An Evolutionary Intelligent Approach for the LTI Systems Identification in Continuous Time

Luis Morales[1(✉)], Oscar Camacho[1], Danilo Chávez[1], and José Aguilar[2]

[1] Dpto. de Automatización y Control Industrial, Escuela Politécnica Nacional, Quito, Ecuador
{luis.moralesec, oscar.camacho, danilo.chavez}@epn.edu.ec
[2] Universidad de Los Andes, Mérida 5101, Venezuela
aguilar@ula.ve

Abstract. Identification and modeling of systems are the first stage for development and design of controllers. For this purpose, as an alternative to conventional modeling approaches we propose using two methods of evolutionary computing: Genetic Algorithms (GA) and Particle Swarm Optimization (PSO to create an algorithm for modeling Linear Time Invariant (LTI) systems of different types. Integral Square Error (ISE) is the objective function to minimize, which is calculated between the outputs of the real system and the model. Unlike other works, the algorithms make a search of the most approximate model based on four of the most common ones found in industrial processes: systems of first order, first order plus time delay, second order and inverse response. The estimated models by our algorithms are compared with the obtained by other analytical and heuristic methods, in order to validate the results of our approach.

Keywords: System modeling · System identification · Genetic algorithms
Particle swarm optimization

1 Introduction

In control systems, identification of invariant models in continuous or discrete time is one of the main steps to be carried out regarding the design of controllers and calibration of parameters. In general, the models can have different structures, depending on the intrinsic characteristics of the process; however, at the level of real applications, complex process approximations can be made to linear and reduced order models (at their operational points). So, there are different methods used to perform the identification of a system, some of them are analytical one, based on principles such as laws of physics, thermodynamics and so on; and the other ones based on experiments, which use particular input signals applied to a system to observe the behavior of the output, and based on it try to determine their respective parameters [1]. This is one of the most common methodologies for the case of reduced order systems. For second order

M. Botto-Tobar et al. (Eds.): CITT 2018, CCIS 895, pp. 430–445, 2019.
https://doi.org/10.1007/978-3-030-05532-5_32

systems, a similar methodology has been developed, however, these analyzes require the measurement of different characteristic parameters of the reaction curve, which is a tedious procedure that takes time [2].

In [3] it is proposed a computer tool that allows the modeling of systems, as mentioned above, using input and output data from the system, in which an analysis of different model estimation methods is done through the toolbox CONTSID (CONtinuous-Time System IDentification). CONTSID uses linear filters, integral methods, modulation functions, among others. However, iterative or recursive methods are not studied, which will be considered in this work. [3–5] are works in which models are estimated by heuristic optimization techniques, such as Genetic Algorithms (GA); nevertheless, they do not consider finding a reduced order model, or they have not been used to model systems with time delay or with inverse response. In works such as [6, 7], the optimization of model parameters is done using Particle Swarm Optimization (PSO) [8]. Both, GA and PSO algorithms, are iterative methods that do not disturb or affect the normal performance of the real plant (since they work offline), so they are very useful for the identification of plant's dynamics using the input and output data of the real system. In these previous works, it is essential to know a priori model from which the parameters will be optimized; however, the algorithms do not have the intelligence to find the simplest model that follows the real system, which is a great disadvantage.

In order to discover these models, this paper proposes an intelligent approach that, unlike works presented in [4–8], looks for the simplest reduced order model that fits a real system, using its input and output data. Thereby, we propose an approach that performs the parameterization of four of the most commonly found models in automatic control focused on industrial processes, these are: first order, first order plus time delay, second order, and inverse response. Our approach makes a search of the most similar model (optimal parameters) to the real system based on the input and output data, starting this search in the simplest model (few variables), until the inverse response model (more variables), with the aim of decreasing the computational cost. Our approach uses GA and PSO, and stops when the objective function, in this case the Integral Square Error "ISE", is lower than the threshold established by the user.

The advantage of our approach is that it reduces the estimation time of the model and does not necessarily require a priori knowledge of the real model. Two intelligent computing techniques have been used to discover the most approximate model, which are compared with the analytical results. The motivation to use PSO and GA, is to perform a comparative analysis of the results obtained in the estimation of the models with each method, as an alternative to conventional modeling approaches.

