MHD Mass Transfer Flow Past an Impulsively Started Semi-Infinite Vertical Plate with Soret Effect and Ramped Wall Temperature

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Abstract An exact solution to the problem of a hydromagnetic natural convective mass transfer flow of an incompressible viscous electrically conducting non-Gray optically thin fluid past an impulsively started semi-infinite vertical plate with ramped wall temperature in presence of appreciable radiation, thermal diffusion and uniform transverse magnetic field is presented. The magnetic Reynolds number is assumed to be small enough to neglect the induced hydromagnetic effects. Closed form Laplace Technique is adopted to get the exact solutions of the resultant non-dimensional governing equations. The influences of thermal radiation, ramped parameter, magnetic field, thermal diffusion and time on the flow and transport characteristics are studied graphically.

Nomenclature

B	Magnetic flux density;
B_0	Strength of the applied magnetic field, Tesla or $\frac{\text{Weber}}{m^2}$;
C	Molar species concentration, $\frac{\text{kmol}}{\text{m}^3}$;
C_p	Specific heat at constant pressure, $\frac{\text{J}}{\text{kg K}}$;
C_∞	Concentration far away from the plate, $\frac{\text{kmol}}{\text{m}^3}$;
C_w	Species concentration at the plate, $\frac{\text{kmol}}{\text{m}^3}$;

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D_M Mass diffusivity, $\frac{\mathrm{m}^2}{\mathrm{s}}$;

D_T Molar thermal diffusivity, $\frac{\mathrm{m}^2\mathrm{kmol}}{\mathrm{K\,s}}$;

$e_{b\lambda}$ Planck function;

g   Gravitational acceleration vector;

g Acceleration due to gravity, $\frac{\mathrm{m}}{\mathrm{s}^2}$;

Gr,   Thermal Grashof number;

Gm   Solutal Grashof number;

J   Current density vector;

K_λ,   Absorption coefficient; $\frac{1}{\mathrm{m}}$

M Magnetic parameter;

p Pressure, $\frac{\mathrm{N}}{\mathrm{m}^2}$;

Pr   Prandtl number;

q   Fluid velocity vector;

q$_\mathbf{r}$ Radiative heat flux vector;

q_r Radiation heat flux, $\frac{\mathrm{W}}{\mathrm{m}^2}$;

Q Radiation parameter;

Ra Ramped parameter;

Sc   Schmidt number;

Sr   Soret number;

t' Time, s;

t_0 Characteristic time, s;

T Temperature, K;

T_w Isothermal temperature, K;

T_∞ Temperature far away from the plate, K;

u', x' Component of fluid velocity, $\frac{\mathrm{m}}{\mathrm{s}}$;

U_0 Plate velocity, $\frac{\mathrm{m}}{\mathrm{s}}$;

(x', y', z') Cartesian coordinate system, (m, m, m);

Greek symbols

σ Electrical conductivity, $\frac{1}{(\mathrm{Ohm}\times\mathrm{m})}$;

ρ Fluid density, $\frac{\mathrm{kg}}{\mathrm{m}^3}$;

ρ_∞ Fluid density far away from the plate, $\frac{\mathrm{kg}}{\mathrm{m}^3}$;

μ Coefficient of viscosity, $\frac{\mathrm{kg}}{\mathrm{ms}}$ or $\frac{\mathrm{Ns}}{\mathrm{m}^2}$;

κ Thermal conductivity, $\frac{\mathrm{W}}{\mathrm{m\,K}}$;

β Coefficient of thermal expansion, $\frac{1}{\mathrm{K}}$;

$\overline{\beta}$ Coefficient of solutal expansion, $\frac{1}{\mathrm{kmol}}$;

υ Kinematic viscosity, $\frac{\mathrm{m}^2}{\mathrm{s}}$;

φ,   Viscous dissipation of energy per unit volume, $\frac{\mathrm{J}}{\mathrm{m}^3}$;

Subscripts

w Refers to physical quantities at the plate;
∞ Refers to physical quantities far away from the plate;

1 Introduction

MHD is a branch of Physics which is concerned with the motion of electrically conducting fluids in presence of magnetic field. MHD pumps, MHD generators and MHD flow meters are some examples of MHD principles. Dynamo and motor is a classical example of MHD principle. The problems dealing with convection in hydromagnetic flows have got importance in Geophysics, Astrophysics, Plasma Physics, Missile technology etc. The MHD principles also find its applications in medicine and biology. The present form of MHD is based on the pioneer contributions of several notable authors like Alfven [9], Cowling [10], Shercliff [28], Ferraro and Plumpton [16] and Crammer and Pai [12].

The natural flow arises in fluid when the change in temperature as well as species concentration causes density variation leading to the existence of buoyancy forces acting on the fluid. Free convection or natural convection is a process of heat or mass transfer in natural flow. The heating of rooms and building by use of radiator is an example of heat transfer by natural convection. On the other hand, the principles of mass transfer are relevant to the working of systems such as a home humidifier and the dispersion of smoke released from a chimney into the environment. The evaporation of alcohol from a container is an example of mass transfer by natural convection. Radiation is also a heat transfer process through electromagnetic waves. Convective flows in presence of radiation are encountered in different industrial and environment processes. At the high operating temperature, the radiation effect can be quite significant. Many processes in engineering and industrial areas occur at high temperatures, and the knowledge of radiation heat transfer becomes very important for design of pertinent equipment. Heat transfer problems become more fruitful from the physical point of view when the simultaneous mass transfer effects on flow are also taken into account. Several authors have carried out model studies on the problems of free convective hydrodynamic and hydromagnetic flows under different flow geometries and physical conditions taking into account of thermal radiation. Some of them are Takhar et al. [35], Mansour [18], Raptis and Perdikis [25], Mankinde [20], Samad and Rahman [27], Prasad et al. [23], Mbeledogu et al. [21], Orhan and Ahmet [22], Seth et al. [34], Ahmed [4, 6], Ahmed et al. [5], Ahmed and Dutta [2, 3]. An analytical solution to the problem of unsteady MHD free-convection flow with Hall effects of a radiating and heat absorbing fluid past a moving vertical plate with variable ramped temperature was obtained by Seth et al. [29]. Seth et al. [30] investigated analytically the problem of heat and mass transfer effects on unsteady MHD natural convection flow of a chemically reactive and radiating

fluid through a porous medium past a moving vertical plate with arbitrary ramped temperature.

In the cases, when both heat and mass transfer occur simultaneously in a moving fluid, the relations connecting the fluxes and driving potentials are of a more intricate nature. It is seen that a mass flux can be generated not only by composition gradient but by temperature gradient as well. The effect concerned with the mass flux under temperature gradient is termed as the Soret effect. The experimental investigation of this effect was first performed by renowned Chemist Charles Soret in 1879 and so this effect is known as the Soret effect in the honour of his name. Soret effect is concerned with the method of separating havier gas molecules from lighter ones by ramped temperature gradient over a given volume of a gas containing particles of different manners. Soret effect is also termed as thermal diffusion effect. Roughly speaking, thermal diffusion effect deals with the mass flux as a result of temperature difference. Comprehensive literature on various aspects of thermal diffusion and diffusion-thermo on different kinds of mass transfer related problems can be found in Eckert and Drake [15], Kafoussias and Williams [17], Postelnicu [24], Ahmed [4, 6, 8], Ahmed and Sengupta [7].

Laplace transform technique is an integral transform technique. It is generally used in case of the problems dealing with impulsively started flow. It also finds its application in case of small Reynolds number flow problems. That is for slow motion or creeping motion. Solutions obtained under the scheme of Laplace transform are generally exact or of closed form. Therefore it does not require stability analysis for checking of validation. Owing to this fact, a good number of researchers have carried out their research works by adopting this method in different kinds of flow problems under different physical and geometrical conditions. Some of them are Ahmed [6], Ahmed et al. [5], Ahmed and Dutta [3], Seth et al. [29–33], Das and Jana [14], Agarwalla and Ahmed [1], Mahanthesh et al. [19], Das et al. [13] and Shah et al. [26].

As the present author is aware, no attempt has been made till now to investigate the problem of natural convective hydromagnetic flow of an electrically conducting viscous incompressible non-Gray optically thin fluid past an impulsively started semi-vertical infinite plate with ramped wall temperature in presence of thermal diffusion, thermal radiation, and uniform transverse magnetic field taking the characteristic time independent of flow properties. Such an attempt has been made in the present work. It is worthwhile to mention that Equations and Solutions of the problem in hand in absence of mass transfer effect are consistent to those of the work of Ahmed and Dutta [3].

2 Gray and Non-gray Gases

The optimal thickness of a material is defined by

$$\tau = \log_e\left(\frac{I_0}{\bar{I}}\right),$$

where I_0 is the original intensity of the beam of light and $\bar{I}$ is the intensity of light after passing through the material.

A gas is called optically thin if $\tau \ll 1$ and it is said to be optically thick if $\tau \gg 1$. Optical thickness is a dimensionless quantity and it measures the capacity of a particular material.

A gas is said to be a Gray gas if the optical thickness τ of the gas is independent of the wave number of electromagnetic radiation. Otherwise, the gas is said to be non-Gray. In general, the commonly found atmospheric gases are non-Gray.

In Cogley's [11] model, the rate of radiation heat flux $\mathbf{q_r}$ in optically thin non-Gray gas near equilibrium is specified by

$$\nabla \cdot \mathbf{q_r} = 4I(T - T_\infty),$$

where

$$I = \int_0^\infty (K_\lambda)_w \left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda.$$

3 Inverse Laplace Transforms of Some Special Functions

   I. $L^{-1}\{e^{-as} f(s)\} = H(t-a)F(t-a),$

   II. $L^{-1}\left(e^{-\sqrt{s}y}\right) = \frac{y}{2\sqrt{\pi} t^{\frac{3}{2}}} e^{-\frac{y^2}{4t}},$

  III. $L^{-1}\left(\frac{1}{\sqrt{s+a}}\right) = \frac{e^{-at}}{\sqrt{\pi t}},$

  IV. $L^{-1}\left(\frac{1}{s\sqrt{s+a}}\right) = \frac{erfc(\sqrt{at})}{\sqrt{a}},$

   V. $L^{-1}\left(\frac{e^{-\sqrt{s}y}}{s}\right) = erfc\left(\frac{y}{2\sqrt{t}}\right),$

  VI. $L^{-1}\left(\frac{e^{-\sqrt{s}y}}{s\sqrt{s}}\right) = 2\sqrt{\frac{t}{\pi}} e^{-\frac{y^2}{4t}} - y\, erfc\left(\frac{y}{2\sqrt{t}}\right),$

 VII. $L^{-1}\left(\frac{e^{-\sqrt{s}y}}{s^2\sqrt{s}}\right) = \sqrt{\frac{t}{\pi}} \frac{e^{-\frac{y^2}{4t}(4t+y^2)}}{s} - \frac{1}{6}y\left(6t + y^2\right)erfc\left(\frac{y}{2\sqrt{t}}\right),$

VIII. $L^{-1}\left\{\frac{e^{-y\sqrt{s\xi}}}{s^2}\right\} = \lambda(\xi, y, t) = t\left[\left(1 + \frac{y^2\xi}{2t}\right)erfc\left(\frac{y\sqrt{\xi}}{2\sqrt{t}}\right) - \frac{y\sqrt{\xi}}{\sqrt{\pi t}} e^{-\frac{y^2\xi}{4t}}\right],$

IX.
$$L^{-1}\left\{\frac{e^{-\sqrt{s+\eta}\,y\sqrt{\xi}}}{s}\right\} = \psi(\xi,\eta,y,t) = \frac{1}{2}\left[e^{\sqrt{\xi\eta}\,y}erfc\left(\frac{\sqrt{\xi}\,y}{2\sqrt{t}} + \sqrt{\eta t}\right)\right.$$

$$\left. +e^{-\sqrt{\xi\eta}\,y}erfc\left(\frac{\sqrt{\xi}\,y}{2\sqrt{t}} - \sqrt{\eta t}\right)\right],$$

$$L^{-1}\left\{\frac{e^{-\sqrt{s+\eta}\sqrt{\xi}\,y}}{s^2}\right\} = f(\xi,\eta,y,t)$$

X.
$$= \left(\frac{t}{2} + \frac{y\sqrt{\xi}}{4\sqrt{\eta}}\right)e^{y\sqrt{\xi\eta}}erfc\left(\frac{y\sqrt{\xi}}{2\sqrt{t}} + \sqrt{\eta t}\right)$$

$$+ \left(\frac{t}{2} - \frac{y\sqrt{\xi}}{4\sqrt{\eta}}\right)e^{-y\sqrt{\xi\eta}}erfc\left(\frac{y\sqrt{\xi}}{2\sqrt{t}} - \sqrt{\eta t}\right),$$

XI.
$$L^{-1}\left(\frac{e^{-y\sqrt{s+a}}}{s^3}\right) = \frac{(\eta^2 + at)t}{4a}\varphi(\eta,a,t) - \frac{\eta\sqrt{t}(1 - 4at)}{8a\sqrt{a}}\bar{\varphi}(\eta,a,t)$$

$$- \frac{\eta t}{2a\sqrt{\pi}}e^{-(\eta^2+at)};$$

where $\eta = \frac{y}{2\sqrt{t}}$, $\varphi(\eta,a,t) = e^{2\eta\sqrt{at}}erfc(\eta + \sqrt{at}) + e^{-2\eta\sqrt{at}}erfc(\eta - \sqrt{at})$ and $\bar{\varphi}(\eta,a,t) = e^{2\eta\sqrt{at}}erfc(\eta + \sqrt{at}) - e^{-2\eta\sqrt{at}}erfc(\eta - \sqrt{at})$.

