Correction to: Gaussian Harmonic Analysis

Correction to:
W. Urbina-Romero, *Gaussian Harmonic Analysis*,
Springer Monographs in Mathematics,
https://doi.org/10.1007/978-3-030-05597-4

This book was inadvertently published without updating the following corrections:

Chapter 2

Page 34 line 2 $\downarrow$ it says

$$(\partial_\gamma^i)^* = -\frac{1}{\sqrt{2}} e^{|x|^2} (\partial_\gamma^i e^{-|x|^2}).$$

it should say

$$(\partial_\gamma^i)^* = -\frac{1}{\sqrt{2}} e^{|x|^2} (\partial_i e^{-|x|^2} I).$$

The updated online versions of the chapters can be found at
https://doi.org/10.1007/978-3-030-05597-4_2
https://doi.org/10.1007/978-3-030-05597-4_4
https://doi.org/10.1007/978-3-030-05597-4_8
https://doi.org/10.1007/978-3-030-05597-4_9

W. Urbina-Romero, *Gaussian Harmonic Analysis*, Springer Monographs in Mathematics, https://doi.org/10.1007/978-3-030-05597-4_10

Page 34 line 12 ↑ it says

$$(-\overline{L}) = \sum_{i=1}^{d} \partial_\gamma^i (\partial_\gamma^i)^* = (-L) + I_d = -\frac{1}{2}\Delta_x + \langle x, \nabla_x \rangle + I_d,$$

it should say

$$(-\overline{L}) = \sum_{i=1}^{d} \partial_\gamma^i (\partial_\gamma^i)^* = (-L) + dI = -\frac{1}{2}\Delta_x + \langle x, \nabla_x \rangle + dI,$$

Page 34 line 10 ↑ it says

$$\overline{L} = L - I_d = \frac{1}{2}\Delta_x - \langle x, \nabla_x \rangle - I_d.$$

it should say

$$\overline{L} = L - dI = \frac{1}{2}\Delta_x - \langle x, \nabla_x \rangle - dI.$$

Chapter 4

Page 157 line 11 ↑ it says

$$\leq \frac{C}{m(c_b)^d} \int_{|y-x|<C_d m(c_b)} \frac{|(f\chi_{\hat{B}(x)})(y)|}{|x-y|^d} dy \leq CM(f\chi_{\hat{B}(\cdot)})(x).$$

it should say

$$\leq \frac{C}{m(c_b)^d} \int_{|y-x|<C_d m(c_b)} |(f\chi_{\hat{B}(x)})(y)| dy \leq CM(f\chi_{\hat{B}(\cdot)})(x).$$

Page 162 line 3 ↓ it says

$$h(v(s))+h(w(s)) \leq \frac{a}{a^2-b^2} + \frac{2a}{(a^2-b^2)^{1/2}\sqrt{s}(a^2-b^2)^{1/4}} \leq \frac{C}{t_0}\frac{1}{(a^2-b^2)^{1/4}}\left(1+\frac{1}{\sqrt{s}}\right),$$

it should say

$$h(v(s))+h(w(s)) \leq \frac{2a}{a^2-b^2} + \frac{2a}{(a^2-b^2)^{1/2}\sqrt{s}(a^2-b^2)^{1/4}} \leq \frac{C}{t_0}\frac{1}{(a^2-b^2)^{1/4}}\left(1+\frac{1}{\sqrt{s}}\right),$$

Page 162 line 10 ↓ it says

$$\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}} \frac{1}{(a^2-b^2)^{1/4}} \int_0^\infty (s^{\eta/2} + u_0^{\eta/2}) e^{-\nu u(s)} \left(\sqrt{s} + \frac{1}{\sqrt{s}}\right) ds$$

it should say

$$\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}}\frac{1}{(a^2-b^2)^{1/4}}\int_0^\infty (s^{\eta/2}+u_0^{\eta/2})e^{-\nu s}\left(\sqrt{s}+\frac{1}{\sqrt{s}}\right)ds$$

