	9	8240	4.021	Many organically grown favelas	

### 4.3 Comparison of Cities

The results in the cross-sectional study can be seemingly grouped into two rather distinct categories, those with  $p$ -value  $< 0.1$  and those with  $p$ -value  $> 0.1$ . This means that approximately half of the selected cities' street network degree distribution does not fit a power law, while the other half has a decent probability of doing that. This finding is interesting since this statistical distribution can tell something about the growth process that has created it. If we identify what differs in the two categories according to Sect. 3.1, there is one main factor that differs: top-down planning or governing during city growth, versus organic growth.

The cities with a  $p$ -value  $< 0.1$  have all been subject to rigorous planning and structuring, even though the purpose, time and type of planning vary (Kostof and Tobias 1991). The other cities that have a  $p$ -value  $> 0.1$  are all to a varying extent what is called organic cities (Kostof and Tobias 1991; Batty and Longley 1994; Hillier 1996), or cities with large areas of organic patterns. These so-called organic cities are those where the city to a large extent has slowly grown over a long period of time and have not been not subject to any comprehensive planning effort. There is a city on the edge of the  $p$ -value limit, Beijing. The value means that it cannot be ruled out that it fits a power law, although it is just 0.01 above the limit. It is unclear why this is the case, but it would be surprising if all data could perfectly fit the dichotomies of way of growth.

Even though the differences in alpha values for the cross-sectional study are not as striking as with the  $p$ -values, the two groups have a significantly ( $p < 0.05$ ) different mean value. Alpha values are on average higher in the group of organic cities than on the group of planned ones, which is in line with the results from the longitudinal study.

Finally, an observation that also deserves some attention is the part of the distribution below  $x$ -min. This part is not included in the distributional analysis, hence the name  $x$ -min for the lower threshold. What is striking is that all cities seem to share the same kind of pattern in this part of the distribution. On average, streets with degree centrality of two (about 30% of total) are around twice as many as the streets with degree centrality of one (roughly 15–20% of total). Then streets with degree three (about 15–20% of total) are approximately half as many as the ones with degree two. Thereafter, the pattern of half as many streets as with the preceding degree number is continuing up until approximately  $x$ -min. This finding (illustrated in Fig. 5) indicates that although cities seem to differ regarding the kind of distribution and scaling factor (alpha), they still share a universal pattern for the finest grained street network, the streets that are connecting directly to the buildings. Whatever the reasons behind this phenomenon, these are so far unexplored.

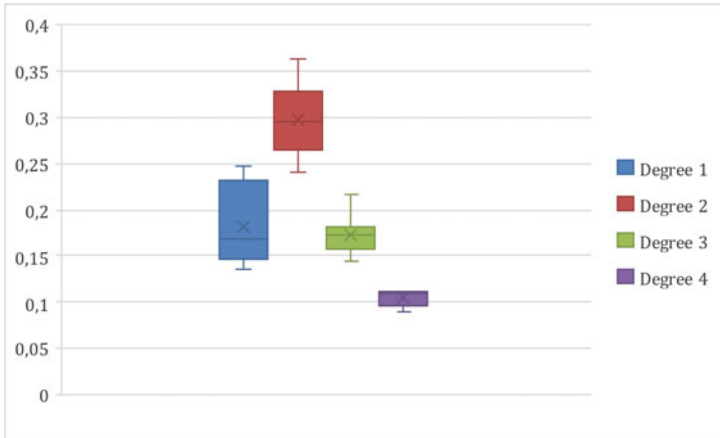


Fig. 5 Proportion of total number for the streets with degree 1–4

## 5 Discussion

### 5.1 Organic and Planned Growth Processes

It seems that, based on these results, whether the degree distribution fits a power law says something about the level of planning in a city. This result can be explained by considering the city as a self-organized scale-free network (Barabasi and Albert 1999). If a city is not subject to large-scale planning, it tends to organize itself according to some growth principle, e.g. preferential attachment or similar processes (Volchenkov and Blanchard 2008). Preferential attachment is an example of a kind of multiplicative growth (Batty and Longley 1994) that results in a scale-free network. This explanation is in line with previous research (Jiang 2007; Volchenkov and Blanchard 2008).

