

5. Relaxometry and Laplace NMR

Distributions of relaxation times Distributions of diffusion coefficients Diffusive diffraction Compact magnets Well-logging NMR 2D Laplace NMR Depth profiling Applications to materials characterization



Acquiring Distributions of Relaxation Times

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NMR Relaxometry

- Relaxation times and diffusion coefficients can be measured in inhomogeneous magnetic fields by analyzing signal decay and build-up in the time domain
- Inhomogeneous fields are inexpensive to generate with permanent magnets
- The fields of permanent magnets are low (up to 2 T) compared to those of most superconducting NMR magnets (up to 24 T)
- NMR relaxometry with compact low-field instruments was commercialized by Bruker in 1973 for food analysis and not much later by Oxford Instruments
- It is used to test the physical properties of materials, because relaxation times and self-diffusion coefficients report on the rotational and translational molecular motion and not the chemical structure
- Longitudinal relaxation is measured with the *inversion recovery sequence* and the *saturation recovery sequence*
- Transverse relaxation in liquids is measured with the *Hahn echo* and the *CPMG sequence*. In solids it is measured with the *solid echo* and Ostroff-Waugh (*OW4*) sequences, which use 90° instead of 180° refocusing pulses
- In simple liquids, T_1 and T_2 relaxation is mono-exponential
- In multi-component mixtures such as emulsions and fluids in porous media, the relaxation signal can be approximated by a sum of exponential functions
- This can be inverted to a relaxation-time distribution by operations similar to *inverse Laplace transformation*
- This is why time-domain NMR is often also called Laplace NMR

Fluids in Porous Media



- The signal amplitude is proportional to porosity at full fluid saturation
- The longitudinal relaxation rate $1/T_1$ and the transverse relaxation rate $1/T_2$ are enhanced by wall relaxation
- The transverse relaxation rate is affected by self diffusion in applied and in internal gradients
- The molecular self-diffusion length is confined by the pore diameter

Relaxation of fluids in in porous media:

$\frac{1}{T_2} = \frac{1}{T_{2,\text{bulk}}} + \rho_2 \frac{S}{V} + \frac{D(\gamma G t_E)^2}{12}$ $\frac{1}{T_1} = \frac{1}{T_{1,\text{bulk}}} + \rho_1 \frac{S}{V}$	 S: surface area V: pore volume ρ: surface relaxivity t_E: echo time D: apparent diffusion coefficient
Restricted diffusion in fluid-filled pores: $\frac{D}{D_0} = 1 - \frac{4}{9\sqrt{\pi}} \frac{S}{V} \sqrt{D_0 \Delta} + O(\Delta)$	<i>D</i> : apparent diffusion coefficient D_0 : bulk diffusion coefficient Δ : diffusion time

Relaxation and Diffusion

- In fluid-filled porous media, relaxation and diffusion reveal essential material properties of fluid and matrix, which can readily be measured by ¹H NMR
- The signal amplitude is proportional to the amount of fluid in the sensitive volume, i.e. to the *fluid saturation* for partially saturated porous media and the porosity for fully saturated porous media
- The pore walls confine the diffusion paths and enhance the relaxation rates of the fluid molecules in the pores
- In the fast diffusion limit the relaxation rate is proportional to the surface-tovolume ratio, i.e. the relaxation time reports pore size
- Contrary to the longitudinal relaxation rate, the transverse relaxation rate is affected by translational diffusion in inhomogeneous fields with a gradient G
- The *field gradient* can arise from the inhomogeneity of the applied field and from susceptibility differences due to the heterogeneity of the porous medium
- Information about the pore size is also obtained from the restrictions in molecular self-diffusion imposed by the pore walls
- NMR relaxation and self-diffusion signals are inverted to distributions of relaxation rates and distributions of apparent diffusion coefficients identifying in a porous medium, for example, bound fluid and producible fluid
- Such distributions from fluid-saturated porous media are interpreted in terms of pore-size distributions, which are evaluated for porosity, permeability, and fluid typing in well-logging NMR



Measuring Diffusion

- Like velocity, *diffusion* is measured in terms of *displacement* $R = r_2 r_1$ within a given time window, where *r* is the position of the molecule
- Position is encoded by the phase φ = γ G r δ = k r of the transverse magnetization evolving for a time δ in an *inhomogeneous field*, which is preferably linear with a constant gradient G. Here k is the position wave number
- Displacement is measured by the phase $\Delta \varphi = k_1 r_1 + k_2 r_2 = k (r_2 r_1) = q R$ accumulated with two gradient pulses at different times having the same wavenumber magnitude *k* but opposite signs, where *q* is the wave number for displacement
- In such an *anti-phase gradient-pulse pair* the position wave number $k = k_1 + k_2$ vanishes after the second gradient pulse so that position is not encoded in the signal phase and the gradient-pulse pair is said to generate a *gradient echo*
- If all spins move coherently in a pulsed linear gradient-field, all acquire the same phase difference, which appears as a *phase shift of the echo*
- If the spins diffuse randomly, positive and negative phase shifts cancel on average, and the echo amplitude is attenuated according to $M_{xy}(t) = M_{xy}(0) \exp\{-t_E/T_2\} \times \exp\{-q^2D(\Delta \delta/3)\}$, where *D* is the *self-diffusion coefficient* and Δ the *diffusion time*
- The transverse magnetization $M_{xy}(t) = M_0 \int P(R) \exp\{-i q R\} dR$ of an ensemble of spins diffusing different distances *R* during the diffusion time without relaxation is determined by the Fourier transform of the *probability density* P(R) *of displacements*, which is known as the *propagator* in NMR



