An Adaptive Code Rate Control of Polar Codes in Time-Varying Gaussian Channel

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Abstract. Under a benchmark of bit error rate (BER) in data transmission, a just perfect trade-off between maximizing code rate (CR) and reliable communication presents a significant coordinated challenge in the time-varying additive white Gaussian noise (T-AWGN) channel. In this paper, based on the guidance of a tight bound as coding parameters of polar code rate R, block length N with the capacity $I(W)$ in channel W of $N \geq \beta/(I(W) - R)^{\mu}$, a criteria of effectively adjusting the size of the parameter μ will achieve a better trade-off between the CR and the reliability, where β depends only on block error probability. In the circumstance of a round-clock traffic light (RTL) simulation, numerical results show that this scheme has a good preference for the guaranteed reliability for the wireless communication.

Keywords: Polar codes · Code rate · Traffic light (RTL) of wireless communication · Time-varying additive white Gaussian noise

1 Introduction

Polar codes are a major breakthrough in coding theory [1]. In binary-input adaptive white-Gaussian-noise channel (BI-AWGN), successive cancellation (SC) decoding is a vital algorithm can provide an approximate state-of-the-art error probability (EP) for polar codes in a communication system, and its complexity of decoding algorithm is acceptable in all decoding methods. In any error controlled coding strategy, code rate (CR) can adjust the EP. Therefore, many applications have used CR of the codeword to match the reliability of communication. This application is called adaptive code rate control (ACRC) polar codes. ACRC polar coding is also a flexible criteria. Characterized by a CR changes with signal-to-noise ratio (SNR) of T-AWGN channel,[1] the CR of polar codes can be modified solely by only one generator matrix.

[1] T-AWGN channel model: $y_i = x_i + n_i(t)$,here x_i is input, y_i is output, and $n_i(t) : N(0, \delta^2(t))$ is a zero mean Gaussian noise and $\delta^2(t)$ is a varying variance with t.

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In the normal SC decoding algorithm of polar codes over DMC, rate R, length N and capacity $I(W)$ of channel W are deduced by an inequation (1) [2]:

$$N \geq \beta/(I(W) - R)^{\mu}, \tag{1}$$

here β is a constant that depends only on block error probability p_e, $R < I(W)$ and "=" implies a critical state with unreliability [2–5]. Where in DMC, the capacity $I(W)$ of channel W is $I(W) = \frac{1}{2T}\log(1 + 2R\frac{E_b}{N_0})$, and $\frac{1}{T}$ is symbol rate of speed. In the inequation (1), one of the three parameters can be calculated from the other two among R, N and p_e. Usually, the calculation method of the inequation is deemed as state of equation. And here "=" also means the state of the maximized CR. Therefore, it is significant to calculate CR in the equation state and weaken a critical state of unreliability. However, as methods of adjustment, the conventional $\mu = 3.627$ for binary erasure channel (BEC) [2] and $\mu = 4.001$ for AWGN channel [3] are edge states for their equations without guaranteeing reliability. In addition, packet loss rates on 10^{-3} order of voice link are generally acceptable; while for the data link, bit error rate (BER) of 10^{-6} is regarded as acceptable. In this paper, under constraint of acceptable link and adjusting μ in inequation (1), we fulfill a maximized CR and achieve a better trade-off between the CR and the communication reliability. And in T-AWGN channel, the CR changes will better match the BER of data communication link constraint.

