Reduction of Contact and Bending Stresses in the Bevel Gear Teeth While Maintaining the Same Overall Dimensions

Vladimir I. Medvedev, Dmitry S. Matveenkov and Andrey E. Volkov

Abstract The proposed article contains a technique for optimizing the parameters of a gear pair with circular teeth, which can be used both in the design of a new gearbox and in the modernization of an existing gearbox. In both cases, the initial data for the design of the transmission, which is part of the gearbox, are its gear ratio, dimensions and transmitted loads. The only difference is that in the second case, as a rule, it is necessary to place a couple in the same housing, which imposes more stringent restrictions on its size. Two examples of the optimal design search by varying the angle of the spiral and by varying the number of teeth while maintaining the same gear ratio are given.

Keywords Spiral bevel gears · Optimization · Instantaneous contact area · Localized bearing contact · Contact pressure · Bending stress · Machine-tool settings

1 Mathematical Model for Research

To optimize the macro-geometry of transmission (spiral angle, face width, number of teeth, tooth height, etc.), it is necessary to be able to select the shape (micro-geometry) of the side surfaces of the transmission teeth, providing optimal contact pressure and bending stress in meshing. Constructive parameters (the macro-geometry) of transmission are assumed to be given. If this problem is solved, optimization by macro-geometric parameters can be performed simply by varying these parameters. The search for micro-geometry problem involves, in turn, solving two problems: the

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problem of constructing the side surface of the teeth (synthesis) and the problem of determining the quality of meshing (analysis). Many studies have been published on improving the methods of analysis of bevel gears [1–5], as well as improving the quality of meshing by improving the parameters of the tool and settings of gear-cutting machines, causing an improvement in the micro-geometry of the tooth [1, 6–13]. A common disadvantage of the mentioned methods of synthesis is the necessity of selection of synthesis parameters interactively. That makes high demands on the qualification of the designer. In this paper, we propose an algorithm to automate the construction of optimal micro-geometry of the tooth on the basis of simple enough to understand the initial data, which are the required distances from the bearing contact to the edges of the teeth at a given load.

Determination of the shape of the tooth is carried out using the synthesis algorithm of gear set [11–13] and algorithms for analyzing the quality of the produced gear [14–16], based on elasticity theory that takes into account:

- the real flank obtained on the gear-manufacturing machine;
- distribution of the bending moment along the length of the tooth obtained by solving the problem of contact of elastic bodies bounded by surfaces of double curvature;
- comparability of tooth sizes in each of the three directions.

The second task is to improve the design of the existing gearbox using a refined test calculation of bending stresses.

To reduce the level of bending stresses in the gear teeth, without changing the design of the transmission, it is possible either by obtaining a more uniform contact load along the length of the tooth or by reducing the load on the tooth. This can be achieved by increasing either the length of the instantaneous contact area or the total contact ratio by selecting the appropriate shape of the contacting surfaces, determined by the machine-tool settings used for processing the teeth. Herewith, contact pressures are also reduced. However, this increase will eventually lead to edge contact and excessive contact pressure. In Fig. 1a, there is a bearing contact on the gear teeth surface, and in Fig. 1b, there is a graph of the maximum contact pressure σ_H, MPa dependence on the meshing phase. The instantaneous contact area extends to the upper edge of the tooth. Even in the absence of technological and assembling errors, there is a concentration of contact pressures on this edge. The location of the maximum contact pressure is indicated in the figure by a circle. Instantaneous contact area in which the maximum pressure is located is highlighted in black. It has the shape of an ellipse cut by the transverse edge of the tooth.

Machine-tool settings, providing a given length $a_{\max} = 0.6b$ of the greater semi-axis of the instantaneous contact area (b is the face width) under the condition of infinity of the tooth surface, were determined by the algorithm [2]. Simulation of the meshing process, the results of which are built in Fig. 1, was made with the help of the "Expert" program [11] to solve the contact problem based on the consideration of teeth as elastic three-dimensional bodies.

This example and many others that can be cited are showing that the minimum contact pressures should be sought among the surfaces that provide some detach-

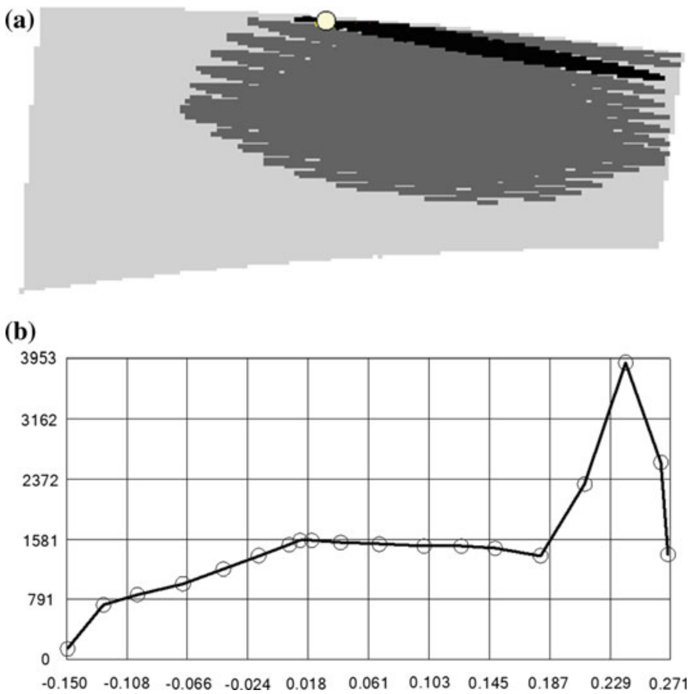

Fig. 1 Solution of the contact problem from the standpoint of elasticity theory: **a** bearing contact and **b** graph of the maximum contact pressure dependence on the meshing phase

ment of the bearing contact from the edges of the teeth, even in the presence of technological and installation errors. This contact we will call a localized contact.