4 Heaviside's Unit Step Function, Error Function, and Complementary Error Function

Heaviside's unit function is defined by

$$H(t - a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}.$$

The Error function is defined as

$$erf(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-u^2}du.$$

The complementary Error function is defined as

$$erfc(t) = 1 - erf(t) = 1 - \frac{2}{\sqrt{\pi}}\int_0^t e^{-u^2}du = \frac{2}{\sqrt{\pi}}\int_t^\infty e^{-u^2}du.$$

5 Some Properties of Error Function and Complementary Error Function

I. $erf'(0) = \frac{2}{\sqrt{\pi}}$,

II. $erfc'(0) = -\frac{2}{\sqrt{\pi}}$,

III. $erf(0) = 0$,

IV. $erfc(0) = 1$,

V. $erf(x) + erf(-x) = 0$,

VI. $erfc(x) + erfc(-x) = 2$,

VII. $erfc(x) - erfc(-x) = -2erf(x)$,

VIII. $erfc'(z) = -\frac{2e^{-z^2}}{\sqrt{\pi}}$,

IX. $erf'(z) = \frac{2e^{-z^2}}{\sqrt{\pi}}$,

X. $erfc'(z) + erfc'(-z) = -4\sqrt{\frac{1}{\pi}}e^{-z^2}$.

6 Mathematical Formulation of the Problem

Equations governing the motion of an incompressible, viscous, electrically conducting radiating fluid in the presence of a uniform transverse magnetic field taking into account of the effect of thermal diffusion are:

Continuity equation

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

Magnetic field continuity equation

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

Ohm's law for moving conductor

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{q} \times \mathbf{B}), \tag{3}$$

MHD momentum equation with buoyancy force

$$\rho\left[\frac{\partial \mathbf{q}}{\partial t'} + (\mathbf{q} \cdot \nabla)\mathbf{q}\right] = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} + \mu \nabla^2 \mathbf{q}, \tag{4}$$

Energy equation

$$\rho\, C_p \left[\frac{\partial T}{\partial t'} + (\mathbf{q} \cdot \nabla)T \right] = \kappa \nabla^2 T + \varphi + \frac{\mathbf{J}^2}{\sigma} - \nabla \cdot \mathbf{q_r}, \tag{5}$$

Species continuity equation

$$\frac{\partial C}{\partial t'} + (\mathbf{q} \cdot \nabla)C = D_M \nabla^2 C + D_T \nabla^2 T. \tag{6}$$

All the physical quantities are defined in the Nomenclature.

Consider a transient natural convective hydromagnetic mass transfer flow of an incompressible, viscous and electrically conducting optically thin non-Gray fluid past a semi-infinite vertical plate in the presence of a transverse magnetic field of uniform strength B_0. Initially the plate and surrounding fluid were at rest at the same temperature T_∞ with constant concentration C_∞ at all the points in the fluid. At time $t' = 0^+$, the plate starts to move suddenly in its own plane with speed U_0. The concentration near the plate rises to C_w and the plate temperature is instantly raised linearly to $T_w + (T_w - T_\infty)\frac{t'}{t_0}$ for $0 < t' \le t_0$, and the constant temperature $T_w(T_w > T_\infty)$ is maintained at $t' > t_0$.

In order to get the mathematical model of the theoretical problem idealized, impose the following restrictions:

  I.   All the fluid properties are constant except the density in the buoyancy force term.
 II.   Viscous and Ohmic dissipation of energy are negligible.
III.   Magnetic Reynolds number is small enough to neglect the induced magnetic field in comparison to the applied magnetic field.
 IV.   Flow is parallel to the plate.
  V.   Plate is electrically non-conducting.
 VI.   Radiation heat flux in the direction of the plate is negligible in comparison to that in the normal direction.
VII.   No external electric field is applied for which the polarization voltage is negligible leading to $\vec{E} = \vec{0}$.

Introduce a rectangular Cartesian space-time coordinate system (x', y', z', t') with X axis along the plate in the upward vertical direction, Y axis normal to the plate directed into the fluid region and Z axis along the width of the plate. Let $\mathbf{q} = (u', 0, 0)$ denote the fluid velocity and $\mathbf{B} = (0, B_0, 0)$ be the applied magnetic field at the point (x', y', z', t') in the fluid.

The radiation heat flux vector $\mathbf{q_r}$ is given by $\mathbf{q_r} = (0, q_r, 0)$.

Equation (1) gives $\frac{\partial u'}{\partial x'} = 0$, which in turn yields,

$$u' = u'(y', t'). \tag{7}$$

The assumption (VII) and Eq. (3) lead the momentum Eq. (4) to take the form

$$\rho \frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} - \rho g - \frac{\partial p}{\partial x'} - \sigma B_0^2 u'. \tag{8}$$

and

$$0 = \frac{\partial p}{\partial y'}. \tag{9}$$

Equation (9) yields that p is independent of y' and so, along a normal to the plate, pressure near the plate is the same as that far away from the plate. Based on this fact, Eq. (8), for the fluid region far away from the plate reduces to

$$\frac{\partial p}{\partial x'} = -\rho_\infty g. \tag{10}$$

By eliminating $\frac{\partial p}{\partial x'}$ from Eqs. (8) and (10), we derive

$$\rho \frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} + (\rho_\infty - \rho)g - \sigma B_0^2 u'. \tag{11}$$

Equation of state according to classical Boussinesq approximation is

$$\rho_\infty = \rho \left[1 + \beta(T - T_\infty) + \bar{\beta}(C - C_\infty) \right]. \tag{12}$$

Elimination of ρ_∞ from Eqs. (11) and (12) gives

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\bar{\beta}(C - C_\infty) - \frac{\sigma B_0^2 u'}{\rho}. \tag{13}$$

The assumption II and Cogley's [11] assumption lead the Eq. (5) to transform to

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y'^2} - 4I(T - T_\infty). \tag{14}$$

The species continuity Eq. (6) becomes

$$\frac{\partial C}{\partial t'} = D_M \frac{\partial^2 C}{\partial y'^2} + D_T \frac{\partial^2 T}{\partial y'^2}. \tag{15}$$

The relevant initial and boundary conditions (Ref. 2, 30) to be satisfied by the Eqs. (13), (14) and (15) are

$$u' = 0, T = T_\infty, C = C_\infty \,\forall\, y' > 0, t' \leq 0, \tag{16}$$

$$u' = U_0, T = T_\infty + \frac{T_w - T_\infty}{t_0} t', C = C_w \text{ at } y' = 0, 0 < t' \le t_0, \qquad (17)$$

$$u' = U_0, T = T_w, C = C_w \text{ at } y' = 0, t' > t_0, \qquad (18)$$

$$u' \to 0, T \to T_\infty, C \to C_\infty \text{ as } y' \to \infty, t' > 0. \qquad (19)$$

Now, for the sake of normalization of the flow model, the following non-dimensional quantities are introduced.

$$u = \frac{u'}{U_0}, y = \frac{y'}{U_0 t_0}, t = \frac{t'}{t_0}, \text{Gr} = \frac{\upsilon g \beta (T_w - T_\infty)}{U_0^3}, \text{Gm} = \frac{\upsilon g \bar\beta (C_w - C_\infty)}{U_0^3}, Sr = \frac{D_T (T_w - T_\infty)}{\upsilon (C_w - C_\infty)},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, \text{Pr} = \frac{\mu C_p}{\kappa}, Q = \frac{4 I \upsilon}{\rho C_p U_0^2}, M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}, \text{Sc} = \frac{\upsilon}{D_M}, Ra = \frac{U_0^2 t_0}{\upsilon}. \qquad (20)$$

In normalized form, the Eqs. (13), (14) and (15) reduce to

$$\frac{\partial u}{\partial t} = \frac{1}{Ra} \frac{\partial^2 u}{\partial y^2} + Ra \text{Gr} \theta + Ra \text{Gm} \phi - Ra M u, \qquad (21)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Ra \, \text{Pr}} \frac{\partial^2 \theta}{\partial y^2} - Ra Q \theta, \qquad (22)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Ra \text{Sc}} \frac{\partial^2 \phi}{\partial y^2} + \frac{Sr}{Ra} \frac{\partial^2 \theta}{\partial y^2}. \qquad (23)$$

The initial and boundary conditions in non-dimensional form are

$$u = 0, \theta = 0, \phi = 0 \quad \forall y \ge 0, t \le 0, \qquad (24)$$

$$u = 1, \theta = t, \phi = 1 \quad \text{at } y = 0, 0 \le t \le 1, \qquad (25)$$

$$u = 1, \theta = 1, \phi = 1 \quad \text{at } y = 0, t > 1, \qquad (26)$$

$$u \to 0, \theta \to 0, \phi \to 0 \text{ as } y \to \infty, t > 0. \qquad (27)$$

7   Method of Solution

On taking Laplace Transforms of the Eqs. (21), (22) and (23) and conditions (24)–(27), the mixed initial and the boundary value problem reduce to a boundary value problem governed by the equations

$$\frac{d^2 \bar u}{dy^2} - (M Ra^2 + s Ra) \bar u = -Ra^2 \text{Gr} \bar\theta - Ra^2 \text{Gm} \bar\phi, \qquad (28)$$

$$\frac{d^2\bar{\theta}}{dy^2} - Ra\,\Pr(Ra\,Q + s)\bar{\theta} = 0, \tag{29}$$

$$\frac{d^2\bar{\phi}}{dy^2} - Ra\mathrm{Sc}\bar{\phi} = -\mathrm{SrSc}\frac{d^2\bar{\theta}}{dy^2}, \tag{30}$$

subject to the conditions

$$\bar{u} = \frac{1}{s}, \bar{\theta} = \frac{1}{s^2}\left(1 - e^{-s}\right), \bar{\phi} = \frac{1}{s}\,\text{at}\,\bar{y} = 0, \tag{31}$$

$$\bar{u} = 0, \bar{\theta} = 0, \bar{\phi} = 0\,\text{as}\,\bar{y} \to \infty. \tag{32}$$

The solutions of the Eqs. (28)–(30) subject to the conditions (31) and (32) are as follows

$$\bar{\theta} = \frac{1}{s^2}\left(1 - e^{-s}\right)e^{-\sqrt{\Pr\,Ra(Ra\,Q+s)}\,y}, \tag{33}$$

$$\bar{\phi} = \bar{\phi}_1 e^{-\sqrt{s\,Ra\mathrm{Sc}}\,y} + \bar{\phi}_2 e^{-\sqrt{\Pr\,Ra(Ra\,Q+s)}\,y}, \tag{34}$$

$$\bar{u} = \begin{cases} \bar{u}_1 e^{-\sqrt{Ra(M\,Ra+s)}\,y} + \bar{u}_2 e^{-\sqrt{\Pr\,Ra(Ra\,Q+s)}\,y} + \bar{u}_3 e^{-\sqrt{Ra\mathrm{Sc}s}\,y} \\ \quad\text{for}\,\Pr \neq 1,\,\mathrm{Sc} \neq 1,\,\Pr \neq \mathrm{Sc} \\ \bar{u}_4 e^{-\sqrt{Ra(M\,Ra+s)}\,y} + \bar{u}_5 e^{-\sqrt{\Pr\,Ra(Ra\,Q+s)}\,y} + \bar{u}_6 e^{-\sqrt{Ra\,\Pr s}\,y} \\ \quad\text{for}\,\Pr \neq 1,\,\mathrm{Sc} \neq 1,\,\Pr = \mathrm{Sc} \\ \bar{u}_7 e^{-\sqrt{Ra(M\,Ra+s)}\,y} + \bar{u}_8 e^{-\sqrt{Ra(Ra\,Q+s)}\,y} + \bar{u}_9 e^{-\sqrt{Ra\mathrm{Sc}s}\,y} \\ \quad\text{for}\,\Pr = 1,\,\Pr \neq \mathrm{Sc} \\ \bar{u}_{10} e^{-\sqrt{Ra(M\,Ra+s)}\,y} + \bar{u}_{11} e^{-\sqrt{\Pr\,Ra(Ra\,Q+s)}\,y} + \bar{u}_{12} e^{-\sqrt{Ra s}\,y} \\ \quad\text{for}\,\mathrm{Sc} = 1,\,\Pr \neq \mathrm{Sc} \\ \bar{u}_{13} e^{-\sqrt{Ra(M\,Ra+s)}\,y} + \bar{u}_{14} e^{-\sqrt{Ra(Ra\,Q+s)}\,y} + \bar{u}_{15} e^{-\sqrt{Ra s}\,y} \\ \quad\text{for}\,\Pr = 1,\,\mathrm{Sc} = 1 \end{cases} ; \tag{35}$$

