Chapter 10
Intentional Nonlinearity in Energy Harvesting Systems

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Abstract. The success of portable electronics, remote sensing, and surveillance equipment is dependent upon the availability of remote power. While batteries can sometimes fulfill this role over short time intervals, batteries are often undesirable due to their finite life span, need for replacement and environmental impact. Instead, researchers have begun investigating methods of scavenging energy from the environment to eliminate the need for batteries or to simply prolong their life. While solar, chemical and thermal sources of energy transfer are sometimes viable, many have recognized the abundance of environmental disturbances that cause either rigid body motion or structural vibrations. This paper describes recent research efforts focused on the intentional use of nonlinearity to enhance the capabilities of energy harvesting systems. In addition, this paper identifies some of the primary challenges that arise in nonlinear harvesters and some new strategies to resolve these challenges. For example, nonlinearities can often result in multiple attractors with both desirable and undesirable responses that may co-exist. I will describe an approach that uses small perturbations to steer the dynamic response to the desirable attractor, thus leveraging the basins of attraction. Other examples will highlight the potential for nonlinear electromechanical transduction and comparisons for single frequency, multi-frequency, and stochastic environments.

10.1 Introduction

The success of portable electronics and remote sensing devices is dependent upon the availability of remote power. While batteries can sometimes fulfill this role over short time intervals, they are often undesirable due to their finite life span, need for replacement, and environmental impact. Instead, researchers are now investigating methods of scavenging energy from the environment to eliminate the need for batteries or to prolong their life [1]. While solar, chemical, and

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thermal sources of energy transfer are sometimes viable, many have recognized the abundance of environmental disturbances that cause either rigid body motion or structural vibrations. This has led to a dramatic increase in the number of studies for vibration-based energy harvesting [2–8].

Most prior works have focused on the power harvested when the response behavior is adequately characterized as a linear oscillator being driven by harmonic excitation. For this type of design, the optimal performance is realized when the natural frequency of the oscillator is nearly identical to a dominant frequency in the ambient environment. Thus, the prototypical approach is to frequency match or to design and fabricate energy harvesting devices to have a natural frequency that coincides with a dominant frequency in ambient environment [5, 9–11]. This equates to building a vibrational harvesters with very specific mass-spring-damper properties that set the resonant frequency to a dominant frequency of their host environment. As such, they can be highly sensitive to uncertainties which may arise from the imprecise characterization of the host environment or, alternatively, from manufacturing defects and tolerances. This design-for-resonance approach places several performance limitations on the energy harvester. Specifically, a linear device will perform poorly when the system's resonance and excitation frequency do not coincide. Additionally, very little energy will be extracted from multi-frequency and/or random excitation sources. Problems also arise in applications where the excitation frequency drifts or changes over time [3, 4].

The vast majority of past research has focused on inertial generators that operate in a linear regime [9, 12–19]. However, it has recently been suggested that the intentional use of nonlinearity enable future harvesters to overcome the limitations of a linear device. More specifically, there is great interest in the concept of intentionally using nonlinearity to enhance performance. In fact, several recent works have suggested the intentional use of nonlinearity might be beneficial to energy harvesting systems [8, 14, 20, 21]. More specifically, several studies have explored the use of nonlinearities broaden the frequency spectrum, to extend the bandwidth, engage nonlinear resonances, and/or to facilitate tuning [14, 20–28]. These efforts take aim at overcoming the limitations of linear devices, which only perform well under very specific circumstances [8].

The content of this paper is organized as follows. The next section summarizes the limitations of a linear harvester by simply examining the response and uncertainty in the response of a linear oscillator. This is followed by a conceptual discussion prior attempts to use nonlinearity in energy harvesting devices. Section 10.3.2 describes several examples where researchers have explored bistability in both piezoelectric and electromagnetic harvesters. This is followed by a discussion of dynamic magnifiers and a summary of potential future research avenues.

