Mechanics-Mathematical Model of Conjugation of a Part of a Trajectory with Conditions of Continuity, Touch and Smoothness

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Abstract. Development process of combined trajectories, in places of joining conic arcs, undesirable intermittent effects inevitably arise due to the second-order non-smoothness. A second order tangency is considered taking into account the curvature and the equality condition of the arcs curvature radii to be joined at the conjugation points. A kinematic method for determining joints on the basis of a rocker mechanism is given, which ensures smooth joints.

1 Introduction

The scientific interests of the creation and formation of complex trajectories locate in the areas of road construction, aviation industry, shipbuilding, textile production, railway and automobile transport. Combined trajectories are created in the form of conjugate contours, the shapes of which are given by curves of different order and mathematically described by analytical equations. The resulting form should provide an improvement of the functional properties of objects. For example, asymmetric planetary vibration exciters with a combined treadmill (trajectory) are used to improve the performance of road vibratory rollers. Similarly, to improve the aerodynamic properties of the aircraft, combined wing shapes are created, and the smooth geometric shapes of the hull greatly improve the seaworthiness of the vessel. Currently, to implement a smooth transition from one arc to another curve arc, methods of patterns, transformations and second-order curves (conics) are used. These methods provide only smoothness of the first order. The study of conics is due to their wide application in science and engineering practice. These curves are the most important components of the contours of double curvature surfaces. A method of analytic determination of the transition section is proposed to ensure second-order smoothness (smoothness). Mathematical patterns that determine the smoothness of the transition

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are found. The process of finding the starting and ending points of the docking is modelled by the movement of the rocker mechanism [1–8].

The contours of the technical product lines are a combination of lines, which in most cases smoothly passing from one to the other.

A smooth transition of one line to another from a transitional line is called conjugation.

The following methods are used to identify intermediate curvilinear sections: (a) template curve, (b) nonlinear transformations, (c) second-order curves. These methods of constructing the curves are widely used in the design of curvilinear sections of the trajectories [9–13].

However, all these methods are approximate.

The process of determining the position of the finishing point is proposed with the condition that the smoothness ratio be simulated by a rocking mechanism. With the motion of the rocking rock of the rocking mechanism, the distances from the conjugate points to the point of intersection of the tangents change simultaneously, i.e. The changes in the lengths of tangents whose relations satisfy the smoothness conditions. The proposed method makes it possible to visually, quickly and effectively determine the position of the finish point on the circumference and ensure a non-collapsible connection of the conical arcs. Using the method of determining the position of the conjugate points based on the kinematics of the rocking mechanism, it is possible to smoothly join the conical arcs satisfying the conditions of continuity, tangency and equality of curvature and to create on their basis new models of treadmills (trajectories) from conical arcs that allow eliminating unwanted impact effects.

2 Problem Statement

To implement the second order smoothness between curve arcs, it is proposed to insert a transition arc, the model of which is a conic arc (transition conic). The functional purposes of the transition conic are as follows:

- the arc of the transition conic must necessarily pass through the connecting points A and B;
- at the points of joining A and B the first derivatives must be equal (there are common tangents at the points of docking);
- at the joining points A and B the radii of curvature should be equal.

The fulfillment of the first two conditions means the smoothness of the connection, and the addition of the third condition to them ensures a smooth connection.

Let the combined trajectory be formed from arcs of a circle $x^2 + y^2 = a^2$ and an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Choose an arbitrary starting point on an elliptical arc A. We calculate the radius of curvature ρ_A at this point and draw a tangent $L_{A\tau} = 0$. It is required to determine the position of the end point B in order to realize a smooth conjugation of circular and elliptical arcs. Thus, the endpoint B is not arbitrarily selected and should provide functional purposes. To solve the problem of determining the final joining point, it is necessary to determine the mathematical relationships between the elements of the transition conic at the points A and B.

3 Mathematical Preliminaries

Take the two points $A\,(x_A,\ y_A)$ and $B\,(x_B,\ y_B)$ on the ellipse (Fig. 1) with the curvature radii ρ_A and ρ_B respectively and draw through them the tangents $L_{A\tau}$ and $L_{B\tau}$ with the normals L_{An} and L_{Bn}. Let A_τ, B_τ and A_n, B_n - points of intersection of tangents and normals with the axis Ox, points A_x, B_x - points of the base of perpendiculars, dropped from points A and B on the axis, and Ox, A_h, B_h and A_d, B_d, are the points of the base of perpendiculars dropped from the points A, B and the center of the ellipse O tangent to $L_{B\tau}$ and $L_{A\tau}$. The tangents $L_{A\tau}$ and $L_{B\tau}$ mutually intersect at the point E [23].

