Vibractivity of Cycle Machinery Drives in the Accounting of the Space Distribution of Working Bodies Characteristics

Iosif I. Vulfson

Abstract The paper deals with new problems related to the excitation of oscillations in the drives of cyclic machines, taking into account the spatial distribution of the characteristics of the working elements and the force closure of systems. Dynamic models based on aggregation and general representations of systems with complex structure are proposed. An original method for studying systems with variable parameters is presented, in which the method of the conditional oscillator is applied to variable spatial and temporal arguments. The paper deals with new problems related to the excitation of oscillations in the drives of cyclic machines, taking into account the spatial distribution of the characteristics of the working elements and the force closure of systems with gaps. Dynamic models based on aggregation and general display of systems with complex structure are proposed. The original method of studying systems with variable parameters is presented, in which the conditional oscillator method is applied to the joint consideration of the variability of spatial and temporal characteristics.

Keywords Vibrations · Cyclic machines · Gaps · Force closure

1 Introduction

The solution of modern problems of machine dynamics is often connected with the analysis of so-called regular oscillatory systems. The term "regularity" means the coincidence of the dynamic structure and parameters of individual subsystems (modules). The theory of regular oscillatory systems is reflected in the works of many prominent scientists. First, a one-dimensional lattice, consisting of the point particles, was studied by Newton, while determining the speed of sound. Further studies are associated with the works of Daniel and Johann Bernoulli, Cauchy, Kelvin, Born,

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Karman, Debye, Brillouin, and others [1, 2]. These works formed the basis of a number of methods, that help to carry out an analytical description of systems with a large number of degrees of freedom, based on the analysis of one structural element of the system. The objects for using the apparatus of the theory of chains were crystal lattices and a number of other problems of theoretical physics.

Among the technical problems of analysis of such class of systems, we can mention the theory of electrical lines, as well as some problems arising in the calculation of deformations and oscillations in frames, girders, etc. In cyclic action machines and automatic lines, one has to deal with regular oscillating systems in connection with the wide spread of dynamically identical modules used for the increased length of the technological process zone, and for the implementation of the same type of technological and transport operations. In such cases, in view of the «natural» desire for unification and interchangeability of individual components or entire units of the machine, arises the certain repeatability of blocks of the dynamic model of the drive. This situation is particularly common in textile machinery, light industry, food, printing, and several other industries [3–6].

The complexity of modern machines and the large dynamic coupling of individual nodes lead to the need to consider vibrational systems with a large number of degrees of freedom. In the analysis, and especially in the dynamic synthesis of such systems, because of the large array of generalized coordinates and variable parameters, certain difficulties arise, for the overcoming of which in mechanics, automatic control, economics, etc., the method of aggregation of the system are used. Applied to the problems of the dynamics of machines in the development of such approaches, continual models were proposed in which the kinematic, elastic, and inertial properties of the mechanisms are displayed by some "pseudo-medium." This allows to operate with a generalized representation of a group of variables and substantially reduce the number of characteristics describing the oscillatory system [3–7].

As a result, the dynamic analysis and synthesis of the system may be simplified, and in many cases to present the solution in analytical form. Note that the use of models with distributed parameters is very effectively combined with the methods of aggregation and decomposition, widely used in solving modern problems of increased complexity in different scientific directions [8, 9]. In this paper, the solution of problems of the dynamics of cyclic machines, considered in a number of works by the author, which are generalized and described in detail in monographs [4, 6], was further developed. As applied to the dynamics of machines, in developing such approaches, continual models are used in which the kinematic, elastic, and inertial properties of mechanisms are replaced by a certain "pseudo-medium". The results of the research make it possible to substantially reduce the vibration activity of the drive and improve the accuracy of reproduction of the specified program movements of the working organs.