Calculations based on the algorithm of the non-Hertz solution [2] are quite time-consuming and do not allow to use it in the process of minimization. As a tool for the tooth contact analysis in the process of minimization, the analysis program based on the solution of H. Hertz is utilized [12]. A large number of numerical experiments showed that if none of the contact ellipses go beyond the surface of the tooth, the contact pressures obtained based on the Hertz solution differ from the pressures obtained using a more accurate solution by no more than 3%. For comparison, in Fig. 2 the results of the analysis of the considered meshing based on Hertz theory are presented. Since the Hertz solution assumes that the contacting surfaces are infinite in size, the instantaneous contact areas (shown in Fig. 2a, various shades of gray) go beyond the boundaries of the lateral surface of the tooth. In addition, there is no concentration of contact stresses, which depend on the phase of meshing as shown in Fig. 2b on the right. In gearing phases where there is no edge contact, the results of the two solutions are almost the same.

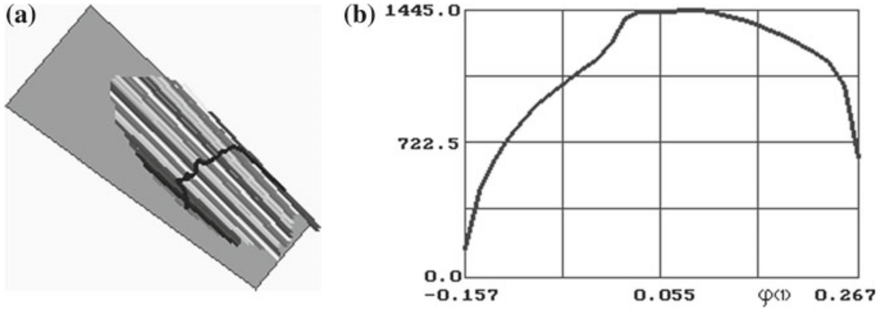

Fig. 2 Some results of the contact problem solution based on Hertz approach: **a** bearing contact and **b** graph of the maximum contact pressure dependence on the meshing phase

2 Formulation of the Optimization Synthesis Problem

The optimal side surfaces of the teeth are the surfaces providing the maximum half of length a_ξ of instantaneous contact area (Fig. 3) in the middle of the meshing interval, which provides localized contact. The degree of contact localization is determined by fourth value (Fig. 3):

- distances (detachment) Δ_{a1}, Δ_{a2} of bearing contact from the tooth upper edge of pinion and gear;
- distances (detachments) Δ_e, Δ_i of bearing contact from the outer and inner transverse edges of the tooth.

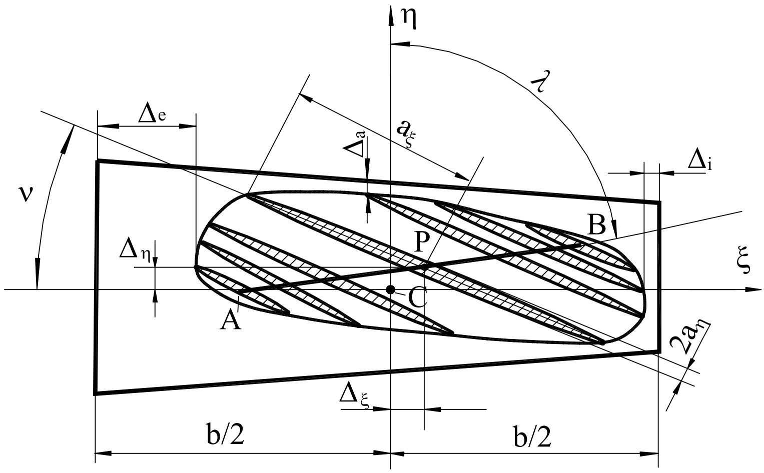

Fig. 3 Location of the bearing contact and the instantaneous contact ellipses in case of localized contact

Distance from the edge is considered positive if all the contact ellipses lie on the tooth surface.

The required values of Δ_{ar}, Δ_{er}, Δ_{ir} of these separation detachments are the input data for solving the problem. The detachment Δ_{ar} is recommended to choose $\Delta_{ar} \geq m_n / 10$, where m_n—mean normal module. The other two detachments should be selected as follows: $\Delta_{er} \geq b / 10$; $\Delta_{ir} \approx 0.25\Delta_{er}$.

Note that the optimal machine-tool settings and the optimal shape of the side surfaces of the teeth depend on the load. At a load greater than the accepted as the maximum permissible at minimization, the edge contact is possible. At the edge contact, the obtained solution becomes not optimal due to a sharp increase in contact pressures.

The optimization problem is reduced to the construction of tooth surfaces providing the maximum half-length a_ξ of the greater semi-axis of the instantaneous contact ellipse in the middle of the meshing interval of a pair of teeth. Clearly, the growth of a_ξ is limited by the size of the flank. As constraints for minimization, we accept

$$|\Delta_{a1} - \Delta_{a2}| < 0.2Y \tag{1}$$

$$Y = \Delta_{a1} + \Delta_{a2} > 1.8\Delta_{ar} \tag{2}$$

$$|\Delta_{i} - \Delta_{ir}| < 0,3\Delta_{ir} \tag{3}$$

$$|\Delta_{e} - \Delta_{er}| < 0.2\Delta_{er} \tag{4}$$

$$f < 0.7w_0 \tag{5}$$

The index "1" refers to the pinion, and the index "2" refers to gear; f is the transmission error; w_0 is the elastic approach of the teeth under load, when the center of contact ellipse is at point P. Distances Δ_{a1}, Δ_{a2}, Δ_e, Δ_i of bearing contact from tooth edges are to be nonnegative.

Restrictions are formulated in the form of inequalities, because the valued: Δ_{a1}, Δ_{a2} Δ_e, Δ_i are determined with considerable error. This is because the algorithm used determines the position of the ends of the instantaneous contact area with an error of 1–2% of the face width b. Thus, the error of determining the distances Δ_{a1}, Δ_{a2} can reach 20%, but this accuracy is sufficient for practical purposes.

The algorithm for solving the optimization synthesis problem is based on the purposeful selection of the synthesis parameter values that ensure the validity of inequalities (1)–(5). The following values are used as synthesis parameters (Fig. 3):

- offset Δ_ξ, Δ_η of teeth meshing interval midpoint P from point C that is located in the tooth in the middle of tooth line;
- length a_ξ of the large semi-axis of the instantaneous contact area at the contact of the teeth in the middle of the meshing interval (point P);
- angle λ between the contact path AB and perpendicular to the gear tooth line (axis ξ);

– maximum transmission error *f*.

