



DOA Estimation of Underwater Acoustic Signals Using Non-uniform Linear Arrays

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Abstract. This paper proposes to use the multiple signal classification (MUSIC) algorithm applied to non-uniform linear array (NLA) to estimate the direction of arrival (DOA) for passive sonar systems. The performance of the DOA estimation obtained by the MUSIC algorithm is investigated by applying different configurations of the antenna array parameters. For the purpose of comparison, uniform linear arrays (ULAs) are also considered. With same number of elements and antenna parameter configuration, the simulation results show, that the NLA significantly provides a better performance in terms of DOA estimation than that obtained by the ULA. In addition, the accuracy and resolution of the DOA, as well as the number of detectable signal sources can be increased, if we enlarge the antenna spacing as well as the number of the antenna elements.

Keywords: Ambiguity · DOA estimation · MUSIC method
Non-uniform linear antenna array · Uniform linear array

1 Introduction

The direction of arrival (DOA) is an important parameter of passive sonar systems [1]. Most of traditional sonar systems usually employ antenna array or passive beam-forming to estimate the DOA of an acoustic signal [2, 3]. In addition, the DOA can be estimated by using the phase difference or the propagation delay difference between the received signals from two different acoustic signal sensors [4–7]. The conventional methods using amplitude, phase or propagation delay of the received signal are suitable for detecting the single source of narrow band acoustic signals. However, these methods are not suitable for the DOA estimation of acoustic signals from multiple sources.

In literature, the multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance technique (ESPRIT) are two well-known high-resolution DOA estimation algorithms. These techniques use the antenna array signal processing for detecting multi-wideband signals [8–10]. In addition, with ULA method, the angular ambiguity can be avoided, if the inter-element spacing d is designed not to be larger than a half of wavelength ($\lambda/2$) of the received signal. For high frequency signals, it is very difficult to set an acoustic sensor array with the condition of $d \leq \lambda/2$.

For understanding this difficulty, we consider the following example. If the frequency of the received acoustic signal is 200 kHz, then the corresponding wavelength in underwater propagation is 0.75 cm. Therefore, the ULA configuration with element spacing fulfilled condition of $d \leq \lambda/2$ is impractical for detecting the DOA of a high frequency acoustic signal. In addition, a small array leads to a lower accuracy and resolution of the DOA estimation [11–13]. For resolving this problem, several NLAs with suitable configurations will be investigated in this study.

In this paper, the MUSIC algorithm is simulated in Matlab environment to evaluate the DOA estimation accuracy and resolution of the NLAs, which can be applied in passive sonar systems. Based on the MUSIC algorithm, the performance of various array configurations for DOA estimation is evaluated. Firstly, the performance of the ULAs using different number of elements is studied. Secondly, by using the same antenna configuration, we compare the performance of the ULA with the NLA one. Finally, the various configurations of the NLA are investigated to find out the optimal antenna configuration for the DOA estimation in terms of accuracy and resolution, as well as, the number of signal sources could be detected. The simulation results show that the NLA provided more precise and better resolution than those obtained by the ULA. The higher SNR and wider NLA aperture, the higher accuracy and resolution can be obtained. These simulation results show that the NLA is promising for application in designing passive sonar systems.

The rest of this paper is organized as follows: Sect. 2 describes briefly the signal model and the MUSIC algorithm. In Sect. 3, the NLAs are compared with each other and with the ULAs in terms of the DOA estimation performance. Finally, conclusion is given in Sect. 4.

2 Signal Model and MUSIC Algorithm

2.1 Signal Model Description

For modelling the signals, a linear array is considered to investigate the MUSIC method. If the signals of far-field sources are mixed at each linear array sensor, then the outputs are modeled by a sum of steering vectors together with the additive noise:

$$\mathbf{x}(t) = \mathbf{A}(\theta) \cdot \mathbf{s}(t) + \mathbf{N}(t) \quad (1)$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_P)]$ and $\mathbf{a}(\theta_i)$ are the $M \times P$ steering matrix, and the $M \times 1$ steering vector corresponding the i^{th} source, respectively. The signal outputs of the antenna array, the corresponding signal amplitudes, and the additive noise are represented by the $M \times 1$ vector \mathbf{x} , the $P \times 1$ vector \mathbf{s} , and the $M \times 1$ vector \mathbf{N} , respectively. The number of sensor elements, M , is usually chosen to be larger than the number of sources, P . In the expression of the steering matrix, θ_i is the DOA of the i^{th} source in the azimuth. In our research work, the elevation angle is not considered.