Time-Invariant Gradient Fields

- If the field gradient results from the sensor or *magnetic susceptibility differences* in the object it is time-invariant and an echo can be generated by rf pulses
- In the maximum of the Hahn echo and the stimulated echo, the dephasing of magnetization components from magnetic fields that are inhomogeneous across the sample is refocused unless the spins are moving
- The refocusing rf pulse of the Hahn echo changes the sign of all *precession* phases φ = k r accumulated before
- This is equivalent to a sign change of all position-encoding gradients active before, and one can write the time variation in terms of the *effective gradient G*_{eff}, which gives the same result but without refocusing rf pulses
- For example, identical gradient pulses before and after the refocusing pulse in a Hahn echo to encode *diffusion* and *coherent flow* are equivalent to an echo with the effective gradient
- In a CPMG train applied in a linear field B(r)−B₀=G r all odd echoes refocus dephasing from different positions, and all even echoes in addition also refocus dephasing from coherent displacement
- The decay of the CPMG echo envelope due to relaxation and diffusion is given by M_{xy}(t) = M_{xy}(0) exp{-n t_E / T₂} exp{-q² D n t_E / 3}
- In stray-field NMR, the rf field is inhomogeneous and all rf pulses are selective, so that the flip angles vary across the sample
- Then the CPMG echoes are mixed Hahn and stimulated echoes, and T_2 becomes an *effective relaxation time* $T_{2\text{eff}}$, which is a mixture of T_1 and T_2

Effects of Displacement



Displacing Spins

- The paths of spins being displaced during a time shorter than T₁ can be probed by different NMR methods
- In a stimulated echo sequence, the echo amplitude is attenuated from displacement by diffusion *D* in a constant gradient and by relaxation following $M_{xy}(t) = \frac{1}{2} M_{xy}(0) \times \exp\{-q^2 D (\Delta + t_F/3)\} \times \exp\{-\Delta/T_1\} \times \exp\{-t_F/T_2\}$
- Other than diffusion, *laminar flow* through a pipe is coherent leading to a phase shift in addition to the attenuation of the echo from diffusion and relaxation
- As $q = \gamma G t_E/2$, the logarithm the CPMG *echo attenuation* depends linearly on t_E^3 for free diffusion, and *D* can be determined from the slope of $\ln\{M_x\}$
- To improve the sensitivity of diffusion measurements, the diffusionattenuated echo may be refocused several times with echo time t_{E2} and the echo sum be detected for a systematic variation of the first echo time t_{E1}
- For sufficiently short echo times *t*_{E2} in the CPMG *detection period*, signal loss in the *CPMG echo train* due to diffusion is suppressed
- Then, without the summation, the scheme gives a 2D data set of Hahn-echo attenuation from diffusion versus *T*₂ relaxation decays
- The *T*₂ relaxation can be neglected in the *diffusion filter* for strong gradients, and a *D*-*T*₂ correlation map is obtained by 2*D* inverse Laplace transformation
- In porous media, diffusion is restricted, so that the signal attenuation by diffusion is lowered, and that from relaxation is enhanced due to *wall relaxation*



Distributions of Velocities and Displacements

- According to the phase table in chapter 4, the magnetization component moving with constant velocity *v* through a linear gradient-field is given by M_{xy}(t) = M_{xy}(0) exp{-t/T₂} exp{-iφ₁₁}
- Here $\varphi_{11}(t) = \gamma_0 \int^t G(t') t' dt' v = \gamma_0 \int^t G(t') t' dt' R/\Delta$, where *R* is the distance travelled during time Δ and G(t) is the *effective gradient*
- For coherent flow φ₁₁(t) = q_ν(t) ν. In the case of a diffusing magnetization component, the notation of Callaghan applies, and φ₁₁(t) = q(t) R with k = q
- For many magnetization components the transverse magnetization is the integral over all velocities v or displacements R,
 M (t) = (1)(0 f M (u) exp(i g u) du = (1)(0 f M (B) exp(i g B) dB
 - $M_{xy}(t) = (1/V) \int M_{xy}(v) \exp\{-i q_v v\} dv = (1/V) \int M_{xy}(R) \exp\{-i q R\} dR$
- $M_{xy}(v)/V$ is the velocity distribution and $M_{xy}(R)/V$ the displacement distribution
- For constant magnetization density, $M_{xy}(R)/(M_0 V) = P(R)$ is the probability density of displacements, also known as the *propagator*
- For diffusion, R depends on the diffusion time Δ and so does the propagator
- For free diffusion, the propagator is a *Gauss function*, which broadens with increasing diffusion time, $P(R,\Delta) = (4\pi D \Delta)^{-3/2} \exp\{-R^2/(4 D \Delta)\}$, the variance $\langle R^2 \rangle = 2 D \Delta$ of which defines the *mean diffusion length* $\langle R^2 \rangle^{1/2} = (2 D \Delta)^{1/2}$
- For *restricted diffusion* in identical pores, the propagator is the *auto-correlation function of the pore shape* in the fast diffusion limit, and *diffraction* peaks are observed in the NMR signal