Because the error in determining the distances Δ_{a1}, Δ_{a2}, Δ_e, Δ_i is large (10% or more), the use of differential methods of minimization is not possible. The minimization is performed by step method; at the same time, the value of only one synthesis parameter is changed at each step. For given values of the synthesis parameters, synthesis of transmission is carried out. After that, analysis of obtained gear set is accomplished. The analysis results are actual distances Δ_{a1}, Δ_{a2}, Δ_e, Δ_i. The actual distances are compared with the required distances. On the basis of comparison we determine the parameter of the synthesis to be changed and its new value. After that, the re-synthesis and analysis are carried out. The process is repeated until conditions (1)–(5) are met.

For the first iteration, it is necessary to set initial approximations of variable parameters.

1. Offset Δ_ξ the contact patch toward the toe in the absence of deformation of the shafts serves to compensate the displacement of the bearing contact in the heel direction under load. As the first approximation, we will take $\Delta_\xi = -(\Delta_{er} - \Delta_{ir})/2$.
2. The value of the displacement Δ_η is determined from the condition of equal detachment of the bearing contact from the tooth head on the pinion and on the gear. Gears, as a rule, have an addendum modification, determined by the addendum modification factor x_n. So, the middle point C of the pinion flank, located on the pitch cone, is shifted to bottom land, and on the surface of the gear tooth—to the top land. If you select the calculated point P on the pitch cone, the bearing contact will be displaced in these directions relative to the middle of the tooth surface profile. For the first iteration, it is possible to take $\Delta_\eta = 0.75 \cdot m_n x_n$.
3. The choice of the initial approximation of values λ and a_ξ as well of the strategy of search of optimum surfaces depends on two parameters:
   - the ratio of the face width *b* and the average height of the active tooth profile $h_w = 2h_a^* m_n$ (here h_a^* is addendum factor);
   - spiral angle β.

The orientation of the contact ellipse is determined by the angle ν (Fig. 3) between the tooth line and the greater half-axis of the ellipse. The estimation of the value of this angle, which does not change much in meshing, is given by the relation

$$\nu = \operatorname{arctg}(\sin\alpha \cdot \operatorname{tg}\beta) \quad (6)$$

The pressure angle α usually is close to 20°. The range of applicable spiral angles is wide. Spiral angle affects strongly on contact ellipse inclination.

If the face width b significantly (more than four times) exceeds the height h_w of active surface (Fig. 4a), then it is possible to ensure the location of the moving instantaneous contact area of large length inside the tooth in all phases of meshing only if the direction of its movement (shown in the figure by the arrow) is close to the

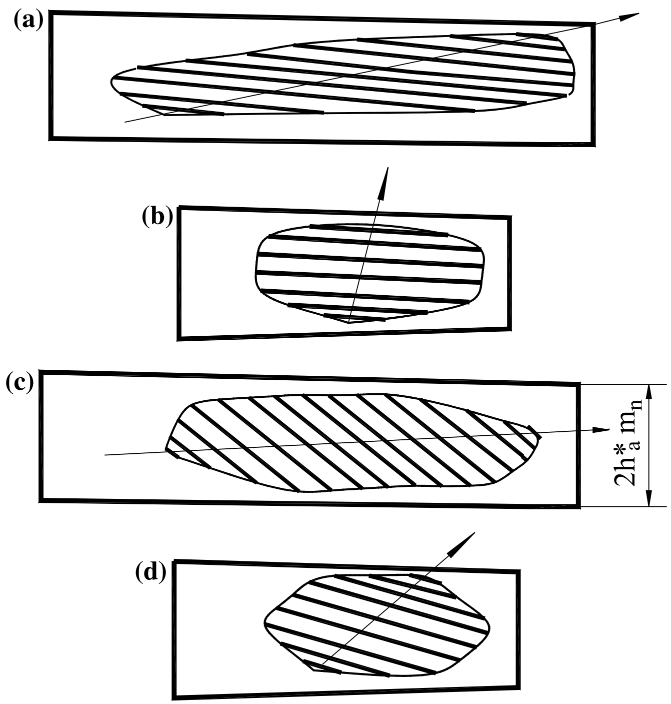

Fig. 4 Expected bearing contacts and the directions of the contact paths: **a** large face width, small spiral angle; **b** small face width, small spiral angle; **c** large face width, large spiral angle; **d** small face width, large spiral angle

direction of the tooth line. In this case, we set $\lambda = 70°$ and relative length of contact area $c = a_\xi / b = 0.33$ as the initial approximation of the parameters.

Otherwise (small gear face width and small spiral angle, see Fig. 4b), movement of the instantaneous contact area in the transverse direction is preferable. In this case, we can increase the sum of distances Y from the longitudinal edges, increasing the transmission error f and thereby reducing the length of the contact path AB (Fig. 3). Only if this is not possible [leads to a violation of condition (5)], further increase of Y is carried out by increasing the angle λ. It is possible to get the localized contact in case of relatively small spiral angles ($\beta \leq 20°$) at the inclination λ of the contact path, close to 20°. This value of λ is taken as the initial approximation.

In case of large spiral angles, the localization of the bearing contact with instantaneous contact areas of considerable length is possible only at significant angles λ (Fig. 4d). As initial approximation can be adopted, angle $\lambda = 50°$. The relative length c of the contact area decreases with the growth of spiral angle. We take $c = 0.3$ as an initial approximation.

At large spiral angles and large relative tooth lengths (Fig. 4c), the main limitation on the length of the instantaneous contact area is the possibility of placing it in the interval between the tip of the tooth and the border of the fillet. The distance between these lines in the middle section is approximately h_w. Therefore, the initial approximation of a_ξ is determined by the relation

$$a_\xi = \min\bigl(cb;\ 0.4h_w\big/\sin\nu\bigr) \tag{7}$$

The coefficient of 0.4 is introduced in order to obtain positive value of Y and to be able to localize the contact in the direction of the contact line, slightly different from the longitudinal direction ($\lambda < 90°$). The recommended initial approximation of spiral angle is 80°.