The rational dynamic synthesis of oscillating systems in this case plays a special role because of the clearances that can lead to large distortions in the programmed motion of the operating organs, as well as to noise, dynamic loads and vibratory activity of the drive. The clearances, as a rule, are the concomitant factor of any kinematic pair that carries out the connection of the links of the mechanism. Often it is the size of the clearances that ultimately limits the performance and performance characteristics, which makes it essential to tighten the precision requirements for their manufacture and assembly. Essentially, the kinematic pair should be considered a non-retentive bond, which is usually referred to as mobile connections of units with one-sided contact. Indeed, although the kinematic pair as a whole realizes a two-way coupling, it performs only partially in the gaps, since in the case of a rearrangement, local discontinuities of the kinematic chain, characteristic of systems with non-retentive bonds, take place in the gap. Such relationships can be characterized as pseudo-restraints.

2 Dynamic Models. Frequency Analysis Technique

Figure 1 a shows a fragment of the kinematic scheme, which consists of the engine M, the transmission mechanism 1(reducer), the main shaft 2, and drives of the actuators with the ring structure 3. Figure 1b presents a dynamical model with discretely specified elements and its modification in the transformation of this model after transition to a system with distributed parameters (continuum model, Fig. 1c). We accept the following conventions: J_0 , J_1 , J_2 are moments of inertia; c, ψ are the reduced coefficients of torsional stiffness and drive dissipation, u—gear ratio; and $\Pi(\varphi)$ is the position function.

The differential equation of free oscillations without taking into account the characteristics of the engine, the forces of resistance, and the force closure is

$$G\frac{\partial}{\partial x}\left[I(x)\frac{\partial\varphi}{\partial x}\right]-\rho(x)\frac{\partial^2\varphi}{\partial t^2}=0, \tag{1}$$

where x is the coordinate of shaft axis; $\rho(x)=\partial J/\partial x$ is the change intensity of the moment of inertia along the shaft axis; and G is the shear modulus.

At uniform distribution of the moment of inertia $\rho = J\big/\ell = \text{const.}$, where ℓ is the length of a shaft. Then

$$GI\frac{\partial^2\eta}{\partial x^2}-\rho\frac{\partial^2\eta}{\partial t^2}-\rho\bar{\omega}^2\frac{\mathrm{d}^2\Pi}{\mathrm{d}\varphi^2}=0.$$

Here $\eta=\varphi_2(x,t)-\Pi(\varphi)$ is the dynamic error in the section x.

The characteristic of the pseudo-medium of one module is the modified transition matrix

$$\tilde{\mathbf{\Gamma}}_*=\prod_s^1\tilde{\mathbf{\Gamma}}_i=\begin{bmatrix}A(p) & B(p)\\ C(p) & D(p)\end{bmatrix}\quad (i=\overline{1,s}), \tag{2}$$