Diffusive Diffraction

• The propagator P(R) can be written as integral over all initial positions r_1 of the product of the probability density $S(r_1)$ of finding a spin at r_1 and the conditional probability density $S(r_1, r_2, \Delta)$ of finding it at $r_2 = r_1 + R$ some time Δ later,

$$P(R) = \int S^{*}(r_{1}) S(r_{1}, r_{1}+R, \Delta) dr_{1}$$

• At long diffusion time Δ , $S(r_1, r_1+R, \Delta) = S(r_2)$. Then, with $q R = k (r_2 - r_1)$,

$$M_{xy}(q) = M_0 \int P(R) \exp\{-i q R\} dR$$
, and rewriting $P(R)$ one obtains

$$M_{xy}(q) = M_0 \int \int S^*(r_1) S(r_2) dr_1 \exp\{-i k (r_2 - r_1)\} d(r_2 - r_1)$$

= $M_0 \int \int S^*(r_1) S(r_2) \exp\{-i k r_2\} \exp\{i k r_1\} d(r_2 - r_1) dr_1$
= $M_0 \int S^*(r_1) \exp\{i k r_1\} dr_1 \int S(r_2) \exp\{-i k r_2\} dr_2$
= $M_0 s^*(k) s(k) = M_0 |s(k)|^2 = M_0 |s(q)|^2$

- This states, that the transverse magnetization measured in a diffusion experiment reports the magnitude square of the Fourier transform of the probability density *S*(*r*) of finding spins at position *r*
- For fluid-filled pores, *S*(*r*) is the normalized projection of the pore shape along the gradient direction. It is commonly called the *pore-shape function*
- In the limit of fast diffusion or long ∆, the transverse magnetization is proportional to the magnitude square of the Fourier transform of the pore-shape function



From B. Blümich, C. Rehorn, W. Zia, in: J. Anders, J. Korvink, eds., Micro and Nano Scale NMR, Wiley-VCH, Weinheim, 2018, Fig. 1.7 with permission

Permanent Magnets for NMR Relaxometry

- The first NMR experiments were done with *electromagnets* and later on also with permanent magnets at ¹H frequencies from 20 MHz up to 90 MHz
- Permanent magnets were hard to shim and drift with temperature so that compact permanent magnets were mostly used for relaxation and diffusion measurements in the past
- There are two basic types of magnets for NMR, stray-field magnets and center-field magnets
- *Center-field magnets* accommodate the sample inside, so that the sample size is limited by the diameter of the magnet opening
- The classical geometry is C-shaped, while mirrored C-shapes and cylinder shapes provide better field homogeneity due to higher symmetry
- Halbach magnets are particularly efficient and ideally are without stray field
- Stray-field magnets produce fields that enter the object from one side
- They were first used for *well logging* to be inserted into the hole of an oil well and to acquire signal from the fluids in the borehole wall
- Compact stray-field instruments were subsequently developed mostly for moisture measurements in buildings and porous media
- The *NMR-MOUSE* (Mobile Universal Surface Explorer) is a small stray-field sensor, which collects the signal from a flat sensitive slice outside the magnet
- It can be used to measure depth profiles into the surface of arbitrarily large objects by changing the distance between sensor and object



NMR Well Logging

- Early on NMR was of interest for *well logging*. Russell Varian had the idea in the early 1950's, and oil companies like Chevron, Mobil, Texaco, Borg-Warner and Schlumberger started to study fluids in porous rock
- The first generation nuclear magnetic logging tools operated in the earth's magnetic field. Later on J. Jackson introduced permanent magnets
- Since the magnet is inside the sample and not the sample inside the magnet, this type of stray-field NMR is also called *inside-out NMR*
- Due to the inhomogeneous stray field and field-gradients inside the rock pores induced by susceptibility differences, the NMR signal is acquired with CPMG-like multi-echo sequences
- Since 1995 NMR logging tools are operated by service companies. Halliburton calls their tool MRIL (Magnetic Resonance Imaging Log) and Schlumberger calls it CMR (Combinable Magnetic Resonance tool)
- The tools of the first and the second generations were operated in *wireline mode* after drilling and measured the NMR information while being pulled up through the hole
- The third generation tools are designed for *logging while drilling*. They are part of the drill string and suitable for *directional drilling*
- Some logging-while-drilling tools explore the original magnet geometry proposed by Jasper Jackson



Sensor Designs

NMR Logging Tools

- The *earth-field tool* of the first generation used a strong DC current through the coil to magnetize the protons and then used the same coil for detection
- The *deadtime* was 30 ms, and the fluid molecules exposed to fast *wall relaxation* could not be measured
- Only *free fluid* could be measured, but not *porosity*. Oil, water, and bore-hole fluid could not be distinguished
- By fitting the sensors with magnets, several problems could be solved:
 - The NMR frequency increased from 2 kHz to about 1 MHz so that the deadtime was shortened and the sensitivity increased
 - The signal dependence on the sensor orientation in the earth's field is avoided
 - The sensitive volume can be localized inside the borehole wall away from the borehole fluid
- The *Jackson tool* has a sensitive volume in the shape of an annulus, which is narrow in the axial direction. Measurements with such a sensor moving along the bore-hole are difficult as the spins find little time to be polarized
- The *Halliburton/Numar tool* has transverse dipolar B₀ and B₁ fields, which are largely perpendicular to each other at all locations. The field magnitude at a given radius is constant along the circumference. The radial gradient helps to select signal from different shells by tuning to different frequencies
- The Schlumberger tool provides an axially extended sweet spot in one angular segment at some distance away from the sensor





