Optimal Beamforming for Multiuser Secure SWIPT Systems (Invited Paper)

Yuqing Su and Derrick Wing Kwan Ng(✉)

School of Electrical Engineering and Telecommunications,
The University of New South Wales, Sydney, Australia
w.k.ng@unsw.edu.au

Abstract. In this paper, we study the beamforming design for simultaneous wireless information and power transfer (SWIPT) downlink systems. The design is formulated as a non-convex optimization problem which takes into account the quality of service (QoS) requirements of communication security and minimum harvested power. In particular, the proposed design advocates the dual use of energy signal to enable secure communication and efficient WPT. The globally optimal solution of the optimization problem is obtained via the semidefinite programming relaxation (SDR). Our simulation results show that there exists a non-trivial tradeoff between the achievable data rate and the total harvested power in the system. Besides, our proposed optimal scheme provides a substantial performance gain compared to a simple suboptimal scheme based on the maximum ratio transmission (MRT).

Keywords: SWIPT · Physical layer security · Energy beamforming

1 Introduction

With the rapidly growing number of wireless communication devices and applications such as automated control in smart cities, monitoring in e-health systems, and remote security sensing [1], it is expected that by 2020 there will be 50 billion wireless devices connected together via the Internet around the world [2]. As a result, the fifth-generation communication systems also target on enhancing mobile broadband (e.g. tactile internet), connecting massive Internet-of-Things (IoT) wireless devices, and enabling new mission-critical controls (e.g. autonomous vehicles) [3]. Among all 5G emerging technologies thriving to fulfill stringent quality of service (QoS) requirements, multiple-input multiple-output (MIMO) technology is a promising solution to reduce the energy consumption caused by the rocketing demand of ultra-fast data rate [4–6]. In addition, the extra degrees of freedom offered by multiple antennas enable an efficient interference management in wireless communication systems. However, due to high computational complexity at receivers, traditional MIMO architecture may not be suitable for portable devices. As an alternative solution, multi-user MIMO with a multiple-antenna transmitter serving multiple receivers equipped with single-antenna [7,8],

T. Q. Duong and N.-S. Vo (Eds.): INISCOM 2018, LNICST 257, pp. 1–14, 2019.
https://doi.org/10.1007/978-3-030-05873-9_1

has been proposed as it shifts the signal processing burden from receivers to the transmitter which allows simple designs and cheap receiver structure.

Recently, an increasing number of mobile devices such as wireless sensors for IoT applications is gaining their popularity among the industry and becoming essential parts of wireless communication networks [9]. In particular, most of these devices are battery-powered with finite energy storage capacity. Hence, the inconvenience/high-cost of battery charging or replacement are the major obstacles in realizing IoT implementation [9]. Consequently, energy harvesting technology is adopted as a viable solution to provide ubiquitous and self-sustainable networks. Although traditional energy harvesting technology, which collects energy from natural renewable energy sources (e.g. solar and tide), can enable self-sustainable communication networks to a certain extent, it is both climate-dependent and location-dependent, which makes renewable energy a perpetual but intermittent energy supply [10,11]. Therefore, directly integrating conventional energy harvesting technology into communication devices may result in unstable communication service. Instead of exploiting renewable energy for energy-limited systems, an emerging solution, RF-based energy harvesting communication [12,13], is considered as a practical approach and is served building block for sustainable wireless systems to unlock the potential of IoT networks.

1.1 Background

Proposed by Nikola Tesla back in the late nineteenth century, wireless power transfer (WPT) was implemented by a magnifying transmitter based on the Tesla coil transmitter [14]. Its ultimate goal was to broadcast wireless power to any location around the globe avoiding shortcomings of conventional cables such as being unaffordable and inconvenient to deploy [14]. Under massive impact of the industrial revolution in late 1800s, WPT was originally designed to apply on high-power machines. With public health concern about harmful electromagnetic radiation caused by large power emission from the transmit tower [1], progress on bringing WPT into practice was hindered. Besides, as antennas in reasonable size are required in practical systems to provide mobility for portable communication devices, the information signal is usually modulated at a high carrier frequency resulting in severe path loss, which leads to a small amount of received power at the receiver side. Therefore, low power transfer efficiency is one of the challenges in implementing WPT. Prevented by these two major challenges, WPT was not able to realize its further development until advancing silicon technology and wireless communication theory bring it back to life recently [1]. Therefore, collecting energy from background RF electromagnetic (EM) wave transmitted from ambient transmitters is feasible via advanced WPT technologies and the application of communication theory. In fact, various proof-of-concepts experiments and prototypes have been developed. For example, it has been shown that practical RF-based energy harvesting circuits are able to harvest microwatt to milliwatt of power over the range of several meters for a transmit power of 1 Watt at a carrier frequency of less than 1 GHz [15].