where
$$\bar{u}_1 = \frac{1}{s} + \frac{Ra\mathrm{Gr}}{\Pr-1}\bar{S}_1 + Ra\mathrm{Gm}\left(\frac{\bar{S}_2}{\mathrm{Sc}-1} - \frac{\bar{S}_3}{\Pr-1}\right), \bar{u}_2 = -\frac{Ra\mathrm{Gr}}{\Pr-1}\bar{S}_1 + \frac{Ra\mathrm{Gm}}{\Pr-1}\bar{S}_3, \bar{u}_3 = -\frac{Ra\mathrm{Gm}}{\mathrm{Sc}-1}\bar{S}_2,$$
$$\bar{S}_1 = \frac{1}{s^2(s-a_2)}\left(1 - e^{-s}\right), \bar{S}_2 = \frac{1}{s-a_3}\left[\frac{1}{s} + \frac{A(s+Ra\,Q)\left(1-e^{-s}\right)}{(\Pr-\mathrm{Sc})s^2(s-a_1)}\right],$$
$$\bar{S}_3 = \frac{1}{(s-a_2)}\frac{A(s+Ra\,Q)\left(1-e^{-s}\right)}{(\Pr-\mathrm{Sc})s^2(s-a_1)}, \bar{u}_4 = \frac{1}{s} + \frac{Ra\mathrm{Gr}}{\Pr-1}\bar{Q}_1 + \frac{Ra\mathrm{Gm}}{\Pr-1}\left(\bar{Q}_2 - \bar{Q}_3\right),$$
$$\bar{u}_5 = -\frac{Ra\mathrm{Gr}}{\Pr-1}\bar{Q}_1 + \frac{Ra\mathrm{Gm}}{\Pr-1}\bar{Q}_3, \bar{u}_6 = -\frac{Ra\mathrm{Gm}}{\Pr-1}\bar{Q}_2\bar{Q}_1 = \frac{1}{s^2(s-a_2)}\left(1 - e^{-s}\right),$$
$$\bar{Q}_2 = \frac{1}{s-a_3}\left[\frac{1}{s} + \frac{\mathrm{Sr}\Pr(s+Ra\,Q)\left(1-e^{-s}\right)}{s^2 Ra\,Q}\right], \bar{Q}_3 = \frac{1}{s-a_2}\left[\frac{\mathrm{Sr}\Pr(s+Ra\,Q)\left(1-e^{-s}\right)}{s^2 Ra\,Q}\right],$$
$$\bar{u}_7 = \frac{1}{s} + \frac{\mathrm{Gr}\bar{P}_1}{Q-M} + \frac{Ra\mathrm{Gm}}{\mathrm{Sc}-1}\left\{\bar{P}_2 + \frac{\mathrm{SrSc}}{Ra(Q-M)}\bar{P}_3\right\},$$
$$\bar{u}_8 = -\frac{\mathrm{Gr}\bar{P}_1}{Q-M} - \frac{Ra\mathrm{GmSrSc}}{Ra(Q-M)(\mathrm{Sc}-1)}\bar{P}_3, \bar{u}_9 = -\frac{Ra\mathrm{Gm}}{\mathrm{Sc}-1}\bar{P}_2,$$
$$\bar{P}_1 = \frac{1}{s^2}\left(1 - e^{-s}\right), \bar{P}_2 = \frac{1}{s-a_3}\left\{\frac{1}{s} - \frac{\mathrm{SrSc}(Ra\,Q+s)\left(1-e^{-s}\right)}{s^2(\mathrm{Sc}-1)(s-\bar{a}_1)}\right\},$$

$$\bar{P}_3 = \frac{(RaQ+s)\left(1-e^{-s}\right)}{s^2(s-\bar{a}_1)}, \bar{u}_{10} = \frac{1}{s} + \frac{RaGr}{Pr-1}\bar{S}_1 - RaGm\left\{\frac{\bar{S}_4}{MRa} + \frac{Pr Sr \bar{S}_5}{(Pr-1)^2}\right\},$$

$$\bar{u}_{11} = -\frac{RaGr}{Pr-1}\bar{S}_1 + \frac{RaGm Pr Sr \bar{S}_5}{(Pr-1)^2}, \bar{u}_{12} = \frac{Gm}{M}\bar{S}_4, \bar{S}_4 = \frac{1}{s} + \frac{Sr Pr(RaQ+s)\left(1-e^{-s}\right)}{s^2(Pr-1)(s-d_1)},$$

$$\bar{S}_5 = \frac{(RaQ+s)\left(1-e^{-s}\right)}{s^2(s-a_2)(s-d_1)}, \bar{u}_{13} = \frac{1}{s} + \frac{Gr}{Q-M}T_1 - \frac{Gm}{M}T_2 - \frac{SrGm}{QRa(Q-M)}T_3,$$

$$\bar{u}_{14} = -\frac{Gr}{Q-M}T_1 + \frac{SrGm}{QRa(Q-M)}T_3, \bar{u}_{15} = \frac{Gm}{M}T_2, T_1 = \frac{1}{s^2}\left(1-e^{-s}\right),$$

$$T_2 = \frac{1}{s} + \frac{Sr(RaQ+s)\left(1-e^{-s}\right)}{QRa s^2}, T_3 = \frac{(RaQ+s)\left(1-e^{-s}\right)}{s^2}.$$

Solving the ordinary 2nd order linear differential equations subject to the conditions (31) and (32) and taking inverse Laplace Transforms of the solutions, the expressions for temperature θ, concentration φ and velocity field u are obtained as follows

$$\theta = \psi_1 - \psi_2, \tag{36}$$

$$\phi = \begin{cases} \phi_1 + \phi_2 - \phi_3 \text{ for } Pr \neq Sc \\ \bar{\phi}_1 + \bar{\phi}_2 - \bar{\phi}_3 \text{ for } Pr = Sc \end{cases}, \tag{37}$$

$$u = \begin{cases} \sum\limits_{\alpha=1}^{7} G_\alpha, Pr \neq 1, Sc \neq 1, Pr \neq Sc \\ \sum\limits_{\alpha=1}^{7} H_\alpha, Pr \neq 1, Sc \neq 1, Pr = Sc \\ \sum\limits_{\alpha=1}^{7} J_\alpha, Pr = 1, Pr \neq Sc \\ \sum\limits_{\alpha=1}^{7} K_\alpha, Sc = 1, Pr \neq Sc \\ \sum\limits_{\alpha=1}^{7} L_\alpha, Pr = 1, Sc = 1 \end{cases} ; \tag{38}$$

where

$\phi_1 = \psi_5, \phi_2 = \frac{A}{Pr-Sc}[A_1(\psi_3 - \psi_4) + A_2(\psi_5 - \psi_6) + A_3(\psi_7 - \psi_8)],$

$\phi_3 = \frac{A}{Pr-Sc}[A_1(\psi_9 - \psi_{10}) + A_2(\psi_{11} - \psi_{12}) + A_3(\psi_1 - \psi_2)], \bar{\phi}_1 = \phi_1,$

$\bar{\phi}_2 = \frac{SrSc}{RaQ}[RaQ(\psi_7 - \psi_8) + \psi_5 - \psi_6], \bar{\phi}_3 = \frac{SrSc}{RaQ}[RaQ(\psi_1 - \psi_2) + \psi_{11} - \psi_{12}],$

$G_1 = \psi_{13}, G_2 = \frac{RaGr}{Pr-1}[A_4(\psi_{14} - \psi_{15}) + A_5(\psi_{13} - \psi_{16}) + A_6(\psi_{17} - \psi_{18})],$

$G_3 = \frac{RaGm}{Sc-1}\left[\psi_{19} + \frac{A}{Pr-Sc}\xi_1\right],$

$\xi_1 = A_7(\psi_{20} - \psi_{21}) + A_8(\psi_{22} - \psi_{23}) + A_9(\psi_{13} - \psi_{16}) + A_{10}(\psi_{17} - \psi_{18}),$

$G_4 = -\frac{RaGmA}{(Pr-1)(Pr-Sc)}\xi_2,$

$\xi_2 = A_{11}(\psi_{20} - \psi_{21}) + A_{12}(\psi_{14} - \psi_{15}) + A_{13}(\psi_{13} - \psi_{16}) + A_{14}(\psi_{17} - \psi_{18}),$

$G_5 = -\frac{RaGr}{Pr-1}\xi_3,$

$$\xi_3 = A_4(\psi_{24} - \psi_{25}) + A_5(\psi_{11} - \psi_{12}) + A_6(\psi_1 - \psi_2),$$

$$G_6 = -\frac{RaGm}{Sc-1}\left[\psi_{26} + \frac{A}{Pr-Sc}\xi_4\right],$$

$$\xi_4 = A_7(\psi_3 - \psi_4) + A_8(\psi_{27} - \psi_{28}) + A_9(\psi_5 - \psi_6) + A_{10}(\psi_7 - \psi_8),$$

$$G_7 = \frac{RaGmA}{(Pr-1)(Pr-Sc)}\xi_5,$$

$$\xi_5 = A_{11}(\psi_{29} - \psi_{30}) + A_{12}(\psi_{26} - \psi_{27}) + A_{13}(\psi_{11} - \psi_{12}) + A_{14}(\psi_1 - \psi_2),$$

$$H_1 = \psi_{13}, \quad H_2 = \frac{RaGr}{Pr-1}\xi_6,$$

$$\xi_6 = A_4(\psi_{14} - \psi_{15}) + A_5(\psi_{13} - \psi_{16}) + A_6(\psi_{17} - \psi_{18}),$$

$$H_3 = \frac{RaGm}{Pr-1}\left(\psi_{19} + \frac{Sr\,Sc}{Ra\,Q}\xi_7\right),$$

$$\xi_7 = C_7(\psi_{22} - \psi_{23}) + C_8(\psi_{13} - \psi_{16}) + C_9(\psi_{17} - \psi_{18}), \quad H_4 = -\frac{Gm\,Sr\,Sc}{Sc-1}\xi_8,$$

$$\xi_8 = C_{10}(\psi_{14} - \psi_{15}) + C_{11}(\psi_{13} - \psi_{16}) + C_{12}(\psi_{17} - \psi_{18}), \quad H_5 = -\frac{Gr\,Ra}{Sc-1}\xi_9,$$

$$\xi_9 = A_4(\psi_{24} - \psi_{25}) + A_5(\psi_{11} - \psi_{12}) + A_6(\psi_1 - \psi_2),$$

$$H_6 = \frac{Gm\,Ra}{Sc-1}\left(\psi_{26} + \frac{Sr\,Sc}{Ra\,Q}\xi_{10}\right),$$

$$\xi_{10} = C_7(\psi_{27} - \psi_{28}) + C_8(\psi_5 - \psi_6) + C_9(\psi_7 - \psi_8), \quad H_7 = \frac{Gm\,Sr\,Sc}{Q(Sc-1)}\xi_{11},$$

$$\xi_{11} = C_{10}(\psi_{24} - \psi_{25}) + C_{11}(\psi_{11} - \psi_{12}) + C_{12}(\psi_1 - \psi_2),$$

$$J_1 = \psi_{13}, \quad J_2 = \frac{Gr}{Q-M}(\psi_{17} - \psi_{18}),$$

$$J_3 = \frac{Gm\,Ra}{Sc-1}(\psi_{19} - Sr\,Sc\,\xi_{12}), \quad J_4 = \frac{Gm}{(Q-M)(Sc-1)}\xi_{13},$$

$$\xi_{12} = \bar{A}_7(\psi_{31} - \psi_{32}) + \bar{A}_8(\psi_{22} - \psi_{23}) + \bar{A}_9(\psi_{13} - \psi_{16}) + \bar{A}_{10}(\psi_{17} - \psi_{18}),$$

$$J_5 = -\frac{Gr}{Q-M}(\psi_{33} - \psi_{34}),$$

$$\xi_{13} = B_1(\psi_{31} - \psi_{32}) + B_2(\psi_{13} - \psi_{16}) + B_3(\psi_{17} - \psi_{18}),$$

$$J_6 = -\frac{RaGm}{Sc-1}\left(\xi_{14} - \frac{SrSc}{Sc-1}\xi_{15}\right), \quad \xi_{14} = \frac{\psi_{29} - \psi_{30}}{a_3},$$

$$\xi_{15} = \bar{A}_7(\psi_{35} - \psi_{36}) + \bar{A}_8(\psi_{27} - \psi_{28}) + \bar{A}_9(\psi_5 - \psi_6) + \bar{A}_{10}(\psi_7 - \psi_8),$$

$$J_7 = -\frac{SrScGm}{(Q-M)(Sc-1)}\xi_{16}, \quad \xi_{16} = B_1(\psi_{37} - \psi_{38}) + B_2(\psi_{39} - \psi_{40}) + B_3(\psi_{33} - \psi_{34}), \quad K_1 = \psi_{13}, \quad K_2 = G_2, \quad K_3 = -\frac{RaGm}{RaM}\left[\psi_{13} + \frac{SrPr}{Pr-1}\xi_{17}\right],$$

$$\xi_{17} = \bar{B}_1(\psi_{41} - \psi_{42}) + \bar{B}_2(\psi_{13} - \psi_{16}) + \bar{B}_3(\psi_{17} - \psi_{18}), \quad K_4 = -\frac{RaGmPrSr}{(Pr-1)^2}\xi_{18},$$

$$\xi_{18} = B_4(\psi_{41} - \psi_{42}) + B_5(\psi_{14} - \psi_{15}) + B_6(\psi_{13} - \psi_{16}) + B_7(\psi_{17} - \psi_{18}),$$

$$K_5 = -\frac{RaGr}{Pr-1}\xi_{19}, \quad K_6 = \frac{Gm}{M}\left[\psi_{45} + \frac{SrPr}{Pr-1}\xi_{20}\right],$$

$$\xi_{19} = A_4(\psi_{24} - \psi_{25}) + A_5(\psi_{11} - \psi_{12}) + A_6(\psi_1 - \psi_2),$$

$$\xi_{20} = \bar{B}_1(\psi_{43} - \psi_{44}) + \bar{B}_2(\psi_{45} - \psi_{46}) + \bar{B}_3(\psi_{48} - \psi_{49}), K_7 = \frac{Ra\mathrm{GmPrSr}}{(\mathrm{Pr}-1)^2}\xi_{21},$$

$$\xi_{21} = B_4(\psi_{49} - \psi_{50}) + B_5(\psi_{24} - \psi_{25}) + B_6(\psi_{11} - \psi_{12}) + B_7(\psi_1 - \psi_2),$$

$$L_1 = \psi_{13}, L_2 = -\frac{\mathrm{Gr}}{Q - M}(\psi_{17} - \psi_{18}), L_3 = -\frac{\mathrm{Gm}}{M}\left[\psi_{13} + \frac{\mathrm{Sr}}{QRa}\xi_{22}\right],$$

$$\xi_{22} = \psi_{13} - \psi_{16} + RaQ(\psi_{17} - \psi_{18}),$$

$$L_4 = -\frac{\mathrm{GmSr}}{QRa(Q - M)}[\psi_{13} - \psi_{16} + RaQ(\psi_{17} - \psi_{18})],$$