Formation Evaluation by NMR

- Modern NMR logging tools acquire data from more than 5 cm deep into the formation of the bore-hole wall
- Typically the transverse magnetization decay from fluid-filled pores is measured as the envelope of a Carr-Purcell-Meiboom-Gill (*CPMG*) echo train
- Its amplitude provides mineralogy-independent porosity $\Phi = V_{\text{pores}}/V_{\text{rock}}$
- The *distribution of relaxation times T*₂ is obtained from the echo-train decay envelope by *inverse Laplace transformation*
- The area under the distribution curve is the amplitude of the CPMG signal
- A model-bound analysis of the relaxation time distribution yields
- irreducible water saturation from clay-bound and capillary-bound water
 amount of producible fluid
- permeability estimates
- hydrocarbon type
- oil-viscosity estimates
- Porosity is estimated traditionally from density values determined by *gamma-ray scattering* or *neutron scattering*, applying formation-dependent corrections
- NMR porosity is formation independent unless the formation is magnetic
- Advanced multi-dimensional Laplace methods provide more detailed information, e.g. they can separate signals from oil and water and characterize pore connectivity

NMR Porosity and Relaxation in Pores



Porosity and Surface Relaxation

- The porosity

 is directly determined from the signal amplitude of the fluid in
 the rock normalized to the signal amplitude of bulk water and corrected for the
 hydrogen index HI, which corresponds to the relative spin density of the fluid
- The fluid molecules diffuse inside the pores of the rock matrix
- To reduce the signal loss within the *deadtime* of the instrument, low measuring fields are advantageous because lower fields induce smaller *field inhomogeneity* at interfaces resulting from *magnetic susceptibility differences* so that signal attenuation from *diffusion in internal gradients* of the pores is low
- NMR porosity scales well with porosity determined by buoyancy
- The *longitudinal magnetization* $M_z(\mathbf{r},t)$ follows the *diffusion equation* with the *bulk relaxation time* $T_{1,\text{bulk}}$ within a pore, and in the direction along the normal \mathbf{n} of the wall it relaxes with the *surface relaxivity* ρ
- A normal mode analysis leads to the slow and the fast diffusion limits
- In the fast diffusion limit, the relaxation rate scales with the surface-tovolume ratio S/V, where S/V = 3/r_{pore} for spherical pores, and the transverse magnetization decay is mono-exponential
- The fast diffusion limit is assumed to be valid in most applications of NMR to *well logging*
- In the *slow diffusion regime*, the magnetization decay is multi-exponential for a single type of pore and depends on the *pore geometry*

Relaxation-Time Distributions and Pore Size



From J. Arnold, C. Clauser, R. Pechnig, S. Anferova, V. Anferov, B. Blümich, **Petrophysics 47** (2006) 306, Figs. 5. 6 with permission

Surface Relaxation

- The surface relaxivity ρ links pore size and relaxation time
- It is determined by matching the *relaxation-time distribution* from NMR with the *pore-throat distribution* determined e.g. by differential capillary pressure measurement such as *mercury intrusion porosimetry*
- The pore-throat distribution scales with the derivative of the *capillary* pressure (CP) curve versus fluid saturation
- The assumptions are that the pore radii are proportional to the throat radii and that all pores have the same surface relaxivity
- By comparing T_1 and T_2 relaxation-time distributions, one finds, that most rocks exhibit T_1/T_2 ratios between 1 and 2.5 with a most probable value of 1.65 at 2 MHz. That value becomes larger at higher measuring fields
- Deviations may arise at short relaxation times from contributions to T_2 by diffusion in internal gradients if the echo time is not chosen short enough
- The shapes of the NMR relaxation-time distributions depend only weakly on pressure and temperature
- For immobile molecules $T_{2\text{eff}}$ derived from a multi-echo decay in a strongly inhomogeneous magnetic field is prolonged by scrambling of longitudinal and transverse magnetization components from resonance-offset related flip-angle distributions
- For mobile molecules T_{2eff} is shortened by diffusion in field gradients



Producing Water from Porous Media

 T_2 distributions of Allermoche sandstones: The bound water decreases with increasing permeability *k* and macro porosity Φ of the rock

From S. Anferova, V. Anferov, J. Arnold, E. Talnishnikh, M.A. Voda, K. Kupferschläger, P. Blümler, C. Clauser, B. Blümich, Magn. Reson. Imag. 25 (2007) 474, Fig. 6 with permission





Fluid Production from Porous Media

- When drying a water-wet rock, the large pores lose water first and the small pores later, i.e. the irreducible water remains in the small pores
- A relaxation-time cut-off serves to estimate bound water from producible fluid
- The T_2 cut-off for water is 33 ms in sandstones and 92 ms in carbonates
- For crude oil with a broad distribution, the relaxation times of bound water and producible oil may overlap, but the fractions may be distinguished by diffusion measurements
- The flow velocity *v* of a fluid with dynamic viscosity η through a porous medium in a pressure gradient $\Delta p/\Delta x$ is determined by the *permeability k*, $v = k/\eta \Delta p/\Delta x$. It is measured in Darcy, where 1 Darcy = 1 D = 0.987×10⁻¹² m²
- The permeability $k = C d^2$ is poportional to the square of the effective pore diameter *d*, where *C* is a constant relating to the flow paths
- It is estimated from the relaxation-time distribution following models of the form $k = a \Phi^4 T_{2,k}^2$, where, depending on the model, $T_{2,k}$ is an average relaxation time derived from all or parts of the relaxation-time distribution
- The Kenyon model fits best for 100% brine saturated rock, the Coates-Timur model applies to hydrocarbons, and the Chang model to vuggy carbonates
- The Chang model has been improved to accommodate a large range of permeabilities by introducing a tortuosity-dependent weight
- The tortuosity $\tau = D_0/D_{\infty}$ is the ratio of the bulk self-diffusion coefficient to the apparent self-diffusion coefficient at long diffusion time $\Delta \rightarrow \infty$