By connecting the A, B and E points, we obtain the ΔAEB triangle. A triangle made up of the tangents $L_{A\tau}$, $L_{B\tau}$, and the chord L_{AB}, and also containing the inside of the arc of the ellipse $\smile AB$ will be the base triangle. EC is the median of the base triangle [23].

We denote by, $n_A = AA_n$, $n_B = BB_n$ - are the lengths of the normals L_{An} and L_{Bn}, $\tau_A = AA_\tau$, $\tau_B = BB_\tau$ - the lengths of the tangents $L_{A\tau}$ and $L_{B\tau}$, $s_A = A_xA_n$, $s_B = B_xB_n$ - are the lengths of the subnormals of the points A and B, $m_A = A_xA_\tau$, $m_B = B_xB_\tau$ - are the lengths of the tangent points A and B, $l_A = AE$, $l_B = BE$- are the lengths of the tangent segments $L_{A\tau}$ and $L_{B\tau}$, prior to their intersection at the point E, $d_A = OA_d$, $d_B = OB_d$ - are the distances of the center O to the tangents $L_{A\tau}$ and $L_{B\tau}$, $h_A = AA_h$, $h_B = BB_h$ - the distances of the points A and B to the tangents L_B and L_A, $\alpha = \angle BAE$, $\beta = \angle ABE$ - are the angles between the tangents $L_{A\tau}$, $L_{B\tau}$ and the chord AB, $\alpha_E = \angle AEC$, $\beta_E = \angle BEC$ - are the angles between the tangents $L_{A\tau}$, $L_{B\tau}$ and the median EC [23,24].

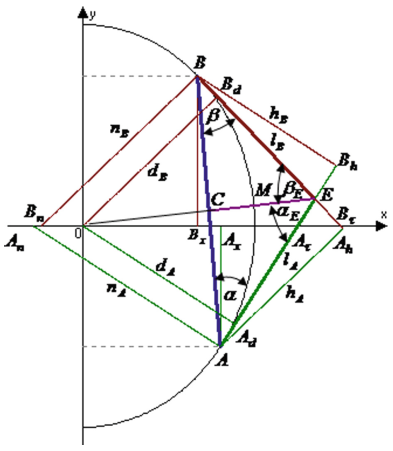

Fig. 1. Basic triangle and its elements

The considered lengths of the segments and the angular quantities will be elements of the basic triangle and find the connecting ratios between them through the radius of curvature [23].

From analytic geometry it is known that the radius of curvature of an ellipse is inversely proportional to the cube of the distance from the center to the tangent at the corresponding point [23]

$$\rho_M = \frac{a^2 b^2}{d_M^3}.$$

We introduce the coefficient defined as the cubic root of the ratio of the radii of curvature [23]:

$$\sqrt[3]{\frac{\rho_A}{\rho_B}} = \frac{d_B}{d_A} = \eta.$$

The coefficient η introduced by us is called the coefficient of curvature [23].

Four points: the center of the ellipse O, the points A_d and B_d of the base of the perpendiculars and the point of E intersection of the tangents are on the same circle [14]. Consider rectangular triangles ΔOA_dE and ΔOB_dE, the vertices of which lie on the intersection circle and apply the sine theorem [23].

Then

$$\frac{\sin\beta_E}{\sin\alpha_E} = \frac{d_B}{d_A} = \eta.$$

Now consider the triangles ΔACE and ΔBCE, which we get from the basic triangle by dividing the median EC, i.e. $AC = BC$, and similarly applying the sine theorem we obtain [23]

$$\frac{\sin\beta_E}{\sin\alpha_E} = \frac{\sin\beta}{\sin\alpha} = \eta.$$

For a basic triangle, we have [23]

$$\frac{\sin\beta}{\sin\alpha} = \frac{l_A}{l_B} = \eta.$$

From rectangular triangles ΔAA_hB and ΔAB_hB with a common hypotenuse AB (chord), we get $\frac{\sin\beta}{\sin\alpha} = \frac{h_A}{h_B} = \eta$.

4 Main Results

Statement. If we have two points A and B an ellipse with radii of curvature ρ_A and ρ_B, then the relationship between the corresponding elements of the basic triangle ΔAEB, made up of the tangents $L_{A\tau}$, $L_{B\tau}$ and the chord L_{AB}, is equal to the smoothness coefficient η [23]:

$$\frac{\sin\beta}{\sin\alpha} = \frac{\sin\beta_E}{\sin\alpha_E} = \frac{d_B}{d_A} = \frac{l_A}{l_B} = \frac{h_A}{h_B} = \frac{n_A}{n_B} = \eta \tag{1}$$

The obtained relations (1), characterizing the properties of the elliptical treadmill, allow determining the position of the point and constructing a transition section [23].

