Fuzzy Gray Relational Analysis for Electromagnetic Parameters of Induction Heating Process

Pei Fu, Ping Zhou, Tianyang Zhao, Zhuo Chen and Chenn Q. Zhou

Abstract Induction heating is widely used for heat treatment, providing fast and precise heating effect. A wide range of electromagnetic parameters, such as the structure parameters of coil and the electrical operating parameters, have significant influences on the temperature distribution of the workpiece in induction heating process, which is important for the subsequent heat treatment process. In this work, the main factors including exciting current, power frequency, coil inner diameter and coil spacing are chosen to be studied by numerical simulation. Meanwhile the single-factor experimental design and the Fuzzy Gray Relational Analysis are combined to investigate the impacts of the four factors on the temperature distribution, providing great reference value for further research of induction heating. The result shows that, for axial temperature difference of specimen, the impacts of the four factors are ranked from the most important to the least important as coil inner diameter, coil spacing, power frequency and exciting current. While for radial temperature difference, the ranking list of importance becomes exciting current, power frequency, coil inner diameter and coil spacing.

Keywords Heat treatment · Induction heating · Temperature distribution · Numerical simulation · Fuzzy gray relational analysis

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Introduction

Induction heating is one of the preferred heating technologies in industrial, domestic and medical applications due to its advantages compared with other classical heating techniques such as flame heating, resistance heating, and traditional ovens or furnaces [1]. It provides contactless, controlled, efficient and clean heating of conductive materials, therefore, it is currently used in smelting, forging and heat treatment, promoting the development of advanced material and metallurgy. During the heat treatment of superalloy, the initial temperature distribution is absolutely necessary, and the temperature distribution and temperature gradient play vital roles for microstructure and mechanical properties [2]. However, the limited current penetration depth and various influence factors lead to nonuniform temperature distribution of workpiece, making it difficult to control the induction heating process and the subsequent heat treatment process. Meanwhile, it is impossible to get the overall temperature distribution of workpiece because of the limited means of measurement. The numerical simulation technique is an efficient way to obtain the temperature distribution which is difficult through the traditional measurement method, making it convenient for the parameter control and optimization [3].

Induction heating is a complex process including electromagnetic, thermal and metallurgic phenomena. In this process, an alternating electric current induces electromagnetic field, which in turn induces eddy currents in the workpiece [4]. The induced eddy currents release energy in the form of heat, which is then distributed throughout the workpiece. It is difficult to solve the electromagnetic thermal coupling problem during induction heating. Based on lumped parameter method, models like equivalent circuit model of power supply and empirical formula of experimental modification are put forward to solve simple induction heating problems [5, 6]. However, the oversimplified models and excessive auxiliary parameters cause computational errors and less understanding of induction heating. Then, distributed parameter method like finite element analysis is adopted to analyse the induction heating [7, 8].

Besides the coupling problem, it is difficult to choose the right shape and position of the induction coil and adjust the electric current properties to attain a desired temperature profile in the workpiece. Huang [9] studied the induction heating process of steel tube and compared the temperature field caused by different power frequency, finally an optimum frequency combination was selected. Shokouhmand [10] analysed the process of moving induction heat treatment, and the effect of velocity, initial position of inductor and inner to outer radius ratio on temperature distribution are investigated. Hammi [11] developed a 2D axisymmetric model of induction heating process to study the parametric and sensitivity effect of machine parameters including current density, heating time, frequency and some dimensional factors. To sum up, many investigations have been done to study the influencing factors in order to obtain an ideal induction heating effect. However, few of them focused on the importance extent of the influencing factors. As there is a wide range of influence factors, it is crucial to compare the different influencing factors and find the main factors to make a better understanding and more precise control of induction heating.

Be different from previous work, this work aims to determine the primary factors that affect the temperature distribution in induction heating process through the numerical simulation. Previously mentioned, the structure parameters of coil (inner and outside diameter, number of turns and coil spacing) and the electrical operating parameters (exciting current, power frequency and heating time) all affect the temperature distribution of workpiece obviously, which are too many to make a comprehensive investigation. In this work, main factors including exciting current, power frequency, coil inner diameter and coil spacing are chosen to be investigated.