If the array is an ULA, then the steering vector can be written as [13]:

$$\mathbf{a}(\theta_i) = [1 \quad e^{-j\pi v \sin \theta_i} \quad \dots \quad e^{-j\pi v(M-1) \sin \theta_i}]^T \quad (2)$$

where, $v = 2d/\lambda$, and d is the spacing between sensor elements.

If the array is an NLA, the steering vector can be described as [13]:

$$\mathbf{a}(\theta_i) = [1 \quad e^{-j2\pi \frac{d}{\lambda} \sin \theta_i} \quad \dots \quad e^{-j2\pi \frac{d}{\lambda} (M-1) \sin \theta_i}]^T \quad (3)$$

where, d_m is the distance between the m^{th} element and the first element.

2.2 A Brief Introduction of the MUSIC Algorithm

According to the signal model, the correlation matrix is represented as following [13]:

$$\mathbf{R}_{xx} = E[\mathbf{x} \cdot \mathbf{x}^H] = \mathbf{A} \cdot E[\mathbf{s} \cdot \mathbf{s}^H] \cdot \mathbf{A}^H + E[\mathbf{N} \cdot \mathbf{N}^H] \quad (4)$$

$$\mathbf{R}_{xx} = \mathbf{A} \cdot \mathbf{R}_{ss} \cdot \mathbf{A}^H + \mathbf{R}_{NN} \quad (5)$$

$$\mathbf{R}_{ss} = E[\mathbf{s} \cdot \mathbf{s}^H] \quad (6)$$

whereby, H denotes the Hermitian conjugate transpose. $E[\cdot]$ represents statistical averaging operation. \mathbf{R}_{ss} is the signal correlation matrix with the dimension of $P \times P$. \mathbf{R}_{NN} corresponds to noise correlation matrix with the size of $M \times M$.

The matrix \mathbf{R}_{xx} has P dominant eigenvalues whose corresponding eigenvectors \mathbf{E}_P represent a signal subspace. The steering vectors $\mathbf{a}(\theta_i)$ span the signal subspace. The remaining $M - P$ eigenvectors \mathbf{E}_N span the noise subspace. The noise subspace eigenvectors are orthogonal to the array steering vectors at the angles of arrival $\theta_1, \theta_2, \dots, \theta_P$. The MUSIC spatial spectrum can be given as:

$$\mathbf{P}_{MUSIC}(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{E}_N \mathbf{E}_N^H \mathbf{a}(\theta)} \quad (7)$$

3 A Comparison of NLA with ULA in Terms of DOA Estimation Performance

To evaluate the performance of the NLA versus ULA, we investigate the following cases: case (1) the ULA is considered for different number of elements (4, 5, ..., 8 elements); case (2) the NLA is simulated with the constant number of elements (e.g. 4 elements), however, the element distance is varied.

3.1 Simulation Results and Discussion of the ULA Performance

The Fig. 1a shows the simulation results in terms of the spectrum power of the received signal obtained by the MUSIC algorithm for the case 4-element and 7-element array, where the inter-element spacing d is $\lambda/2$. The system is simulated at the condition of SNR = 10 dB. It can be seen, that the ULA with 4 elements can detect up to 3 signal sources (full curve line), whereas the ULA with 7 elements can detect up to 6 signal sources (dash curve line).