A hypothesis for organic growth could be that new roads attach first and foremost to the existing roads, and these existing roads gain higher degree up to a certain limit (when there is no more space). This means that the organic growth process results in the evolution of the fine-grained minor road (background) network connecting houses to the closest street, while the high degree main roads (foreground) network is more stationary or undergoes a very slow development. The higher alpha values (steeper slope) found in the organic street networks supports this explanation. Since there are mainly local bottom-up forces that shape these (organic) cities, relatively few primary highly connected streets are built because these require a more central initiative and a larger amount of resources (Wang 2015). This also makes sense since it is a more effective use of land (less proportion of roads relative other land use), according to the space filling principle. (Kuffner 2009), which is logical from the

perspective that the forces driving the bottom-up growth (often residential and firms) are acting economically.

But, what the result in this study also indicates is that not all cities' development can be explained in this way; there seems to be a dividing line between planning and self-organizing processes. In planned cities, a common trait is the creation of many high degree streets such as highways, boulevards and main streets. This can have various reasons: political, e.g. it is said that Haussmann's boulevards were created to facilitate the movement of police in case of unrest; traffic management, e.g. new high degree roads are often arterial roads that can handle increased traffic; or to make the road network more redundant. It is probably the case that many of the planned cities started with an organic growth process or have grown on an organic core, but then had a modern road system imposed on them, like in the case of Paris.

## 5.2 *The Classification of Cities*

Planned cities can consist of many different kinds and structures. The selection of cities in this study includes those that are planned in one go (e.g. Brasilia, Fig. 5), as well as cities that have grown on a planned substrate (e.g. New York grown on a regular grid), or cities that have got a new structured superimposed (e.g. Hausmann's Paris). The city of Moscow (Fig. 6) was also subject to repeated and extensive restructuring by centralized regimes. The results from this study are inconclusive regarding planned cities; some seem to be exponential while others might be log-normal, and some do not even seem to have a good fit for any heavy-tailed distribution.

This is not surprising if we consider the kinds of distributions that some ideal city forms have. In a perfect grid, all streets in one direction will have the same degree value while the perpendicular streets will all have another degree value, which gives a distribution with only two values. The same occurs in the case of radial cities with concentric rings. A deeper and more systematic investigation into the many outcomes of city planning actions and their consequences for the street network structure could probably present very fruitful work (Fig. 7).

Whether a city is organic or planned is not a black and white classification either. Cities can be founded and grow an organic network and then suddenly planning measures are put in place affecting the organic pattern, or further growth outside the core is subject to new planning measures. This is quite common for European cities, that a medieval city core is surrounded with more planned patterns. Especially areas built during the twentieth century are to a very large extent planned. The opposite, organic growth on a planned pattern, can also happen. A good example is Herat in Afghanistan (Kostof and Tobias 1991). Where a more fine-grained organic network has grown on the original roman grid. In this way, cities alternate back and forth between these two types of structure. This temporal alternation is often slow for the organic pattern while planning interventions and new planned areas often represent fast change.