In addition, Gray Relational Analysis (GRA) is a useful method to evaluate the impacts of various factors without knowing the mathematical relationship among the investigated factors and the results. Meanwhile the single-factor experimental design and Fuzzy Gray Relational Analysis (FGRA) are combined to investigate the impacts of various factors on the temperature distribution, providing great reference value for further research of induction heating.

Evaluating Method

Single-Factor Experimental Design

The Single-factor experimental design is a popular method to deal with the test, including multiple factors and levels. It has been widely applied to many fields since it is simple and distinct [12]. In this work, four factors are investigated, including exciting current, power frequency, coil inner diameter and coil spacing.

Fuzzy Gray Relational Analysis

Gray relational analysis (GRA) is an effective statistical method for measuring the degree of approximation among the sequences using a gray relational grade. It was developed by Deng [13] and has been successfully applied in other fields [14–17]. In this work, this method is improved and employed to evaluate the impact of various factors on the temperature uniformity of the induction heating process. The steps are as follows:

Step 1: List the reference matrix and comparison matrix. The reference matrix y and the comparison matrix x are expressed as follows:

$$y = [y(1)y(2)\ldots y(n)] \tag{1}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{bmatrix} = \begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(n) \\ x_2(1) & x_2(2) & \cdots & x_2(n) \\ \vdots & \vdots & \vdots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(n) \end{bmatrix} \tag{2}$$

where m is the number of investigated factors and n is number of investigated conditions.

Step 2: Nondimensionalize original matrix. The original matrix should be nondimensionalized because of the different dimensions of the investigated factors and the reference variable by the following equation:

$$x_i(k)' = \frac{x_i(k) - \min x_i(k)}{max x_i(k) - \min x_i(k)} \tag{3}$$

Step 3: Calculate the cosine value of fuzzy membership. The similarity of the two factors is related to the include dangle cosine of the two factors, which is expressed as follows:

$$r_1 = \frac{\sum_{k=1}^{n} y(k)x(k)}{\sqrt{\sum_{k=1}^{n} y(k)^2}\sqrt{\sum_{k=1}^{n} x(k)^2}} \tag{4}$$

Step 4: Calculate the gray relational grade.

$$\xi_i(k) = \frac{\Delta min + l\Delta \max}{\Delta k + l\Delta \max} \tag{5}$$

where l is the resolution coefficient, which represents the weight of the maximum absolute difference and is used to satisfy the integrity and anti-interference of the relational grade. Absolute difference $\bar{\Delta}$ is determined as follows:

$$\bar{\Delta} = \frac{1}{m \cdot n}\sum_{j=1}^{m}\sum_{k=1}^{n}\left|y_t(k) - x_{tj}(k)\right| \tag{6}$$

The resolution coefficient is determined as follows:

$$l \in \begin{cases} [c, 1.5c] & c < 1/3 \\ (1.5c, 2c] & c \geq 1/3 \end{cases} \tag{7}$$

where ratio $c = \bar{\Delta}/\Delta\max$. If $c < 1/3,\ \ l = 1.5c$, else if $c \geq 1/3,\ l = 1.75c$.

Step 5: Calculate the Euclidean gray relational grade. The difference between the reference matrix and the comparison matrix can be evaluated by the Euclidean distance, which improves the evaluation accuracy. The Euclidean gray relational grade r_2 is expressed as follows:

$$r_2 = 1 - 2\sqrt{\sum_{k=1}^{n}\left[w(1-\xi_i(k))\right]^2} \tag{8}$$

Where w is weight factor of different factors in the comparison matrix.

Step 6: Calculate the fuzzy gray relational grade. The fuzzy relational grade is a combination of the fuzzy membership coefficient and the Euclidean gray relational grade, which is calculated by the following formula:

$$r = \sqrt{\frac{r_1^2 + r_2^2}{2}} \tag{9}$$

Step 7: Rank. Based on the value of the fuzzy gray relational grade, impacts of the investigated factors are ranked.