1.2 Communication Security

On the other hand, communication security is a critical issue that needed to be taken into account in wireless communication systems. Nowadays, most of the conventional cryptographic encryption methods are implemented in the application layer to ensure secure communication [16]. However, this technology relies on perfect secret key information protection and distribution which are not practical in some of wireless communication networks [1]. Furthermore, secret-key cryptography algorithms usually assume that potential eavesdroppers have bounded computational capabilities, which might pose a future threat on itself as computers with ultra-high computational capabilities would be the developing trend (e.g. quantum computers). In fact, the security issue is more prominent in RF-based energy harvesting communication systems. In particular, RF-based energy harvesting receivers are generally located closer to the transmitter than an information receiver for more efficient energy harvesting. Thus, such scenarios raise about communication security due to the broadcast nature of wireless channels and the relatively high transmit powers needed for SWIPT. Thus, the emerging and stringent QoS requirements on guaranteeing secure communication have drawn significant attention in SWIPT beamforming design [1]. For example, information-theoretic physical layer (PHY) security aims at protecting secure communication by making use of wireless communication channels' physical natures (e.g. channel fading, etc.) [17], has been proposed for SWIPT systems. In particular, in order to guarantee secure communication, an energy signal is generated deliberately not just for degrading the quality of eavesdroppers' channels, but also to help facilitate an efficient energy transfer to ERs, e.g. [18–25]. In this paper, we study the resource allocation algorithm design to enable secure SWIPT systems via multiple antennas.

The remainder of this paper is organized as follows: In Sect. 2, we introduce the SWIPT downlink system model. In Sect. 3, we formulate the resource allocation design as an optimization problem and solve it optimally. Section 4 presents the numerical performance results for the proposed optimal algorithm. In Sect. 5, we conclude the paper with a summary.

Notation

Key mathematical notations are given in Table 1. Boldface lower and capital case letters are used to denote vectors and matrices, respectively. $\text{Rank}(\mathbf{A})$, $\text{Tr}(\mathbf{A})$, and $\mathbf{A}^{\text{H}}$ are the rank, the trace, and Hermitian transpose of matrix $\mathbf{A}$, respectively. $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is a positive semi-definite matrix. $\mathbb{H}^{\text{N}}$ represents all Hermitian matrix sets. $\mathbb{C}^{\text{N}\times\text{M}}$ and $\mathbb{R}^{\text{N}\times\text{M}}$ represent all $N \times M$ sets with complex and real entries, respectively. The circularly symmetric complex Gaussian (CSCG) distribution is denoted by $\mathcal{CN}(\mathbf{m}, \mathbf{\Sigma})$ with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$; $\sim$ indicates "distributed as"; $\mathcal{E}\{\cdot\}$ denotes statistical expectation; $|\cdot|$ represents the absolute value of a complex scalar. $[x]^+$ stands for $\max\{0, x\}$.

Table 1. Nomenclature adopted in this paper.

Notation	Description
$\mathbf{h}$	Channel vector between the information receiver (IR) and the transmitter
$\mathbf{g}_j$	Channel vector between energy harvesting receiver (ER) j and the transmitter
$\mathbf{w}$	Information beamforming vector
$\mathbf{w}_{\mathrm{E}}$	Energy signal beamforming vector
σ^2_{ant}, σ^2_{s}	Antenna and signal processing noise power
N_{T}	Number of transmit antennas
$R^{\mathrm{Tol}}_{\mathrm{ER}}$	Maximum tolerable data rate
$P_{\max}$	Maximum transmit power of the transmitter
$P_{\min}$	Minimum required power transfer to ERs

2 System Model

In this paper, we focus on a downlink SWIPT system. There is a transmitter equipped with N_{T} antennas, one information receiver (IR), and J energy harvesting receivers (ERs). In particular, both the IR and ERs are single-antenna devices [17], cf. Fig. 1. In the following, it is assumed that both the transmitter and IR know the perfect channel state information (CSI) for resource allocation. Table 1 shows the nomenclature adopted in this paper.

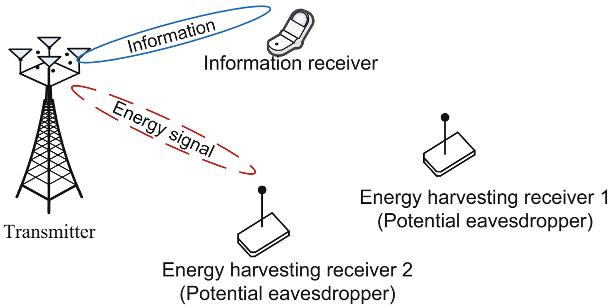

Fig. 1. A downlink SWIPT model with one IR and $J = 2$ ERs. For guaranteeing communication security, the ERs are also treated as potential eavesdroppers eavesdropping information transmitted from the transmitter to the IR.