10.2 Linear Energy Harvester Limitations

Oscillators are often designed to operate within a linear regime in vibratory energy harvesters. While restricting the oscillator to operate in a linear regime can greatly simplify the math analysis, it also limits the harvester's performance in several ways. To illustrate these points, we consider the following contrived example of a dimensionless linear oscillator

$$y'' + \mu y' + y = \Gamma \sin \eta\tau \,, \tag{10.1}$$

where y is the dimensionless displacement, a $()'$ denotes a derivative with respect to dimensionless time, μ is a damping coefficient, η is the ratio of the excitation frequency to the natural frequency, and Γ is the excitation level. For the typical case where $\mu > 0$, the steady-state response of Eq. (10.1) is given by

$$y = r \cos(\eta\tau - \phi) \,, \tag{10.2}$$

where the amplitude of the response, r, is given by

$$r = \frac{\Gamma}{\sqrt{\left(1-\eta^2\right)^2 + \left(\mu\eta\right)^2}} \,. \tag{10.3}$$

Here, it is important to note that the power harvested will be proportional to the response amplitude. To both quantify and unveil the robustness of the linear oscillator's response to parameter variations, an expression for total uncertainty in the oscillator's response U_r is introduced

$$U_r^2 = \left(\frac{\partial r}{\partial \mu}\right)^2 U_\mu^2 + \left(\frac{\partial r}{\partial \eta}\right)^2 U_\eta^2 + \left(\frac{\partial r}{\partial \Gamma}\right)^2 U_\Gamma^2 \tag{10.4}$$

where U_{x_i} represents the uncertainty in the variable x_i at the same confidence level. It is common to express the uncertainty at the 95% confidence level (or 20:1 odds) and, consequently, 95% of the physical realizations can be expected to lie within the confidence intervals [29].

Figure 10.1 shows the nominal response amplitude and clearly affirms a large nominal response near resonance. A more in-depth study of Fig. 10.1 also reveals that the response away from this narrow-band peak is rather small. While these result highlight the importance of aligning the natural frequency with the excitation frequency, a more complete understanding of the robustness of the frequency matching strategy is obtained by also considering the uncertainty in the oscillator's response for uncertainties in the system's parameters. As noted previously, uncertainties in these parameters are quite common and arise from the imprecise characterization of the host environment or, alternatively, from imperfections in manufacturing and/or tolerances. The dashed lines of Fig. 10.1 show the confidence intervals or expected deviation in the oscillators response. Note that the dashed lines were obtained by first determining the uncertainty in the response U_r; next, the upper and lower confidence intervals were determined from $r_u = r + U_r$ and $r_l = r - U_r$ where r_u is the upper confidence interval and r_l is the lower. In essence, the confidence intervals provide a measure of the robustness in the response of the system when parameter uncertainty is considered.

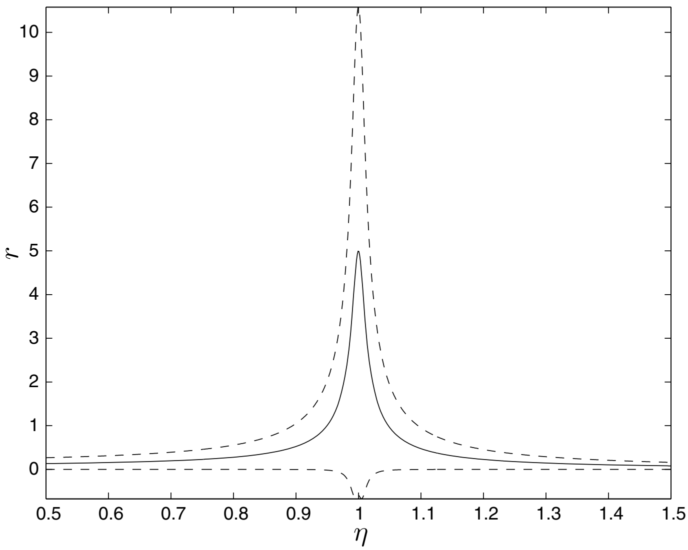

Fig. 10.1. Nominal response (solid line) and confidence intervals (dashed lines) of a linear oscillator for $\mu = 0.02$ and $\Gamma = 0.1$ and parameter uncertainties $U_\mu = \mu/5$, $U_\eta = 0.02$, and $U_\Gamma = \Gamma/10$. Confidence intervals show a lack of robustness in the nominal response in the vicinity of resonance

The confidence intervals highlight the lack of robustness in a frequency-matching strategy, since even small parameter variations, or uncertainty, can cause large differences in the expected response. More specifically, the upper and lower confidence intervals, dashed lines in Fig. 10.1, show the uncertainty in the oscillator response can sometimes be as large as the nominal value (solid line). Armed with this understanding, we now focus our attention on the intentional use of nonlinearity to address the limitations imposed by the linear oscillator.