4. On the basis of selected values of the variable parameters Δ_ξ, Δ_η, λ and a_ξ, the transmission is synthesized. Since the transmission error before the synthesis is not known and the value of w_0 is slightly dependent on f, the synthesis is performed at $f = 0$. In the process of synthesis, the elastic approach w_0 of the teeth at a single-pair contact at the point P is calculated; after that, the initial approximation of the value f is determined as follows

$$f = \bigl(2w_0 + f_{pt1} + f_{pt2}\bigr)\big/k. \tag{8}$$

Here, f_{pt1}, f_{pt2} are the permissible pinion and gear circular pitch errors; $k = 8$ in case $b > 4h_w$ and $k = 6$ otherwise. The coefficient k is chosen from the condition that at the most unfavorable combination of pitch errors the total contact ratio $\varepsilon < 2$.

The algorithm execution includes the following actions:

1. creation of the required sum Y of distances from the bearing contact boundary to tip edges of pinion and gear due to the increase of transmission error f or the angle λ of inclination of the contact path;
2. alignment of distances from bearing contact to the tooth tips of pinion and gear by changing the parameter Δ_η to satisfy inequality (1);
3. increasing the length of the instantaneous contact area if the sum $X = \Delta_e + \Delta_i$ of the distances from the transverse edges significantly exceeds the required value $X_r = \Delta_{er} + \Delta_{ir}$;
4. displacement of the bearing contact along the tooth line to create the required detachment from the transverse edges of the tooth. Additional displacement $\delta\Delta_\xi$ is determined from the system of equations

$$\Delta_e - \delta\Delta_\xi = \Delta_{er},\ \Delta_i + \delta\Delta_\xi = \Delta_{ir}. \tag{9}$$

Equation (9) is the condition that after the displacement $\Delta_\xi + \delta\Delta_\xi$ the required distances from tooth heel and from tooth toe will be obtained. By solving the system (9), we have

$$\delta\Delta_\xi = (\Delta_{ir} - \Delta_{er} + \Delta_e - \Delta_i)\big/2. \tag{10}$$

To automate the optimization process, a computer program has been developed, the initial data of which are the design parameters of the gear pair, the design load and the parameters of the tool for processing the teeth, as well as the required distances of bearing contact from the edges of the teeth. The results are the machine-tool settings and the meshing characteristics.

3 The Optimization of Transmission by Variation Spiral Angle and Tooth Thickness Modification Factor

Optimization preserves the previous values of the following characteristics:

- mechanical properties of materials of gear and pinion;
- numbers z_p, z_g of pinion and gear teeth;
- face width b;
- face angles and root cone angles;
- addendum modification factor x_{n} and radial clearance c^*;
- parameters of tools (grinding wheels or cutter heads) used in the processing of teeth, except for the diameters;
- mean cone distance L_{m}.

In addition, it is assumed that the deviation of the pinion and gear outer tip diameters d_{ae1}, d_{ae2} from their original values do not exceed several tenths of a millimeter, which allows to place the transmission in the former housing.

Let β_0 be the initial spiral angle and m_{n0} be the initial value of the transmission mean normal module; conserving the original value of mean cone distance can be achieved by changing the mean normal module. The new module is determined by the relation

$$m_{\mathrm{n}} = m_{\mathrm{n0}} \cos\beta \big/ \cos\beta_0. \tag{11}$$

Note that the decrease in the angle of the spiral leads to an increase in the thickness of the tooth according to the normal and a decrease in the load on the tooth while maintaining the torque on the shaft.

To preserve the pinion and gear outer tip diameters during the optimization process, the addendum factor h_{a}^* is recounted as follows

$$h_{\mathrm{a}}^* = h_{\mathrm{a0}}^* m_{\mathrm{n0}} \big/ m_{\mathrm{n}}, \tag{12}$$

where h_{a0}^* is the initial value of the factor.

As the target function for optimization, we take the durability t of the transmission, which is equal to the smallest of the three values: contact durability t_{H}, bending durability of the pinion t_{FP} and bending durability of the gear t_{FG}. The input data for calculating the values of t_{H}, t_{FP} and t_{FG} are:

– loading cyclogram, which determines the fraction ν_i of the total time of loading, falling on the ith mode (i = 1,2,…, k), the torque M_i on the gear shaft and the number of N_i revolutions of the gear shaft per minute; k is the number of loading modes;
– contact pressure $\sigma_{\mathrm{H}i}$ and bending stress $\sigma_{\mathrm{FP}i}$, $\sigma_{\mathrm{FG}i}$ in loading mode number i;
– permissible contact pressure $[\sigma_{\mathrm{H}}]$ at the base number N_0H of loading cycles, which can be estimated using the expression (Russian standard GOST 21354-87)

$$[\sigma_{\mathrm{H}}] = \sigma_{\mathrm{H\,lim}} Z_{\mathrm{R}} Z_{\mathrm{V}} \big/ S_{\mathrm{H}}; \tag{13}$$

– permissible bending stress $[\sigma_{\mathrm{F}}]$ at the base number N_0F of the loading cycles, defined as follows

$$[\sigma_{\mathrm{F}}] = \sigma_{\mathrm{F\,lim}} Y_{\mathrm{A}} Y_{\mathrm{R}} Y_{\mathrm{V}} \big/ S_{\mathrm{F}}. \tag{14}$$

Here, $\sigma_{\mathrm{H\,lim}}$, $\sigma_{\mathrm{F\,lim}}$ are the limits of contact and bending endurance, respectively (for simplicity, we assume that the pinion and gear are made of the same materials), S_{H}, S_{F} are the safety factors, Z_{V}, Y_{V} are the dynamic coefficients, Z_{R}, Y_{R} are the coefficients of roughness, and Y_{A} is the coefficient of reversibility of rotation.