Page 162 line 11 ↓ it says

$$\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}}\left(1+\frac{u_0^{\eta/2}}{(a^2-b^2)^{1/4}}\right)\int_0^\infty \left(\sqrt{s}+\frac{1}{\sqrt{s}}\right)\,ds$$

it should say

$$\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}}\left(1+\frac{u_0^{\eta/2}}{(a^2-b^2)^{1/4}}\right)\int_0^\infty e^{-\nu s}\left(s+\frac{1}{\sqrt{s}}\right)\,ds$$

Page 163 line 2–3 ↓ it says

$$\int_0^1 (u(t))^{\eta/2}e^{-\nu u(t)}\frac{dt}{t^{3/2}}\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}}\frac{1}{(a^2-b^2)^{1/4}}\int_0^\infty (s^{\eta/2}+u_0^{\eta/2})e^{-\nu u(s)}\left(\sqrt{s}+\frac{1}{\sqrt{s}}\right)ds.$$

it should say

$$\int_0^1 (u(t))^{\eta/2}e^{-\nu u(t)}\frac{dt}{t^{3/2}}\leq C\frac{e^{-\nu u_0}}{t_0^{1/2}}\frac{1}{(a^2-b^2)^{1/4}}\int_0^\infty (s^{\eta/2}+u_0^{\eta/2})e^{-\nu s}\left(\sqrt{s}+\frac{1}{\sqrt{s}}\right)\sqrt{1-v(s)}\,ds.$$

Chapter 8

Page 353 line 1 ↑ it says

$$\bar{I}_\beta \mathbf{H}_\nu(x) = \frac{1}{(|\nu|+1)^{\beta/2}}\mathbf{H}_\nu(x).$$

it should say

$$\bar{I}_\beta \mathbf{H}_\nu(x) = \frac{1}{(|\nu|+d)^{\beta/2}}\mathbf{H}_\nu(x).$$

Page 354 line 2 ↓ it says "$\{T_t^{(1)}\}_t = \{e^{-t}T_t\}_t$, the 1-translated"

it should say "$\{T_t^{(d)}\}_t = \{e^{-td}T_t\}_t$, the d-translated"

Page 354 line 4 ↓ it says

$$\begin{aligned}\bar{I}_\beta f(x) = (-\bar{L})^{-|\beta|/2} f(x) &= \frac{1}{\Gamma(|\beta|/2)} \int_0^\infty t^{\frac{|\beta|-2}{2}} T_t^{(1)} f(x)\, dt \\ &= \frac{1}{\Gamma(|\beta|/2)} \int_0^\infty t^{\frac{|\beta|-2}{2}} e^{-t} T_t f(x)\, dt \\ &= C_\beta e^{|x|^2} \int_{\mathbb{R}^d} \left(\int_0^1 (-\log r)^{\frac{|\beta|-2}{2}} \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}}\, dr \right) f(y)\, \gamma_d(dy).\end{aligned}$$

it should say

$$\begin{aligned}\bar{I}_\beta f(x) = (-\bar{L})^{-\beta/2} f(x) &= \frac{1}{\Gamma(\beta/2)} \int_0^\infty t^{\frac{\beta-2}{2}} T_t^{(d)} f(x)\, dt \\ &= \frac{1}{\Gamma(\beta/2)} \int_0^\infty t^{\frac{\beta-2}{2}} e^{-dt} T_t f(x)\, dt \\ &= C_\beta e^{|x|^2} \int_{\mathbb{R}^d} \left(\int_0^1 (-\log r)^{\frac{\beta-2}{2}} r^d \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}} \frac{dr}{r} \right) f(y)\, \gamma_d(dy). \\ &= C_\beta \int_{\mathbb{R}^d} \left(\int_0^1 (-\log r)^{\frac{\beta-2}{2}} r^d \frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}} \frac{dr}{r} \right) f(y)(dy).\end{aligned}$$

Chapter 9

Page 360 line 6 ↓ it says

$$R_j = \frac{\partial}{\partial x_j} (-L)^{-1/2}$$

it should say

$$R_j = \frac{\partial}{\partial x_j} (-\Delta)^{-1/2}$$

Page 361 line 3 ↑ it says

$$\mathscr{R}_j = \partial_j^\gamma I_{1/2} = \frac{1}{\sqrt{2}} \frac{\partial}{\partial x_j} (-L)^{-1/2},$$

it should say

$$\mathscr{R}_j = \partial_\gamma^j I_{1/2} = \frac{1}{\sqrt{2}} \frac{\partial}{\partial x_j} (-L)^{-1/2},$$