10.3 Nonlinear Examples

Despite the fact that nonlinearities are inherent in many natural and engineered systems, it is common for engineers to remove, or attempt to remove, all nonlinearity from their designs. Although this simplifies the performance analyses, it also overlooks a wide array of phenomena, that could potentially enable the harvesting of more energy. Improving the performance of inertial harvesters requires that they become more robust to uncertainties and/or subtle changes in their environment. More specifically, the ideal harvester would perform well in a variety of settings and could scavenge energy from a broad range of frequencies. This means the harvester must be able to adjust, adapt, or tune into its current environment. Furthermore, it is essential that future harvesters have a broader frequency response - thus enabling energy to be scavenged over a wider range of frequencies.

This section will discuss select past works that sought to use nonlinear behavior to improve the performance of energy harvesting systems. The section starts with some examples of using some common structural nonlinearities and describes their potential benefits and pitfalls for different environments. This followed by a discussion of some past works that used nonlinearity in the electromechanical coupling of a harvester device. It is important to note that many of the provided examples will show a benefit to the intentional use of nonlinearity; however, as one might expect, nonlinearity must be intelligently designed into a device to reap these benefits. Furthermore, the mere introduction of nonlinearity into these systems also introduces new problems to consider, such as the presence multiple attractors, i.e. both a high and low energy response. Additional works, which have considered different types of random excitation, such as broadband white noise and colored noise, are also discussed in Sect. 10.3.2.

10.3.1 Hardening and Softening Systems

Several researchers have studied energy harvesting systems with either hardening or softening-spring-like behavior. For example, Ref. [8] considered a electromagnetic inductions system with nonlinear restoring forces that were created from a magnet levitation system. The restoring force in that system was a hardening type spring and it showed the ability to tune by peak in its frequency response by changing the relative magnet positions. However, a hardening system can only alter its peak response to one side of linear resonance. Systems displaying similar hardening type behavior have been investigated in many other references. Upon comparing the peak response of the linear oscillator to that of the hardening system, it may seem problematic that the linear oscillator has a larger response for single frequency excitation. However, an uncertainty analysis on the frequency response of the hardening system has shown its response is more robust [30].

To help cover a broader range of frequencies, some investigators have sought to combine hardening and softening type effects into a single device. For example, Fig. 10.2 shows a harvester that demonstrated the potential of adding nonlinearity from magnet-magnet interactions to create either a hardening and softening effect [21]. More specifically, positioning the adjustable magnets behind the tip mass creates a hardening frequency response - thus extending the region of a relatively large response to higher frequencies. If the adjustable magnets are pushed forward of the tip mass, a softening type behavior is created, thus the region of relatively large responses switches directions and extends to frequencies lower than the linear natural frequency.

10.3.2 Bistable Systems

The concept of a bistable system can be brought into focus by considering the motion of a small ball rolling on the surface under the influence of gravity, see Fig. 10.3, where the ball height is proportional to the potential energy. Consider first the potential energy of a linear oscillator, shown in Fig. 10.3a. This system has a linear relationship between the restoring force and deflection which

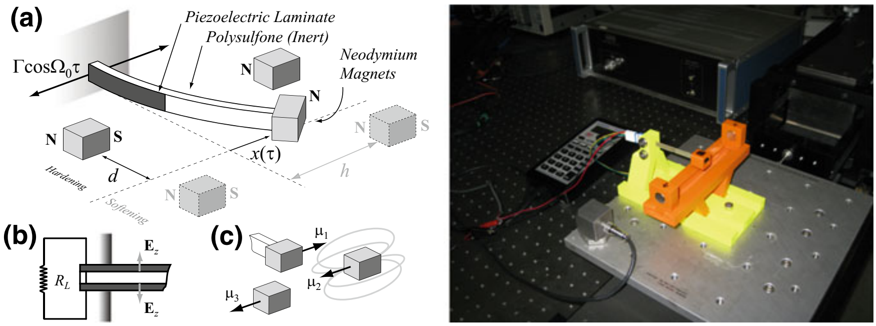

Fig. 10.2. Illustration of an experimental system from Ref. [21] that demonstrated that the nonlinear restoring forces enable tuning and a broader range of frequencies with a large amplitude response