Investigated Model

Geometric Model

In this work, the computational domain includes specimen, insulated cotton, inductance coil, cooling water and ambient air. The model is simplified in the following ways:

a. The model can be simplified to be axisymmetric since the cylindrical specimen and the inductance coil are coaxial;
b. The 3D model can be simplified to be 2D as the materials are all isotropic.

The schematic diagram of the induction heating process is presented in Fig. 1. The specimen is 12.5 mm wide and 100 mm long, while the air section is 400 mm wide and 800 mm long. The material of the specimen is FGH96 and the physical properties are determined by former experiment. Point A is set as temperature control point, which is the mid-point of the side surface of specimen. While point B is set as temperature measure point, which is 10 mm from the top of the side surface.

Mathematical Model

The mathematical model of induction heating process consists of the electromagnetic field equation and temperature field equation.

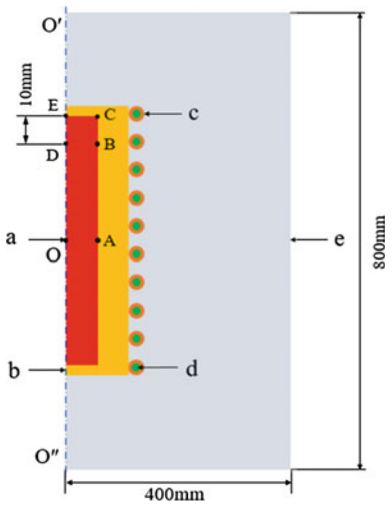


Fig. 1 Schematic diagram of the induction heating process **a**—specimen; **b**—insulated cotton; **c**—inductance coil; **d**—cooling water; **e**—ambient air

For the electromagnetic field, combined with Coulomb gauge and auxiliary quantity, like magnetic vector potential $\vec{A}$, and electric scalar potential $\phi^{*}\left(\phi = \int \phi^{*} dt\right)$, the Maxwell's equations are described as:

$$\vec{\boldsymbol{J}} = -\sigma\left(\frac{\partial \vec{\boldsymbol{A}}}{\partial t} + \nabla \frac{\partial \phi}{\partial t}\right) + \overrightarrow{\boldsymbol{J}_e} \tag{10}$$

$$\nabla \times \frac{1}{\mu} \nabla \times \vec{\boldsymbol{A}} - \nabla \frac{1}{\mu}\left(\nabla \cdot \vec{\boldsymbol{A}}\right) + \sigma\left(\frac{\partial \vec{\boldsymbol{A}}}{\partial t} + \nabla \frac{\partial \phi}{\partial t}\right) = \overrightarrow{\boldsymbol{J}_e} \tag{11}$$

$$-\nabla \cdot \sigma\left(\frac{\partial \vec{\boldsymbol{A}}}{\partial t} + \nabla \frac{\partial \phi}{\partial t}\right) = 0 \tag{12}$$

where $\vec{\boldsymbol{J}}$, σ, ∇, $\overrightarrow{\boldsymbol{J}_e}$ are the total current density, total conductivity, Hamilton operator and current density of external exciting source respectively.

For the temperature field, the Joule heat source caused by induced eddy current can be determined according to Joule's law. The heat conduction equation is:

$$\frac{\partial}{\partial t}\left(\rho C_p T\right) = \frac{J^2}{\sigma} + \nabla \cdot (\lambda \nabla T) \tag{13}$$

where ρ, C_p, λ is density, specific heat capacity and thermal conductivity of the specimen, respectively.