Page 366 line 3 ↑ it says

$$\mathscr{R}_\beta = \partial_\beta^\gamma (-L)^{-|\beta|/2},$$

it should say

$$\mathscr{R}_\beta = \partial_\gamma^\beta (-L)^{-|\beta|/2},$$

Page 368 lines 3–7 ↓ it says

$$\begin{aligned}\mathscr{K}_\beta(x,y) &= \partial_\beta^\gamma N_{|\beta|/2}(x,y)\\ &= \frac{1}{\pi^{d/2}\Gamma(\beta)}\int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{|\beta|}\mathbf{H}_\beta\left(\frac{y-rx}{\sqrt{1-r^2}}\right)\frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{d/2+1}}\frac{dr}{r}.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathscr{R}_\beta f(x) &= p.v.\int_{\mathbb{R}^d}\mathscr{K}_\beta(x,y)f(y)dy\\ &= p.v.\frac{1}{\pi^{d/2}\Gamma(\beta)}\int_{\mathbb{R}^d}\int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{|\beta|}\mathbf{H}_\beta\left(\frac{y-rx}{\sqrt{1-r^2}}\right)\\ &\qquad \frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{d/2+1}}\frac{dr}{r}f(y)dy.\end{aligned}$$

it should say

$$\begin{aligned}\mathscr{K}_\beta(x,y) &= \partial_\gamma^\beta N_{|\beta|/2}(x,y)\\ &= \frac{1}{\pi^{d/2}\Gamma(|\beta|/2)}\int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{|\beta|}\mathbf{H}_\beta\left(\frac{y-rx}{\sqrt{1-r^2}}\right)\frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{d/2+1}}\frac{dr}{r}.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathscr{R}_\beta f(x) &= p.v.\int_{\mathbb{R}^d}\mathscr{K}_\beta(x,y)f(y)dy\\ &= p.v.\frac{1}{\pi^{d/2}\Gamma(|\beta|/2)}\int_{\mathbb{R}^d}\int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{|\beta|}\mathbf{H}_\beta\left(\frac{y-rx}{\sqrt{1-r^2}}\right)\\ &\qquad \frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{d/2+1}}\frac{dr}{r}f(y)dy.\end{aligned}$$

Page 379 line 7 ↓ it says

$$(-\overline{L}) = \sum_{i=1}^d \partial_\gamma^i(\partial_\gamma^i)^* = (-L) + I_d = -\frac{1}{2}\Delta + \langle x, \nabla_x\rangle + I_d.$$

it should say

$$(-\overline{L}) = \sum_{i=1}^{d} \partial_\gamma^i (\partial_\gamma^i)^* = (-L) + dI = -\frac{1}{2}\Delta + \langle x, \nabla_x \rangle + dI.$$

Page 379 line 12 ↓ it says

$$(\partial_\gamma^\beta)^* = \frac{(-1)^{|\beta|}}{2^{|\beta|/2}} e^{|x|^2} (\partial_\gamma^\beta e^{-|x|^2})$$

it should say

$$(\partial_\gamma^\beta)^* = \frac{(-1)^{|\beta|}}{2^{|\beta|/2}} e^{|x|^2} (\partial^\beta e^{-|x|^2} I)$$

Page 379 line 6 ↑ it says

$$\overline{\mathscr{R}}_\beta \mathbf{H}_\nu = \frac{1}{2^{|\beta|/2}(|\nu|+1)^{|\beta|/2}} \mathbf{H}_{\nu+\beta},$$

it should say

$$\overline{\mathscr{R}}_\beta \mathbf{H}_\nu = \frac{1}{2^{|\beta|/2}(|\nu|+d)^{|\beta|/2}} \mathbf{H}_{\nu+\beta},$$

Page 379 line 4 ↑ it says

$$(-\overline{L})^{-|\beta|/2} H_\nu = \frac{1}{(|\nu|+1)^{|\beta|/2}} \mathbf{H}_\nu,$$

it should say

$$(-\overline{L})^{-|\beta|/2} H_\nu = \frac{1}{(|\nu|+d)^{|\beta|/2}} \mathbf{H}_\nu,$$