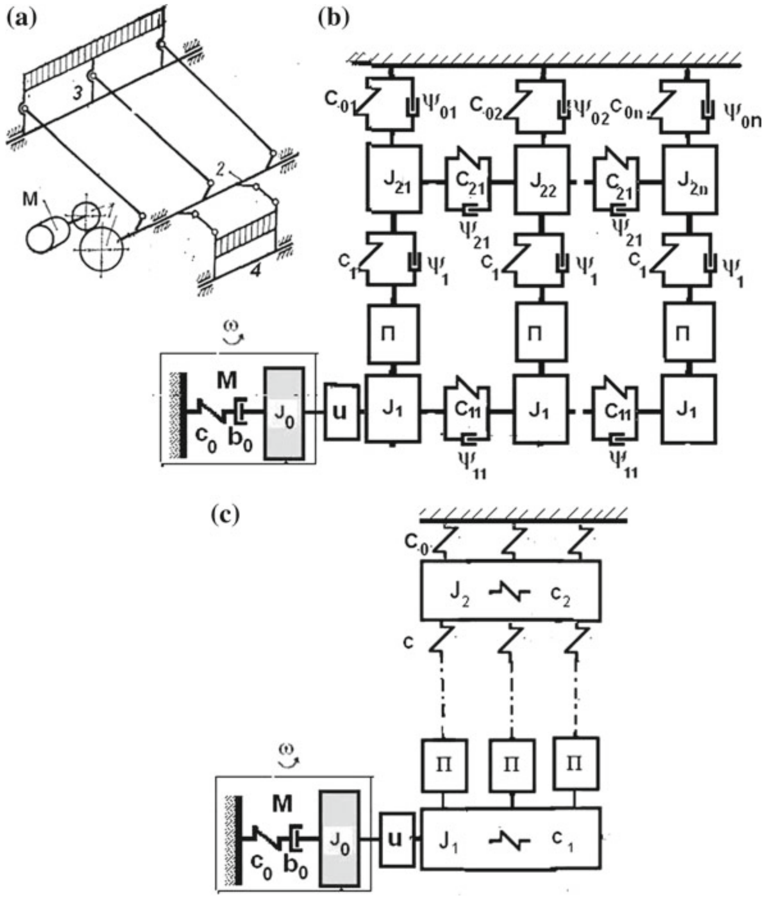

Fig. 1 Kinematic scheme and modifications of the dynamic drive models of the regular structure: **a** 1—reducer; 2—main shaft; 3, 4—mechanisms of drive of working bodies

where s is the number of elements of the mechanism, $\tilde{\mathbf{\Gamma}}_i$ is the transition matrix of the element i.

So, in particular, if the module is formed by a combination of inertial, elastic, and kinematic elements, then

$$\tilde{\mathbf{\Gamma}}_J = \begin{bmatrix} 1 & 0 \\ -p^2 J_*/\ell & 1 \end{bmatrix}; \quad \tilde{\mathbf{\Gamma}}_c = \begin{bmatrix} 1 & \ell/(cn) \\ 0 & 1 \end{bmatrix};$$

$$\tilde{\mathbf{\Gamma}}_\Pi = \begin{bmatrix} \Pi'(\varphi_1^*) & 0 \\ 0 & \Pi'(\varphi_1^*)^{-1} \end{bmatrix}. \quad (3)$$

Here, $\Pi'(\varphi^*) = d\varphi_2^*/d\varphi_1^*$ is the first geometric transfer function (below we will take $\Pi' = h_0 \sin\varphi_1$); $J_* = nJ$.

With reference to the analysis and dynamic synthesis of oscillatory systems of the working organs of knitted machines, the problem in question is partially reflected in the work [10].

3 Analysis of Oscillations with Variable Intensity of the Spatial Distribution of Characteristics

In contrast to the above case $\rho(x) \neq \text{const}$. Further, following the Fourier method, we seek the solution in the form

$$\varphi(x, t) = X(x, \tau)T(t), \tag{4}$$

where τ is "slow time".

After substituting (4) into (3) and some transformations, we obtain

$$\begin{aligned} &X'' + 2P(x, \tau)X' + K^2 X(x, \tau) = 0; \\ &\ddot{T} + p^2(\tau)T = 0. \end{aligned} \tag{5}$$

Here it is customary:

$$P(x) = 0,5\gamma'(x)/\gamma(x);\ K^2(x, \tau) = p^2(\tau)a(x, \tau)^{-2}\gamma(x)^{-1};$$

$$\begin{aligned} &a^2(x, \tau) = GI(0)/\rho(x, \tau);\ \gamma(x) = I(x)/I(0); \\ &()' = \partial/\partial x;\ (\dot{}) = \partial/\partial t. \end{aligned}$$

On the basis of the method of the conditional oscillator, we have

$$\exp\left\{-\int_0^x P(\xi)d\xi - 0,5[z(x) - z(0)]\right\}(C_1 \cos\Phi + C_2 \sin\Phi), \tag{6}$$

where $\Phi(\tau) = \int_0^\ell U(\xi, \tau)d\xi$; $U(x, \tau) = \sqrt{K^2 - P^2 - P'}$; C_1, C_2 are the arbitrary constants.