$$L_5 = -\frac{\mathrm{Gm}}{Q - M}[\psi_{33} - \psi_{34}],$$

$$L_6 = \frac{\mathrm{Gm}}{M}\left[\psi_{45} + \frac{\mathrm{Sr}}{QRa}\xi_{23}\right], \xi_{23} = \psi_{45} - \psi_{46} + RaQ(\psi_{47} - \psi_{48}),$$

$$L_7 = \frac{\mathrm{SrGm}}{QRa(Q - M)}[\psi_{33} - \psi_{34} + RaQ(\psi_{33} - \psi_{34})], A = \mathrm{SrScPr}, B = \mathrm{Pr}\ RaQ,$$

$$a_1 = \frac{-B}{\mathrm{Pr}-\mathrm{Sc}}, A_1 = \frac{RaQ + a_1}{a_1^2}, A_2 = -\frac{RaQ + a_1}{a_1^2}, A_3 = -\frac{RaQ}{a_1^2},$$

$$a_2 = \frac{Ra(M - \mathrm{Pr}\,Q)}{\mathrm{Pr}-1}, A_4 = \frac{1}{a_2^2}, A_5 = -A_4, A_6 = -\frac{1}{a_2}, A_7 = \frac{RaQ + a_1}{a_1^2(a_1 - a_3)},$$

$$A_8 = \frac{RaQ + a_3}{a_3^2(a_3 - a_1)}, A_9 = -(A_7 + A_8), A_{10} = \frac{RaQ}{a_1 a_3}, A_{11} = \frac{RaQ + a_1}{a_1^2(a_1 - a_2)},$$

$$A_{12} = \frac{RaQ + a_2}{a_2^2(a_2 - a_1)}, A_{13} = -(A_{11} + A_{12}), A_{14} = \frac{RaQ}{a_1 a_2}, a_3 = \frac{MRa}{\mathrm{Sc}-1},$$

$$C_7 = \frac{RaQ + a_3}{a_3^2}, C_8 = -C_7, C_9 = -\frac{RaQ}{a_3}, C_{10} = \frac{RaQ + a_2}{a_2^2}, C_{11} = -C_{10},$$

$$C_{12} = -\frac{RaQ}{a_2}, \bar{a}_1 = \frac{RaQ}{\mathrm{Sc}-1}, \bar{A}_7 = \frac{RaQ + \bar{a}_1}{\bar{a}_1^2(a_1 - a_3)}, \bar{A}_8 = \frac{RaQ + a_3}{a_3^2(a_3 - \bar{a}_1)},$$

$$\bar{A}_9 = -(\bar{A}_7 + \bar{A}_8), \bar{A}_{10} = \frac{RaQ}{\bar{a}_1 a_3}, B_1 = \frac{RaQ + \bar{a}_1}{\bar{a}_1^2}, B_2 = -B_1,$$

$$B_3 = -\frac{RaQ}{\bar{a}_1}, d_1 = \frac{RaQ\,\mathrm{Pr}}{1 - \mathrm{Pr}}, \bar{B}_1 = \frac{RaQ + d_1}{d_1^2}, \bar{B}_2 = -\bar{B}_1, \bar{B}_3 = -\frac{RaQ}{d_1},$$

$$B_4 = \frac{d_1 + RaQ}{d_1^2(d_1 - a_2)}, B_5 = \frac{a_2 + RaQ}{a_2^2(a_2 - d_1)}, B_6 = -(B_4 + B_5), B_7 = \frac{RaQ}{d_1 a_2},$$

$$\psi_1 = f(\mathrm{Pr}\,Ra, QRa, y, t), \psi_2 = f(\mathrm{Pr}\,Ra, QRa, y, t - 1)H(t - 1),$$

$$\psi_3 = e^{a_1 t}\psi(\mathrm{Sc}Ra, a_1, y, t), \psi_4 = e^{a_1(t-1)}\psi(\mathrm{Sc}Ra, a_1, y, t - 1)H(t - 1),$$

$$\psi_7 = \lambda(Ra\mathrm{Sc}, y, t), \psi_8 = \lambda(Ra\mathrm{Sc}, y, t - 1)H(t - 1),$$

$$\psi_5 = erfc\left(\frac{y}{2}\sqrt{\frac{Ra\text{Sc}}{t}}\right), \psi_6 = erfc\left(\frac{y}{2}\sqrt{\frac{Ra\text{Sc}}{t-1}}\right)H(t-1),$$

$$\psi_9 = e^{a_1 t}\psi(\text{Pr }Ra, RaQ + a_1, y, t),$$

$$\psi_{10} = e^{a_1(t-1)}\psi(\text{Pr }Ra, RaQ + a_1, y, t-1)H(t-1),$$

$$\psi_{11} = \psi(\text{Pr }Ra, QRa, y, t), \psi_{12} = \psi(\text{Pr }Ra, QRa, y, t-1)H(t-1),$$

$$\psi_{13} = \psi(Ra, MRa, y, t), \psi_{16} = \psi(Ra, MRa, y, t-1)H(t-1),$$

$$\psi_{14} = e^{a_2 t}\psi(Ra, RaM + a_2, y, t),$$

$$\psi_{15} = e^{a_2(t-1)}\psi(Ra, RaM + a_2, y, t-1)H(t-1),$$

$$\psi_{17} = f(Ra, MRa, y, t), \psi_{18} = f(Ra, MRa, y, t-1)H(t-1),$$

$$\psi_{19} = \frac{1}{a_3}\left[e^{a_3 t}\psi(Ra, MRa + a_3, y, t) - \psi_{13}\right],$$

$$\psi_{20} = e^{a_1 t}\psi(Ra, MRa + a_1, y, t),$$

$$\psi_{21} = e^{a_1(t-1)}\psi(Ra, MRa + a_1, y, t-1)H(t-1),$$

$$\psi_{22} = e^{a_3 t}\psi(Ra, MRa + a_3, y, t),$$

$$\psi_{23} = e^{a_3(t-1)}\psi(Ra, MRa + a_3, y, t-1)H(t-1),$$

$$\psi_{24} = e^{a_2 t}\psi(\text{Pr }Ra, RaQ + a_2, y, t),$$

$$\psi_{25} = e^{a_2(t-1)}\psi(\text{Pr }Ra, RaQ + a_2, y, t)H(t-1),$$

$$\psi_{26} = \frac{1}{a_3}\left[e^{a_3 t}\psi(\text{Sc}Ra, a_3, y, t) - \psi_5\right],$$

$$\psi_{27} = e^{a_3 t}\psi(\text{Sc}Ra, a_3, y, t), \psi_{28} = e^{a_3(t-1)}\psi(\text{Sc}Ra, a_3, y, t-1)H(t-1),$$

$$\psi_{29} = e^{a_1 t}\psi(\text{Pr }Ra, RaQ + a_1, y, t),$$

$$\psi_{30} = e^{a_1(t-1)}\psi(\text{Pr }Ra, RaQ + a_1, y, t)H(t-1),$$

$$\psi_{31} = e^{\bar{a}_1 t}\psi(Ra, MRa + \bar{a}_1, y, t),$$

$$\psi_{32} = e^{\bar{a}_1(t-1)}\psi(Ra, MRa + \bar{a}_1, y, t-1)H(t-1),$$

$$\psi_{33} = f(Ra, QRa, y, t), \psi_{34} = f(Ra, QRa, y, t-1)H(t-1),$$

$$\psi_{35} = e^{\bar{a}_1 t}\psi(Ra\text{Sc}, \bar{a}_1, y, t), \psi_{36} = e^{\bar{a}_1(t-1)}\psi(Ra\text{Sc}, \bar{a}_1, y, t-1)H(t-1),$$

$$\psi_{37} = e^{\bar{a}_1 t}\psi(Ra, QRa + \bar{a}_1, y, t),$$

$$\psi_{38} = e^{\bar{a}_1(t-1)}\psi(Ra, QRa + \bar{a}_1, y, t-1)H(t-1),$$

$$\psi_{39} = \psi(Ra, QRa, y, t), \psi_{40} = \psi(Ra, QRa, y, t-1)H(t-1),$$

$$\psi_{41} = e^{d_1 t}\psi(Ra, MRa + d_1, y, t),$$

$$\psi_{42} = e^{d_1(t-1)}\psi(Ra, MRa + d_1, y, t-1)H(t-1),$$

$$\psi_{43} = e^{d_1 t}\psi(Ra, d_1, y, t), \psi_{44} = e^{d_1(t-1)}\psi(Ra, d_1, y, t)H(t-1),$$

$$\psi_{47} = \lambda(Ra, y, t), \psi_{48} = \lambda(Ra, y, t-1)H(t-1),$$

$$\psi_{45} = erfc\left(\frac{y}{2}\sqrt{\frac{Ra}{t}}\right), \ \psi_{46} = erfc\left(\frac{y}{2}\sqrt{\frac{Ra}{t-1}}\right)H(t-1),$$

$$\psi_{49} = e^{d_1 t}\psi(\Pr Ra, RaQ + d_1, y, t),$$

$$\psi_{50} = e^{d_1(t-1)}\psi(\Pr Ra, RaQ + d_1, y, t)H(t-1);$$

$$\psi(\xi, \eta, y, t) = \frac{1}{2}\left[e^{\sqrt{\xi\eta}\,y}erfc\left(\frac{y}{2}\sqrt{\frac{\xi}{t}} + \sqrt{\eta t}\right) + e^{-\sqrt{\xi\eta}\,y}erfc\left(\frac{y}{2}\sqrt{\frac{\xi}{t}} - \sqrt{\eta t}\right)\right],$$

$$f(\xi, \eta, y, t) = \left(\frac{t}{2} + \frac{y}{4}\sqrt{\frac{\xi}{\eta}}\right)e^{\sqrt{\xi\eta}\,y}erfc\left(\frac{y}{2}\sqrt{\frac{\xi}{t}} + \sqrt{\eta t}\right)$$

$$+ \left(\frac{t}{2} - \frac{y}{4}\sqrt{\frac{\xi}{\eta}}\right)e^{-\sqrt{\xi\eta}\,y}erfc\left(\frac{y}{2}\sqrt{\frac{\xi}{t}} - \sqrt{\eta t}\right),$$

$$\lambda(\xi, y, t) = \left(t + \frac{y^2\xi}{2}\right)erfc\left(\frac{y}{2}\sqrt{\frac{\xi}{t}}\right) - \frac{y\sqrt{\xi t}}{\sqrt{\pi}}e^{-\frac{\xi y^2}{4t}};$$

$$H(t-1) = \begin{cases} 0, & t < 1 \\ 1, & t > 1 \end{cases} \text{ is the unit step function.}$$

8 Rate of Momentum Transfer

The viscous drag per unit area on the plate in the direction of the plate velocity is quantified by Newton's law of viscosity as follows

$$\tau' = -\mu\frac{\partial u'}{\partial y'}\bigg]_{y'=0} = -\frac{\mu}{t_0}\frac{\partial u}{\partial y}\bigg]_{y=0}. \tag{39}$$

The coefficient of skin friction or the coefficient of the rate of momentum transfer at the plate is given by

$$\tau = \frac{\tau'}{\mu/t_0} = -\frac{\partial u}{\partial y}\bigg]_{y=0} = \begin{cases} -\sum\limits_{\alpha=1}^{7}M_\alpha, \Pr \neq 1, \text{Sc} \neq 1, \Pr \neq \text{Sc} \\ -\sum\limits_{\alpha=1}^{7}N_\alpha, \Pr \neq 1, \text{Sc} \neq 1, \Pr = \text{Sc} \\ -\sum\limits_{\alpha=1}^{7}P_\alpha, \Pr = 1, \Pr \neq \text{Sc} \\ -\sum\limits_{\alpha=1}^{7}Q_\alpha, \text{Sc} = 1, \Pr \neq \text{Sc} \\ -\sum\limits_{\alpha=1}^{7}S_\alpha, \Pr = 1, \text{Sc} = 1 \end{cases} ; \tag{40}$$

where

$$M_1 = \Omega_{13}, \quad M_2 = \frac{Ra\,Gr}{Pr - 1}[A_4(\Omega_{14} - \Omega_{15}) + A_5(\Omega_{13} - \Omega_{16}) + A_6(\Omega_{17} - \Omega_{18})],$$

$$M_3 = \frac{Ra\,Gm}{Sc - 1}\left[\Omega_{19} + \frac{A}{Pr - Sc}\eta_1\right], \quad M_4 = -\frac{Ra\,Gm\,A}{(Pr - 1)(Pr - Sc)}\eta_2,$$

$$\eta_1 = A_7(\Omega_{20} - \Omega_{21}) + A_8(\Omega_{22} - \Omega_{23}) + A_9(\Omega_{13} - \Omega_{16}) + A_{10}(\Omega_{17} - \Omega_{18}),$$

$$\eta_2 = A_{11}(\Omega_{20} - \Omega_{21}) + A_{12}(\Omega_{14} - \Omega_{15}) + A_{13}(\Omega_{13} - \Omega_{16}) + A_{14}(\Omega_{17} - \Omega_{18}),$$

$$M_5 = -\frac{Ra\,Gr}{Pr - 1}[A_4(\Omega_{24} - \Omega_{25}) + A_5(\Omega_{11} - \Omega_{12}) + A_6(\Omega_1 - \Omega_2)],$$

$$M_6 = -\frac{Ra\,Gm}{Sc - 1}\left[\Omega_{26} + \frac{A}{Pr - Sc}\{A_7(\Omega_3 - \Omega_4) + A_8(\Omega_{27} - \Omega_{28})\right.$$
$$\left. + A_9(\Omega_5 - \Omega_6) + A_{10}(\Omega_7 - \Omega_8)\}\right],$$