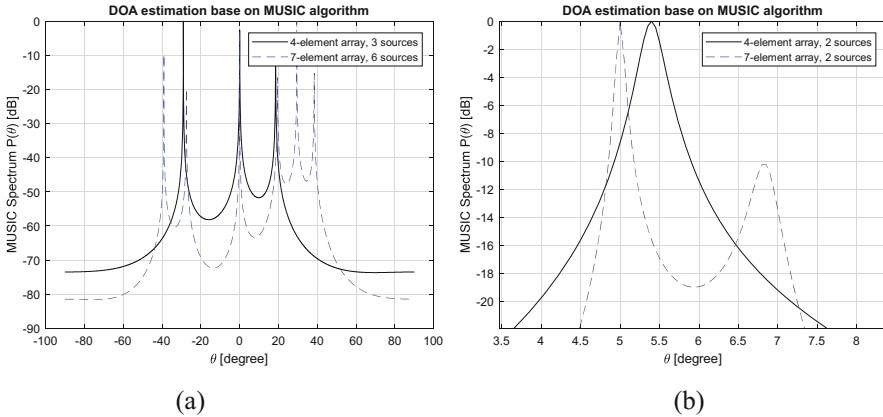


Fig. 1. The MUSIC spectrum function obtained using four-element array (full line), and 7-element array (dash line) with inter-element spacing $d = \lambda/2$; SNR = 10 dB; The simulation results are depicted for (a) maximum number of signal sources, which can be detected; (b) the DOA resolution possibility obtained by the ULAs with different number of elements

Figure 1b validated the DOA resolution possibility of array. The 4-element ULA (full curve line) cannot distinguish two signal sources in the conditions of DOA = 5° and 7° , while the 7-element ULA (dash curve line) can estimates the DOAs of these two sources with relatively high accuracy. It can be concluded that a larger number of sensor elements provides better a DOA resolution.

Regarding to the DOA estimation accuracy of the ULA array, the ULAs with different number of elements were considered. Specifically, the ULAs with 4, 5, 6 and 8 elements were chosen for simulation and comparison. Figure 2 illustrated the dependence of DOA accuracy on the SNR values. It is obvious that the ULA with 4 elements gives the lower accuracy than other array, and the ULA with 8 elements provides the higher accuracy than other array. It also demonstrated, that the larger number of sensor elements as well as the wider aperture of array, the higher accuracy of estimated DOA can be obtained.

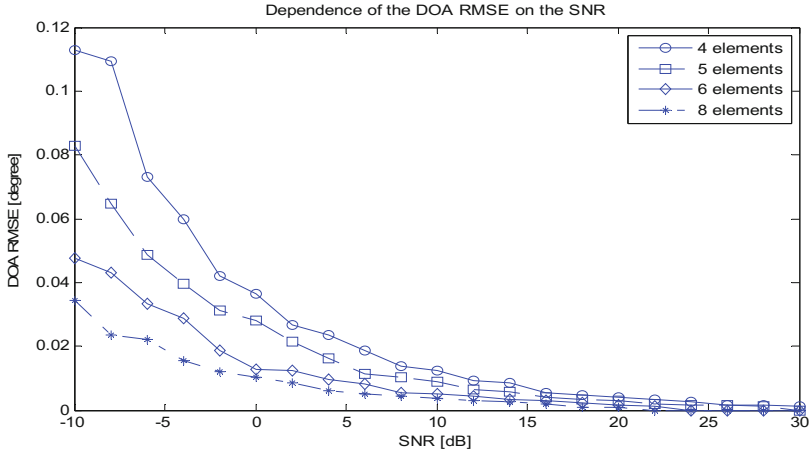


Fig. 2. The obtained DOA RMSE versus the SNR

3.2 Simulation Results and Discussion of the 4-Element NLA Performance

This section investigates the performance of 4-element array configured by different inter-element spacing. We conducted several trials. In the first trial, three array configurations of $\mathbf{d}_1 = [0 \ 1 \ 2 \ 3] \lambda/2$, $\mathbf{d}_2 = [0 \ 2 \ 4 \ 6] \lambda/2$ and $\mathbf{d}_3 = [0 \ 3 \ 6 \ 9] \lambda/2$ are compared to each other. The first trial simulation result plotted in Fig. 3a shows, that the ULAs generates ambiguity of the estimated DOA, if the inter-element spacing is greater than a half of wavelength. Particularly, the first array \mathbf{d}_1 is estimated appropriately unambiguous DOA = 19°; the second array \mathbf{d}_2 with $d = 2 \lambda/2$ is estimated appropriately ambiguous DOA = 19° and 42°; and the third array \mathbf{d}_3 with $d = 3 \lambda/2$ provides appropriately three DOA = -20°, 19° and 83°.