The function is a particular solution (exact or approximate) of the equation of the conditional oscillator [11], which in this case is defined as

$$z'' - 0,5z'^2 + 2U(0, \tau)^2 e^{2z} = 2U(x, \tau)^2. \tag{7}$$

For definiteness, we assume that on the end of the shaft is located a disk (the rotor of the motor, the flywheel), the moment of inertia of which considerably exceeds the reduced moment of inertia of the main shaft, i.e., $J_0 \gg J^*_{\max}$ (see Fig. 1b, model 2). In this case, the boundary conditions for frequency analysis coincide with the conditions for embedding. Then $X(0) = 0$; $X'(\ell) = 0$. In addition, when $I = \text{const.}$ we have $\gamma = 1$; $P = 0$; $U(x, \tau) = K(x, \tau) = p^2/a^2(x, \tau)$. On the basis of (6) and (7), we find that for a slow change in the parameters, the solution coincides with the WKB approximation of the first order [12] and has the form

$$X \approx A\sqrt{a(x,\tau)/a(0,\tau)}\,\sin[p\int_0^x \frac{\mathrm{d}x}{a(x,\tau)} + \alpha]. \tag{8}$$

Hence, after substituting the boundary conditions, we obtain the frequency equation

$$\tan[p\int_o^\ell \mathrm{d}x/a(x,\tau)] + 2p/a'(\ell) = 0. \tag{9}$$

If there is a sufficiently compliant element between the motor and the main shaft (for example, a belt drive, an elastic coupling, etc.), then the following boundary conditions are valid:

$$GIX'(0) = cX(0);\ \ X'(\ell) = 0. \tag{10}$$

Here, c is the stiffness coefficient of the intermediate elastic element.

On the basis of (9) and (10), we obtain

$$\tan\left[p\int_0^\ell \frac{\mathrm{d}x}{a(x,\tau)}\right] - \left[\frac{0,5a' + (a\sigma - 0,5a')}{p - 0,5a'p^{-1}(a\sigma - 0,5a')}\right]_{x=\ell} = 0, \tag{11}$$

where $\sigma = c/GI$.

If the assumption that the function is slow is not right-dimensional, we can use the following modification of the conditional oscillator method. We take as an approximating function

$$a = a_0\big(\zeta x/\ell + 1\big). \tag{12}$$

Here, ζ is some constant determined by concrete conditions [3–6].

It is easy to see that Eq. (7) is satisfied when substituting (12) and

$$z = \ln\frac{b}{\beta\tau + 1}, \tag{13}$$

where $\beta = \zeta/\tau_*$; $\tau_* = pl/a_0$; $\tau = px/a_0$; $b = \sqrt{1 - 0,25\beta^2}$.

Further, using the boundary conditions and eliminating the trivial solution from consideration $A \neq 0$, we write the frequency equation

$$\tan\left[\frac{b}{\beta}\ln(1+\zeta)\right] = -\frac{2b}{\beta}. \tag{14}$$

On the basis of (14) we find, after which we define b/β, β and $\tau_* = \zeta/\beta$. The required eigenfrequencies are found as

$$p_r = a_0\tau_{*r}/\ell \quad (r = 1, 2, 3, \ldots). \tag{15}$$

4 Force Closure

The method considered above can be applied with a change in the stiffness coefficient of the closing spring as a function of the coordinate, as well as in the study of forced oscillations. It should, however, be taken into account that in solving this problem, the Fourier method cannot be directly applied, since the boundary conditions are not homogeneous. This difficulty is eliminated by using a substitution

$$X(x, \tau) = X_1(x, \tau) + w(x, \tau),$$

where $w(x, \tau) = X(0, \tau) + [X(\ell, \tau) - X(0, \tau)]x/\ell$.