$$M_7 = -\frac{A\,Ra\,Gm}{(Pr - 1)(Pr - Sc)}[A_{11}(\Omega_{29} - \Omega_{30}) + A_{12}(\Omega_{26} - \Omega_{27})$$
$$+ A_{13}(\Omega_{11} - \Omega_{12}) + A_{14}(\Omega_1 - \Omega_2)],$$

$$N_1 = \Omega_{13}, \quad N_2 = \frac{Ra\,Gr}{Pr - 1}[A_4(\Omega_{14} - \Omega_{15}) + A_5(\Omega_{13} - \Omega_{16}) + A_6(\Omega_{17} - \Omega_{18})],$$

$$N_3 = \frac{Ra\,Gm}{Pr - 1}\left[\Omega_{19} + \frac{Sr\,Sc}{Ra\,Q}\{C_7(\Omega_{22} - \Omega_{23}) + C_8(\Omega_{13} - \Omega_{16}) + C_9(\Omega_{17} - \Omega_{18})\}\right],$$

$$N_4 = -\frac{Gm\,Sr\,Sc}{Sc - 1}[C_{10}(\Omega_{14} - \Omega_{15}) + C_{11}(\Omega_{13} - \Omega_{16}) + C_{12}(\Omega_{17} - \Omega_{18})],$$

$$N_5 = -\frac{Ra\,Gr}{Sc - 1}[A_4(\Omega_{24} - \Omega_{25}) + A_5(\Omega_{11} - \Omega_{12}) + A_6(\Omega_1 - \Omega_2)],$$

$$N_6 = \frac{Ra\,Gm}{Sc - 1}\left[\Omega_{26} + \frac{Sr\,Sc}{Ra\,Q}\{C_7(\Omega_{27} - \Omega_{28}) + C_8(\Omega_5 - \Omega_6) + C_9(\Omega_7 - \Omega_8)\}\right],$$

$$N_7 = \frac{Gm\,Sr\,Sc}{Q(Sc - 1)}[C_{10}(\Omega_{24} - \Omega_{25}) + C_{11}(\Omega_{11} - \Omega_{12}) + C_{12}(\Omega_1 - \Omega_2)],$$

$$P_1 = \Omega_{13}, \quad P_2 = \frac{Gr}{Q - M}(\Omega_{17} - \Omega_{18}),$$

$$P_3 = \frac{Ra\,Gm}{Sc - 1}\left[\Omega_{19} - Sc\,Sr\{\bar{A}_7(\Omega_{31} - \Omega_{32}) + \bar{A}_8(\Omega_{22} - \Omega_{23})\right.$$
$$\left. + \bar{A}_9(\Omega_{13} - \Omega_{16}) + \bar{A}_{10}(\Omega_{17} - \Omega_{18})\}\right],$$

$$P_4 = \frac{Gm}{(Q - M)(Sc - 1)}[B_1(\Omega_{31} - \Omega_{32}) + B_2(\Omega_{13} - \Omega_{16}) + B_3(\Omega_{17} - \Omega_{18})],$$

$$P_5 = -\frac{Gr}{Q - M}(\Omega_{33} - \Omega_{34}), \quad P_6 = -\frac{Ra\,Gm}{Sc - 1}\left[\frac{1}{a_3}(\Omega_{29} - \Omega_{30}) - \frac{Sc\,Sr\,\Omega_{51}}{Sc - 1}\right],$$

$$P_7 = -\frac{Sr\,Sc\,Gm}{(Q - M)(Sc - 1)}[B_1(\Omega_{37} - \Omega_{38}) + B_2(\Omega_{39} - \Omega_{40}) + B_3(\Omega_{33} - \Omega_{34})],$$

$$Q_1 = \Omega_{13}, \ Q_2 = \frac{RaGr}{Pr-1}[A_4(\Omega_{14}-\Omega_{15})+A_5(\Omega_{13}-\Omega_{16})+A_6(\Omega_{17}-\Omega_{18})],$$

$$Q_3 = -\frac{Gm}{M}\left[\Omega_{13}+\frac{SrPr}{Pr-1}\{\bar{B}_1(\Omega_{41}-\Omega_{42})+\bar{B}_2(\Omega_{13}-\Omega_{16})+\bar{B}_3(\Omega_{17}-\Omega_{18})\}\right],$$

$$Q_4 = -\frac{RaGmPrSr}{(Pr-1)^2}[B_4(\Omega_{41}-\Omega_{42})+B_5(\Omega_{14}-\Omega_{15})$$
$$+B_6(\Omega_{13}-\Omega_{16})+B_7(\Omega_{17}-\Omega_{18})],$$

$$Q_5 = -\frac{RaGr}{Pr-1}[A_4(\Omega_{24}-\Omega_{25})+A_5(\Omega_{11}-\Omega_{12})+A_6(\Omega_1-\Omega_2)],$$

$$Q_6 = \frac{Gm}{M}\left[\Omega_{45}+\frac{SrPr}{Pr-1}\{\bar{B}_1(\Omega_{43}-\Omega_{44})+\bar{B}_2(\Omega_{45}-\Omega_{46})+\bar{B}_3(\Omega_{48}-\Omega_{49})\}\right],$$

$$Q_7 = \frac{RaGmPrSr}{(Pr-1)^2}[B_4(\Omega_{49}-\Omega_{50})+B_5(\Omega_{24}-\Omega_{25})$$
$$+B_6(\Omega_{11}-\Omega_{12})+B_7(\Omega_1-\Omega_2)],$$

$$S_1 = \Omega_{13}, \ S_2 = -\frac{Gr}{Q-M}(\Omega_{17}-\Omega_{18}),$$

$$S_3 = -\frac{Gm}{M}\left[\Omega_{13}+\frac{Sr}{QRa}\{\Omega_{13}-\Omega_{16}+RaQ(\Omega_{17}-\Omega_{18})\}\right],$$

$$S_4 = -\frac{SrGm}{QRa(Q-M)}[\Omega_{13}-\Omega_{16}+RaQ(\Omega_{17}-\Omega_{18})],$$

$$S_5 = -\frac{Gr}{Q-M}(\Omega_{33}-\Omega_{34}),$$

$$S_6 = \frac{Gm}{M}\left[\Omega_{45}+\frac{Sr}{QRa}\{\Omega_{45}-\Omega_{46}+RaQ(\Omega_{47}-\Omega_{48})\}\right],$$

$$S_7 = \frac{SrGm}{QRa(Q-M)}[\Omega_{33}-\Omega_{34}+RaQ(\Omega_{33}-\Omega_{34})].$$

9 Rate of Heat Transfer

The heat flux q^* from the plate to the fluid specified by the Fourier law of conduction is given by

$$q^* = -\kappa\frac{\partial T}{\partial y'}\bigg]_{y'=0} = -\frac{\kappa(T_w-T_\infty)}{U_0 t_0}\frac{\partial\theta}{\partial y}\bigg]_{y=0}. \tag{41}$$

The coefficient of the rate of heat transfer at the plate in terms of Nusselt number Nu is expressed as

$$Nu = \frac{U_0 t_0 q^*}{\kappa(T_w-T_\infty)} = -\frac{\partial\theta}{\partial y}\bigg]_{y=0} = -(\Omega_1-\Omega_2). \tag{42}$$

10 Rate of Mass Transfer

The mass flux M_w at the plate determined by Fick's law of diffusion is given by

$$M_w = -D_M \left. \frac{\partial C}{\partial y'} \right]_{y'=0} = -\frac{D_M(C_w - C_\infty)}{U_0 t_0} \left. \frac{\partial \phi}{\partial y} \right]_{y=0}. \tag{43}$$

The coefficient of the rate of mass transfer at the plate in terms of Sherwood number is given by

$$\text{Sh} = \frac{M_w U_0 t_0}{D_M(C_w - C_\infty)} = -\left. \frac{\partial \phi}{\partial y} \right]_{y=0} = \begin{cases} -(R_1 + R_2 - R_3), \text{Pr} \neq \text{Sc} \\ -(\bar{R}_1 + \bar{R}_2 - \bar{R}_3), \text{Pr} = \text{Sc} \end{cases}, \tag{44}$$

where

$$R_1 = \Omega_5, \ R_2 = \frac{A}{\text{Pr} - \text{Sc}}[A_1(\Omega_3 - \Omega_4) + A_2(\Omega_5 - \Omega_6) + A_3(\Omega_7 - \Omega_8)],$$

$$R_3 = \frac{A}{\text{Pr} - \text{Sc}}[A_1(\Omega_9 - \Omega_{10}) + A_2(\Omega_{11} - \Omega_{12}) + A_3(\Omega_1 - \Omega_2)], \ \bar{R}_1 = \Omega_5,$$

$$\bar{R}_2 = \frac{\text{SrSc}}{Ra Q}[Ra Q(\Omega_7 - \Omega_8) + \Omega_5 - \Omega_6],$$

$$\bar{R}_3 = \frac{\text{SrSc}}{Ra Q}[Ra Q(\Omega_1 - \Omega_2) + \Omega_{11} - \Omega_{12}],$$

$$\Omega_1 = \varphi(\text{Pr} \, Ra, Ra Q, t), \ \Omega_2 = \varphi(\text{Pr} \, Ra, Ra Q, t - 1)H(t - 1),$$

$$\Omega_3 = e^{a_1 t}\Omega(RaSc, a_1, t), \ \Omega_4 = e^{a_1(t-1)}\Omega(RaSc, a_1, t - 1)H(t - 1),$$

$$\Omega_5 = -\sqrt{\frac{RaSc}{\pi t}}, \ \Omega_6 = -\sqrt{\frac{RaSc}{\pi(t-1)}}H(t-1),$$

$$\Omega_7 = -2\sqrt{\frac{RaSct}{\pi}}, \ \Omega_8 = -2\sqrt{\frac{RaSc(t-1)}{\pi}}H(t-1),$$

$$\Omega_9 = e^{a_1 t}\Omega(Ra \, \text{Pr}, Ra Q + a_1, t),$$

$$\Omega_{10} = e^{a_1(t-1)}\Omega(Ra \, \text{Pr}, Ra Q + a_1, t - 1)H(t - 1),$$

$$\Omega_{11} = \Omega(\text{Pr} \, Ra, Ra Q, t), \ \Omega_{12} = \Omega(\text{Pr} \, Ra, Ra Q, t - 1)H(t - 1),$$

$$\Omega_{13} = \Omega(Ra, RaM, t), \ \Omega_{16} = \Omega(Ra, RaM, t - 1)H(t - 1),$$

$$\Omega_{14} = e^{a_2 t}\Omega(Ra, RaM + a_2, t), \ \Omega_{15} = e^{a_2(t-1)}\Omega(Ra, RaM + a_2, t - 1)H(t - 1),$$

$\Omega_{17} = \varphi(Ra, RaM, t), \Omega_{18} = \varphi(Ra, RaM, t - 1)H(t - 1),$

$\Omega_{19} = \dfrac{1}{a_3}\big[e^{a_3 t}\Omega(Ra, RaM + a_3, t) - \Omega_{13}\big],$

$\Omega_{20} = e^{a_1 t}\Omega(Ra, RaM + a_1, t), \Omega_{21} = e^{a_1(t-1)}\Omega(Ra, RaM + a_1, t - 1)H(t - 1),$

$\Omega_{22} = e^{a_3 t}\Omega(Ra, RaM + a_3, t), \Omega_{23} = e^{a_3(t-1)}\Omega(Ra, RaM + a_3, t - 1)H(t - 1),$

$\Omega_{24} = e^{a_2 t}\Omega(\text{Pr } Ra, RaQ + a_2, t),$

$\Omega_{25} = e^{a_2(t-1)}\Omega(\text{Pr } Ra, RaQ + a_2, t - 1)H(t - 1),$

$\Omega_{26} = \dfrac{1}{a_3}\big[e^{a_3 t}\Omega(\text{Sc}Ra, a_3, t) - \Omega_5\big],$

$\Omega_{27} = e^{a_3 t}\Omega(\text{Sc}Ra, a_3, t), \Omega_{28} = e^{a_3(t-1)}\Omega(\text{Sc}Ra, a_3, t - 1)H(t - 1),$

$\Omega_{29} = e^{a_1 t}\Omega(\text{Pr } Ra, RaQ + a_1, t),$

$\Omega_{30} = e^{a_1(t-1)}\Omega(\text{Pr } Ra, RaQ + a_1, t - 1)H(t - 1),$

$\Omega_{31} = e^{\bar{a}_1 t}\Omega(Ra, RaM + \bar{a}_1, t), \Omega_{32} = e^{\bar{a}_1(t-1)}\Omega(Ra, RaM + \bar{a}_1, t - 1)H(t - 1),$

$\Omega_{33} = \varphi(Ra, RaQ, t), \Omega_{34} = \varphi(Ra, RaQ, t - 1)H(t - 1),$

$\Omega_{35} = e^{\bar{a}_1 t}\Omega(Ra\text{Sc}, \bar{a}_1, t), \Omega_{36} = e^{\bar{a}_1(t-1)}\Omega(Ra\text{Sc}, \bar{a}_1, t - 1)H(t - 1),$

$\Omega_{37} = e^{\bar{a}_1 t}\Omega(Ra, RaQ + \bar{a}_1, t), \Omega_{38} = e^{\bar{a}_1(t-1)}\Omega(Ra, RaQ + \bar{a}_1, t - 1)H(t - 1),$

$\Omega_{39} = \Omega(Ra, RaQ, t), \Omega_{40} = \Omega(Ra, RaQ, t - 1)H(t - 1),$

$\Omega_{41} = e^{d_1 t}\Omega(Ra, RaM + d_1, t), \Omega_{42} = e^{d_1(t-1)}\Omega(Ra, RaM + d_1, t - 1)H(t - 1),$

$\Omega_{43} = e^{d_1 t}\Omega(Ra, d_1, t), \Omega_{44} = e^{d_1(t-1)}\Omega(Ra, d_1, t - 1)H(t - 1),$