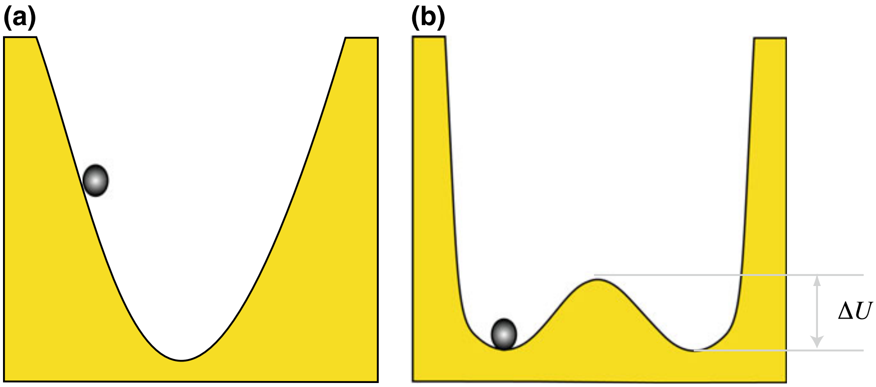

Fig. 10.3. Potential energy curves for: **a** the quadratic potential well of a linear oscillator and **b** a nonlinear oscillator with two stable equilibria separated by an unstable equilibrium position. The energy difference between the potential energy barrier and the stable equilibria, labeled ΔU, is an important factor for determining the threshold for an escape

results in a quadratic potential energy well with a single equilibrium. Regardless of where the ball placed, it will eventually come to rest at the bottom of the potential energy well. Shaking the parabola laterally yields the linear harmonic oscillator with the largest response occurring when it is shaken at its resonance frequency.

Consider next the same ball under the influence of a nonlinear restoring force where the potential energy description may be more complex - see Fig. 10.3b. Consider again the same ball under the influence of small lateral excitations. This results in a system that behaves linearly for small-amplitude motions with oscillations that remain confined to a single well. For increasingly large excitations, motion amplitudes grow until the threshold for a potential well escape occurs (i.e. where an escape is imminent for energy levels above the threshold criteria ΔU in Fig. 10.3b). Once exceeding the threshold criteria, the small ball would then escape from the potential well and traverse both potential wells, sometimes called well-mixing behavior, with large-amplitude displacements and velocities. Acknowledging the dramatic increase in the energetic response of the oscillator in the post-escape regime [31], several researchers have become interested in this type of system [32].

Figure 10.4 shows example responses from a prototypal bistable harvester. In contrast to the hardening and softening cases, the bistable system shows the emergence of additional solution branches. More specifically, these solutions are associated with the oscillations within a single potential well and those that cross the center potential well barrier and are the result of a potential well escape phenomenon. This system can exhibit similar P_a (dimensionless power) values to those of the linear system, but, as in the case of the softening and hardening system, displays more complex scaling in its response behavior as Γ, the dimensionless excitation, is increased. In addition, the plots of ρ vs. P_a, where ρ is the dimensionless electrical load, show the system can have even more local maxima. Further examples of bistable energy harvesters can be found in references [14,20,24,27,33,34].

As a summary, a bistable harvester introduces some new considerations. For example, while the strategy of matching the natural frequency of the device to a frequency in the environment still exists, an alternative strategy also exists. In particular, one can instead focus on designing the potential energy curves to ensure a potential well escape. Similar to the hardening and softening cases, the responses of the bistable system can be more robust than the linear system (see reference [30] for further details).

The bistable system has also been studied for other forms of excitation, such as random excitation [35–37]. One result worth mentioning is the finding of reference [35]. In this study, it was shown that a bistable harvester could outperform a linear harvester in an environment with colored noise.