The number of revolutions N_{T} of gear before failure (resource) is determined on the basis of the hypothesis of linear summation of damage as follows

$$N_{\mathrm{T}} \sum_i \frac{\nu_i}{N_i} = 1, \quad (i = 1, 2, \ldots, k) \tag{15}$$

where each of the terms in the left part of the equation determines the contribution of the ith loading mode to the degree of damage to the part, which at the time of failure of the part is assumed to be equal to one. The values of N_i represent the number of cycles to failure during operation from the beginning of operation to failure only in the ith mode. The relation (15) is used to compute the transmission contact endurance N_{TH} and to calculate resources N_{TFP}, N_{TFG} of pinion and gear on the flexural durability.

In the first case, instead of N_i in the ratio (15) should be the number $N_{\mathrm{H}i}$ of cycles to failure as a result of contact fatigue, which will be considered associated with the permissible contact stress at the base number of cycles as follows

$$[\sigma_{\mathrm{H}}]^{q_{\mathrm{H}}} N_0\mathrm{H} = \sigma_{\mathrm{H}i}^{q_{\mathrm{H}}} N_{\mathrm{H}i}. \tag{16}$$

According to GOST 21354-87, $q_{\mathrm{H}} = 6$ at $\sigma_{\mathrm{H}i} \le [\sigma_{\mathrm{H}}]$ and $q_{\mathrm{H}} = 20$ otherwise.

In the second case, in the relation (15) instead of N_i is substituted by the number of $N_{\mathrm{F}i}$ cycles up to failure as a result of flexural fatigue, which is determined by a relation similar to the relation (16)

$$[\sigma_{\mathrm{F}}]^{q_{\mathrm{F}}} N_0\mathrm{F} = \sigma_{\mathrm{F}i}^{q_{\mathrm{F}}} N_{\mathrm{F}i}, \tag{17}$$

where $q_F = 9$ for cemented or nitrided gears with a non-grinded fillet surface. In other cases, $q_F = 6$.

According to the relations (16) and (17), even for the approximate estimation of the transmission durability, it is necessary to calculate the contact and bending stresses with a very high degree of accuracy. For example, an error of σ_H of 3% leads to an error of N_{TH} approximately equal to 100% $(1.03^{20} - 1) = 80\%$. Therefore, even when using the most precise methods of calculation of stresses only a rough estimate of the durability of the transmission may be obtained. However, the purpose of the proposed work is not so much to determine the durability itself, as to determine the parameters of the optimal in terms of durability of the bevel gear structure. At the same time, it can be assumed with a high degree of confidence that the stress reduction will cause an increase in the life durability.

Consider a specific example. Suppose you want to optimize an orthogonal transmission that has the following parameters: Numbers of teeth are $z_p = 31$; $z_g = 73$; mean normal module is $m_n = 6.0$ mm; spiral angle is $\beta = 30°$; face width is $b =$ 70 mm. The face angles are $\delta_{a1} = 24° \; 58'$, $\delta_{a2} = 68° \; 02'$; the root cone angles are $\delta_{f1} = 21° \; 58'$ and $\delta_{f2} = 65° \; 02'$, respectively. The addendum factor is $h_a^* = 1.0$; the radial clearance coefficient is $c^* = 0.25$; the addendum modification factor is $x_n = 0.34$; tooth thickness modification factor is $x_\tau = 0.06$. A backlash of 0.2 mm is formed by reducing the thickness of the pinion tooth. Tooth-to-tooth composite tolerance we will take one and the same for pinion and gear $f_{pt} = 0.02$ mm.

Both the gear and the wheel are made of chrome–nickel steel. The hardness of the cemented surfaces of the teeth HRC = 64. Young's modulus $E = 2.0 \times 10^5$ MPa, and Poisson's ratio $\mu = 0.3$. Limits of contact and flexural endurance are: $\sigma_{H\,\lim} =$ 1470 MPa; $\sigma_{F\,\lim} = 800$ MPa at numbers of cycle $N_0H = 1.2 \times 10^8$ and $N_0F = 4.0 \times 10^6$. Allowable stress at the base number of cycles is $[\sigma_H] = 1220$ MPa; $[\sigma_F] =$ 600 MPa.

The loading cyclogram includes two modes: $M_1 = 1.0 \cdot 10^4$ Nm, $\nu_1 = 0.001$; $n_1 = 3000$ rpm and $M_2 = 0.75 \cdot 10^4$ Nm, $\nu_2 = 0.999$; $n_2 = 3000$ rpm.

The required bearing contact distances from the edges at maximum load M_1 are $\Delta_{ar} = 0$; $\Delta_{er} = 9$ mm; $\Delta_{ir} = 3$ mm.

When finishing the surfaces of the teeth of the pinion and gear, grinding wheels of nominal diameter 500 mm with a profile angle of the inner side equal to 20° 45′ and profile angle 20° 45′ outside surface are used. Modification of the roll motion is not used during processing.

Changes in the design parameters of the wheels, the length of the instantaneous contact areas, contact pressures and bending stresses, contact and bending durability at the angles of the spiral in the range from 10° to 35° are shown in Table 1.

Figure 5 shows how the contact pressure σ_{H2} (upper continuous curve) and bending stresses σ_{FP2} (lower continuous curve), σ_{FG2} (lowest dashed line) in the loading mode 2, depend on the spiral angle β.

An increase of spiral angle is the cause of decrease of the mean normal module, and hence the thickness of the tooth decreases. It leads to an increase in the level of bending stress. In addition, to transfer the same moment at a greater angle of the spiral a larger contact force is required.