Now for the new variable, we have zero boundary conditions: $X_1(0, \tau) = 0$; $X_1(\ell, \tau) = 0$. In the framework of this article, we confine ourselves to an analysis of some qualitative results illustrating the effectiveness of the rational use of force closure in relation to the oscillation system under consideration. For definiteness, let us take the ideal law of motion (without taking into account vibrations and gaps), described by the geometric characteristics given in Fig. 2. Dynamic models (see Fig. 1) do not show operators that display gaps, since by force closure their effect is eliminated. However, in the case of force closure and taking into account the elastic-dissipative characteristics, distortions of the given motion law arise (Fig. 3).

In the general case, the static deformation consists of a constant component due to the preload tention and a variable component proportional to the position function. Curves 2 * and 3 * correspond to two levels of rigidity. Since these curves simultaneously display a slowly varying equilibrium position, order to avoid impactsic excitation the additional inertia force (curves 1, 2, 3), counted from this position, should not cross the gap zone (curves 2, 3). Violation of this condition leads to the excitation of intense oscillations (Fig. 3b, curve 1). When the conditions corresponding to the curves 2 *, 2 are fulfilled (see Fig. 3a, curve 2), the amplitude of the oscillations is substantially reduced. At the same time, the force closure can be lead to additional dynamic errors due to force and parametric vibrations of drive. This

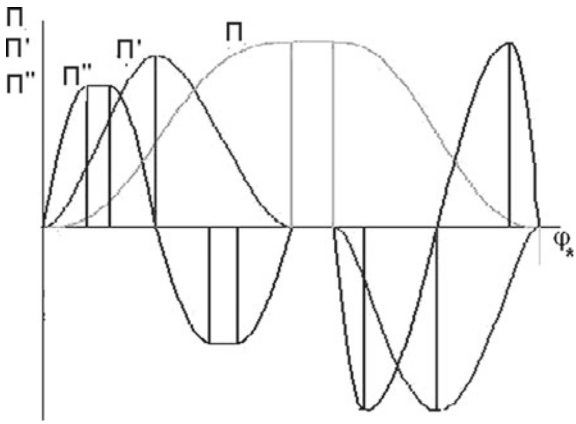

Fig. 2 Geometric characteristics of the law of motion

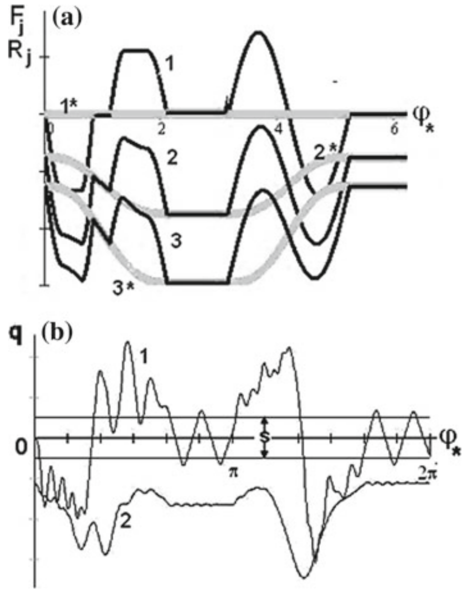

Fig. 3 To the analysis of the force closure in the case of kinematic excitation

problem, the study of which requires a joint consideration of the characteristics of the engine and the mechanical system of the cyclic machine, is partially reflected in works [13–17], that requires further study.

5 Conclusion

The article explores a number of issues related to the further improvement of high-speed cyclic machines, widely used in the textile, light, polygraphic, and a number of other industries. One of the important tasks in this area is to reduce the vibroactivity of machines, with which the accuracy of reproduction of the laws of program movement of working bodies and the quality of output are directly related. A feature of many machines is the large length of the processing zones. This requires increased attention to the spatial distribution of the dynamic characteristics and suppression of oscillations caused by the collisions in the gaps. In the development of known works in this field, new dynamic models and methods for solving this class of problems are proposed.

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