10.3.3 Coupling Nonlinearity

The work of Ref. [22] was the first to consider the influence of nonlinear electromechanical coupling in PZT systems. Since then, the inherent nonlinearities in piezoelectric harvesters have been studied in greater detail [38]. Outside of piezoelectric systems, inherent nonlinearities have also been studied in electromagnetic induction systems [28]. One interesting finding worth mentioning is that nonlinear coupling appears to be particularly suited to multi-frequency excitation [28]. However, further research needs to be done to further explore the potential benefits and pitfalls of nonlinear coupling.

10.3.4 Dynamic Magnifier

The use of a dynamic magnifier has been another area of inquiry for linear and nonlinear systems. A dynamic magnifier is a dummy oscillator, essentially an

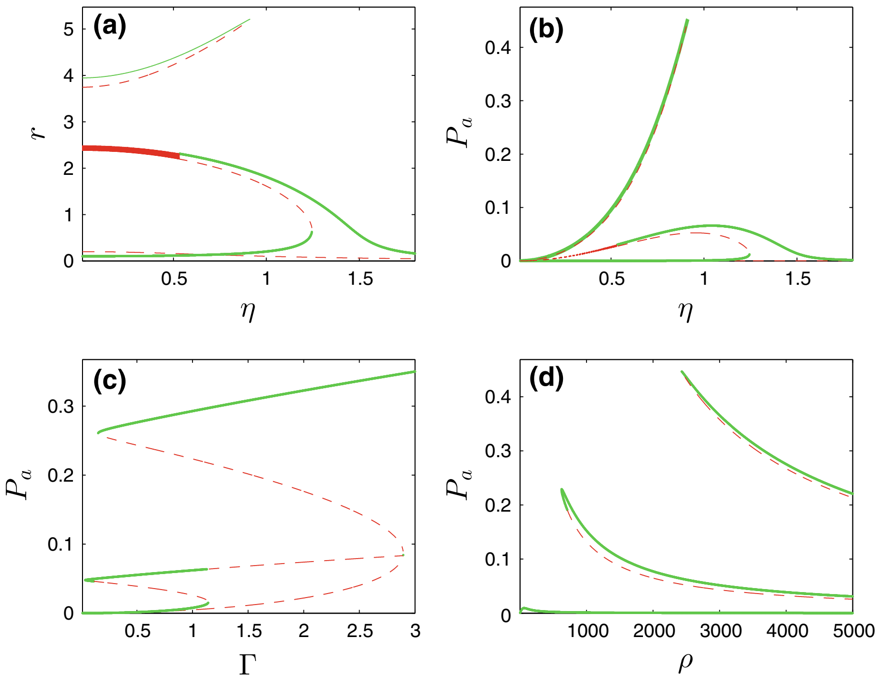

Fig. 10.4. Plots showing the stable (green dots) and unstable (red dots) response trends for a harvester with a bistable potential well. Graphs show frequency responses for **a** the oscillation amplitude of the mechanical system and **b** the dimensionless average power; graphs (**c**) and **d** plot the dimensionless average power for changes in Γ and ρ, respectively

oscillator without any electromechanical coupling, that is used to magnify the response of the primary oscillator, i.e. the one with electromechanical coupling. As a brief summary, several researchers have now shown that a dynamic magnifier can successfully increase the energy harvested from the primary oscillator and even be used to modify the corresponding basins of attraction [39].

10.4 Further Considerations

Many recent works have explored the use of nonlinearity in vibratory energy harvesters, e.g. see [8, 14, 18, 22, 30, 34, 35, 38, 40–42]. While these investigations, along with many other recent works, have advanced the current understanding on the beneficial use of nonlinearity, the introduction of nonlinearity can also cause many additional difficulties. Paramount amongst these challenges, and a common issue in nearly all nonlinear harvesting systems, is the presence of coexisting solutions. To illustrate the problem, Fig. 10.5a shows the frequency response for a Duffing Oscillator with coexisting solutions over the dimensionless frequency range of $\approx 1.25 < \eta < 2$. Assuming the environmental excitation