2 Coding and Reliable CR

2.1 Polar Coding

Polar codes, as a linear block coding scheme, have been proved to achieve the channel capacity at a low encoding and decoding complexity [1]. For polar codes, the generator matrix G_2 of size 2×2 can create the two independent copies of a probability W with double channels. Where $G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Apply the transform matrix $G_2^{\otimes n}$ (where "$\otimes$" denotes the n-th Kronecker power) to constructed a codeword with length $N = 2^n$. Data will be changed into polar codes as follow: data bits in the information set $\mathcal{A}$ together with known bits in frozen set $\mathcal{A}^c$ compose the un-encoded word $U = u_1^N \triangleq (u_1, u_2, \ldots, u_n)$. The encoded code word is generated as the code word: $UG_2^{\otimes n} = X = x_1^N \triangleq (x_1, x_2, \ldots, x_n)$. After over T-AWGN channel, the received code word $Y = y_1^N \triangleq (y_1, y_2, \ldots, y_n)$ will be decoded by SC decoder. This procedure is a process of bit-channel combining. This encoding procedure corresponds to transforming N copies of channel W into a channel combining: $W_N(Y|U) = \prod_{i=1}^{N} W(y_i|x_i)$. Then, in the decoding process, its decomposed bit-channels and channel splitting probability $P(\cdot)$ are given by [1]:

$$W_N^{(i)}(y_0^{N-1}, u_0^{i-1}|u_i) \triangleq P(y_0^{N-1}, u_0^{i-1}|u_i) = \sum\nolimits_{u_{i+1}^{N-1}} \frac{P(y_0^{N-1}|u_0^{i-1})P(u_0^{i-1})}{P(u_i)} = \frac{1}{2^{N-1}}$$

$\sum_{u_{i+1}^{N-1}} P(y_0^{N-1}|u_0^{i-1})$. And SC decoding with the variable format of log-likelihood ratio (LLR) can be derived from [7]:

$$\mathcal{L}_N^{(2i-1)}(y_1^N, \hat{u}_1^{2i-2}) \simeq \text{sign}(\phi)\text{sign}(\varphi)\min(|\phi|, |\varphi|), \tag{2}$$

$$\mathcal{L}_N^{(2i)}(y_1^N, \hat{u}_1^{2i-2}) = (-1)^{\hat{u}_{2i-1}}\phi + \varphi, \tag{3}$$

where $\phi = L_{N/2}^{(i)}(y_1^{N/2}, \hat{u}_{1,o}^{2i-2} \oplus \hat{u}_{1,e}^{2i-2})$ and $\varphi = L_{N/2}^{(i)}(y_{N/2+1}^{N}, \hat{u}_{1,e}^{2i-2})$. According to formula (1) and (2), the front decoded bits are used to deduce the sequel bits. Then, the LLR of each decoding bit can be decided as [8]. $\mathcal{L}(\hat{u}_i) = ln(\frac{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|0)}{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|1)})$. In the recursive process, the decoding result is decided by

$$\hat{u}_i = \begin{cases} 1 & if \quad \mathcal{L}(\hat{u}_i) < 0 \\ 0 & if \quad \mathcal{L}(\hat{u}_i) \geq 0 \\ u_i & if \quad i \in \mathcal{A}^c \end{cases}.$$

2.2 An Equation and an Inequation

Consider a binary-input DMC channel W. The channel capacity is $I(W)$ with any CR of $R < I(W)$ and strictly block error probability p_e, a coding scheme is constructed that allows transmission at R with an EP not exceeding p_e. Ideally, given a family of polar coding, the constraint relationship among the three parameters above. There have been two relationships among these three parameters.

The first one is a universal equation for channel coding as a fellow [6]:

$$I_{AWGN} - \sqrt{\frac{V}{N}}Q^{-1}(p_e/RN) + \frac{\log(N)}{2N} = R. \tag{4}$$

where $V \triangleq \frac{P(P+2)}{2(P+1)^2}\log_2^2(e)$, $Q^{-1}(\cdot)$ is the inverse of Q-function. Due to (4) fitting AWGN channel, the construction polar codes utilized by Gaussian (normal) approximation can well match (4). Inequation (1) is the relation of CR and SNR. μ is a defined parameter: Consider the block error probability function $p_e(\cdot)$ of SC decoding algorithm. If there is a function f with a constant $\mu > 0$ as Eq. (4), μ will be defined to satisfy (4) by [3],

$$\lim_{N\to\infty, N^{1/\mu}(I_{AWGN}-R)=\ell} P_N(R, I_{AWGN}) = f(\ell). \tag{5}$$

μ is provided by a heuristic method for computing in BEC by [2]. In [3], the error probability $E(W)$ of channel W of the information set $\mathcal{A}$ satisfies $\sum_{\exists\varepsilon > 0, i\in\mathcal{A}} E(W_N^{(i)}) \leq \delta$.