Table 1 Characteristics of the gear transmission at different angles of spiral

β	10°	15°	20°	22°	25°	30°	35°
m_n (mm)	6.823	6.692	6.510	6.468	6.279	6.00	5.675
h_a^*	0.879	0.897	0.922	0.934	0.956	1.00	1.057
L_m (mm)	274.74	274.74	274.74	274.74	274.74	274.74	274.74
d_{ae1} (mm)	259.65	259.57	259.46	259.40	259.31	259.14	258.93
d_{ae2} (mm)	573.56	573.60	573.65	573.67	573.71	573.78	573.87
a_ξ (mm)	24.50	24.50	24.50	24.50	24.50	22.45	18.69
σ_{H1} (MPa)	1264	1234	1224	1201	1184	1199	1301
σ_{H2} (MPa)	1119	1111	1091	1076	1066	1089	1190
σ_{FP1} (MPa)	379	384	400	410	420	475	572
σ_{FG1} (MPa)	398	411	420	431	441	486	577
σ_{FP2} (MPa)	298	305	324	326	337	374	451
σ_{FG2} (MPa)	317	324	331	345	349	385	451
t_{FP} (h)	11,990	9740	5650	5350	3970	1550	290
t_{FG} (h)	16,210	13,300	10,980	8620	6820	2820	680
t_H (h)	3760	4340	6240	8230	9920	6470	1100

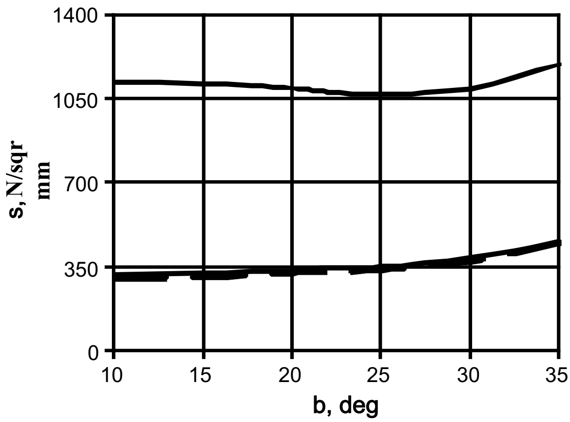

Fig. 5 Stress dependences on the spiral angle

Despite the fact that the coefficient of the height of the tooth with the increase of the spiral angle also increases, the height of the tooth remains virtually unchanged, which makes it impossible to increase the length a_ξ of the instantaneous contact area. Starting from the angle $\beta \approx 25°$, the contact area decreases, and this together with the growth factor of the normal force to the tooth surface leads to an increase in contact pressures. The mean cone distance, as well as the internal and external cone distances, remains unchanged since the normal module is recalculated by the ratio (11). Changes in the outer diameter of the pinion and gear are tenths of a millimeter that makes it possible to place the transmission in the former housing.

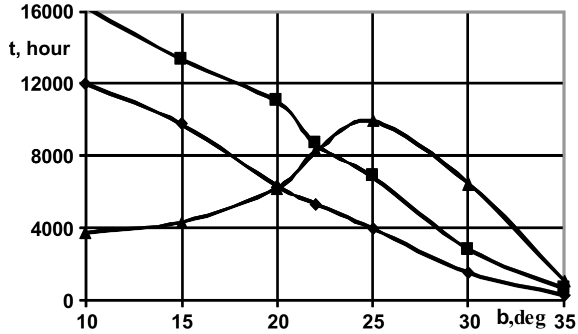

Fig. 6 Longevity dependences on the spiral angle

In Fig. 6, the curve, marked with markers in the form of triangles, gives the dependence of the contact durability of the transmission on the angle of the spiral; the curves of the bending durability of the pinion and gear are marked with markers in the form of diamonds and squares, respectively. It can be seen that relatively small changes in contact pressures (about 10%) and bending stresses (about 40%) have a very large impact on the durability of the transmission.

Figure 6 shows that the highest longevity occurs in the interval $20° < \beta < 22°$. Assume that the optimal value $\beta = 21°$ is in the middle of this interval. When the adopted value of the tooth thickness modification factor is $x_\tau = 0.06$, the bending durability of the gear greatly exceeds the bending durability of the pinion (Fig. 6). This is because the number of loading cycles of the pinion tooth for the same period is 2.35 times greater. Varying the values of the factor x_τ, we obtain that at $x_\tau = 0.12$ the bending durability of the pinion and gear takes close values: $t_{\mathrm{FP}} = 7080$ h, $t_{\mathrm{FG}} = 6990$ h. Contact durability $t_{\mathrm{H}} = 7500$ h at the same time does not change. It is not possible to obtain a more accurate coincidence of the values of all three considered durations due to errors in the determination of stresses having an order of 2–3%.

Thus, as a result of optimization it was possible to increase the estimated transmission life from 1550 h (see Table 1) up to 6690 h, i.e., more than four times.

4 The Optimization of Transmission by the Number of Teeth

In order that the new gear pair with the increased loading capacity could be placed in the free space of the old reducer shown in Fig. 7 (would be interchangeable with a pair of the former construction), the equity of the following inequalities is sufficient:

$$x_p \min \geq x_p^* \min; \; x_p \max \leq x_p^* \max; \; R_g \min \geq R_g^* \min;$$
$$R_g \max \leq R_g^* \max; \; x_g \max \leq x_g^* \max; \; x_g \min \geq x_g^* \min;$$