In each time slot, the transmit signal vector $\mathbf{x}$ is given as

$$\mathbf{x} = \mathbf{w}\, s + \mathbf{w}_{\mathrm{E}}, \tag{1}$$

where $\mathbf{w}_{\mathrm{E}}$ is a pseudo-random energy signal and modeled as a complex Gaussian random vector with zero-mean and covariance matrix $\mathbf{W}_{\mathrm{E}}$, i.e., $\mathbf{w}_{\mathrm{E}} \sim$

$\mathcal{CN}(\mathbf{0}, \mathbf{W}_\mathrm{E})$, $\mathbf{w}$ is the beamforming vector of the information signal, and s is the information signal. Hence, the signals received at IR and ER $j \in \{1, \ldots, J\}$ are given by

$$y = \mathbf{h}^H(\mathbf{w}s + \mathbf{w}_\mathrm{E}) + n \text{ and} \tag{2}$$

$$y_{\mathrm{ER}_j} = \mathbf{g}_j^H(\mathbf{w}s + \mathbf{w}_\mathrm{E}) + n_{\mathrm{ER}_j}, \forall j \in \{1, \ldots, J\}, \tag{3}$$

respectively, where $\mathbf{h}$ and $\mathbf{g}_j$ are channel vectors between the transmitter and IR, transmitter and ER j, respectively, capturing the impact of small scale fading and large scale fading of the channels. Variables n, n_{ER_j} are additive white Gaussian noise (AWGN) of IR and ER j from the receiving antenna, respectively, with zero-mean and variance σ_s^2 and σ_j^2, respectively. We also assume without loss of generality that $\mathbb{E}\{|s|^2\} = 1$.

2.1 Energy Harvesting Model

In practice, an energy harvesting circuit consists of various hardware components, such as bandpass filter, receifying circuit, etc., cf. Fig. 2. In the SWIPT literature [26–32], for simplicity, the total harvested power at ER j is typically modelled by a linear equation[1]:

$$\Phi_{\mathrm{ER}_j}^{\mathrm{Linear}} = \eta_j P_{\mathrm{ER}_j} \quad \text{where} \tag{4}$$

$$P_{\mathrm{ER}_j} = \mathbb{E}\{|\mathbf{g}_j^H \mathbf{x}|^2\} = \mathrm{Tr}\Big((\mathbf{W}_\mathrm{E} + \mathbf{w}\mathbf{w}^H)\mathbf{G}_j\Big) \tag{5}$$

denotes the harvested power from the channel, $\eta_j \in [0, 1]$ is the RF-to-DC power conversion efficiency, and $\mathbf{G}_j = \mathbf{g}_j\mathbf{g}_j^H$.

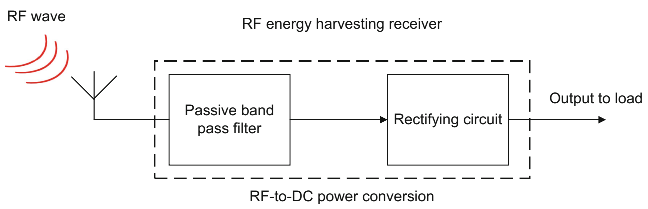

Fig. 2. Block diagram of an ER.

[1] Recently, various non-linear energy harvesting models have been proposed in the literature, e.g. [33,34]. However, in this paper, we adopt the conventional linear energy harvesting model for the ease of illustration.

2.2 Achievable Rate and Secrecy Rate

Given perfect CSI at the receiver, the achievable rate (bit/s/Hz) between the transmitter and the IR is given by

$$R = \log_2\Big(1+\frac{\mathbf{w}^H\mathbf{H}\mathbf{w}}{\sigma_{\mathrm{s}}^2}\Big), \tag{6}$$

where $\mathbf{H}=\mathbf{h}\mathbf{h}^H$. Note that since the pseudo-random energy signal, $\mathbf{w}_{\mathrm{E}}$, is a deterministic sequence which is known by the IR, its impact on the achievable rate, i.e., $\mathrm{Tr}(\mathbf{H}\mathbf{W}_{\mathrm{E}})$, has been canceled with interference cancelation techniques before decoding the desired signal.

On the other hand, the capacity between the transmitter and ER j for decoding the signal of the IR can be expressed as

$$R_j^{\mathrm{ER}} = \log_2\Big(1+\frac{\mathrm{Tr}(\mathbf{W}\mathbf{G}_j)}{\mathrm{Tr}(\mathbf{G}_j\mathbf{W}_{\mathrm{E}})+\sigma_{\mathrm{s}_j}^2}\Big). \tag{7}$$

Thus, the achievable secrecy rate of IR is given by

$$R_{\mathrm{sec}} = \Big[R-\max_{\forall j}\{R_{\mathrm{ER}_j}\}\Big]^+. \tag{8}$$

3 Problem Formulation

The considered system design objective is to maximize the achievable rate of the IR while guaranteeing secure communication and facilitating efficient wireless power transfer. To this end, the resource allocation design is formulated as the following optimization problem:

Resource Allocation Design:

$$\begin{aligned}
\underset{\mathbf{W},\mathbf{W}_{\mathrm{E}}\in\mathbb{H}^{N_{\mathrm{T}}}}{\mathrm{maximize}}\quad & R=\log_2\Big(1+\frac{\mathrm{Tr}(\mathbf{W}\mathbf{H})}{\sigma_{\mathrm{s}}^2}\Big) \\
\mathrm{s.t.}\quad & \mathrm{C1}:\ \mathrm{Tr}((\mathbf{W}+\mathbf{W}_{\mathrm{E}})\mathbf{G})\geq P_{\min}, \\
& \mathrm{C2}:\ \mathrm{Tr}(\mathbf{W}+\mathbf{W}_{\mathrm{E}})\leq P_{\max}, \\
& \mathrm{C3}:\ R_j^{\mathrm{ER}}\leq R_{\mathrm{tol}}^{\mathrm{ER}}, \forall j\in\{1,\dots,J\}, \\
& \mathrm{C4}:\ \mathbf{W}\succeq\mathbf{0}, \\
& \mathrm{C5}:\ \mathbf{W}_{\mathrm{E}}\succeq\mathbf{0}, \\
& \mathrm{C6}:\ \mathrm{Rank}(\mathbf{W})\leq 1.
\end{aligned} \tag{9}$$

where $\mathbf{G}=\sum_{j=1}^J\mathbf{G}_j\mathbf{G}_j^H$. Constants $P_{\min}$ and $P_{\max}$ in constraints C1 and C2 are the minimum required total harvested power and the maximum transmit power budget offered by transmitter, respectively. Constant $R_{\mathrm{ER}}^{\mathrm{Tol}} > 0$ in C3 is the maximum tolerable data rate which restricts the capacity of ER j if it attempts to decode the message of the IR. In practice, the transmitter sets $R_{\mathrm{ER}}^{\mathrm{Tol}} \to 0$, to ensure secure communication. If the above optimization problem is feasible, the adopted problem formulation guarantees that the achievable secrecy rate is bounded below by $R_{\mathrm{sec}} \geq R - R_{\mathrm{ER}}^{\mathrm{Tol}} > 0$.

3.1 Optimal Solution

The optimization problem in (9) is non-convex due to the constraint C3 and the rank-one matrix constraint C6. In general, there might be multiple local maximums [35] in a non-convex optimization problem. Although exhaustive search can be used to obtain the globally optimal solution, the incurred computational complexity increases exponentially with respect to the numbers of transmit antennas and the ERs. In order to design a computational efficient resource allocation, we apply the SDP relaxation to the considered problem. In particular, we remove constraint C6 and after some mathematical manipulation which yields:

Problem Transformation

$$\begin{aligned} \underset{\mathbf{W},\mathbf{W}_{\mathrm{E}}\in\mathbb{H}^{N_{\mathrm{T}}}}{\text{minimize}} \quad & -\mathrm{Tr}(\mathbf{W}\mathbf{H}) \qquad (10)\\ \text{s.t.} \quad & \mathrm{C1}: P_{\min} - \mathrm{Tr}((\mathbf{W}+\mathbf{W}_{\mathrm{E}})\mathbf{G}) \leq \mathbf{0},\\ & \mathrm{C2}: \mathrm{Tr}(\mathbf{W}+\mathbf{W}_{\mathrm{E}}) - P_{\max} \leq \mathbf{0},\\ & \mathrm{C3}: \mathrm{Tr}(\mathbf{W}\mathbf{G}_j) - (2^{R_{\mathrm{tol}}^{\mathrm{ER}}} - 1)\,\mathrm{Tr}(\mathbf{G}_j\mathbf{W}_{\mathrm{E}} + \sigma_{\mathrm{s}_j}^2) \leq \mathbf{0}, \forall j,\\ & \mathrm{C4}: -\mathbf{W} \preceq \mathbf{0},\\ & \mathrm{C5}: -\mathbf{W}_{\mathrm{E}} \preceq \mathbf{0},\\ & \mathrm{C6}: \mathrm{Rank}(\mathbf{W}) \leq 1. \end{aligned}$$

Note that the transformed problem is convex and can be solved efficiently and optimally via standard convex program solvers, such as CVX. Yet, the rank constraint relaxation may not be tight $\mathrm{Rank}(\mathbf{W}) > 1$ occurs. Therefore, we reveal the tightness of the adopted SDP relaxation in (10) in the following theorem:

Theorem 1. *If channels* $\mathbf{H}$ *and* $\mathbf{G}_j$ *are statistically independent and the transformed problem in (10) is feasible, then the optimal information beamforming matrix* $\mathbf{W}$ *is at most rank-one with probability one, i.e.,* $\mathrm{Rank}(\mathbf{W}) \leq 1$.

Proof: Please refer to the appendix.

Thus, the adopted SDP relaxation is tight as long as the channel conditions in Theorem 1 are fulfilled. Therefore, the considered information beamforming is optimal for the maximization of achievable data rate.

3.2 Suboptimal Solution

To further reduce the computational complexity, suboptimal scheme is proposed in this section. In particular, maximum ratio transmission (MRT) with respect to the direction of IR is adopted for information transmission, i.e., $\mathbf{w} = \frac{\mathbf{h}^*}{\|\mathbf{h}\|}$. Then, the transmit power of $\mathbf{w}$ and the covariance matrix of $\mathbf{W}_\mathrm{E}$ are optimized to maximize the achievable data rate of the system subject to the same constraint set as in (10).