An inequation $R < I(W) - \beta N^{-\frac{1}{\mu}}$ can revise a size of μ. In [4], because of a 0.5 switching probability of the Arikan's recursions between $Z_{i+1} = Z_i^2$ and $Z_{i+1} = 2Z_i -$

Z_i^2, the eigenvalues of the expanded matrix of the recursions help to calculate μ in the BEC. Where Z is Bhattacharyya parameter and δ is a constant. μ in aforementioned studies can be estimated under the ideal condition. The definition of μ in Eq. (3) is also based on the ideal polarization of polar codes. Hence, the sizes of μ of the inequation (1) are in critical states when the "=" is established. However, Eq. (2) should aid inequation (1) to enhance the reliability of the transmission.

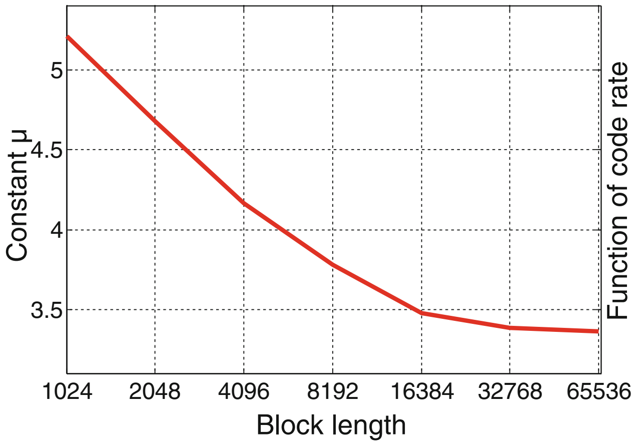

Fig. 1. μ calculation aided by Eq. (4).

3 μ Calculation with Verification

3.1 Polar Coding

Given an encoding and decoding couple, the key of reliable transmission is characterized by CR. Obviously, the more small CR is adjusted that means the more reliability in transmission. However, this trade-off is subtly based on maximal CR and reliability. Define $\mathcal{R}$ as a CR calculated by inequation (1), $\mathcal{R}'$ s CR of Eq. (3). Under the same block error probability p_e constraint, $\mathcal{R} \leq \mathcal{R}'$ of the same condition will be explained by [10] in AWGN channel. However, based on "=" in (1), $\mathcal{R}$ in (1) means the edge of unreliable transmission. Hence, suppose $\mathcal{R}_x$ presents a CR of (1) to escape from the unreliable edge of transmission adjusted by μ and based on "=" calculation, there should be $\mathcal{R}_x \leq \mathcal{R} \leq \mathcal{R}'$ for ACR criteria of polar codes and $\mathcal{R}_x$ is more reliable than others. From the (1) and (2), the deduction is as follows.

Deducing: First, difference of channel capacity:

$$\Delta I \triangleq I_{AWGN} - R \Rightarrow$$

$$\Delta I = (\beta/N)^{1/\mu},\ \mu > 0,\ \mu_{\mathcal{R}} = \mu, \beta > 0, \tag{6}$$

Similarly:

$$2\Delta I' \triangleq \sqrt{V/N}Q^{-1}(p_e/\mathcal{R}'N), \tag{7}$$

From (6) and (7), we get: $\Delta I' \leq \Delta I$, and then we can get result of $\mathcal{R}_x \leq \mathcal{R} \leq \mathcal{R}'$ as follows:

$$\begin{aligned}
&\Delta I' \leq \Delta I \\
&\quad \Rightarrow \mu_{\mathcal{R}} = \ln(\beta/N)/\ln(\Delta I) \leq \ln(\beta/N)/\ln(\Delta I') = \mu_{\mathcal{R}'} \\
&\quad \Rightarrow \mathcal{R}_x = I_{AWGN} - (\beta/N)^{1/\mu_{\mathcal{R}'}} \leq \mathcal{R} = I_{AWGN} - (\beta/N)^{1/\mu_{\mathcal{R}'}} \leq \mathcal{R}' \\
&\quad \Rightarrow \mathcal{R}_x \leq \mathcal{R} \leq \mathcal{R}'
\end{aligned}$$

Over deducing.