$\Omega_{45} = -\sqrt{\dfrac{Ra}{\pi t}}, \Omega_{46} = -\sqrt{\dfrac{Ra}{\pi(t - 1)}}\, H(t - 1),$

$\Omega_{47} = -2\sqrt{\dfrac{Rat}{\pi}}, \Omega_{48} = -2\sqrt{\dfrac{Ra(t - 1)}{\pi}}\, H(t - 1),$

$\Omega_{49} = e^{d_1 t}\Omega(Ra \text{ Pr}, RaQ + d_1, t),$

$\Omega_{50} = e^{d_1(t-1)}\Omega(Ra \text{ Pr}, RaQ + d_1, t - 1)H(t - 1),$

$\Omega_{51} = \bar{A}_7(\Omega_{35} - \Omega_{36}) + \bar{A}_8(\Omega_{27} - \Omega_{28}) + \bar{A}_9(\Omega_5 - \Omega_6) + \bar{A}_{10}(\Omega_7 - \Omega_8);$

$\Omega(\xi, \eta, t) = -\left[\sqrt{\dfrac{\xi}{\pi t}}e^{-\eta t} + \sqrt{\xi\eta}\, erf\left(\sqrt{\eta t}\right)\right],$

$\varphi(\xi, \eta, t) = -\left[\sqrt{\dfrac{\xi}{4\eta}}erf\left(\sqrt{\eta t}\right) + t\sqrt{\xi\eta}erf\left(\sqrt{\eta t}\right) + \sqrt{\dfrac{\xi t}{\pi}}e^{-\eta t}\right].$

11   Results and Discussion

Numerical computations for the dimensionless velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number have been carried out by assigning some specific values (arbitrarily chosen) to the physical parameters involved like Soret number Sr, Ramped parameter Ra, radiation parameter Q, magnetic parameter M, solutal Grashof number Gm, thermal Grashof number Gr, Schmidt number Sc and Prandtl number Pr. Thought the investigation, the values of the Pradtl number Pr, thermal Grashof number Gr and Schmidt number Sc have been chosen as 0.71, 30, 0.60 respectively as the numerical calculations are concerned. It is worthwhile to mention that Pr = 0.71 corresponds to air at the temperature of 25 °C and 1 atmospheric pressure and Sc = 0.60 refers to water vapor diffused in dry air. It is to be noted that in the most of the cases of mass transfer related problems, dry air is considered as an ideal medium of diffusion (solvent). Because, all most all the natural gases diffuse in dry air easily. In the present investigation, the fluid is to be electrically conducting. Dry air is a poor conductor. In contrast, air with moisture is electrically conducting to a reasonable extent. In the present work, a fluid is required which is electrically conducting as well as a good solvent. On the basis of this fact, air is taken as solvent and water vapor as solute. This phenomenon justifies the fact of taking Pr = 0.71 and Sc = 0.60. Also Gr > 0 presents the externally cool case. Further both the cases of $t < 1$ and $t > 1$ are incorporated in performing the computations from the exact solutions of the problem under consideration. It is recalled that $t < 1$ represents the ramped wall temperature and in contrast $t > 1$ refers to isothermal plate.

The concentration profiles for $t < 1$ and for $t > 1$ under the influence of Sr, Ra, Q and normal coordinate y are exhibited in Figs. 1, 2, 3, 4, 5 and 6. These figures show a comprehensive growth in fluid concentration for increasing the Soret number Sr and the radiation parameter Q. An interesting observation on the behavior of ϕ under the ramped parameter Ra is marked in Figs. 3 and 4. These two figures depict that the concentration level of the fluid goes up marginally in a thin layer adjacent to the plate and thereafter it takes a reverse turn. All the figures uniquely simulate that the concentration distribution ϕ first increases in a thin layer closed to the wall and after that it falls asymptotically as $y \to \infty$. This is due to the buoyancy force, which is very much effective near the plate and its influence diminishes in fluid region way from the plate. As the effects of Sr, Ra and Q on ϕ are concerned, it is immaterial whether the plate is isothermal or the temperature of the plate is ramped. However in case of $t > 1$(isothermal), the asymptotic fall of $\phi \to 0$ as $y \to \infty$ is delayed in comparison to the case of $t < 1$.

The influences of Ra and Q on the temperature field θ are displaced in Figs. 7 and 8 for $t < 1$ and $t > 1$. These two figures indicate a substantial fall in the temperature field under the effect of Ra as well as the radiation parameter Q. It is worthwhile to mention that, in case of Cogley et al. [11]'s model of radiation, radiation acts like a heat sink. Hence, in the present case, radiation which is similar to heat sink, absorbs heat from the fluid and as a consequence, the fluid temperature falls under
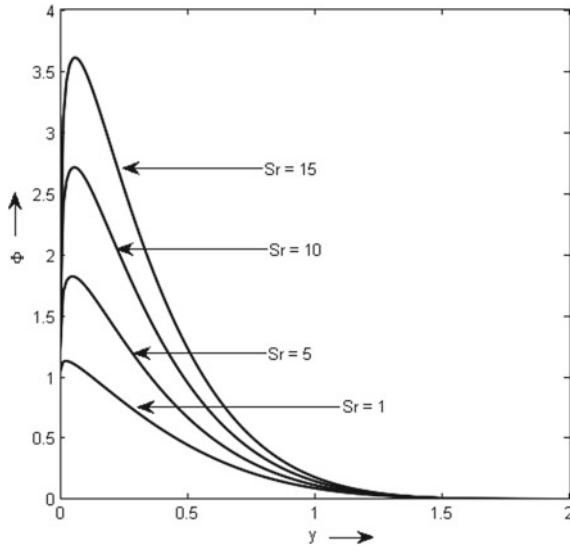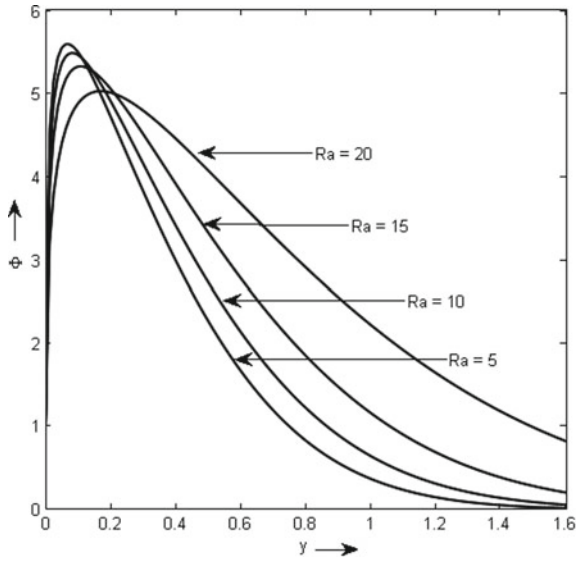
Fig. 1 Concentration ϕ versus y for Pr $= 0.71$, $Ra = 10$, Sc $= 0.6$, $Q = 5$, $t = 0.5$
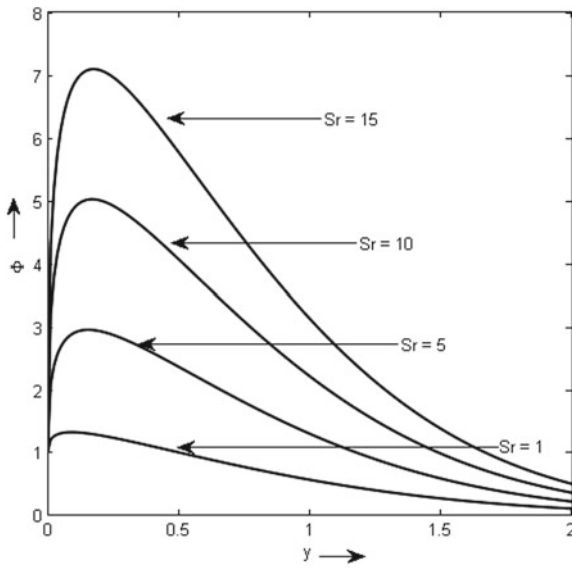
Fig. 2 Concentration ϕ versus y for Pr $= 0.71$, $Ra = 10$, Sc $= 0.6$, $Q = 5$, $t = 2$
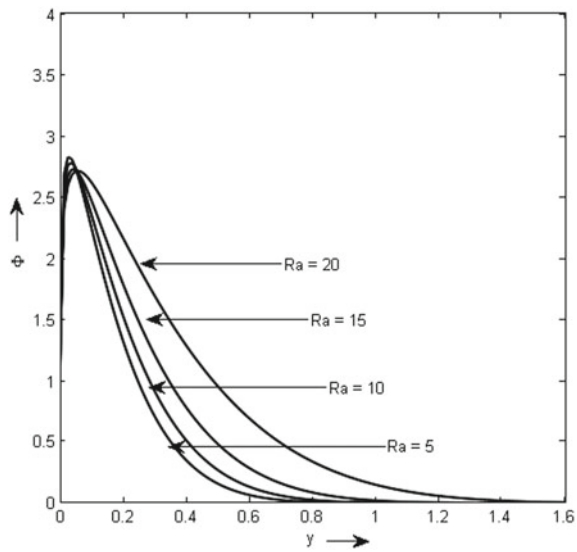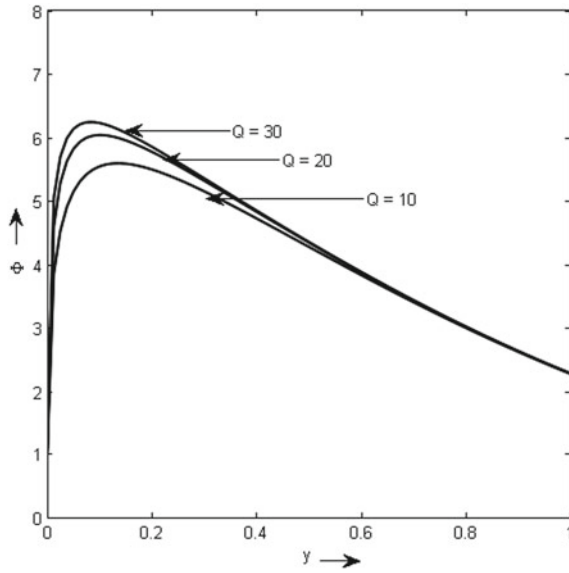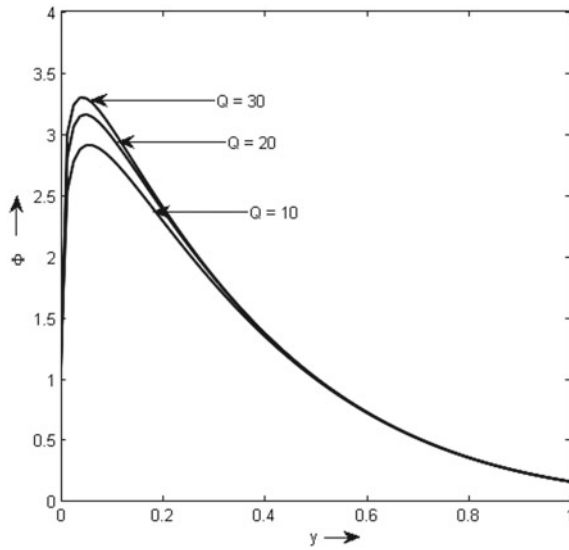
Fig. 3 Concentration ϕ versus y for $Pr = 0.71$, $Sr = 10$, $Sc = 0.6$, $Q = 5$, $t = 0.5$

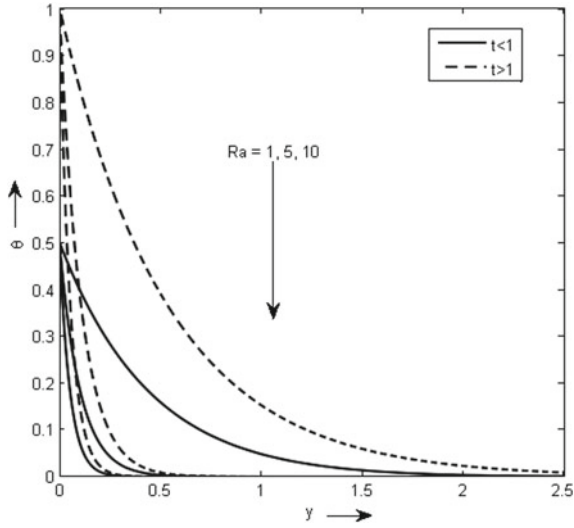**Fig. 4** Concentration ϕ versus y for $Pr = 0.71$, $Sr = 10$, $Sc = 0.6$, $Q = 5$, $t = 2$

Fig. 5 Concentration ϕ versus y for Pr $= 0.71$, Sr $= 10$, Sc $= 0.6$, $Ra = 10$, $t = 0.5$

Fig. 6 Concentration ϕ versus y for Pr $= 0.71$, Sr $= 10$, Sc $= 0.6$, $Ra = 10$, $t = 2$