Table 2. Parameters in simulation.

Centre frequency of carrier signal	915 MHz
Bandwidth	200 kHz
Gain of transceiver antenna	10 dBi
Transmit antenna number N_T	$3, 6, 9$
Noise power σ^2	-95 dBm
Maximum transmit power $P_{\max}$	1 W
$\mathbf{G}_j$ fading distribution	Ricean with Ricean factor 3 dB
$\mathbf{h}$ fading distribution	Rayleigh
$R_\mathrm{ER}^\mathrm{Tol}$	0.1375 bit/s/Hz
η_j RF-to-DC power conversion efficiency	0.5

4 Simulation Results

In this section, we demonstrate the performance of the proposed optimal resource allocation via simulations. Unless further specified, the important simulation parameters are listed in Table 2. Figure 3 shows the non-trivial trade-off between the total system data rate and the total system harvested power for the SWIPT model under proposed optimal beamforming scheme. In general, the area enclosed by the curve of a certain transmit antenna number is the achievable region. In other words, all the points lie inside or on the curve can be achieved by tuning the associated system parameters of the optimal scheme. By comparing intersecting points of both y and x-axis for different transmit antenna numbers, it can be observed that with increasing number of transmitter antennas, the maximum total system data rate of IR and the maximum total system harvested power of ERs increase due to extra spatial degrees of freedom supplied by multiple transmit antennas which improve the accuracy in beamforming. On the other hand, by comparing the two tradeoff regions (i.e., region between N_T $= 3$ and $N_\mathrm{T} = 6$, region between $N_\mathrm{T} = 6$ and $N_\mathrm{T} = 9$), it is manifest that with increasing number of transmitter antennas, the increasing rate of the enlarged region decreases due to channel hardening [36]. For comparison, we also show the performance of the suboptimal MRT-based transmission scheme. It can be seen that the suboptimal scheme cannot achieve the maximum system data rate as communication security is taken into account. Also, the suboptimal scheme can only achieve a smaller tradeoff region compared to the proposed optimal scheme due to its less flexibility in beamforming.

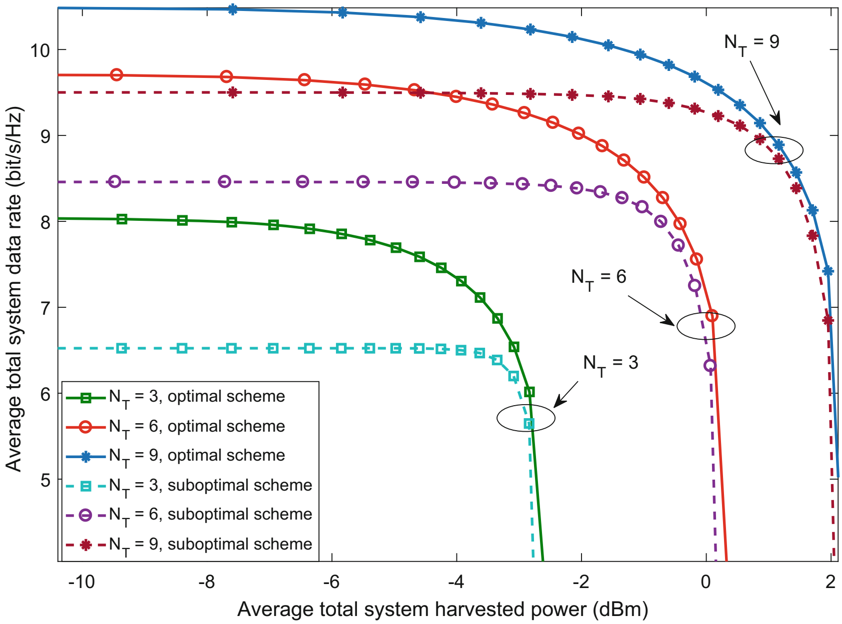

Fig. 3. Average total system data rate (bit/s/Hz) versus average total system harvested power (dBm).

Figure 4 shows the average data rate versus the total transmit power budget for different numbers of ERs of the proposed optimal scheme. It can be observed that for a given amount of total transmit power budget, the average total system data rate decreases when there are more ERs in the system. In fact, when there are more ERs in the system, both the QoS requirements on communication security and minimum required harvested power become more stringent. Particularly, the transmitter is forced to steer the direction of information signal towards the ERs. This will decrease the received signal strength of the desired signal at the IR. Besides, the transmitter would also increase the transmit power of energy signal to neutralize the higher potential of information leakage, which leads to a further reduction in the data rate. Furthermore, the proposed suboptimal scheme is able to guarantee both the QoS requirements of minimum required harvested power and communication security.