Page 379 lines 1–2 ↑ it says

$$\begin{aligned}\overline{\mathscr{R}}_\beta \mathbf{H}_\nu(x) &= (\partial_\gamma^\beta)^* (-\overline{L})^{-|\beta|/2} \mathbf{H}_\nu(x) = \frac{(-1)^{|\beta|}}{(|\nu|+1)^{|\beta|/2}} e^{|x|^2} \partial_\gamma^\beta (e^{-|x|^2} \mathbf{H}_\nu(x)) \\ &= \frac{(-1)^{|\beta+\nu|}}{2^{|\beta|/2}(|\nu|+1)^{|\beta|/2}} e^{|x|^2} \partial^{\beta+\nu}(e^{-|x|^2}) = \frac{1}{2^{|\beta|/2}(|\nu|+1)^{|\beta|/2}} \mathbf{H}_{\nu+\beta}(x);\end{aligned}$$

it should say

$$\begin{aligned}\overline{\mathscr{R}}_\beta \mathbf{H}_\nu(x) &= (\partial_\gamma^\beta)^* (-\overline{L})^{-|\beta|/2} \mathbf{H}_\nu(x) = \frac{(-1)^{|\beta|}}{(|\nu|+d)^{|\beta|/2}} e^{|x|^2} \partial^\beta (e^{-|x|^2} \mathbf{H}_\nu(x)) \\ &= \frac{(-1)^{|\beta+\nu|}}{2^{|\beta|/2}(|\nu|+d)^{|\beta|/2}} e^{|x|^2} \partial^{\beta+\nu}(e^{-|x|^2}) = \frac{1}{2^{|\beta|/2}(|\nu|+d)^{|\beta|/2}} \mathbf{H}_{\nu+\beta}(x);\end{aligned}$$

Page 380 line 2 ↓ it says

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{(|\nu|+1)^{|\beta|/2}} \Big[\prod_{i=1}^{d} (\nu_i+\beta_i)(\nu_i+\beta_i-1)\cdots(\nu_i+1)\Big]^{1/2} \mathbf{h}_{\nu+\beta}(x),$$

it should say

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{(|\nu|+d)^{|\beta|/2}} \Big[\prod_{i=1}^{d} (\nu_i+\beta_i)(\nu_i+\beta_i-1)\cdots(\nu_i+1)\Big]^{1/2} \mathbf{h}_{\nu+\beta}(x),$$

Page 380 lines 4–7 ↓ it says

$$\begin{aligned}
\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) &= \overline{\mathscr{R}}_\beta\Big(\frac{\mathbf{H}_\nu(x)}{(2^{|\nu|}\nu!)^{1/2}}\Big) = \frac{1}{(2^{|\nu|}\nu!)^{1/2}} \overline{\mathscr{R}}_\beta \mathbf{H}_\nu(x)\\
&= \frac{1}{(2^{|\nu|}\nu!)^{1/2}} \frac{1}{2^{|\beta|/2}(|\nu|+1)^{|\beta|/2}} \mathbf{H}_{\nu+\beta}(x) = \frac{1}{(\nu!)^{1/2}(|\nu|+1)^{|\beta|/2}} \frac{\mathbf{H}_{\nu+\beta}(x)}{2^{|\nu|/2+|\beta|/2}}\\
&= \frac{1}{(|\nu|+1)^{|\beta|/2}} \Big(\frac{(\nu+\beta)!}{\nu!}\Big)^{1/2} \frac{\mathbf{H}_{\nu+\beta}(x)}{(2^{|\nu+\beta|}(\nu+\beta)!)^{1/2}}\\
&= \frac{1}{(|\nu|+1)^{|\beta|/2}} \Big[\prod_{i=1}^{d} (\nu_i+\beta_i)(\nu_i+\beta_i-1)\cdots(\nu_i+1)\Big]^{1/2} \mathbf{h}_{\nu+\beta}(x).
\end{aligned}$$