2 Theoretical Framework

In the field of control systems, it is very important for the design of controllers, to know the dynamic behavior of the plant and its mathematical model. In this section is presented a brief description of the most common system models in control.

2.1 Commonly Models in Control Systems

A classical SISO (single-input, single-output) model of continuous time LTI systems can be represented by Eq. (1).

$$y(s) = G(s)u(s) \tag{1}$$

With:

$$G(s) = \frac{b(s)}{a(s)} = \frac{b_0 + b_1 s + \ldots + b_j s^j}{a_0 + a_1 s + \ldots + a_k s^k} e^{-t_0 s}; k \geq j \tag{2}$$

Where: b_i and a_i are the parameters of the transfer function, and t_0 is the dead time. These parameters are unknown, and must be identified by our intelligent algorithm, $u(s)$ is the input, $y(s)$ is the output of the system, and "s" is the operator d/dt.

The identification of the system is the first step to be considered in the design of a controller, and different methods can be used. Model estimation consists of two stages: the selection of an appropriate model (which can often be complicated), and the estimation of its parameters [6]. Currently, many model structures are known that describe the behavior of different types of systems in a very precise way; therefore, the aim is to determine their respective parameters. The dynamic properties of systems can be approximated by the temporal characteristics of simpler systems. Simple models are understood as those that define their dynamics by linear differential equations of first or second order. Reduced order models are commonly found in the field of industrial process, robotics, etc. In this section are presented the models used in this work.

2.2 First-Order Systems (FO)

The order of a system corresponds to the degree of its characteristic polynomial. The transfer function of a first-order system is:

$$G(s) = \frac{K}{Ts + 1} \tag{3}$$

Where K is the static gain, and T the response time (time constant).

In a first-order system, its parameters can be determined experimentally by means of the observation of the response produced in the system by a step input. The static gain "K" will be the final value of the output signal, and the time constant "T" approximates the time that output reaches about 63% of the value of the stationary gain. Figure 1 shows the characteristic response of a first-order system by a step input.

In real cases, when the value of the input variable is modified, the effect of that change on the dynamic response of the system is not immediately observed, it can take some time for the system to start responding to the effect of the change made (see

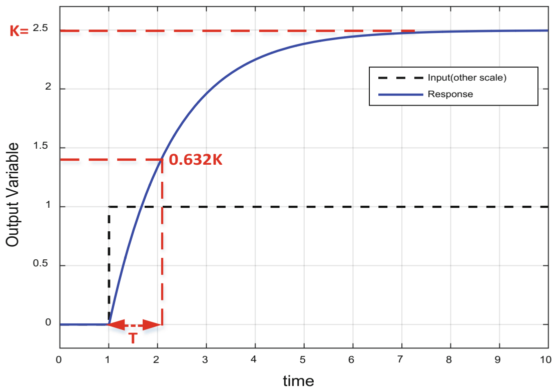

Fig. 1. First-order system response

Fig. 2), which is known as the "first order plus time delay" system (FOPTD). t_O is the time that system takes to respond. In this case, the transfer function is given by:

$$G(s) = \frac{Ke^{-t_0 s}}{Ts + 1} \tag{4}$$

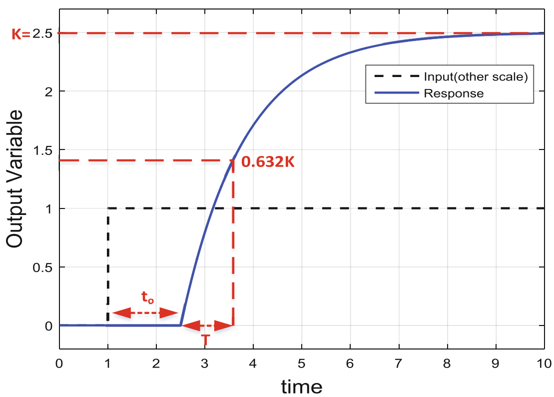

Fig. 2. First-order plus time delay system response

2.3 Second-Order Systems (SO)

The dynamics of a LTI system can be defined by the roots of the denominator. The nature of this expression can be real or complex. If it is real, the response to the input to the step will be defined by the next expression.