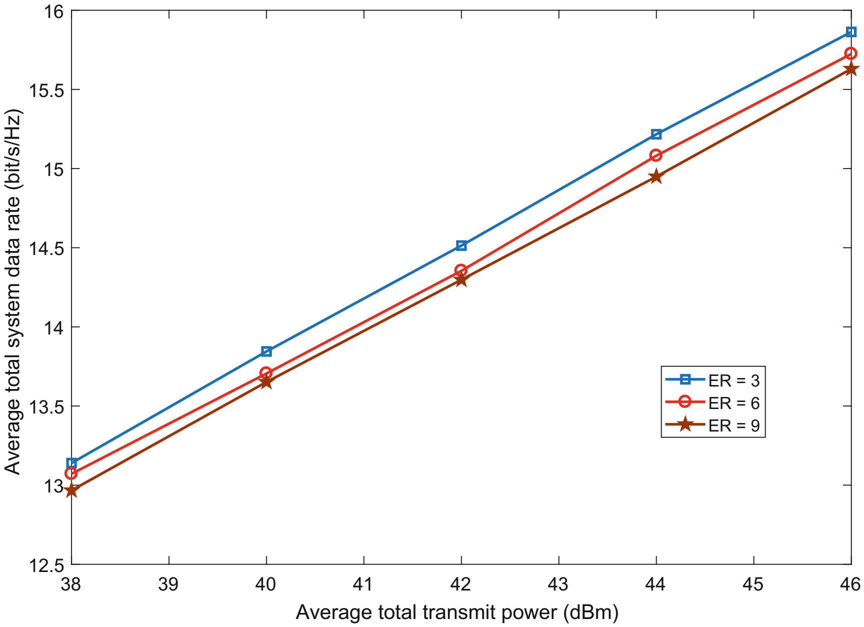

Fig. 4. Average total system data rate (bit/s/Hz) versus the total transmit power (dBm).

5 Conclusions

In this paper, we designed a resource allocation algorithm to enable secure SWIPT which is of fundamental importance for RF-based energy harvesting communication networks. In particular, the algorithm design was formulated as a non-convex optimization problem for the maximization of the achievable data rate of the IR while taking into account the QoS of efficient WPT and communication security. By exploiting the SDP relaxation, we solve the non-convex optimization problem optimally. Numerical results showed the potential gains in both data rate and harvested power enabled by the proposed optimization and its robustness against eavesdropping.

6 Appendix – Proof of Theorem 1

It can be verified that the transformed optimization problem in (10) is convex and satisfies the Slater's constraint qualification, hence, strong duality holds. In other words, solving its dual problem is equivalent to solving the primal problem. In this section, we intend to prove Theorem 1 via first defining the Lagrangian function:

$$\begin{aligned} L = & -\mathrm{Tr}(\mathbf{WH}) - \lambda_{\mathrm{C1}}\,\mathrm{Tr}((\mathbf{W}+\mathbf{W}_{\mathrm{E}})\mathbf{G}) + \lambda_{\mathrm{C2}}\,\mathrm{Tr}(\mathbf{W}+\mathbf{W}_{\mathrm{E}}) \\ & + \lambda_{\mathrm{C3}}\,\mathrm{Tr}(\mathbf{W}\mathbf{G}_j) - \mathrm{Tr}(\mathbf{YW}) + \varDelta, \end{aligned} \tag{11}$$

where Δ represents the variables and the constants that are independent of $\mathbf{W}$ and therefore irrelevant in the proof. $\mathbf{Y}$ and λ_{C1}, λ_{C2}, λ_{C3} are dual variables related to the constraints C4 and C1, C2, C3, respectively. Now, we can express the dual problem of (10) as

$$\max_{\mathbf{Y},\lambda_{\mathrm{C1}},\lambda_{\mathrm{C2}},\lambda_{\mathrm{C3}}} \ \min_{\mathbf{W},\mathbf{W}_{\mathrm{E}}\in\mathbb{H}^{N_{\mathrm{T}}}} \quad L. \tag{12}$$

Then, we study the structure of $\mathbf{W}$ via applying the Karush-Kuhn-Tucker (KKT) conditions:

$$\mathbf{Y} \succeq \mathbf{0}, \lambda_{\mathrm{C1}}, \lambda_{\mathrm{C2}}, \lambda_{\mathrm{C3}} \geq 0, \tag{13}$$
$$\mathbf{Y}\mathbf{W} = \mathbf{0}, \tag{14}$$
$$\mathbf{Y} = -\mathbf{H} + \mathbf{B}, \tag{15}$$

where (14) is obtained by taking the derivative of the Lagrangian function with respect to $\mathbf{W}$ and $\mathbf{B} = -\lambda_{\mathrm{C1}}\mathbf{G} + \lambda_{\mathrm{C2}}\mathbf{I} + \lambda_{\mathrm{C3}}\mathbf{G}_j$. Equation (13) is the complementary slackness property which implies that the columns of matrix $\mathbf{W}$ fall into the null-space spanned by $\mathbf{Y}$ for $\mathbf{W} \neq \mathbf{0}$. Hence, if we can prove that $\mathrm{Rank}(\mathbf{Y}) \geq N_{\mathrm{T}} - 1$, the optimal beamforming matrix $\mathbf{W}$ is a rank-one matrix or a zero matrix. Now, we study the structure of $\mathbf{Y}$ via examining (15). First, we prove by contradiction that $\mathbf{B}$ is a positive definite matrix with probability one. Suppose not, $\mathbf{B}$ is a positive semi-definite matrix. Then, there exist at least one zero eigenvalue and we denote the associated eigenvector as $\mathbf{v}$. Without loss of generality, we create a matrix $\mathbf{V} = \mathbf{v}\mathbf{v}^H$ from the eigenvector. By multiplying both sides of (15) with $\mathbf{V}$ and applying the trace operation, we obtain