it should say

$$\begin{aligned}
\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) &= \overline{\mathscr{R}}_\beta\Big(\frac{\mathbf{H}_\nu(x)}{(2^{|\nu|}\nu!)^{1/2}}\Big) = \frac{1}{(2^{|\nu|}\nu!)^{1/2}} \overline{\mathscr{R}}_\beta \mathbf{H}_\nu(x)\\
&= \frac{1}{(2^{|\nu|}\nu!)^{1/2}} \frac{1}{2^{|\beta|/2}(|\nu|+d)^{|\beta|/2}} \mathbf{H}_{\nu+\beta}(x) = \frac{1}{(\nu!)^{1/2}(|\nu|+d)^{|\beta|/2}} \frac{\mathbf{H}_{\nu+\beta}(x)}{2^{|\nu|/2+|\beta|/2}}\\
&= \frac{1}{(|\nu|+d)^{|\beta|/2}} \Big(\frac{(\nu+\beta)!}{\nu!}\Big)^{1/2} \frac{\mathbf{H}_{\nu+\beta}(x)}{(2^{|\nu+\beta|}(\nu+\beta)!)^{1/2}}\\
&= \frac{1}{(|\nu|+d)^{|\beta|/2}} \Big[\prod_{i=1}^{d} (\nu_i+\beta_i)(\nu_i+\beta_i-1)\cdots(\nu_i+1)\Big]^{1/2} \mathbf{h}_{\nu+\beta}(x).
\end{aligned}$$

Page 380 line 12 ↓ it says

$$\overline{\mathscr{K}}_\beta(x,y) = C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} \mathbf{H}_\beta\left(\frac{x-ry}{\sqrt{1-r^2}}\right) \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{n}{2}+1}}\, dr.$$

it should say

$$\overline{\mathscr{K}}_\beta(x,y) = C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{d-1}\mathbf{H}_\beta\left(\frac{x-ry}{\sqrt{1-r^2}}\right) \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}+1}}\, dr.$$

Page 380 lines 10 ↑ it says

$$(-\overline{L})^{-|\beta|/2} f(x) = \frac{1}{\Gamma(|\beta|/2)} \int_0^\infty t^{\frac{|\beta|-2}{2}} T_t^{(1)} f(x)\, dt$$
$$= C_\beta e^{|x|^2} \int_{\mathbb{R}^d} \Big(\int_0^1 (-\log r)^{\frac{|\beta|-2}{2}} \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}}\, dr \Big)\, f(y)\, \gamma_d(dy).$$

it should say

$$(-\overline{L})^{-|\beta|/2} f(x) = \frac{1}{\Gamma(|\beta|/2)} \int_0^\infty t^{\frac{|\beta|-2}{2}} T_t^{(d)} f(x)\, dt$$
$$= C_\beta e^{|x|^2} \int_{\mathbb{R}^d} \Big(\int_0^1 (-\log r)^{\frac{|\beta|-2}{2}} r^d \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}} \frac{dr}{r} \Big)\, f(y)\, \gamma_d(dy)$$
$$= C_\beta \int_{\mathbb{R}^d} \Big(\int_0^1 (-\log r)^{\frac{|\beta|-2}{2}} r^d \frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}}} \frac{dr}{r} \Big)\, f(y)\, dy.$$

Page 381 line 3 ↓ it says

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{(|\nu|+1)^{|\beta|/2}} \Big[\prod_{j=1}^d (\nu_j+\beta_j)\cdots(\nu_j+1) \Big]^{1/2} \mathbf{h}_{\nu+\beta}(x).$$

it should say

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{(|\nu|+d)^{|\beta|/2}} \Big[\prod_{j=1}^d (\nu_j+\beta_j)\cdots(\nu_j+1) \Big]^{1/2} \mathbf{h}_{\nu+\beta}(x).$$

Page 381 lines 7–8 ↓ it says

$$\overline{\mathscr{R}}_1^{\beta_1} \overline{\mathscr{R}}_2^{\beta_2} \ldots \overline{\mathscr{R}}_d^{\beta_d} \mathbf{h}_\nu(x) = \prod_{j=1}^d \Big(\prod_{i=1}^{\beta_j} \Big(\frac{\nu_j+i}{|\nu|+i} \Big) \Big)^{1/2} \mathbf{h}_{\nu+\beta}(x)$$
$$= \Big[\prod_{j=1}^d \frac{(\nu_j+\beta_j)\cdots(\nu_j+1)}{(|\nu|+\beta_j)\cdots(|\nu|+1)} \Big]^{1/2} \mathbf{h}_{\nu+\beta}(x)$$