5 Simulation of Connection

The process of finding the position of a point B on a circle that satisfies the $\frac{l_A}{l_B} = \eta$ relation can be modeled by a link mechanism (Fig. 2) [23].

In the rocking mechanism, the guides AE and BE correspond to the directions of the tangents $L_{A\tau}$, $L_{B\tau}$, and the stone E represents the point of their intersection. The movement of the stone leads to a change in the length of the tangents. Value d_A - the distance from the center O to the tangent $L_{A\tau}$, m_A - the distance from the center O to the normal L_{An}, γ - the varying angle of inclination of the tangent $L_{B\tau}$, r - the radius of the connected arc of the circle [23].

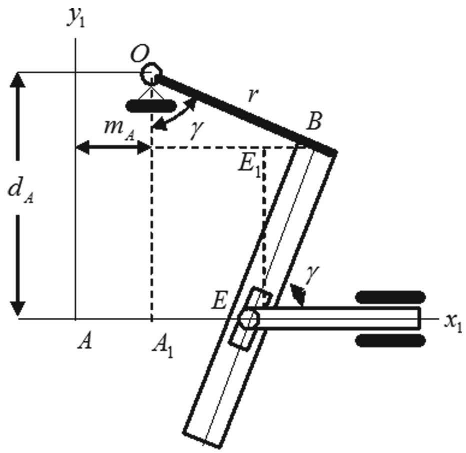

Fig. 2. Rocking mechanism

The equation of motion of the rock of the wings as a function of the angle γ [23]:

$$x_E = (m_A + r\sin\gamma) - \frac{d_A - r\cos\gamma}{tg\gamma}.$$

Similarly, for the point B we have [23]

$$\begin{cases} x_B = m_A + r\sin\gamma \\ y_B = d_A - r\cos\gamma \end{cases}$$

Further, using the dependence $l_A = \eta l_B$, we obtain the equation for determining $k = tg\gamma$ [23]:

$$\frac{m_A + r\sin\gamma + \frac{r}{k\sqrt{1+k^2}} - \frac{d_A}{k}}{m_A^2 + r^2 + d_A^2 + 2r\left(m_A\frac{k}{\sqrt{1+k^2}} - d_A\frac{1}{\sqrt{1+k^2}}\right)} = \eta$$

Applying this method, one can find a conic section having two given tangents at two points with given radii of curvature and passing through the third point in the form of the Lyming equations [15–19, 23]:

$$F\left(x,\ y\right) = \left(1-\lambda\right) L_{A\tau} L_{B_\tau} + \lambda L_{AB}^2 = 0.$$

The third point is found from the condition for determining the engineering discriminant for a known radius of curvature [23]

$$f = \frac{1}{1+\sqrt{\frac{2l_A^2}{h_A \rho_A}}}.$$

Using a concrete example, obtaining the equation of a smooth transition conic is shown. It is needed to find a transition curve connecting lemniskats arcs $\left[x^2 + (y+y_0)^2\right] - 2c^2\left[x^2 - (y+y_0)^2\right] = 0$ and circumference $x^2 + y^2 = r^2$, where $y_0 = 7$, $c = 5$, $r = 6$.

Choose a lemniscate on the arc starting point $A\,(6.8; -8.1)$ the radius of curvature is equal to $\rho_A = \frac{2c^2}{3\rho} = 2.4195$, as well as draw a tangent through it $L_{A\tau} = 1.9155x - y - 21.1258 = 0$. Smoothness coefficient $\eta = \sqrt[3]{\frac{\rho_A}{\rho_B}} = 0.7388$. Using the kinematic method we find the angular coefficient between the tangent ones, conducted through the starting point and the desired finishing point B:

$$\eta = \frac{m_A k - d_A + r\sqrt{1+k^2}}{d_A\sqrt{1+k^2} - r} \quad \Rightarrow \quad k = 2.$$

Point B defined as a tangency point with a circle $B\,(4.8621; 3.5157)$ and the tangent equation at the finish point is: $L_{B\tau} = -1.383x - y + 10.2399$. Chord equation $L_{AB} = -5.994x - y + 32.6589 = 0$.

Find the length of the tangent $l_A = 5.859$ and distance $h_A = 5.238$ from the starting point A up to the tangent $L_{B\tau}$, which are necessary to calculate the engineering discriminant $f = \frac{1}{1+\sqrt{\frac{2l_A^2}{h_A\rho_A}}} = 0.333$.