$$\mathrm{Tr}(\mathbf{Y}\mathbf{V}) = -\,\mathrm{Tr}(\mathbf{H}\mathbf{V}) + \mathrm{Tr}(\mathbf{B}\mathbf{V}) = -\,\mathrm{Tr}(\mathbf{H}\mathbf{V}). \tag{16}$$

Since $\mathbf{H}$ and $\mathbf{G}_j$ are statistically independent, we have $\mathrm{Tr}(\mathbf{H}\mathbf{V}) > 0$. This leads to contradiction as $\mathrm{Tr}(\mathbf{Y}\mathbf{V}) \geq 0$. Hence, matrix $\mathbf{B}$ is a positive definite matrix[2], i.e., $\mathrm{Rank}(\mathbf{B}) = N_{\mathrm{T}}$. To further proceed the proof, we introduce the following rank inequality:

Lemma 1. *Let* $\mathbf{A}$ *and* $\mathbf{B}$ *be two matrices with same dimension. The inequality of matrix* $\mathrm{Rank}(\mathbf{A}+\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A}) - \mathrm{Rank}(\mathbf{B})$ *holds.*

Proof: By basic rule of inequality for the rank of matrix, $\mathrm{Rank}(\mathbf{A})+\mathrm{Rank}(\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A}+\mathbf{B})$ with both matrices of same dimension. Thus we have $\mathrm{Rank}(\mathbf{A}+\mathbf{B})+\mathrm{Rank}(-\mathbf{B}) \geq \mathrm{Rank}(\mathbf{A})$. Since $\mathrm{Rank}(\mathbf{B}) = \mathrm{Rank}(-\mathbf{B})$, the lemma is proved. □

[2] It can be verified that matrix $\mathbf{B}$ is not a negative definite or a negative semi-definite matrix.

Now, we apply Lemma 1 on (14) which yields:

$$\begin{aligned}\text{Rank}(\mathbf{Y}) &= \text{Rank}(-\mathbf{Y}) = \text{Rank}(-\mathbf{B}+\mathbf{H})\\ &\geq \text{Rank}(-\mathbf{B}) - \text{Rank}(\mathbf{H}) = N_{\text{T}} - 1\end{aligned} \tag{17}$$

As $\text{Rank}(\mathbf{Y}) \geq N_{\text{T}} - 1$, we have $\text{Rank}(\mathbf{W}) \leq 1$ which completes the proof. □

References

1. Ng, D.W.K., Leng, S., Schober, R.: Multiple Antennas and Beamforming for SWIPT Systems, pp. 170–216. Cambridge University Press, Cambridge (2016)
2. Ding, Z., et al.: Application of smart antenna technologies in simultaneous wireless information and power transfer. IEEE Commun. Mag. **53**(4), 86–93 (2015)
3. Wong, V., Schober, R., Ng, D.W.K., Wang, L.-C.: Key Technologies for 5G Wireless Systems. Cambridge University Press, Cambridge (2017)
4. Goldsmith, A.: Wireless Communications. Cambridge University Press, Cambridge (2005)
5. Ng, D.W.K., Lo, E.S., Schober, R.: Energy-efficient resource allocation in OFDMA systems with large numbers of base station antennas. IEEE Trans. Wirel. Commun. **11**(9), 3292–3304 (2012)
6. Wu, Q., Li, G.Y., Chen, W., Ng, D.W.K., Schober, R.: An overview of sustainable green 5G networks. IEEE Wirel. Commun. **24**(4), 72–80 (2017)
7. Marzetta, T.: Noncooperative cellular wireless with unlimited numbers of base station antennas. IEEE Trans. Wirel. Commun. **9**, 3590–3600 (2010)
8. Ng, D.W.K., Lo, E.S., Schober, R.: Energy-efficient resource allocation in multi-cell OFDMA systems with limited backhaul capacity. IEEE Trans. Wirel. Commun. **11**, 3618–3631 (2012)
9. Zorzi, M., Gluhak, A., Lange, S., Bassi, A.: From today's INTRAnet of things to a future INTERnet of Things: a wireless- and mobility-related view. IEEE Wirel. Commun. **17**, 44–51 (2010)
10. Ahmed, I., Ikhlef, A., Ng, D.W.K., Schober, R.: Power allocation for an energy harvesting transmitter with hybrid energy sources. IEEE Trans. Wirel. Commun. **12**, 6255–6267 (2013)
11. Ng, D.W.K., Lo, E.S., Schober, R.: Energy-efficient resource allocation in OFDMA systems with hybrid energy harvesting base station. IEEE Trans. Wirel. Commun. **12**, 3412–3427 (2013)
12. Chen, X., Zhang, Z., Chen, H.-H., Zhang, H.: Enhancing wireless information and power transfer by exploiting multi-antenna techniques. IEEE Commun. Mag. **4**, 133–141 (2015)
13. Varshney, L.: Transporting Information and Energy Simultaneously. In: Proceedings of IEEE International Symposium on Information Theory, pp. 1612–1616, July 2008
14. Tesla Memroial Society of New York, "Nikola Tesla's Idea of Wireless Transmission of Electrical Energy is a solution for World Energy Crisis" (2011). [Online]. http://www.teslasociety.com/tesla_tower.htm
15. Powercast Coporation: "RF Energy Harvesting and Wireless Power for Low-Power Applications" (2011). [Online]. http://www.mouser.com/pdfdocs/Powercast-Overview-2011-01-25.pdf