it should say

$$\overline{\mathscr{R}}_1^{\beta_1} \overline{\mathscr{R}}_2^{\beta_2} \ldots \overline{\mathscr{R}}_d^{\beta_d} \mathbf{h}_\nu(x) = \prod_{j=1}^d \Big(\prod_{i=1}^{\beta_j} \Big(\frac{\nu_j+i}{|\nu|+d+(i-1)} \Big) \Big)^{1/2} \mathbf{h}_{\nu+\beta}(x)$$
$$= \Big[\prod_{j=1}^d \frac{(\nu_j+\beta_j)\cdots(\nu_j+1)}{(|\nu|+d+\beta_j-1)\cdots(|\nu|+d)} \Big]^{1/2} \mathbf{h}_{\nu+\beta}(x)$$

Page 381 lines 10–15 ↓ it says

$$
\begin{aligned}
T_\beta \mathbf{h}_\nu(x) &= \Big[\frac{\prod_{j=1}^d (|\nu|+\beta_j)\cdots(|\nu|+1)}{(|\nu|+1)^{|\beta|}}\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\frac{\prod_{j=1}^d (|\nu|+\beta_j)\cdots(|\nu|+2)}{(|\nu|+1)^{|\beta|-d}}\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(\frac{|\nu|+\beta_j)\cdots(|\nu|+2)}{(|\nu|+1)^{\beta_j-1}}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(\frac{(|\nu|+1)+(\beta_j-1)}{|\nu|+1}\Big)\cdots\frac{(|\nu|+1)+1)}{|\nu|+1}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(\frac{(|\nu|+1)+(\beta_j-1)}{|\nu|+1}\Big)\cdots\Big(\frac{(|\nu|+1)+1}{|\nu|+1}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(1+\frac{(\beta_j-1)}{|\nu|+1}\Big)\cdots\Big(1+\frac{1}{|\nu|+1}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)
\end{aligned}
$$

it should say

$$
\begin{aligned}
T_\beta \mathbf{h}_\nu(x) &= \Big[\frac{\prod_{j=1}^d (|\nu|+d+\beta_j-1)\cdots(|\nu|+d)}{(|\nu|+d)^{|\beta|}}\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\frac{\prod_{j=1}^d (|\nu|+d+\beta_j-1)\cdots(|\nu|+2)}{(|\nu|+d)^{|\beta|-d}}\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(\frac{|\nu|+d+\beta_j-1)\cdots(|\nu|+d+1)}{(|\nu|+d)^{\beta_j-1}}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(\frac{(|\nu|+d)+(\beta_j-1)}{|\nu|+d}\Big)\cdots\frac{(|\nu|+d)+1)}{|\nu|+d}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)\\
&= \Big[\prod_{j=1}^d \Big(1+\frac{(\beta_j-1)}{|\nu|+d}\Big)\cdots\Big(1+\frac{1}{|\nu|+d}\Big)\Big]^{1/2}\mathbf{h}_\nu(x)
\end{aligned}
$$

Page 382 line 4 ↑ it says

$$
= C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} \mathbf{H}_\beta\left(\frac{x-ry}{\sqrt{1-r^2}}\right)\frac{e^{-\frac{|y-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{n}{2}+1}}\,dr.
$$

it should say

$$
= C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{d-1}\mathbf{H}_\beta\left(\frac{x-ry}{\sqrt{1-r^2}}\right)\frac{e^{-\frac{|y-rx|^2}{1-r^2}}}{(1-r^2)^{\frac{n}{2}+1}}\,dr.
$$

Page 382 line 2 ↑ it says

$$\frac{\partial \mathscr{K}}{\partial y_j}(x,y) = 2C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} \left[\frac{-r\beta_j}{\sqrt{1-r^2}}\mathbf{H}_{\beta-\mathbf{e_j}}\left(\frac{x-ry}{\sqrt{1-r^2}}\right)\right.$$

it should say

$$\frac{\partial \mathscr{K}}{\partial y_j}(x,y) = 2C_\beta \int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} r^{d-1} \left[\frac{-r\beta_j}{\sqrt{1-r^2}}\mathbf{H}_{\beta-\mathbf{e_j}}\left(\frac{x-ry}{\sqrt{1-r^2}}\right)\right.$$