Example of a Gas-Well Log



Courtesy of L. Xiao, China University of Petroleum, Changping, China

Elementary Well Logging

- Elementary NMR *well logging* is based on the analysis of *CPMG decays* and associated *distributions of relaxation times* from fluid saturated pores
- The amplitude of the total signal estimates the porosity
- The amplitude of the slowly relaxing part estimates the amount of *producible fluid* (free fluid index FFI) corresponding to the integral of the relaxation time distribution to the right of the relaxation cut-off
- The amplitude of the rapidly relaxing components estimates the amount of capillary and clay-bound water also called the bulk volume irreducible BVI corresponding to the integral of the relaxation time distribution to the left of the relaxation cut-off
- Permeability is estimated from the porosity following different models
- Information about the hydrocarbon type is obtained from T₁ weights introduced by different polarization times t₀ and from diffusion weights introduced by different echo times t_E when the signal is acquired in the presence of a field gradient
- All NMR data are interpreted in the context of other logging data from measurements of *electrical resistivity*, *gamma-ray scattering*, *neutron scattering*, etc.
- Detailed measurements by 2D Laplace NMR and other more advanced methods are performed in *logging-while-drilling* applications and on core samples in the laboratory
- Well logging data are validated by data from laboratory studies

Hydrogen Index and Relaxation of Dead Oils



NMR Properties of Fluids in Porous Media

- The NMR signals from fluids in porous media relate to the bulk fluids and the pore space. The signal decay is affected by *bulk relaxation*, *wall relaxation*, and by *diffusion* across regions with different magnetic field strengths
- The *signal amplitude* scales with the total number of protons within the sensitive volume. At full fluid saturation it reports formation *porosity*
- Natural fluids saturating rock are water of variable salinity, gas, and oil
- The water salinity ranges from zero to full saturation with mostly NaCl
- The most important gas in well logging is methane
- When the oil is free of gas, it is called *dead oil*, otherwise it is called *live oil*
- Live oils exist at elevated pressures and temperatures below the bubble point. Their relaxation times depend on the the gas/oil ratio
- Synthetic oils are used in drilling mud and can mix with reservoir fluids
- The amount of a fluid is quantified from the NMR signal amplitude via the proton density relative to that of water. This quantity is the *hydrogen index HI*
- Since the beginning of the relaxation curve is hidden within the *dead time* of the instrument, *HI* determined by NMR is an *apparent hydrogen index*
- Most rocks are water wet, so that the detected water signal exhibits relaxation times shortened by *wall relaxation*
- Because the relaxation time of bulk water is long (1 to 3 s), the bulk-water signal can be suppressed with short repetition times





Bulk Relaxation and Diffusion

- In water-wet rock the oil is not in direct contact with the pore wall, so that T₁ and T₂ of oil in the pores are the same as in bulk
- For pure compounds both relaxations are mono-exponential, but *crude oils* consist of many components and exhibit distributions of relaxation times
- From the T₂ distribution x(T₂), the *logarithmic mean relaxation time* T_{2,LM} is calculated as log{T_{2,LM}/s} = [Σ_i x_i log{T_{2i}/s}]/[Σ_i x_i]. It scales with the *viscosity*
- For dead oils T_{2.LM} scales linearly with the ratio of viscosity/temperature
- Unlike T_1 of liquids, T_1 of gases is shortened by the *spin rotation interaction*, i.e. the coupling between the proton spins and the field produced by the rotation of the molecule, so that T_1 of gases increases with increasing pressure
- In inhomogeneous fields the magnetization decays by T₂ and diffusion
- The diffusion of water (D = 2.23 10⁻⁹ m²/s at RT) is insensitive to pressure but sensitive to temperature
- The diffusion of methane (D ≈ 10⁻³ m²/s at RT) is much faster than that of water and increases with increasing temperature and decreasing density
- For a melt of linear polymers, the diffusion coefficient D_i = N_i^{-ν} A <N>^{-β} of component *i* relates to the average degree <N> of polymerization through a power law with degrees of polymerization N_i and exponents ν, β
- The molecular weight distribution of the melt is mapped by the distribution of diffusion coefficients, i.e. the chemical composition of crude oil can be mapped by the distribution of a physical NMR parameter



General Two-Dimensional NMR

- Two-dimensional NMR was introduced for *high-resolution spectrosocpy* and subsequently extended to 3D and *multi-dimensional NMR spectroscopy*
- With the availability of a fast 2D inverse Laplace transformation algorithm, 2D Laplace NMR became popular
- Similar to the 1D inverse Laplace transformation, also the 2D inverse Laplace transformation is unstable in the presence of noise and needs to be regularized, so that the transforms are not unique but, depending on the regularization algorithm, they are probable solutions
- The scheme for 2D Laplace NMR is the same as that for 2D Fourier NMR including MRI, except that instead of a free-induction decay (FID), a multiecho decay envelope is recorded during the detection period t_2
- It's amplitude is modulated by the events during the preceding *mixing period*, *evolution period*, *preparation period* and *recovery period*
- The most popular 2D Laplace experiments are the T_1 - T_2 correlation experiment, the T_2 - T_2 exchange experiment and the *D*- T_2 correlation experiment
- Diffusion-ordered spectroscopy (DOSY) can be understood as a mixed 2D Laplace-Fourier NMR experiment, which correlates distributions of diffusion coefficients with distributions of chemical shifts, i.e. with the NMR spectrum. It is used for chemical identification of components in complex solutions