16. Ng, D., Schober, R.: Max-min fair wireless energy transfer for secure multiuser communication systems. In: IEEE Information Theory Workshop (ITW), pp. 326–330, November 2014
17. Ng, D.W.K., Schober, R., Alnuweiri, H.: Secure layered transmission in multicast systems with wireless information and power transfer. In: Proceedings of IEEE International Communication Conference, pp. 5389–5395, June 2014
18. Wyner, A.D.: The Wire-Tap Channel. Technical report, October 1975
19. Zhu, J., Schober, R., Bhargava, V.: Secure transmission in multicell massive MIMO systems. IEEE Trans. Wirel. Commun. **13**, 4766–4781 (2014)
20. Goel, S., Negi, R.: Guaranteeing secrecy using artificial noise. IEEE Trans. Wirel. Commun. **7**, 2180–2189 (2008)
21. Wang, H.M., Wang, C., Ng, D., Lee, M., Xiao, J.: Artificial noise assisted secure transmission for distributed antenna systems. IEEE Trans. Sig. Process. **64**(15), 4050–4064 (2016)
22. Chen, J., Chen, X., Gerstacker, W.H., Ng, D.W.K.: Resource allocation for a massive MIMO relay aided secure communication. IEEE Trans. Inf. Forensics Secur. **11**(8), 1700–1711 (2016)
23. Ng, D.W.K., Lo, E.S., Schober, R.: Efficient resource allocation for secure OFDMA systems. IEEE Trans. Veh. Technol. **61**, 2572–2585 (2012)
24. Wang, H.M., Wang, C., Ng, D.W.K.: Artificial noise assisted secure transmission under training and feedback. IEEE Trans. Sig. Process. **63**(23), 6285–6298 (2015)
25. Chen, X., Ng, D.W.K., Chen, H.H.: Secrecy wireless information and power transfer: challenges and opportunities. IEEE Wirel. Commun. **23**(2), 54–61 (2016)
26. Zhou, X., Zhang, R., Ho, C.K.: Wireless information and power transfer: architecture design and rate-energy tradeoff. In: Proceedings of IEEE Global Telecommunication Conference, December 2012
27. Ng, D.W.K., Lo, E.S., Schober, R.: Energy-efficient resource allocation in multiuser OFDM systems with wireless information and power transfer. In: Proceedings of IEEE Wireless Communication and Networking Conference (2013)
28. Leng, S., Ng, D.W.K., Schober, R.: Power efficient and secure multiuser communication systems with wireless information and power transfer. In: Proceedings of IEEE International Communication Conference, June 2014
29. Ng, D.W.K., Xiang, L., Schober, R.: Multi-objective beamforming for secure communication in systems with wireless information and power transfer. In: Proceedings of IEEE Personal Indoor and Mobile Radio Communication Symposium (2013)
30. Ng, D.W.K., Schober, R.: Resource allocation for coordinated multipoint networks with wireless information and power transfer. In: Proceedings of IEEE Global Telecommunication Conference, pp. 4281–4287, December 2014
31. Chynonova, M., Morsi, R., Ng, D.W.K., Schober, R.: Optimal multiuser scheduling schemes for simultaneous wireless information and power transfer. In: 23rd European Signal Processing Conference (EUSIPCO), August 2015
32. Wu, Q., Tao, M., Ng, D.W.K., Chen, W., Schober, R.: Energy-efficient transmission for wireless powered multiuser communication networks. In: Proceedings of IEEE International Communication Conference, June 2015
33. Boshkovska, E., Ng, D., Zlatanov, N., Schober, R.: Practical non-linear energy harvesting model and resource allocation for SWIPT systems. IEEE Commun. Lett. **19**, 2082–2085 (2015)
34. Boshkovska, E., Ng, D.W.K., Zlatanov, N., Koelpin, A., Schober, R.: Robust resource allocation for MIMO wireless powered communication networks based on a non-linear EH model. IEEE Trans. Commun. **65**(5), 1984–1999 (2017)

35. Floudas, C.A.: Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications, 1st edn. Oxford University Press, Oxford (1995)
36. Tse, D., Viswanath, P.: Fundamentals of Wireless Communication, 1st edn. Cambridge University Press, Cambridge (2005)