Fig. 7 Temperature θ versus y for Pr $= 0.71$, $Q = 5$, $t = 0.5$, $t = 2$
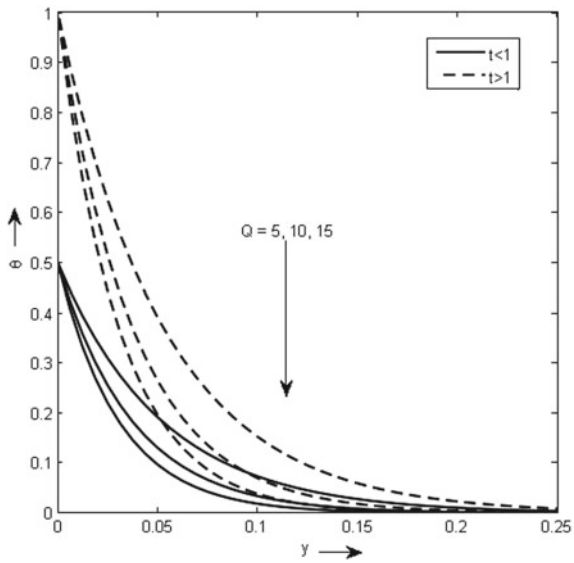
Fig. 8 Temperature θ versus y for Pr $= 0.71$, $Ra = 10$, $t = 0.5$, $t = 2$

the radiation effect. This phenomenon establishes the fact that the observation on the effect of Q on θ is consistent with the physical reality. Further the role of time t on θ is almost similar to its role on ϕ.
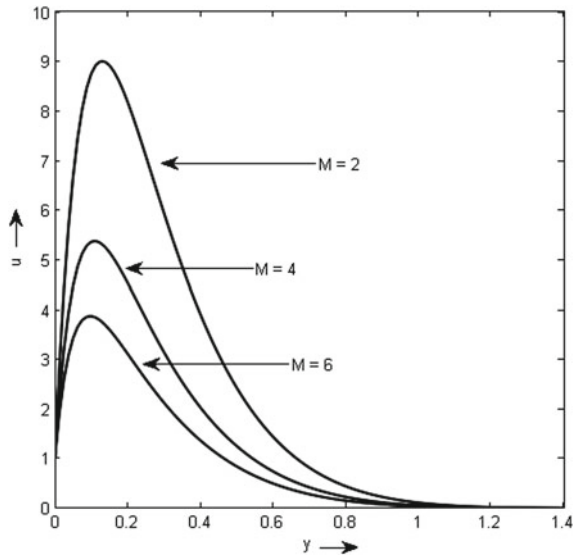
Fig. 9 Velocity u versus y for Gr $= 30$, Gm $= 20$, $Ra = 10$, $Q = 5$, Sr $= 10$, Sc $= 0.6$, Pr $= 0.71$, $t = 0.5$

Figures 9, 10, 11, 12, 13 and 14 present the variation of the velocity field u versus y under the effects of M, Q, Sr for the cases $t < 1$ and $t > 1$. A comprehensive fall in the fluid velocity for increasing the parameter M is marked in Figs. 9 and 10. It registers the fact that the flow is retarded due to the imposition of the transverse magnetic field. In real situation, a magnetic body force called Lorentz force gets generated on interaction of the magnetic field and electrically conducting fluid in motion. This induced force acts as a resistive force to the flow, which in turn causes the flow to decelerate. It confirms the consistency of our graphical observation with the actual physical situation. It is inferred from Figs. 11 and 12 that the thermal radiation has also some contribution in retarding the fluid motion. Figures 13 and 14 clearly indicate that an increase in the parameter Sr leads the fluid motion to accelerate to a remarkable extent. All the Figs. 9, 10, 11, 12, 13 and 14 show that the fluid velocity first increases in a thin layer adjacent to the plate and thereafter it falls asymptotically as $y \to \infty$. It reveals the fact that the buoyancy force is very much effective near the plate and its influence on the fluid velocity slowly and steadily gets nullified as moved away from the plate. Like the concentration field ϕ and the temperature field θ, the asymptotic fall of the fluid velocity gets delayed for $t > 1$ rather than $t < 1$. Thus the asymptotic fall of temperature θ or concentration field ϕ or the velocity field u is faster in case of ramped wall temperature than that of the isothermal plate temperature.

Figures 15, 16 and 17 demonstrate how the skin friction τ at the plate is affected by the magnetic parameter M, Soret number Sr, solutal Grashof number Gm and time t. It is inferred from the Fig. 15 that the shear stress at the plate is diminished for
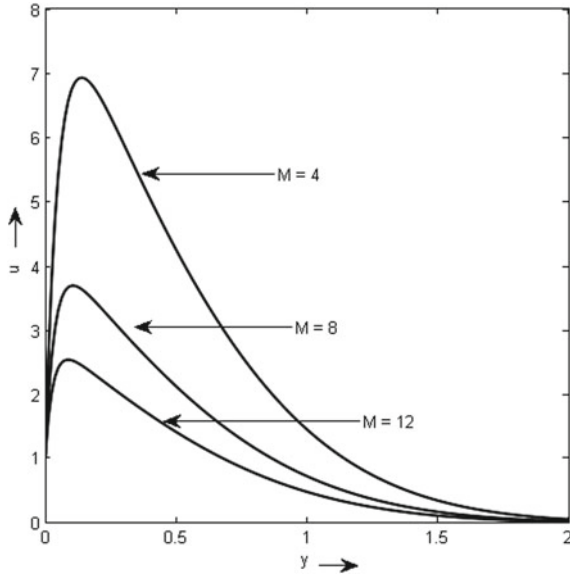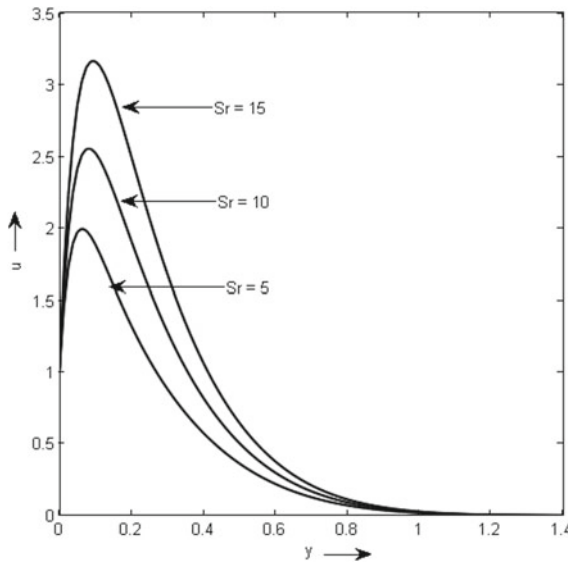
Fig. 10 Velocity u versus y for $Gr = 30$, $Gm = 20$, $Ra = 10$, $Q = 5$, $Sr = 10$, $Sc = 0.6$, $Pr = 0.71$, $t = 2$
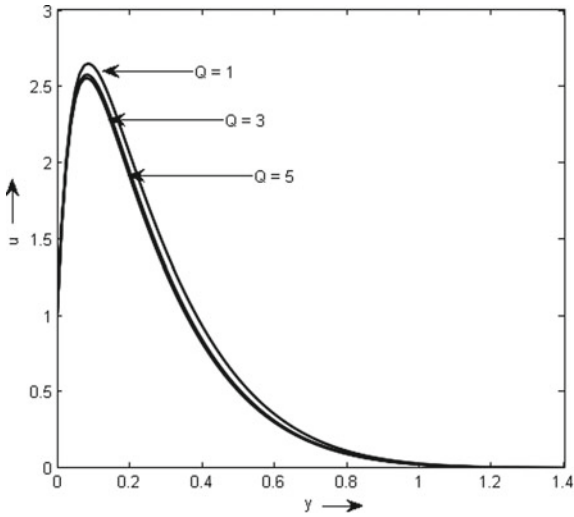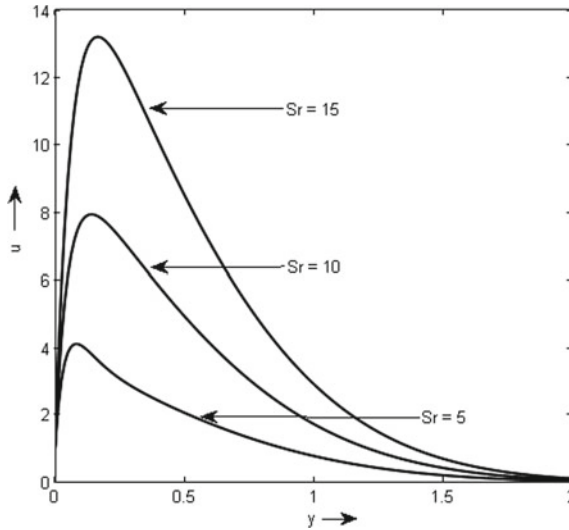

Fig. 11 Velocity u versus y for $Gr = 30$, $Gm = 20$, $Ra = 10$, $M = 4$, $Sr = 10$, $Sc = 0.6$, $Pr = 0.71$, $t = 0.5$
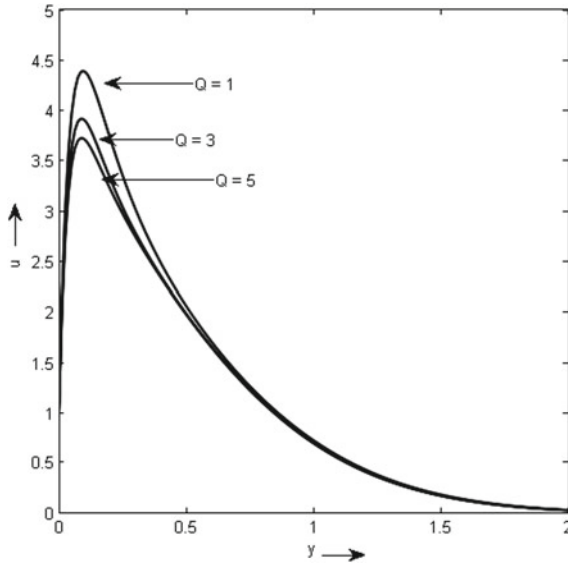
**Fig. 12** Velocity u versus y for Gr $=$ 30, Gm $=$ 20, $Ra = 10$, $M = 4$, Sr $=$ 10, Sc $=$ 0.6, Pr $= 0.71$, $t = 2$

Fig. 13 Velocity u versus y for Gr $= 30$, Gm $= 20$, $Ra = 10$, $M = 4$, $Q = 5$, Sc $= 0.6$, Pr $=$ 0.71, $t = 0.5$

Fig. 14 Velocity u versus y for Gr $= 30$, Gm $= 20$, $Ra = 10$, $M = 4$, $Q = 5$, Sc $= 0.6$, Pr $= 0.71$, $t = 2$

increasing M. It reflects the fact that the viscous drag at the plate gets suppressed on the imposition of the magnetic field. As such strong magnetic field may be applied in operations for successful inhabitation of the frictional resistance at the plate. The drag force at the plate due to viscosity seems to be enhanced under the thermal diffusion effect or buoyancy force (solutal) effect as visualized in Figs. 16 and 17. These two Figures further predict that the skin friction sharply increases as time progresses in case of ramped wall temperature ($t < 1$) and it becomes stationary for isothermal plate ($t > 1$).

The effects of the ramped parameter Ra and radiation parameter Q on the Nusselt number Nu are demonstrated in Figs. 18 and 19. It is observed from these two figures that the rate of heat transfer at the plate rises significantly for increasing the ramped parameter Ra as well as radiation parameter Q. It leads to conclude that the low viscosity or high heat absorption raises the heat flux at the plate. The same figures also register the fact that for $t < 1$, Nu increases linearly at a very fast rate as t increases and it becomes uniform in case of $t > 1$.

Figures 20 and 21 depict that an increase in Sr or Ra results in a substantial rise in the Sherwood number. The two figures uniquely establish that the rate of mass transfer at the plate falls initially and as the time progresses, it rises up linearly for $t < 1$. Like the Nusselt number, the Sherwood number also becomes stationary for $t > 1$. It may be concluded from the Figs. 15, 16, 17, 18, 19, 20 and 21 that the rate of momentum transfer, rate of heat transfer and the rate of mass transfer seem to be uniform in case of isothermal plate. In contrast, these transport properties get enhanced almost straight way for ramped wall temperature.
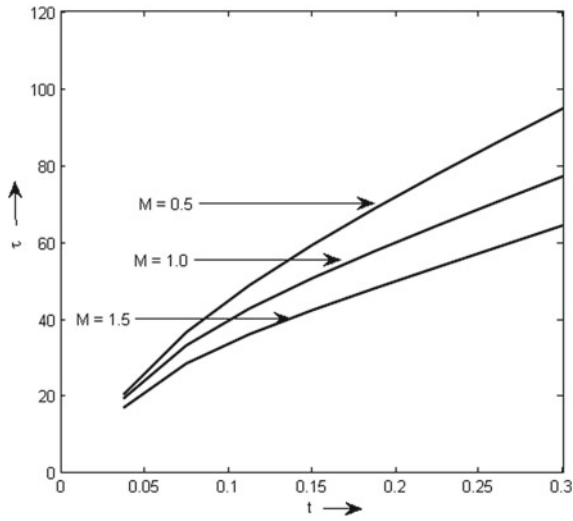
Fig. 15 Skin friction τ versus t for Gr $= 30$, Gm $= 20$, $Ra = 10$, Sr $= 10$, Sr $= 10$, $Q = 5$, Sc $= 0.6$, Pr $= 0.71$
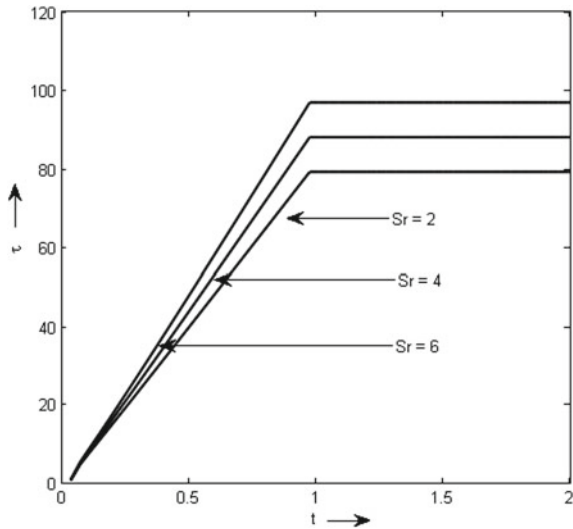
Fig. 16 Skin friction τ versus t for Gr $= 30$, Gm $= 20$, $Ra = 10$, $M = 4$, $Q = 5$, Sc $= 0.6$, Pr $= 0.71$
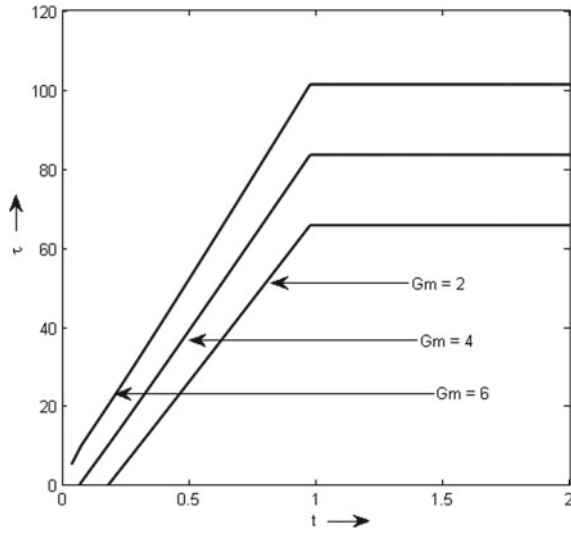
Fig. 17 Skin friction τ versus t for $Gr = 30$, $Sr = 10$, $Ra = 10$, $M = 4$, $Q = 5$, $Pr = 0.71$
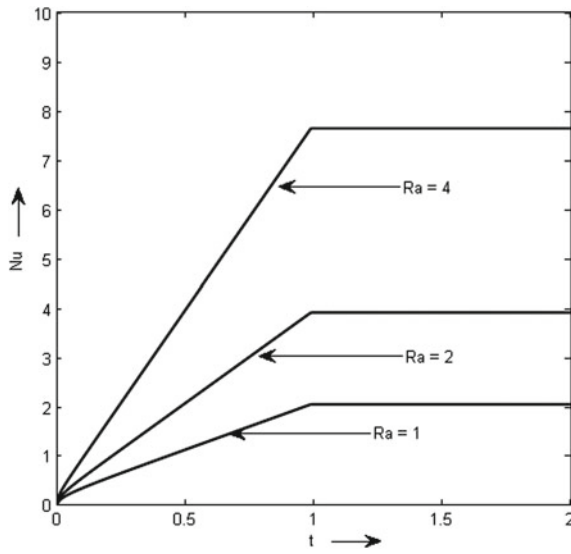