Two-Dimensional Laplace NMR

- Two-dimensional Laplace NMR concerns correlation maps of distributions of relaxation times and diffusion coefficients. It can be executed on fluids in porous media exhibiting internal gradients and even with stray-field instruments
- The signal is detected with a multi-echo sequence giving the T_2 distribution
- With T_1 - T_2 correlation NMR, T_1 is encoded in the evolution period
- The T_1 - T_2 correlation map of water in Allermöhe sandstone reports the impact of diffusion in internal gradients *G* on the transverse signal decay: Due to diffusive attenuation, the T_1/T_2 ratio varies with pore size 1/(S/V)
- With D-T₂ correlation NMR, D is encoded in the evolution period
- While the relaxation times of oil and water in porous rock overlap on the T₂ scale, the two types of fluid can be separated along the diffusion axis D
- D-T₂ correlation NMR is good for quantifying different fluids such as oil and water in porous media based on differences in diffusion coefficients and T₂
- *T*₂ -*T*₂ *exchange NMR* probes diffusion without application of field gradients by molecules diffusing between different relaxation sites
- T_2 is encoded in both, the evolution period and the detection period, which are separated by a *mixing time t*_m
- The exchange map reveals connectivity of pores by diffusion-mediated magnetization transfer between pores similar to 2D NOESY spectroscopy, which reveals distances between chemical sites by relaxation-mediated magnetization transfer between different chemical groups in molecules





2D Fourier-Laplace NMR

- Mixed 2D Fourier-Laplace methods correlate the NMR spectrum with a distribution of relaxation times or diffusion coefficents, where the spectral information is detected directly during t₂
- The most powerful method is diffusion-ordered spectroscopy (DOSY)
- It separates a crowded spectrum of a multi-component liquid into a set of subspectra of components with similar diffusion coefficients
- In the pulse sequence, the FID is diffusion encoded in the evolution periods of a stimulated echo with two anti-phase field-gradient pulses of amplitude *G* and duration δ . With the diffusion time Δ the signal of one component follows the decay $M_{xv}(t) = \frac{1}{2} M_{xv}(0) \times \exp\{-q^2 D (\Delta \delta/3)\} \times \exp\{-\Delta/T_1\} \times \exp\{-T_E/T_2\}$
- Instead of successive Fourier and Laplace transformations a dedicated DOSY transformation provides better inversion stability
- Slices through diffusion peaks of the DOSY map S(D,δ) parallel to the chemical shift axis δ produce separate spectra for molecules with distinct diffusion coefficients D
- DOSY can be understood as a non-destructive form of *spectral editing* by fractionation through diffusion

Depth Profiling with the NMR-MOUSE



Depth Profiling

- The *NMR-MOUSE* (MObile Universal Surface Explorer) is a *stray-field NMR relaxometer* with a thin, flat slice, which defines the sensitive volume at a fixed distance up to 25 mm away from the surface of the device
- The spins in the sensitive slice produce the NMR signal
- Typically CPMG-type multi-echo trains are measured at different distances of the sensor surface from the object surface
- The NMR signal $M_{xy}(t) = \sum_i M_{xy,i}(0) \exp\{-t/T_{2\text{eff},i}\}$ is analyzed for relative component amplitudes $x_i = M_{xy,i}(0)/M_{xy}(0)$ and relaxation times $T_{2\text{eff},i}$ to define amplitudes of profiles as a function of depth into the object (1D images)
- The initial magnetizations $M_{xy,i}(0)$ can be sensitized to diffusion or longitudinal relaxation by means for suitable pulse sequences preceding signal detection
- For fluids in porous media, the initial amplitude of the echo envelope gives *volumetric fluid content* when normalized to the amplitude of the bulk fluid
- The normalized integral of the *CPMG echo envelope* defines a spin-density weighted average relaxation time, $\int_{x_v}^{\infty} M_{xv}(t) dt / M_{xv}(0) = \sum_i x_i T_{2eff,i} = \langle T_{2eff} \rangle$
- The empirical weight function $w = (t_2-t_1)t_1 \int_{0}^{t_1} M_{xy}(t) dt /_{t_1} \int_{0}^{t_2} M_{xy}(t) dt$ provides good contrast as depth-profile amplitude at favorable signal-to-noise ratio



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Emulsions and Suspensions

- *Emulsions* are mixtures of immiscible fluids, where one phase is dispersed as droplets in another continuous phase. Examples are mixtures of oil and water like milk, salad dressing, and skin cream
- In strong field gradients the signal from the freely diffusing molecules decays fast, so that for oil-in-water emulsions, the oil signal survives the water signal
- After 1 to 2 ms diffusion time ∆ in a stimulated echo sequence, the water signal in salad dressing has decayed in the stray-field gradient of the order of 20 T/m from the NMR-MOUSE, and the decay of the surviving signal is dominated by the *restricted diffusion* in the more viscous oil droplets
- The decay observed as a function of the diffusion time ∆ is typically fitted with a decay modeled for a logarithmic Gaussian *droplet-size distribution*
- Suspensions are solid particles suspended in a liquid. They are found in processed food and fat
- The FID from the solid decays fast, that from the liquid much slower
- NMR relaxometry is an official method to determine the *solid fat content*, which specifies the ratio of protons from solid fat to those from total fat
- With the *indirect method*, the amount of liquid fat is given by the room-temperature amplitude of fat at 25 μ s deadtime, and the amount of total fat by the signal amplitude at elevated temperature of 60° C where all fat is molten