With $\mu_{\mathcal{R}'}$ to govern the CR changes, $\mathcal{R}_x$ means more reliable communication than that of the conventional μ. Hence, $\mathcal{R}_x$ is a suitable CR. Gratified p_e by the transmission over the un-encoded channel, the channel environment will be deemed a noiseless channel on a certain SNR. Hence, if the CR equals to 1, Eq. (4) is still valid, and the size of μ is verified by the known CR with SNR. As polar coding scheme, the strategy is the "provably capacity achieving" code scheme and the CR must show $R \neq 1$ because of the inevitable redundancy.

3.2 μ Calculation

Due to $\Delta I' \leq \Delta I$, the gap to capacity of $\Delta I'$ guarantees to reliably govern changes of ACRC by a size of μ in inequation (1). If $\Delta I'$ substitutes into inequation (1), R equals to 1 and $\beta = \chi p_e$, as a fixed coefficient, and $\chi \in \mathbb{R}$ is aided to verify the point of $R = 1$. μ calculation of polar codes is represented by:

$$\hat{\mu} = \left.\frac{\log(p_e/k)}{\log(\Delta I')}\right|_{k=N,R=1}$$

On $E_b/N_0 = 13\,\text{dB}$ of the AWGN channel, the BER of on-off keying (OOK) modulation, as a modulation scheme, is less than 10^{-6} which is an acceptable communication data link. μ equals to 4.639 by calculation of (4) when block length is 2048. As shown in Fig. 1, the calculated μ also keeps an independent constant character while the block length approaches infinity in AWGN channel. In Fig. 2, the calculated value of μ has shown to govern changes of ACRC as a function $\beta/(I_{AWGN} - R)^{\hat{\mu}} - N$ equals to 0. In [3], Gaussian approximation deduces that μ equals to 4.001. However, from Fig. 2, there is not a large difference between the proposed μ and the $\mu = 4.001$ for the CR changes in AWGN channel. Thus, as block length belongs to finite-block length regime, μ will govern the CR changes with a slight preference for the reliability.

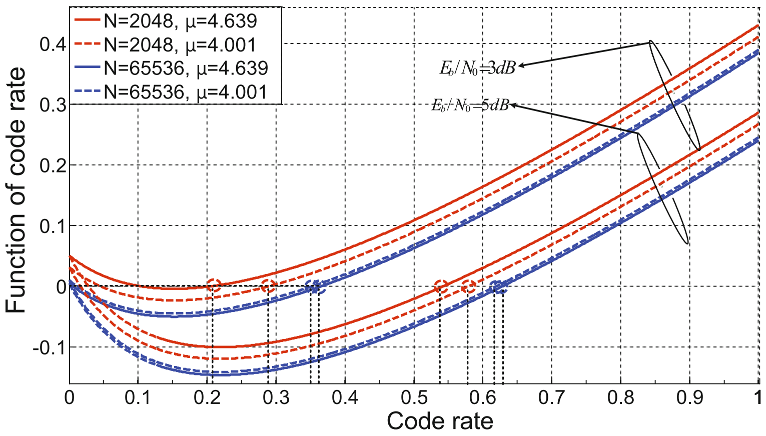

Fig. 2. Changing CR estimation

4 Utilizing RTL Scene to Simulate Polar Codes

More than 70% of total global people will be expected to live in cities by 2050 [9], then, greenhouse gas emissions and energy consumption will increase greatly. As a green communication, the RTL in cities can provide a sufficient light communication service for the waiting, the passing, and other users nearby to alleviate these problems above. Then, a simulation demonstrates an ACRC control of polar codes for RTL wireless transmission in T-AWGN channel environment.