The second simulation trial shows a comparison in performance between NLA and ULA. Following that, three arrays are chosen to simulate and compare with array \mathbf{d}_1 . The three arrays were $\mathbf{d}_4 = [0 \ 1 \ 3 \ 8] \lambda/2$, $\mathbf{d}_5 = [0 \ 3 \ 8 \ 13] \lambda/2$ and $\mathbf{d}_6 = [0 \ 9 \ 12 \ 16] \lambda/2$. These chosen configurations are satisfied the condition of the remainder theorem [14]. The array \mathbf{d}_6 is chosen following the principle of $[0 \ m^2 \ m \cdot n \ n^2] \lambda/2$, where m and n are integer numbers whose greatest common divisor is 1 and $m < n$ [4, 5]. A signal is emitted from one source, which is placed in the direction 9° with SNR = 10 dB. The MUSIC spectrum for this case is shown in Fig. 3b.

Figure 3b shows that NLAs achieved higher accuracy than the ULA in case of same number of sensor elements. To evaluate the accuracy of different array configurations Monte-Carlo simulation is performed with 10000 trials for each array. The RMSEs of the DOA estimation for each array were shown in Fig. 4a.

Figure 4a indicates that three NLAs \mathbf{d}_4 , \mathbf{d}_5 and \mathbf{d}_6 have better RMSEs than the ULA \mathbf{d}_1 . The NLA \mathbf{d}_6 has highest accuracy, the second is NLA \mathbf{d}_5 and the third is \mathbf{d}_4 . This is due to the difference of array aperture. The NLA \mathbf{d}_4 has aperture equal to $8 \lambda/2$, the

NLA \mathbf{d}_5 has aperture $13 \lambda/2$ and the aperture of the NLA \mathbf{d}_6 is $16 \lambda/2$. So, the NLA has larger aperture could estimate DOA with more accuracy.

In the next simulation trial, the uncorrelated narrowband signals incoming from two DOAs 21° and 29° are considered. Based on the MUSIC spectrum shown in Fig. 4b, we can conclude, that the NLA using MUSIC method could resolve two uncorrelated signals in the DOA 21° and 29° with $\text{SNR} = 10 \text{ dB}$.

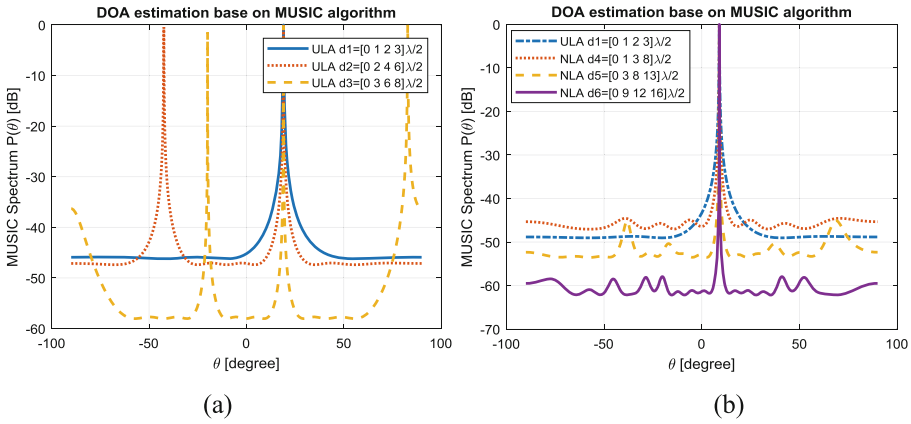


Fig. 3. (a) Ambiguity of DOA estimation using MUSIC algorithm for ULA with $d > \lambda/2$; (b) The MUSIC spectrum for various NLAs ($\text{SNR} = 10 \text{ dB}$)