From D. Oligschläger, C. Rehorn, S. Lehmkuhl, M. Adams, A. Adams, B. Blümich, J. Magn. Reson. 278 (2017) 80, Figs. 4, 5 with permission

Biological Tissues

- Biological tissues are assemblies of cells, which are found in animals and plants
- The most important application of NMR is the study of human tissue by MRI
- The *contrast* in medical MRI is largely determined by spin density, relaxation times and diffusion coefficients, which are all accessible with Laplace NMR
- The *NMR-MOUSE* has been modified to work with a 2 mm thick sensitive slice with a homogeneous gradient for frequency encoding of depth
- This version has been employed to measure depth-resolved distributions of diffusion coefficients through human *skin*. They bear the characteristic signature of the individual
- With a simple permanent center-field magnet, small *plants* can be studied and the water in the pant tissue be characterized
- The amount of water in different parts of the plant is given by the NMR signal amplitude
- The water transport is high when it is dark and the light stress is off
- *Distributions of relaxation times* measured in the water-concentration extrema during the dark and bright periods identify different locations of the water in the plant tissue

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150

Rubber and Elastomers

180

• *Rubber* is a melt of entangled macromolecules

1.6

- Elastomers are chemically crosslinked macromolecules in the molten state
- Sulfur is the most common crosslinking agent
- The chemical crosslinking process is called vulcanization
- Technical elastomers are statistical products from macromolecules, fillers like carbon black, and processing aids, made by mixing, diffusion, and reaction
- In rubber and elastomers the ¹H relaxation times are determined by the chain stiffness and the residual dipole-dipole coupling among ¹H nuclei
- A fraction of the dipole-dipole interaction remains unaveraged due to the motional anisotropy of chain segments between entanglements and crosslinks
- At 60 to 80 K above the glass temperature T_{a} the motion is fast, and T_{2eff} is proportional to the crosslink density determined by measuring strain and shear
- Closer to $T_{\rm q}$ the chain stiffness also determines the relaxation times
- The signal amplitude of foamed rubber scales with the foam density
- For strained elastomers the relaxation rate depends on the orientation angle
- It is the sum of an isotropic and an anisotropic component. The latter can be approximated by a Gaussian distribution of second Legendre polynomials
- For NR both relaxation rates change like first-order and second-order phase transitions at the point where strain-induced crystallization sets in

10

8

 $\Lambda = L/L_c$

Properties of Elastomers

- *T*_{2eff} changes with the *curing time*, e.g. for a moisture-curing poly(urethane) adhesive used for mounting windshields in cars
- Changes in T_{2eff} can be observed well beyond the nominal curing time
- Aging of rubber in the presence of oxygen is mostly a combination of chain scission, cross-linking, and oxidation of the polymer
- In many cases the rubber hardens upon aging
- UV aging affects only the surface of the rubber, while thermo-oxidative aging also affects deeper layers
- Technical rubber products are inhomogeneous due to their statistical nature
- Depending on the miscibility of compounds and the processing conditions, some compounds may agglomerate as revealed by *NMR imaging*
- The reproducibility of T_{2eff} in measurements at one point is better than 1 %, but may be worse than 10 % for measurements at different points due to the statistical nature of the material and the small volume of the NMR-MOUSE
- The *NMR-MOUSE* collects signal from a volume element much smaller than the sample size for the rubber process analyzer (RPA), giving rise to a larger coefficient of variation
- For *quality control of rubber products* several measurements need to be conducted at equivalent spots of the product. The mean fit parameters characterize the *average material properties* and the standard deviation the *material heterogeneity*

Non-Destructive Tire Testing

Quality Control of Elastomer Products

- At room temperature, the relaxation times of rubber and elastomers strongly depend on temperature
- For *quality control* of *rubber products*, experimental values of *T*_{2eff} need to be extrapolated to a reference temperature and correlated with material properties by means of calibration curves
- Calibration curves are determined with small samples in physical testing laboratories by measuring swelling, rheology, and dynamic-mechanical relaxation
- By means of calibration curves NMR relaxation times report parameters like the *glass temperature*, the *elastic modulus*, and the *crosslink density* non-destructively in selected spots from production intermediates and the final product
- A standard application of the NMR-MOUSE is for tire testing
- The depth profile of a tire reveals all layers from the tire tread, including the tread base, the steel-belt overlay, and the belt coat up to the first steel belt
- At each depth, the amplitude of the depth profile is calculated in different ways from the *CPMG decay*
- The *distributions of relaxation times* obtained by Laplace transformation of the CPMG decays fingerprint the properties of the material at the given depth