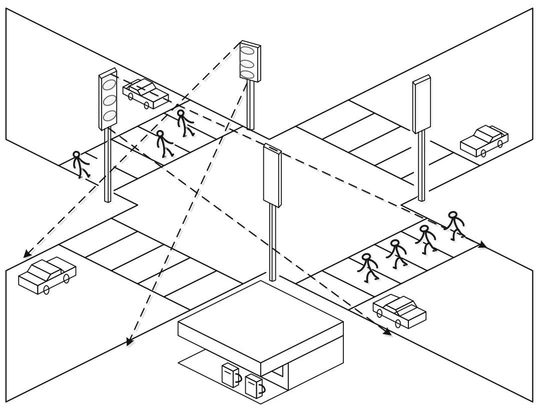

Fig. 3. RTL physical scene.

4.1 RTL Physical Scene

The physical scene of the RTL communication system can be shown in Fig. 3. The desired optical and undesirable optical signals travel through air before reaching RTL receiver. Optical filter is used to minimize the ambient-light noise. In RTL system, error performance variation is occurred in whole daytime. Due to daylight wavelength variation on multi-color channel, the daylight passing the optical filter and the sun can be removed but the color channels of green. Then yellow and red from diffuse reflectance from sun are all distributed itself color-light noise.

4.2 Numerical Results from RTL Simulation

Primarily, we assume that sender and receiver of RTL are ideal. The visible light from the sun is the very significance of photosynthesis, but the visible red light, yellow light and green light in the sunlight are interferences to the visible light transmissions over Gaussian channel model [11]. The light wavelength of green is from 455 nm to 492 nm, yellow is from 577 nm to 597 nm and red is from 622 nm to 780 nm. A fluctuating SNR in one day can be estimated by [10]. Based on the SNR, the frozen set is confirmed with the CR from $N = \beta/(I_{AWGN} - R)^{\hat{\mu}}$ calculation. As a visible light communication in varying channel environment, RTL wireless transmission can be OOK modulation [11]. In the paper of [10], Hu, Wang, and Liu took more than two years to observe and measure the photosynthetic photon flux density (Q_p). In this paper, the literature [10] will be utilized to estimate the varying variance of optical noise in the daytime. In [10], the maximum $Q_{p\_\max}$ can be obtained in every hour. Hence, the wireless transmission of RTL is considered as a T-AWGN channel model. The users can receive signal power of $S = (E_r/1.09\pi d^2)A_{ph}$ (A fan-shaped angle of RTL wireless emission is about 50°) and the ambient light noise power of $N = \kappa\,\mathrm{var}(Q_p)\hbar v A_{ph} N_A$, meanwhile the variance can be estimated. Where $\hbar$ is Planck constant, v is green, red or yellow light frequency, N_A is the Avogadro constant and E_r is the rated power of RTL. κ is ratio coefficient (based on sundown or sunrise illumination, the non-direct by sunlight efficiency estimation is $\kappa = Q_{p\_\min}/(Q_{p\_}\max(hour) - Q_{p\_\min})$, all kinds of wavelength from 400 nm to 700 nm, κ is about $10/(1500 - 10) \times 1/(700 - 400)$. The luminous power of red, green or yellow RTL is about limited by $E_r = 10\,\mathrm{W}$. From 80 m to RTL, if the receiving area of user antenna of visible light is $A_{ph} = 0.8\,\mathrm{cm}^2$, the signal power of $S = 3.98 \times 10^{-7}$ W can be received. Algorithm 1 can calculate the estimated code rate of $\hat{R}$ACRC by the iterative method.