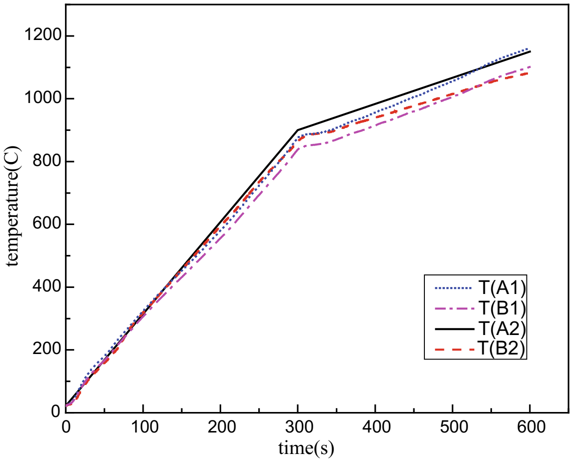


Fig. 2 Comparison between numerical and experimental results of point A and B

Based on the geometric model and the equations above, the numerical computation is solved using the COMSOL Multiphysics 5.2a. The boundary conditions in the numerical model are listed as follows: the temperature of cooling water is 20 °C, the air is at room temperature and the side surface of the specimen is adiabatic as the specimen is wrapped in insulation cotton.

Model Verification

To verify the numerical model, the simulation results (line A1, line B1) and the experimental results (line A2, line B2) of specimen are shown in Fig. 2. Overall, the simulated results agree with experimental results very well in terms of the magnitude and the trend. The maximum relative error is less than 3%, which is acceptable. As a result, the numerical model is adopted to investigate the effects of electromagnetic parameters on the temperature distribution of induction heating process.

Results and Discussion

Results Based on Single-Factor Experimental Design

The investigated factors are exciting current, power frequency, coil inner diameter and coil spacing, and they all have five levels, respectively. In this work, distance between adjacent turns from middle to the end is designed as arithmetic progression, and the coil spacing is described as the common difference of the arithmetic

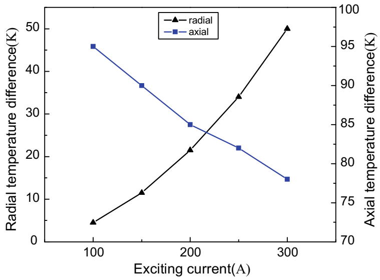
Table 1 Factors and levels

Levels	Factors			
	Exciting current (A)	Power frequency (kHz)	Coil inner diameter (mm)	Coil spacing (mm)
1	100	50	40	−4
2	150	75	45	−2
3	200	100	50	0
4	250	125	55	2
5	300	150	60	4

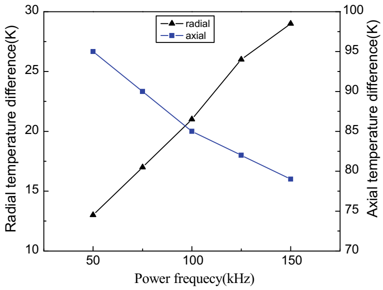
progression. The investigated factors and their levels are presented in Table 1. On basic condition, exciting current is 200 A, power frequency is 100 kHz, coil inner diameter is 50 mm and coil spacing is 0. Temperature difference equals to the temperature in the middle minus the temperature at the end of the specimen. The simulation conditions and results of test cases are shown in Table 2.

Table 2 Simulation conditions and results of test cases

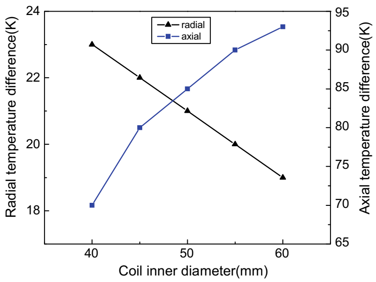
Case	Factors				Results	
	Exciting current (A)	Power frequency (kHz)	Inner diameter of coil (mm)	Coil spacing (mm)	Axial temperature difference (K)	Radial temperature difference (K)
1	100	100	50	0	95	4.5
2	200	100	50	0	90	11.5
3	300	100	50	0	85	21.5
4	400	100	50	0	82	34
5	500	100	50	0	78	50
6	200	50	50	0	95	13
7	200	75	50	0	90	17
8	200	100	50	0	85	21
9	200	125	50	0	82	26
10	200	150	50	0	79	29
11	200	100	40	0	70	23
12	200	100	45	0	80	22
13	200	100	50	0	85	21
14	200	100	55	0	90	20
15	200	100	60	0	93	19
16	200	100	50	−4	525	65
17	200	100	50	−2	180	25
18	200	100	50	0	90	22
19	200	100	50	2	14	20
20	200	100	50	4	−130	17