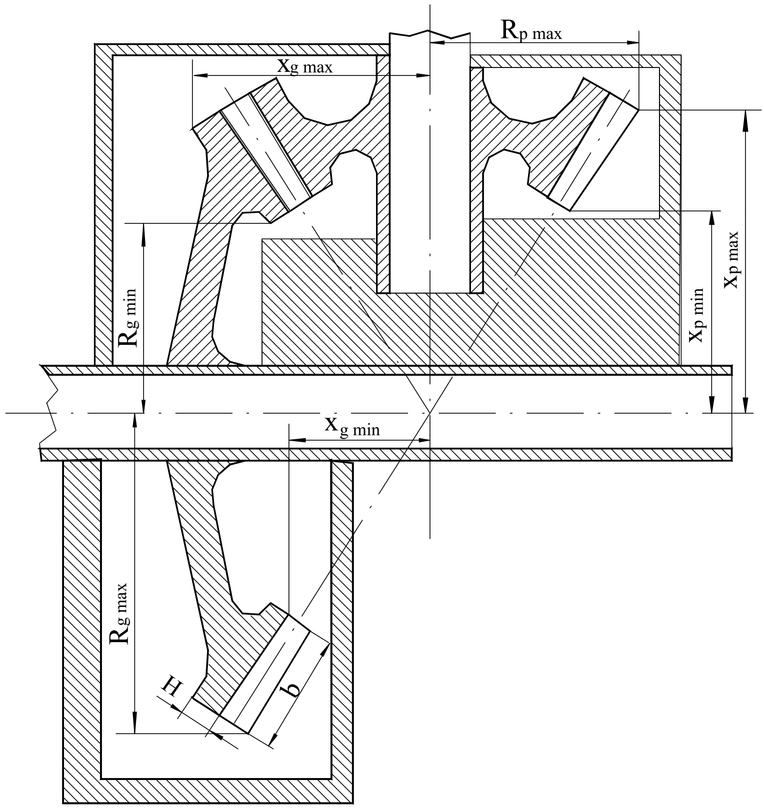

Fig. 7 Gear housing design

$$R_{p\,\min} \geq R^*_{p\,\min};\; R_{p\,\max} \leq R^*_{p\,\max}; \tag{18}$$

The index p in the inequalities (18) determines the parameter of pinion; the index g determines the parameter of gear. Dimensions $R_{\max}$, $R_{\min}$, $x_{\max}$ and $x_{\min}$ are shown in Fig. 7. The upper index "asterisk" marked values related to the original transmission design. The values contained in the inequalities (18) are expressed through the external L_{e} and the inner L_{i} cone distances, outer and inner h_{ae}, h_{ai} addendums, and external and internal dedendums h_{fe}, h_{fi} as follows

$$\begin{aligned} R_{g\,\max} &= L_{\mathrm{e}} \sin\delta_g + h_{\mathrm{ae}} \cos\delta_g; \quad R_{g\,\min} = L_{\mathrm{i}} \sin\delta_g - (h_{\mathrm{fe}} + H)\cos\delta_g; \\ R_{p\,\max} &= L_{\mathrm{e}} \sin\delta_p + h_{\mathrm{ae}} \cos\delta_p; \quad R_{p\,\min} = L_{\mathrm{i}} \sin\delta_p - (h_{\mathrm{fe}} + H)\cos\delta_p; \end{aligned} \tag{19}$$

$$\begin{aligned} x_{g\,\max} &= L_{\mathrm{e}} \cos\delta_g + (h_{\mathrm{fe}} + H)\sin\delta_g; \quad x_{g\,\min} = L_{\mathrm{i}} \cos\delta_g - h_{\mathrm{ae}} \sin\delta_g; \\ x_{p\,\max} &= L_{\mathrm{e}} \cos\delta_p + (h_{\mathrm{fe}} + H)\sin\delta_p; \quad x_{p\,\min} = L_{\mathrm{i}} \cos\delta_p - h_{\mathrm{ae}} \sin\delta_p, \end{aligned} \tag{20}$$

where (Fig. 7) H is the thickness of the tooth rim; according to the recommendations of [9], this value is taken to be equal to the total external tooth height $H = h_e = h_{fe} + h_{ae}$; δ_p and δ_g are angles of the pinion and gear pitch cones.

We define addendums using relations

$$h_{ag} = m_n(h_a^* - x_n);\ h_{ap} = m_n(h_a^* + x_n) \tag{21}$$

Expressions for gear and pinion dedendums are

$$h_{fg} = m_n(h_a^* + c^* - x_n);\ h_{fp} = m_n(h_a^* + c^* + x_n) \tag{22}$$

The remaining values present in the ratios (19) and (20) are defined as follows (since the ratios are the same for the pinion and the gear, the indices p or g are omitted):

$$\begin{aligned} &L_e = L_m + b/2;\ L_i = L_m - b/2;\ h_{ae} = h_a + b \cdot \mathrm{tg}\ \theta_a/2; \\ &h_{fe} = h_f + b \cdot \mathrm{tg}\ \theta_f/2;\ h_{ai} = h_a - b \cdot \mathrm{tg}\ \theta_a/2;\ h_{fi} = h_f - b \cdot \mathrm{tg}\ \theta_f/2, \end{aligned} \tag{23}$$

where the addendum angle is $\theta_a = \delta_a - \delta$ and the dedendum angle is $\theta_f = \delta - \delta_f$.

The mean cone distance L_m is equal

$$L_m = \tfrac{1}{2} m_n \sqrt{z_p^2 + z_g^2}. \tag{24}$$

Since the bending stresses of the cantilever beam are inversely proportional to its thickness, it should be expected that the bending stresses in the tooth decrease with an increase in the normal module m_n of the gear.

The number z_g of gear teeth when changing the number z_p of pinion teeth is determined from the condition of obtaining the closest to the original gear ratio as follows: $z_g = u \cdot z_p$, where $u = z_g^*/z_p^*$, then rounded to the nearest entire number. To conserve the initial cone distance, the mean normal m_n modulus with decreasing number of z_g teeth is determined by the relation resulting from the first of the relationship (24):

$$m_n = m_n^* \sqrt{\frac{z_p^{*2} + z_g^{*2}}{z_p^2 + z_g^2}}, \tag{25}$$

where m_n^* is the normal module and z_p^*, z_g^* are the numbers of teeth of the original pair.

Next, we find the maximum value of the parameter b, at which all the inequalities (18) are true. Since one cannot satisfy all the conditions at the same time, you have to choose the most important one. In case of gearbox housing shown in Fig. 7, these conditions should be imposed on $x_{p\ \max}$, $x_{p\ \min}$, $R_{g\ \min}$ and $R_{g\ \min}$. The remaining conditions can be met by a small design change.

Table 2 Characteristics of the gear transmission at different numbers of teeth

z_p	31	29	27	25	23	21
z_g	73	68	64	59	54	49
Δu (%)	0%	−0.4	0.66	0.2	−0.3	−0.9
m_n (mm)	5.946	6.378	6.789	7.359	8.035	8.846
b (mm)	70.0	68.4	67.0	65.8	64.0	62.2
x_p max (mm)	291.9	291.6	291.9	291.8	291.8	291.9
$x_{p\,\min}$ (mm)	217.0	217.4	218.3	218.5	218.9	219.2
R_p max (mm)	125.8	126.1	125.2	125.8	126.4	125.2
x_g max (mm)	137.9	139.0	138.7	140.2	142.0	144.2
$x_{g\,\min}$ (mm)	84.8	84.8	83.5	83.3	83.1	82.9
R_g max (mm)	286.8	286.1	286.1	285.8	285.2	284.6
$R_{g\,\min}$ (mm)	211.8	211.9	212.6	212.4	212.2	211.9
L_{+}m (mm)	272.32	272.32	272.32	272.32	272.32	272.32
ρ_s (mm)	2.3	2.4	2.6	2.7	2.8	3.0
σ_H (MPa)	1507	1491	1499	1492	1496	1508
σ_{Fp} (MPa)	685	641	591	563	519	468
σ_{Fg} (MPa)	671	640	593	567	517	462
a_ξ (mm)	22.0	22.0	22.0	21.5	21	20.3
a_η (mm)	1.07	1.08	1.09	1.12	1.16	1.21