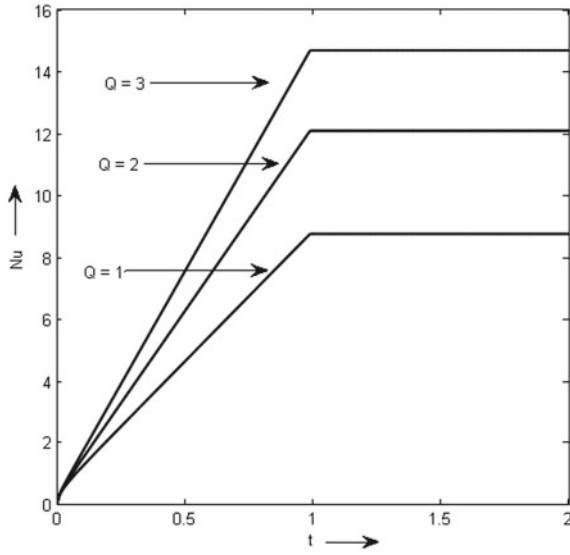**Fig. 18** Nusselt number Nu versus t for $Q = 5$, $Pr = 0.71$

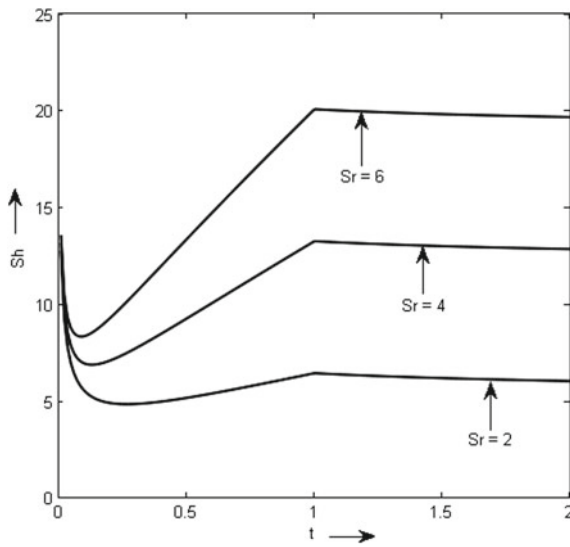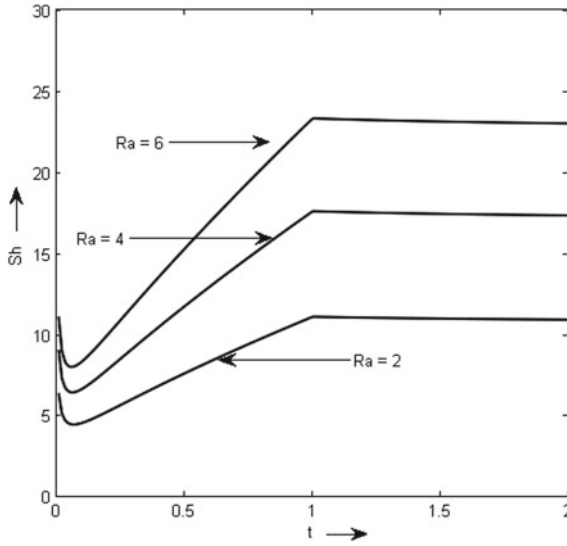**Fig. 19** Nusselt number Nu versus t for $Ra = 10$, $Pr = 0.71$

Fig. 20 Sherwood number Sh versus t for $Q = 5$, $Sc = 0.6$, $Ra = 10$, $Pr = 0.71$

Fig. 21 Sherwood number Sh versus t for $Q = 5$, $Sc = 0.6$, $Sr = 10$, $Pr = 0.71$

12 Conclusions

The present investigation may be summarized to the following conclusions:

- Drag force at the plate due to viscosity seems to be enhanced under the thermal diffusion effect or buoyancy force (solutal) effect.
- Skin friction sharply increases as time progresses in case of ramped wall temperature $(t < 1)$ and it becomes stationary for isothermal plate $(t > 1)$.
- Low viscosity or high heat absorption raises the heat flux at the plate.
- For $t < 1$, Nu increases linearly at a very fast rate as t increases and it becomes uniform in case of $t > 1$.
- An increase in Sr or Ra results in a substantial rise in rate of mass transfer at the plate.
- Rate of mass transfer at the plate falls initially and as the time progresses, it rises up linearly for $t < 1$.

13 Future Scope

This chapter deals with the study of an impulsively started convective flow problem. The mathematical model of the problem is idealized to a considerable extent on imposition of some physically realistic constraints. A set of closed form solutions of the resultant governing equations are obtained by adopting the Laplace transform

technique. There is wide scope of reinvestigation of the same problem numerically by reducing the number of constraints. In this regard, Crank-Nicolson type implicit finite difference scheme may be suggested.

References

1. Agarwalla, S., Ahmed, N.: MHD mass transfer flow past an inclined plate with variable temperature and plate velocity embedded in a porous medium. Heat Transf. Asian Res. **47**, 27–41 (2018)
2. Ahmed, N., Dutta, M.: Natural convection in transient MHD dissipative flow past a suddenly started infinite vertical porous plate: a finite difference approach. Heat Transf. Res. **49**(6), 491–508 (2018)
3. Ahmed, N., Dutta, M.: Analytical analysis of Magneto-hydrodynamic (MHD) transient flow past a suddenly started infinite vertical plate with thermal radiation and ramped wall temperature. J. Heat Transf. (ASME) **136**, 0411703(1-8) (2014)
4. Ahmed, N.: Effect of chemical reaction on a transient MHD flow past a suddenly started infinite vertical plate with thermal diffusion and radiation. J. Calcutta Math. Soc. **10**(1), 9–36 (2014)
5. Ahmed, N., Sengupta, S., Datta, D.: An exact analysis for MHD free convection mass transfer flow past an oscillatory plate embedded in a porous medium. Chem. Eng. Commun. **200**, 494–513 (2013)
6. Ahmed, N.: Soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate. J. Heat Transf. (ASME) **134**, 06270(1-9) (2012)
7. Ahmed, N., Sengupta, S.: Thero-diffusion and diffusion-thermo effects on a three dimensional MHD mixed convection flow past an infinite vertical porous plate with thermal radiation. Magnetohydrodynamics **47**, 41–60 (2011)
8. Ahmed, N.: MHD convection with Soret and Dufour effects in a three dimensional flow past an infinite vertical porous plate. Can. J. Phys. **88**, 663–674 (2010)
9. Alfven, H.: Discovery of Alfven waves. Nature **150**, 405–406 (1942)
10. Cowling, T.G.: Magnetohydrodynamics. Wiley Inter Science, New York (1957)
11. Cogley, A.C.L., Vincenti, W.G., Gilles, E.S.: Differential approximation for radiative heat transfer in a Gray gas near equilibrium. Am. Inst. Aeronaut. Astronaut. **6**(3), 551–553 (1968)
12. Crammer, K.R., Pai, S.I.: Magneto Fluid Dynamics for Engineers and Applied Physicist. Mc Graw Hill Book Co., New York (1973)
13. Das, S., Jana, R.N., Ghosh, S.K.: Hall effects on unsteady MHD natural convective flow past an impulsively moving plate ramped temperature and concentration. Indian J. Pure Appl. Phys. **54**, 517–534 (2016)
14. Das, S., Jana, R.N.: Natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate. Alexandria Eng. J. **54**, 55–64 (2015)
15. Eckert, E.R.G., Drake, R.M.: Analysis of Heat and Mass Transfer. McGraw-Hill Book Co., New York (1972)
16. Ferraro, V.C.A., Plumpton, C.: An Introduction to Magneto-Fluid Mechanics. Clarendon Press, Oxford (1966)
17. Kafoussias, N.G., Williams, E.W.: Thermal diffusion and diffusion thermo effects on mixed free forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Int. J. Eng. Sci. **33**, 1369–1384 (1998)
18. Mansour, M.A.: Radiation and free convection effects on the oscillatory flow past a moving vertical porous plate. Astrophys. Space Sci. **166**(2), 269–275 (1990)
19. Mahanthesh, B., Gireesha, B.J., Gorla, R.S.R.: Heat and mass transfer effects on the mixed convective flow of chemically reacting nanofluid past a moving/stationary vertical plate. Alexandria Eng. J. **55**, 569–581 (2016)

20. Mankinde, O.D.: Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Int. Commun. Heat Mass Transf. **32**(10), 1411–1419 (2005)
21. Mbeledogu, I.U., Amakiri, A.R.C., Ogulu, A.: Unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. Int. J. Heat Mass Transf. **50**(9–10), 326–331 (2007)
22. Orhan, A., Ahmet, K.: Radiation effect on MHD mixed convection flow about a permeable vertical plate. Heat Mass Transf. **45**, 239–246 (2008)
23. Prasad, N.R., Reddy, N.B., Muthucumaraswamy, R.: Transient Radiative Hydro-Magnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux. Theor. Appl. Mech. **33**(1), 31–63 (2006)
24. Postelnicu, A.: Influence of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous media considering Soret and Dufour effects. Int. J. Heat Mass Transf. **47**, 1467–1472 (2004)
25. Raptis, A., Perdikis, C.: Radiation and free convection flow past a moving plate. Int. J. Appl. Mech. Eng. **4**(4), 817–821 (1999)
26. Shah, N.A., Zafar, A.A., Akhtar, S.: General solution for MHD-free convection flow over a vertical plate with ramped wall temperature and chemical reaction. Arab. J. Math. **7**(1), 49–60 (2018)
27. Samad, M.A., Rahman, M.M.: Thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. J. Nav. Archit. Mar. Eng. **3**(1), 7–14 (2006)
28. Shercliff, J.A.: A Text Book of Magnetohydrodynamics. Pergamon Press, London (1965)
29. Seth, G.S., Kumbhakar, B., Sharma, R.: Unsteady MHD free convection flow with Hall effect of a radiating and heat absorbing fluid past a moving vertical plate with variable ramped temperature. J. Egypt. Math. Soc. **24**(3), 471–478 (2016)
30. Seth, G.S., Sharma, R., Kumbhakar, B.: Heat and mass transfer effects on unsteady MHD natural convection flow of a chemically reactive and radiating fluid through a porous medium past a moving vertical plate with arbitrary ramped temperature. J. Appl. Fluid Mech. **9**(1), 103–117 (2016)
31. Seth, G.S., Sharma, R., Sarkar, S.: Natural convection heat and mass flow with Hall current, rotation and heat absorption past an accelerated moving vertical plate with ramped temperature. J. Appl. Fluid Mech. **8**(1), 7–20 (2015)
32. Seth, G.S., Sarkar, S., Hussain, S.M., Mahato, G.K.: Effects of Hall current and rotation on hydromagnetic natural convection flow with heat and mass transfer of a heat absorbing fluid past an impulsively moving vertical plate with ramped temperature. J. Appl. Fluid Mech. **8**(1), 159–171 (2015)
33. Seth, G.S., Hussain, S.M., Sarkar, S.: Hydromagnetic natural convection flow with heat and mass transfer of a chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium. J. Egypt. Math. Soc. **23**, 197–207 (2015)
34. Seth, G.S., Ansari, M.S., Nandkeolyar, R.: MHD Natural convection flow with radiative heat transfer past an impulsively moving plate with ramped temperature. Heat Mass Transf. **47**, 551–561 (2011)
35. Takhar, H.S., Gorla, R.S.R., Soundalgekar, V.M.: Radiation effects on MHD free convection flow of a radiating gas past a semi infinite vertical plate. Int. J. Numer. Meth. Heat Fluid Flow **6**(2), 77–83 (1996)