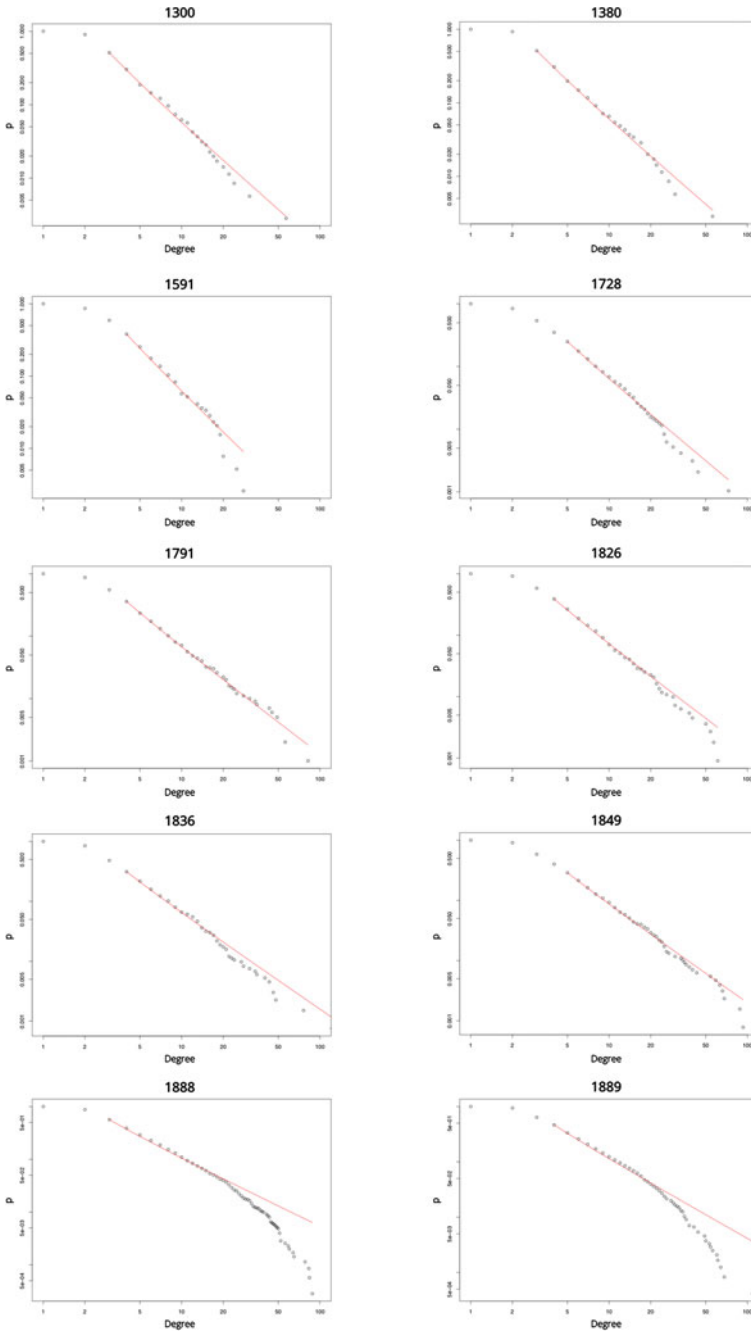


Fig. 6 CCDF of degree distribution in the longitudinal study (best-fit line drawn)