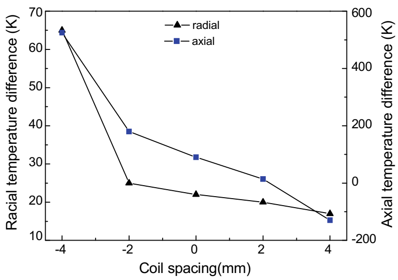
a. Temperature difference with exciting current



b. Temperature difference with power frequency



c. Temperature difference with coil inner diameter



d. Temperature difference with coil spacing

Fig. 3 Radial and axial temperature difference of the specimen with different factors

Through various investigations, the temperature difference of the 20 cases is presented in Fig. 3. It can be clearly observed that all the four factors have obvious influences on the radial and axial temperature difference of the specimen. The tendency of radial and axial temperature difference is opposite for the exciting current, power frequency and coil inner diameter. The larger exciting current, higher power frequency and smaller inner distance of coil lead to shorter heating time, causing shorter depth of penetration and smaller heat loss at the end, which means radial temperature difference gets larger and axial temperature difference gets smaller. When the coil spacing is sparse in the middle and dense at the end, it is beneficial to decrease the temperature difference due to the less heat loss at the end. However, if the coil spacing is over sparse in the middle, the temperature difference becomes negative, meaning the temperature in the middle is lower than that at the end, which has a negative effect on the temperature uniformity.

Results Based on Fuzzy Gray Relational Analysis

The results above have shown that the investigated factors have various impacts on temperature difference. As a result, Fuzzy Gray Relational Analysis, an improved method based on the gray relational theory, is adopted in this work.

Firstly, the axial and radial temperature difference under various conditions is considered as reference matrix $y_1(k)$ and $y_2(k)$. Exciting current, power frequency, coil inner diameter and coil spacing are taken to be the element $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$ in comparison matrix, respectively. The reference matrix and comparison matrix are processed by (3).

Secondly, the cosine value of fuzzy membership is calculated by (4). Figure 4 illustrates the fuzzy membership grades of four factors to the temperature difference. It indicates that the fuzzy membership grades of the four factors have a huge difference. For axial temperature difference, the fuzzy membership grade of coil inner diameter is the largest, while the coil spacing is the smallest. Based on the analysis of (4), the higher the fuzzy membership grade, the better the similarity of the changing trends of the investigated factors and axial temperature difference, which means the coil inner diameter has the most noticeable effect on axial temperature difference, while the coil spacing has the lowest effect. Similarly, the exciting current has the most remarkable effect on radial temperature difference, while the coil spacing has the lowest effect.

Thirdly, the Euclidean gray relational grade is computed by (5)–(8). In (8), the weights are considered to be equal due to the mutual independence of the cases. Figure 5 depicts the Euclidean gray relational grades of the four factors to the temperature difference. It shows that the Euclidean gray relational grades of the four factors have a small difference. Based on the analysis of (5)–(8), it still can be attributed to that the coil inner diameter has great effect on axial and radial temperature difference.