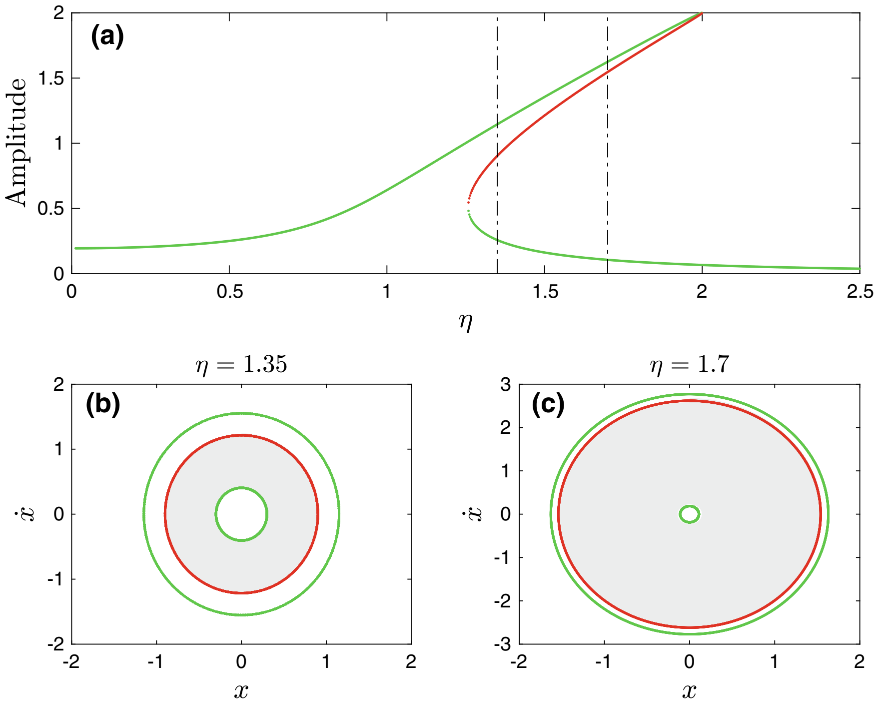

Fig. 10.5. Illustrative example of a system with coexisting periodic solutions, i.e. two or more stable periodic solutions for the same system parameters. Plots illustrate the challenge of attractor selection in energy harvesting systems. Plot **a** shows a bifurcation diagram illustrating a hardening spring nonlinearity in response to dimensionless frequency η, and plots (**b**) and **c** show the corresponding periodic attractors and repellers in phase space for two different values of η. Curves are labeled stable (green) and unstable (red)

remains constant, only the initial conditions determine whether a higher or lower energy solution is obtained. Furthermore, if the basins of attraction are studied for this range of η, one finds that the more desirable response (higher amplitude) is unlikely to be obtained when the excitation frequency is closer to the peak response. Thus a fundamental challenge prevalent in nearly all nonlinear energy harvesting approaches is a strategy to select a desired attractor.

Vibratory energy harvesters convert mechanical energy into electrical energy with electromechanical coupling, e.g. piezoelectric, electromagnetic, or capacitive. While these transduction schemes allow some form of control to be applied to alter the response of the mechanical system, a number of challenges prevent the use of continuous control. To elaborate, the power required to apply continuous control is typically larger than the power harvested. It is also common that the electromechanical coupling is not strong enough to drastically alter the response of the mechanical system in a single application of control, unless external energy is provided. Thus methods to choose the desired attractor present an on-going area of research.

10.5 Conclusions

This paper discusses select past works on the intentional use of nonlinear behavior in inertial energy harvesters. Many forms of nonlinearity have been investigated and many have shown some potential benefit. However, the fact remains that analyzing these nonlinear systems can be much more difficult than their linear counterpart. The introduction of nonlinearity adds an interesting feature that can allow effecting device tuning in a semi-active or passive way to overcome uncertainties in the environmental excitation or physical parameters of the system.

Nonlinear energy harvesting systems often have co-existing solutions. When one of the responses is desirable and the other undesirable, it becomes critically important to have methods to select the desired attractor with minimal energy expenditure. A great solution to this problem should be the target of future investigations.

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