Algorithm 1: Calculation ACRC

Input: E_b/N_0

Output: $\hat{R}$

1 : $SNR(dB)_i, i = 0$;

// initialize the link of OOK SNR and the variable

2:**if** $E_b/N_0 > SNR(dB)_i$ **then**

3 : *Give up any channel coding regimes*;

4 : **else**

5 : Set initial $R(0) = 0.5, R(1) = 1, \frac{E_b}{N_0}\{R(0) \text{ or } R(1)\} = 10\log(\frac{1}{2R(0 \text{ or } 1)(\text{var of nois.})^2})$;

// initialize the CR and utilize SNR calculation

6: **While** $(|\hat{R}(i+1) - \hat{R}(i)| > 1/N) \cap (|\frac{E_b}{N_0}(i+1) - \frac{E_b}{N_0}(i)| > 0.1)$ **do**

7 : $\frac{E_b}{N_0}(i) = 10\log(\frac{1}{2R(i)(\text{var of nois.})^2})$;

8 : $\hat{R}(i) - \frac{1}{T}\log(1 + 2\hat{R}(i)\frac{E_b}{N_0}(i)) + (p_e/N)^{1/\hat{\mu}} = 0$; // Calculation $\hat{R}(i)$

$i = i + 1$;

9 : **end while**

10 : **end if**

In the background of solar irradiance and the data in [10], the diurnal SNR (dB) of green, yellow and red light can be estimated. Under the constraint of BER 10^{-6}, the Fig. 4 shows the simulation of the fixed CR and ACRC of polar codes in the RTL wireless transmission. Based on reliable data communication, if CR is 0.54 of polar

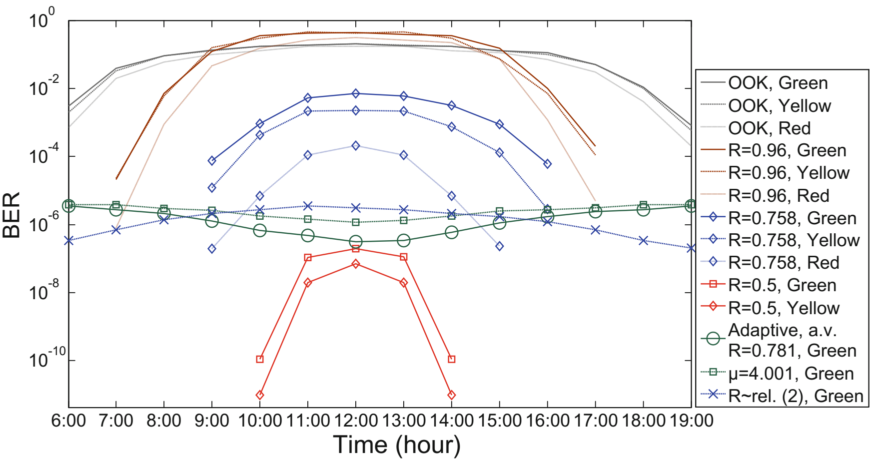

Fig. 4. Seamlessly ACRC by a generator of polar codes in RTL.

codes with block length 2048 in the RTL system, the communication data link works availably for 20 h a day. If CR is fixed 0.758 for polar codes of length 2048, the communication link can't be set up from 8:00 to 17:00 in the sunny days of RTL wireless transmission. However, if the ACRC varies with SNR under the error rate of 10^{-6} constraint, the communication is more perfect with SC decoding algorithm of parameter μ in this paper than that of $\mu = 4.001$. In fact, the polar codes of block length 2048 with fixed 0.758 CR does not match communication link for most of the daytime, but an average polar CR of 0.781 in the adaptive regime can be obtained in an uninterrupted wireless communication connection of a day. Hence, the criteria proposed in this paper can achieve a high average CR in time-varying channel.

5 Conclusion

The trade-off between reliability and CR (or throughput) for frame length is significance. Polar codes have only one generator matrix to cater the variable CR. Nonetheless, in the fluctuating channel environment, the conventional "=" state of single inequation (1) cannot guarantee the reliable communication with any CR from the channel state. However, we find and create a criterion to control variable CR. In T-AWGN channel, μ has been adjusted to satisfy the perfect and reliable mapping for any CR, while preserving the largest CR.

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