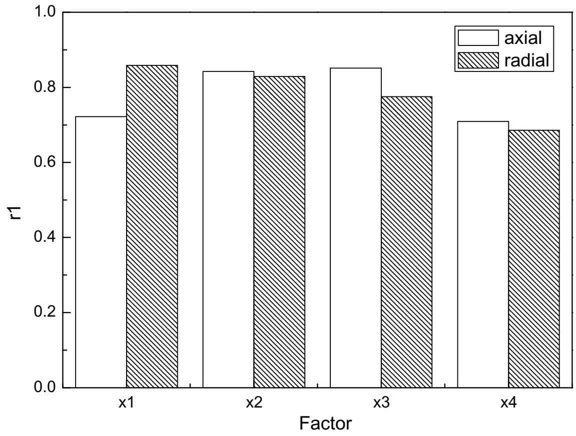


Fig. 4 Fuzzy membership grades

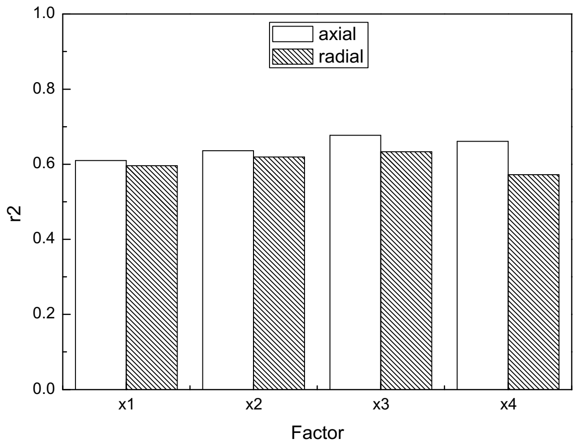


Fig. 5 Euclidean gray relational grades

Finally, the fuzzy gray relational grade is computed by (9), as shown in Fig. 6. It offers us comprehensive consideration for evaluating impacts of the four factors. It shows that, for axial temperature difference, the fuzzy gray relational grades of the four factors, exciting current, power frequency, coil inner diameter and coil spacing, are 0.6684, 0.7464, 0.7691 and 0.6853, respectively, meaning that the impacts of the four factors are ranked from the most important to the least important as coil inner diameter, coil spacing, power frequency and exciting current. While the ranking list of importance becomes exciting current, power frequency, coil inner diameter and coil spacing for radial temperature difference, since the fuzzy gray relational grades are 0.7389, 0.7311, 0.7078 and 0.6315, respectively. As a result,

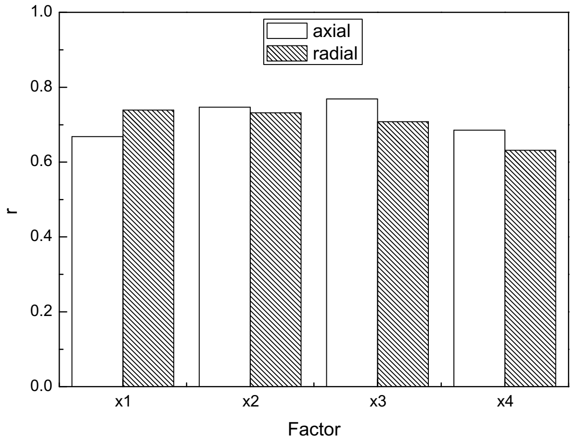


Fig. 6 Fuzzy gray relational grades

coil inner diameter has the most important effect on the axial temperature difference. The exciting current has the most important effect on the radial temperature difference, while it is not important for the axial temperature difference. This work provides us great reference value for optimizing the temperature uniformity of the specimen in induction heating process. As few studies investigated the importance extent of the influencing factors before, the results will be instructive and provide great reference for further research of induction heating.

Conclusions

In this work, numerical investigation of induction heating process was conducted. The single-factor experimental design and the Fuzzy Gray Relational Analysis (FGRA) were employed to evaluate the impacts of four electromagnetic parameters on temperature uniformity. As a result, the major conclusions are summarized as follows:

(1) The larger exciting current, the higher power frequency and the smaller inner distance of coil led to larger radial temperature difference and smaller axial temperature difference. Also, it was beneficial to the temperature uniformity of specimen to make the coil spacing sparser in the middle and denser at the end properly.

(2) The fuzzy membership grades and the Euclidean gray relational grades both showed that coil inner diameter had the most noticeable effect on axial temperature difference, while coil spacing had the lowest effect on radial temperature difference.

(3) For axial temperature difference of specimen, the impacts of the four factors were ranked from the most important to the least important as coil inner diameter, coil spacing, power frequency and exciting current, since the fuzzy gray relational grades were 0.7691, 0.7464, 0.6853 and 0.6684, respectively. While for radial temperature difference, the ranking list of importance became exciting current, power frequency, coil inner diameter and coil spacing, since the fuzzy gray relational grades were 0.7389, 0.7311, 0.7078 and 0.6315, respectively

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