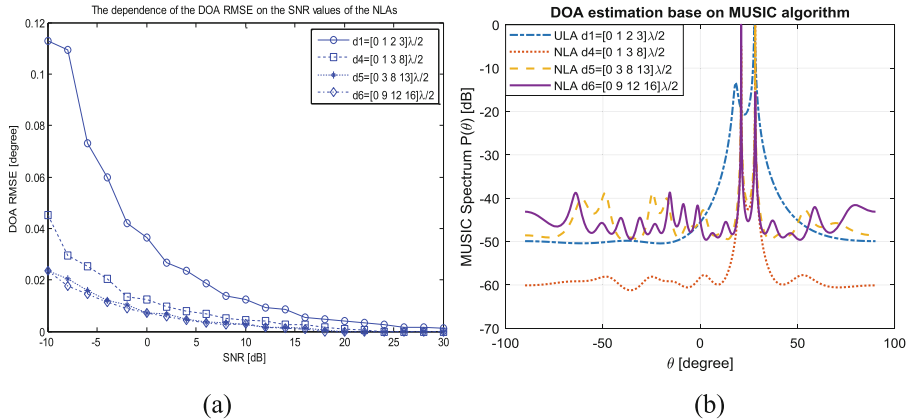


Fig. 4. (a) The dependence of the DOA RMSE on the SNR values of the NLA; (b) The DOA resolution possibility of the NLAs ($\text{SNR} = 10 \text{ dB}$)

In the following, we compare the probability of DOA resolution for different configurations. The probability of DOA resolution represents the possibility of resolution of two close signal sources in dependence on the SNR values. In the simulations,

the DOAs of -2° and 2° are considered. The Monte-Carlo simulations conducted for 10000 trials. This comparison results are illustrated in Fig. 5. It reveals that the NLAs has a higher probability of DOA resolution than that obtained by the ULA. Moreover, the array with wider aperture will provide a higher probability as well as higher accuracy in terms of the estimated DOA.

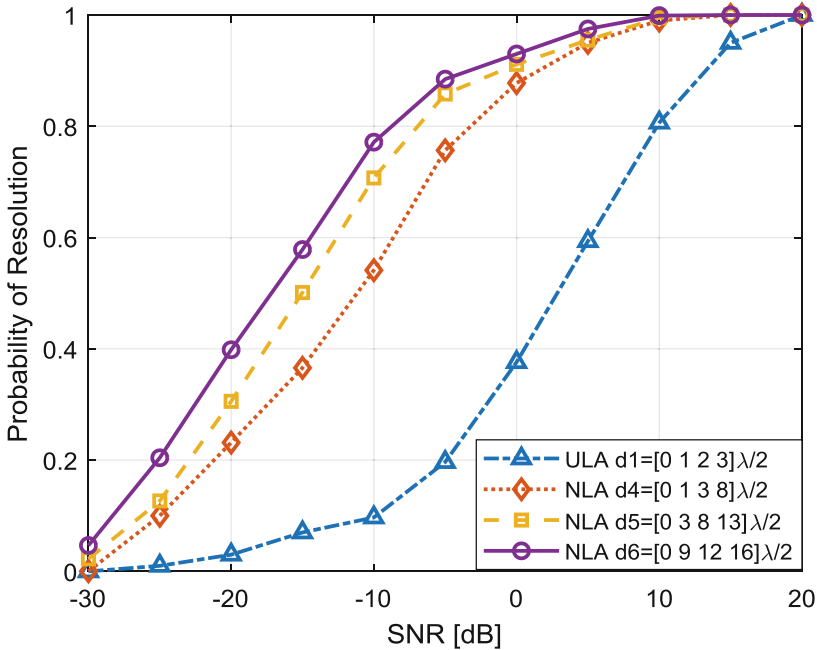


Fig. 5. The probability of DOA resolution of different NLAs

4 Conclusion

The paper analyzes the performance of the MUSIC algorithm, which is applied to estimate the DOA of acoustic signal. The DOA estimation possibility of the ULAs and NLAs are evaluated from points of view of the accuracy and resolution. The MUSIC algorithm is simulated to compare the performance of the various array configurations with the purpose of the DOA estimation. The simulation results show that the NLA provides more precise and better resolution than the ULA. The higher signal to noise ratio (SNR) and/or the wider aperture of NLA, the higher accuracy and resolution can be obtained. These simulation results are promising for developing and designing a NLA in passive sonar systems, in which the DOA of the acoustic signal can accurately be estimated.

Acknowledgment. This work has been partially supported by the project of Faculty of Communication and Radar, Naval Academy, Nha Trang City, Khanh Hoa Province, Vietnam.

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