The coordinates of the point M through which the smooth transitional conic passes is determined by

$$\sigma = \frac{f}{1-f} = 0.4993$$

$$\begin{cases} x_M = \frac{x_C + \sigma x_E}{1+\sigma} = 7.0556 \\ y_M = \frac{y_C + \sigma y_E}{1+\sigma} = -2.498 \end{cases}$$

The coefficient λ in the Lyming equation is calculated by the formula

$$\lambda = \left.\frac{L_{A\tau} L_{B\tau}}{L_{A\tau} L_{B\tau} - L_{AB}^2}\right|_{\substack{x = x_M \\ y = y_M}} = 0.23035.$$

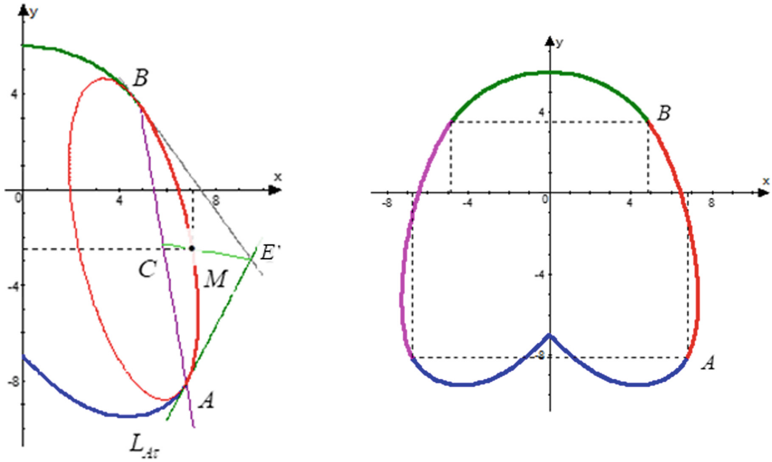

Fig. 3.

Equation of transition conic with continuity, tangency and smoothness conditions

$$F(x, y) = (1-\lambda)(1.9155x - y - 21.1258)(-1.383x - y + 10.2309) +$$

$$+\lambda(-5.994x - y + 32.6589)^2 = 0$$

or

$$F(x, y) = 27.0768x^2 + 10.2088xy + 4.3412y^2 - 228.4166x - 28.9158y + 344.4504 = 0.$$

The desired conic is represented as a displaced rotated ellipse [23]

$$\frac{[(x-a_0)\cos\alpha + (y-b_0)\sin\alpha]^2}{a_k^2} + \frac{[-(x-a_0)\sin\alpha + (y-b_0)\cos\alpha]^2}{b_k^2} = 1 \quad (2)$$

where a_k and b_k the semiaxes of the desired conic a_0, b_0 are the coordinates of the center of the conic displacement, and α is the angle of rotation of the focal axis [23].

The movement of the center of the vane of the exciter is described by changing the distance from the center of the runner to the anchoring point of the carrier, i.e. polar radius R. Therefore, all the component curves (circle, ellipse and transition conic) with which we form the roll must be described in polar coordinates (R, φ). The anchoring point of the carrier (the origin) is in the common geometric center of the circular and elliptical parts of the roll [23].

The components - the circular and elliptical parts - of the roller in polar coordinates are described by the equations $R = a$ and $R = \frac{b}{\sqrt{1-e^2\cos^2\varphi}}$, respectively, where a and b are the semiaxes of the given ellipse are, $e = \frac{a^2-b^2}{a^2}$ the eccentricity of this ellipse.

Having made mathematical transformations, from the Cartesian equation (2) it is possible to obtain the polar equation of the transition conic [23].

Then

$$R^2[1-e^2\cos^2(\varphi-\alpha)]+2Rq\sin(\varphi-\alpha)-p\cos(\varphi-\alpha)(1-e^2)]+[p^2(1-e^2)+q^2-b^2]=0,$$

where

$$p=a_0\cos\alpha+b_0\sin\alpha,\ q=a_0\sin\alpha-b_0\cos\alpha,\ g=p^2(1-e^2)+q^2-b^2.$$

The movement of the center of the slider C will be considered along sections divided by the joints of the curve arcs. The polar angle φ is measured from the abscissa axis against the clockwise direction (Fig. 3) [23].