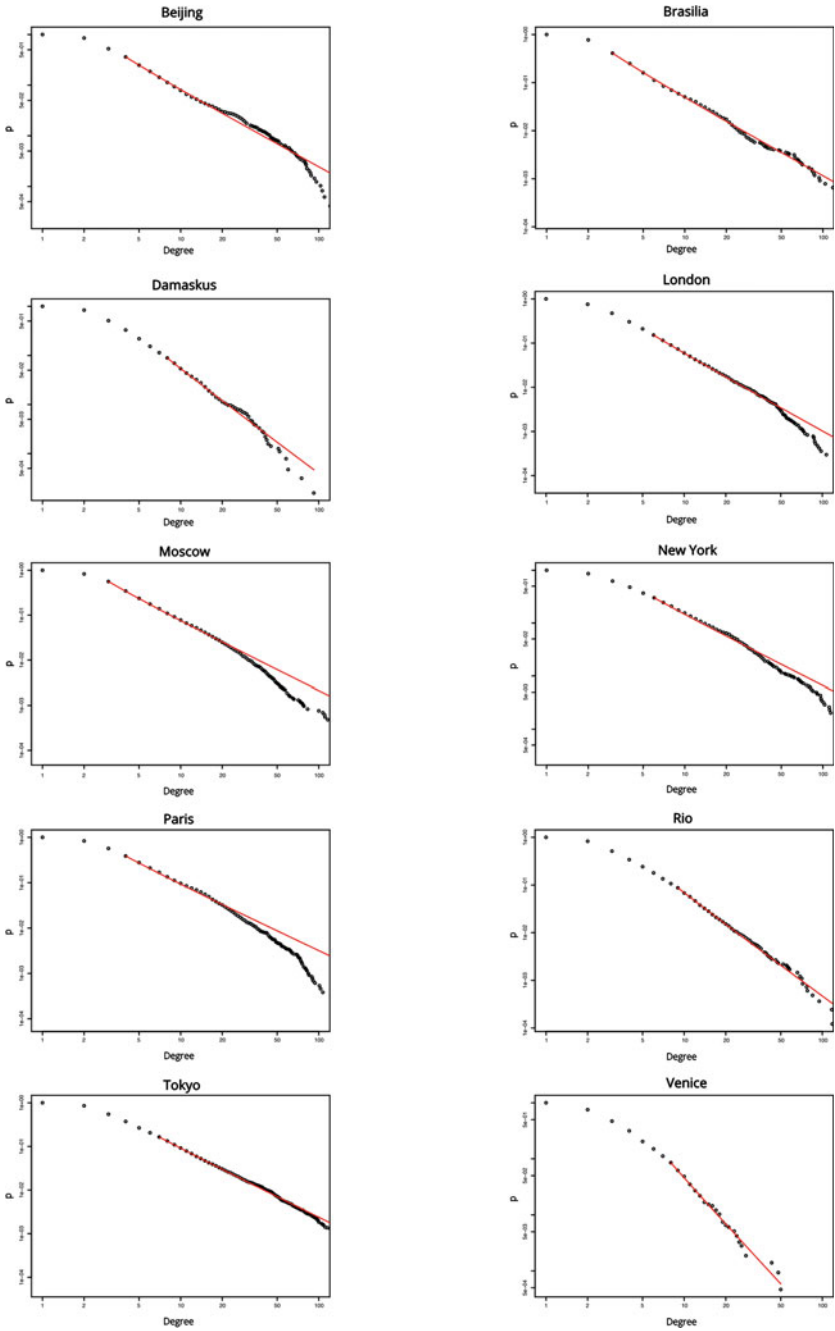


Fig. 7 CCDF of degree distribution in the cross-sectional study (best-fit line drawn)

The temporal alternation also goes in hand with spatial alternation of areas grown organically and other planned. As a consequence, the degree distribution of cities suffers disruptions in the middle range from an increased number of planned streets representing smaller areal interventions, as opposed to the few whole city interventions. While these disruptions deviate from a power law distribution representative of organic growth they do not completely rule out this fit, as occurs with entirely planned cities.

Overall, these results are interesting because they can provide an explanation to the earlier, sometimes contradictory, results (Crucitti et al. 2006a, b; Porta et al. 2006a, b; Jiang 2007; Volchenkov and Blanchard 2008), where some conclusions stated that cities adhered to a power law while others did not. It can be that the studied cities simply were of different kinds (i.e. organic/planned). Another possible explanation is in the methods used; as described in the methods section, there are several criteria that can be used to judge whether a distribution is a power law or not. All together these results show that more studies are required in order to make firmer inferences.

## 6 Conclusions

This study has revealed similarities and differences in the degree distribution of the street network among cities with varying cultural, topographical, geographical or societal organizational circumstances. It has also revealed a striking pattern in the longitudinal study of Paris, which is to a large extent a different city, before and after Hausmann's rebuilding of the city. The factor that seems to explain the striking differences found in the degree distributions is the growth process, organic bottom-up or planned top-down. It seems that cities with a resulting organic street pattern are likely to follow a power law in their degree distribution. On the other hand, the extensively planned cities do not follow a power law distribution. These results can be useful in several ways.

First, they offer an explanation for the disparate results from other studies on the properties of the street networks, which have tried to find universal patterns in cities, often power law distributions. The answer to the question whether a universal pattern exists is: no, because it depends on the growth process of the city. Rather, it seems that there are at least two kinds of patterns.

Second, they elegantly verify the categorization and judgment of cities that urban planning historians and theorists such as Kostof and Tobias have done on a qualitative basis, through visual analysis of the morphology. This opens up the possibility for urban historians, theorists and morphologists to support their work with quantitative analysis on a larger scale than the usual qualitative judgements allow.

Third, for a long time there have been attempts to plan cities or neighbourhoods with an organic morphology. These attempts have often been unsuccessful or failed to produce the kind of urban fabric that was intended. The insights in this study might help such attempts, opening up the possibility to test various designs. The results can

be used to see if the designs have the same properties as the organically grown cities that serve as models.

Although this study is too small to draw conclusive lessons, it provides clues and starting points for further research. It also raises the question of whether there is some differentiated universal pattern in cities. It has shown that one measure of the street network can reveal such patterns and relations that seem to depend on the city's history of growth and planning.

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