Page 383 line 9 ↓ it says

$$\left|\frac{\partial \mathscr{K}}{\partial y_j}(x,y)\right| \leq C\left|e^{-|x|^2+|y|^2}\frac{\partial \mathscr{K}}{\partial y_j}(x,y)\right|.$$

it should say

$$\left|\frac{\partial \mathscr{K}}{\partial y_j}(x,y)\right| \leq C\left|e^{-|x|^2+|y|^2}\frac{\partial \overline{\mathscr{K}}}{\partial y_j}(x,y)\right|.$$

Page 384 line 4 ↓ it says

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{2^{|\beta|/2}}\frac{\prod_{j=1}^d[(\nu_j+1)\cdots(\nu_j+\beta_j)]^{\frac{1}{2}}}{(|\nu|+1)^{|\beta|/2}}h_{\nu+\beta}(x).$$

it should say

$$\overline{\mathscr{R}}_\beta \mathbf{h}_\nu(x) = \frac{1}{2^{|\beta|/2}}\frac{\prod_{j=1}^d[(\nu_j+d)\cdots(\nu_j+\beta_j)]^{\frac{1}{2}}}{(|\nu|+d)^{|\beta|/2}}h_{\nu+\beta}(x).$$

Page 384 line 6 ↓ it says

$$\|\overline{\mathscr{R}}_\beta f\|^2_{L^2(d\gamma)} = \sum_\nu \frac{\prod_{j=1}^d[(\nu_j+1)\cdots(\nu_j+\beta_j)]}{2^{|\beta|}(|\nu|+1)^{|\beta|}}|\widehat{f}_\gamma(\nu)|^2$$

it should say

$$\|\overline{\mathscr{R}}_\beta f\|^2_{L^2(d\gamma)} = \sum_\nu \frac{\prod_{j=1}^d[(\nu_j+1)\cdots(\nu_j+\beta_j)]}{2^{|\beta|}(|\nu|+d)^{|\beta|}}|\widehat{f}_\gamma(\nu)|^2$$

Page 384 line 10 ↑ it says

$$|\overline{\mathscr{K}}_\beta(x,y)| = \left|\int_0^1 \left(\frac{-\log r}{1-r^2}\right)^{\frac{|\beta|-2}{2}} \mathbf{H}_\beta\left(\frac{x-ry}{\sqrt{1-r^2}}\right)\frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{d}{2}+1}}\,dr\right|$$

$$\leq C\int_0^{\frac{3}{4}}(-\log r)^{\frac{|\beta|-2}{2}}\frac{e^{-\frac{|x-ry|^2}{2(1-r^2)}}}{(1-r^2)^{\frac{d}{2}}}\,dr$$

it should say

$$|\overline{\mathscr{K}}_\beta(x,y)| = \left| \int_0^1 \left(\frac{-\log r}{1-r^2} \right)^{\frac{|\beta|-2}{2}} r^{d-1} \mathbf{H}_\beta \left(\frac{x-ry}{\sqrt{1-r^2}} \right) \frac{e^{-\frac{|x-ry|^2}{1-r^2}}}{(1-r^2)^{\frac{n}{2}+1}} \, dr \right|$$

$$\leq C \int_0^{\frac{3}{4}} (-\log r)^{\frac{|\beta|-2}{2}} \frac{e^{-\frac{|x-ry|^2}{2(1-r^2)}}}{(1-r^2)^{\frac{n}{2}}} \, dr$$

Page 384 line 5 ↑ it says

$$|\overline{\mathscr{K}}_\beta(x,y)| = C \left(\overline{\mathscr{K}}^1_\beta(x,y) + \overline{\mathscr{K}}^2_\beta(x,y) + \overline{\mathscr{K}}^3_\beta(x,y) \right),$$

it should say

$$|\overline{\mathscr{K}}_\beta(x,y)| \leq C \left(\overline{\mathscr{K}}^1_\beta(x,y) + \overline{\mathscr{K}}^2_\beta(x,y) + \overline{\mathscr{K}}^3_\beta(x,y) \right),$$