As an example, consider the same gear with the numbers of teeth $z_p = 31$, $z_g = 73$, as in the previous example, but now $m_n = 5.464$ mm, $h_a^* = 0.9537$ and $\beta = 30°\ 1'$. The direction of the gear spiral is left. The coefficient of change in the calculated thickness of the tooth $x_\tau = 0.054$. The backlash is created by reducing the thickness of the pinion tooth at $\Delta_p = 0.1$ mm and the gear tooth at $\Delta_g = 0.14$ mm.

The load is the torque $M_g = 1.5 \times 10^4$ N m on the gear shaft. Under this load, the distance Δ_{ar} of bearing contact from the tip should be 0.6 mm, and the distances Δ_{er}, Δ_{ir} of bearing contact from the heel and the toe are to be equal to 9.0 and 3.0 mm, respectively.

The characteristics of the gears obtained from the initial gear by decreasing the number of pinion teeth and the recalculation of the mean normal module m_n and the face width b while maintaining the values of all other initial design parameters are given in Table 2.

Table 2 shows that the deviation Δu of the transmission ratio from the original value does not exceed one percent. The overall dimensions of the modified gears are not much different from the size of the initial transmission.

If you reduce the numbers of teeth, the major semi-axis a_ξ of the instantaneous contact ellipse is slightly reduced, but this increases the length of the smaller semi-axis a_η. The total area of the ellipse varies little, and therefore the maximum contact pressures are very little. As expected, bending stresses decrease monotonically with decreasing number of teeth of the pair. When $z_p = 21$, flexural stress is less than at $z_p = 31$ approximately 30%. Due to the fact that a further decrease in the number of teeth is associated with an increase in the sliding speed at the contact of teeth and contact pressures, a further decrease in the number of teeth of the gear, although it leads to a decrease in bending stresses, should be considered impractical. The number of teeth $z_p = 21$, $z_g = 49$ gears and wheels can be considered optimal.

The analysis of the tooth shape made by the algorithm showed that there is no undercutting in the considered range of tooth numbers, so the addendum modification factor can be left unchanged. The point width tooth taper and thickness tooth taper are satisfactory, so the correction of the face angle and the root cone angle is not necessary.

5 Conclusion

The proposed algorithm of synthesis of spiral bevel gears of the given dimensions is designed to build the shape of the surfaces of the teeth, providing the minimum contact pressures and bending stresses. The algorithm of shape optimization is implemented in the form of software that has simple enough to understand the source data. The search for the required tooth surfaces and machine-tool settings to ensure the bearing contact, spaced from the edges of the teeth at the given distances, is carried out automatically. It is shown that the software can be used to improve the existing transmission without changing its dimensions. The new gear transmission with decreased stress level and increased longevity may be placed in housing of prior transmission. In the considered examples, this is achieved by choosing the optimal spiral angle and reducing the number of teeth of the gear and the pinion while maintaining gear ratio close to the previous one.

References

1. Litvin F.L., Fuentes A. (2004) Gear geometry and applied theory. Cambridge University Press, Cambridge
2. Sheveleva GI, Volkov AE, Medvedev VI (2007) Algorithms for analysis of meshing and contact of spiral bevel gears. Mech Mach Theory 129:198
3. Simon V (2007) Load distribution in spiral bevel gears. ASME J Mech Des 129:201
4. Wilcox LE. (1981) An exact analytical method for calculating stresses in bevel and hypoid gear teeth. In: Proceedings of international symposium on gearing and power transmissions, Tokyo, II, p 115
5. Gosselin C, Cloutier L, Nguyen QD (1995) A general formulation for the calculation of the load sharing and transmission error under load of spiral bevel and hypoid gears. Mech Mach Theory 30:433
6. Simon V (2009) Head-cutter for optimal tooth modifications in spiral bevel gears. Mech Mach Theory 44:1420
7. Medvedev VI, Volkov AE (2007) Synthesis of spiral bevel gear transmissions with a small shaft angle. ASME J Mech Des 129:949

8. Wang PY, Fong YH (2006) Fourth-order kinematic synthesis for face milling spiral bevel gears with modified radial motion (MRM) correction. ASME J Mech Des 128:457
9. Zhang J, Fang F, Cao X, Deng X (2007) The modified pitch cone design of the hypoid gear: manufacture, stress analysis and experimental tests. Mech Mach Theory 42:147
10. Argyris J, Fuentes A, Litvin FL (2002) Computerized integrated approach for design and stress analysis of spiral bevel gears. Comput Methods Appl Mech Eng 191:1057
11. Medvedev VI (2001) Programs packet for quality analysis of the conical and hypoid teeth pairs. Problemy Mashinostraeniya i Nadezhnos'ti Mashin 3:77
12. Medvedev VI, Sheveleva GI (2002) Spiral-conical gears synthesis by the contact teeth strength conditions. Problemy Mashinostraeniya i Nadezhnos'ti Mashin 4:75
13. Medvedev VI (1999) Synthesis of generated non-orthigonal bevel and hypoid gear sets. J Mach Manuf Reliab 5:1
14. Medvedev VI, Volkov AE, Volosova MA, Zubelevich OE (2015) Mathematical model and algorithm for contact stress analysis of gears with multi-pair contact. Mech Mach Theory 86:156
15. Volkov AE, Medvedev VI, Skorodumov OI (2008) Increasing the loading capacity of circular-arc teeth of conic gears without a significant change in dimensions. J Mach Manuf Reliab 37:152
16. Volkov AE, Matveenkov DS, Medvedev VI (2010) Optimizing the structural parameters of conical pairs with circular teeth. Russ Eng Res 10:1060