Morphology of Semi-Crystalline Polymers

Semi-Crystalline Polymers

- Semi-crystalline polymers are solids from macromolecules with mobile amorphous and rigid crystalline domains. Sometimes a rigid amorphous interface can be identified by transverse relaxation measurements
- In the *crystalline domains*, the chains are ordered and packed densely, the degrees of freedom for molecular motion are restricted, and *T*₂ is short
- In the amorphous domains, the chains are disordered, the packing density is lower, the motional degrees of freedom are less restricted, and T₂ is longer
- Semi-crystalline polymers like poly(ethylene), poly(propylene), and Nylon, solidify from the melt in lamellar stacks of crystalline and amorphous domains
- Upon drawing, the macromolecular chains realign
- Upon annealing, a fibrillar structure forms with alternating layers of chainfolded lamellae and amorphous domains arranged perpendicular to the drawing direction
- Depending on the temperature history and the shear fields applied during processing, different overall *crystallinity*, different size distribution of the crystallites, and different order in the amorphous domains are generated
- A crude fit to the *transverse relaxation decay* of such polymers is a *tri-exponential function*, where the rapidly, intermediate and slowly relaxing components are assigned to the crystalline, the interfacial and the amorphous domains, respectively

From B. Blümich, A. Adams-Buda, M. Baias, GWF Gas Erdgas 148 (2007) 95, Figs. 2, 3, 4 with permission

Heterogeneity and Annealing of PE

- Polymer products are inhomogeneous for a number of reasons
- They are made from macromolecules with distributions in molecular porperties such as molecular weight, branching, and copolymer statistics
- Polymer processing often involves cooling from the melt. Rapid cooling at the surface leads to lower crystallinity than slow cooling inside the bulk
- In semi-crystalline polymers also the domain sizes are distributed
- The transverse relaxation decays contain signal contributions from crystalline, interfacial and amorphous domains
- The rapidly decaying signal from the crystalline domains is partially lost in the deadtime of the NMR instrument
- A higher value of the *weight parameter w* extracted from the CPMG decay indicates more rigid material or higher crystallinity
- In a melt-extruded HDPE pipe the *crystallinity* increases from the surface to the interior due to the temperature gradient during cooling
- The small sensitive volume of the NMR-MOUSE maps fluctuations in crystallinity. Their distribution shifts to higher crystallinity upon annealing for 24 h at a temperature of 80° C well below the melting temperature of 136° C
- An old LDPE steel-pipe shell has become brittle due to chemical corrosion. The elastic properties could be restored by annealing for 24 h at a temperature of 60° C

From A. Adams, M. Adams, B. Blümich, J.H. Kocks, O. Hilgert, S. Zimmermann, 3R Int. 49 (2010) 216, Figs. 8, 20 with permission

Mechanical Properties of Semi-Crystalline Polymer Materials

- The *mechanical properties* of semi-crystalline polymer materials are usually explained in terms of the degree of *crystallinity*
- This is so, because crystallinity can easily be determined by *X-ray diffraction* and thermo-analytical methods, whereas NMR is a less common method
- In contrast to X-ray diffraction, NMR relaxation directly measures the signal from the *amorphous domains*
- At small loads in the linear regime of the *stress-strain curve*, deformations are reversible and largely affect only the amorphous but not the crystalline domains
- This is evidenced by the changes in relaxation time T_{2eff} of a slightly curved, carbon-black filled section of *water pipe* cut from a pipe delivered on a large spool
- Upon bending the section straight, the *T*_{2eff} values of the amorphous and interface domains change over the whole length of the pipe section but not those of the crystalline domains
- Upon bending the section in reverse over a pin, the relaxation times of the amorphous and interface domains change only locally at the site of the pin

Physical and Chemical Aging of LDPE

Aging of Semi-Crystalline Polymers

- Aging of semi-crystalline polymers involves at least two competitive processes: Physical aging (crystallization) and chemical aging (corrosion)
- A third process can be the loss of low molecular weight compounds by evaporation and leaching through *solvent exposure*
- During *thermo-oxidative aging* the material is exposed to elevated temperature in air
- At short aging times the material predominantly crystallizes so that the fraction of crystalline domains increases, and the mobility, i.e. *T*₂, of the amorphous domains decreases
- At longer aging times, chemical *degradation* dominates along with chain scission causing the rigid fraction to decrease and the mobility in the amorphous domains to increase
- *Wax* is generated in most polymerization processes as evidenced by the number fraction in the *Schulz-Flory distribution* of molecular weights
- The presence of wax accelerates the physical aging process
- Solvents are taken up essentially only by the amorphous domains in semicrystalline polymers
- The chain mobility gained by the amorphous chains upon swelling leads to solvent-induced crystallization reminiscent of temperature-induced crystallization. This process may be impacted by the leaching of wax

Cultural Heritage

- Non-destructive testing by stray-field relaxation and diffusion NMR finds more and more applications for studying objects of value to *cultural heritage*
- A proton density higher than expected in depth profiles often indicates the presence of a *conservation agent* like wax in mortar or varnish in bones
- The stratigraphy of paintings reveals the thickness of planar layers with an accuracy of 10 μm and material properties from relaxation times and diffusion constants
- For example, in the 1473 AD painting on wood 'Adoration of the Magi' by Perugino, depth profiles at two spots showed a thicker textile layer at the joint between two wooden boards than elsewhere
- The paintings 'The Dinner' and 'The Dance' from the 'Pipenpoyse Wedding' were made in 1616. One was restored and solvent cleaned, the other never restored. T_{2eff} depth profiles through the paint and canvas regions reveal low T_{2eff} throughout for the cleaned painting and higher T_{2eff} for the original painting at deeper depth, suggesting differences in leaching of low molecular weight constituents by *solvent cleaning* and *evaporation*, respectively
- *Frescoes* are painted on wet *mortar* applied in strata of finer and finer grain size towards the surface. The *moisture distribution* across the mortar layers reports the manufacturing technology and skills of the trade from former times such as in the walls of the Villa of the Papyri in Herculaneum