1. Section 1 - the arc B_0B_1. A point $B_0(x_0,\,0)$ is a point of intersection of an arc of a conic with a positive abscissa, a point $B_1(x_1,\,y_1)$ is a point of connection of an arc of a conic with an arc of a circle $0\le x\le x_1$, $0\le y\le y_1$. The polar angle is $0\le\varphi\le\varphi_1$ [23].

$$R=\frac{1}{1-e^2\cos^2(\varphi-\alpha)}\{-A\cos(\varphi-\beta)+\sqrt{A^2\cos^2(\varphi-\beta)-g[1-e^2\cos^2(\varphi-\alpha)]}\}$$

where [23]

$$q\sin(\varphi-\alpha)-p(1-e^2)\cos(\varphi-\alpha)=A\cos(\varphi-\beta),$$

$$A=\sqrt{p^2(1-e^2)^2+q^2},\ \cos\beta=\frac{p(1-e^2)}{A},\ \sin\beta=\frac{q}{A}.$$

2. Section 2 - an arc B_1B_2. Point $B_2(x_2,\ y_2)$- the point of connection of the arc of a circle with an arc of a conic $x_2\le x\le x_1$ [23], $0\le y\le y_2$;
The polar angle $\varphi_1\le\varphi\le\varphi_2$. Equation of motion $R=a$.
3. Section 3 - the arc B_2B_3. The point $B_3(x_3,\ y_3)$ is the point of joining the arc of the conic with the arc of the ellipse $x_2\le x\le x_3$, $0\le y\le y_3$; The polar angle $\varphi_2\le\varphi\le\varphi_3$ [23].

$$R=\frac{1}{1-e^2\cos^2(\varphi+\alpha)}\{A\cos(\varphi-\beta)+\sqrt{A^2\cos^2(\varphi-\beta)-g[1-e^2\cos^2(\varphi+\alpha)]}\}$$

4. Section 4 - arc B_3B_4. Point $B_4(x_4,\ y_4)$- the point of connection of the arc of an ellipse with an arc of a conic $x_3\le x\le x_4$, $0\le y\le y_4$ [23]; The polar angle $\varphi_3\le\varphi\le\varphi_4$.

$$R=\frac{b}{\sqrt{1-e^2\cos^2\varphi}}$$

5. Section 5 - the arc B_5B_0 is part of the right conic.

The diagram of a smooth change in the polar coordinate R, describing the movement of the center of the slider of the reclosure on the combined treadmill, is shown in Figs. 4 and 5 [23].

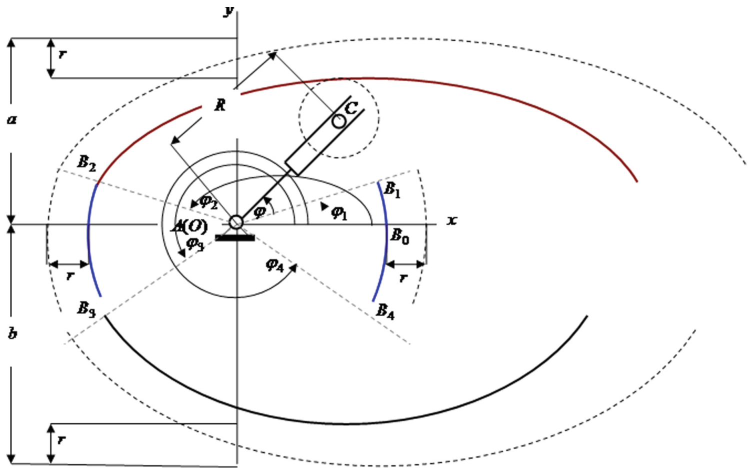

Fig. 4. Combined roll

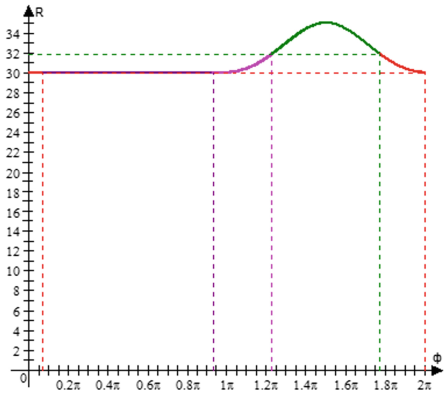

Fig. 5. The diagram of the movement of the center of the runner

6 Conclusion

Thus, the original way of connecting the treadmill of a planetary vibration exciter, obtained by connecting the arc of an ellipse with a circular arc with a radius equal to one of the semi-axes, is obtained. Moreover, the connection point has a common tangent, and does not have a jump along the curvature. The presented method and analytical dependencies of the smooth connection of two curves described by different equations allows making a dynamic calculation of the planetary vibration exciter, the treadmill is described by the intrusion curves (conic). The result is a smooth connection of treadmill sections, which ensures that the inertial runner of the planetary exciter moves along it evenly without drops and jumps at the junction points of various curves.

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