$$G(s) = \frac{K\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} e^{-t_0 s} \tag{5}$$

Where ω_n is the natural oscillation frequency and ξ is the damping coefficient. For a second order system, there are different cases for this coefficient:

– Underdamped system $0 < \xi < 1$:

$$s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2} \tag{6}$$

– Critically damped system $\xi = 1$:

$$s_1, s_2 = -\omega_n \tag{7}$$

– Overdamped system $\xi > 1$:

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}; \tag{8}$$

– Undamped system $\xi = 0$:

$$s_1, s_2 = \pm j\omega_n \tag{9}$$

In Fig. 3, the output of different systems is shown in response to a step input, considering the different damping coefficients.

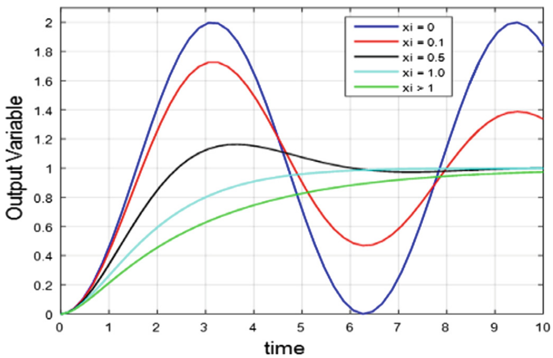

Fig. 3. Second order system response

2.4 Inverse Response Systems (IR)

In these systems, when a step input is applied, the response of the system first decreases until it reaches a minimum, and then begins to “rise” until it reaches the new stationary state, as is shown in Fig. 4. An inverse response system can be represented by:

$$G(s) = \frac{K(-T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \tag{10}$$

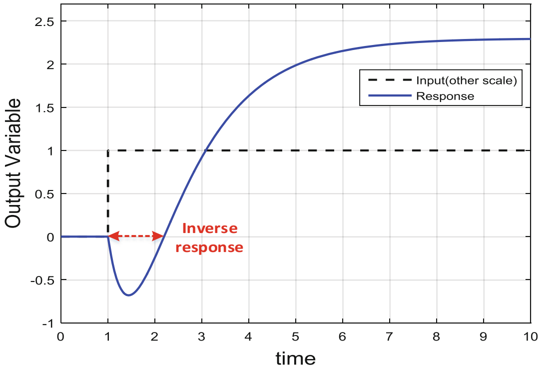

Fig. 4. Inverse response system

These models can be understood as the interaction of slow and fast dynamics, and can be represented as the difference of two systems of first order [9]:

$$G(s) = \frac{K_1}{T_1 s + 1} - \frac{K_2}{T_2 s + 1} \tag{11}$$

With the condition:

$$\frac{T_1}{T_2} > \frac{K_1}{K_2} > 1 \tag{12}$$

2.5 System Identification Problem

The estimation of the aforementioned models can be performed analytically (mathematically), which is well defined in the literature for the case of first and second order systems [1, 9], and for inverse response systems [10], by means of a graphic estimation and calculations that in some cases require a lot of time, and do not guarantee the best possible approximation. Intelligent computing has been used in the field of control systems, both for the identification of systems (estimation) [11, 12], and in the control itself; for example, to determine the optimal parameters of a controller [3].

In this work is proposed the identification of reduced order models based on two well-known techniques of evolutionary computation, such as: GA [13] and PSO [14]. For that, it is necessary to have the input and output data as system information, and based on this, estimate the model closest to the real one.

The algorithms proposed by other authors have the disadvantage that the user must know a priori the type of model, and based on this, it calculates its parameters, which does not guarantee to obtain the most suitable model. Hence, the need for an intelligent approach that approximates the real system to the model of reduced order that most resembles, for example of first order, first order plus time delay, second order, or inverse response, which are the most frequently found in real processes. In our approach is making a search, starting from the simplest model (FO), going through FOPTD, SO, until it ends with the more complex model IR.

3 Our Evolutionary Approach for System Identification

In this paper, unlike other works [5–7, 15, 16], our approach will automatically search for the most approximate model (FO, FOPTD, SO or RI) to the real system. Our approach is based on the system response to an excitation at input, due to that each system type has a characteristic output (see Fig. 5).

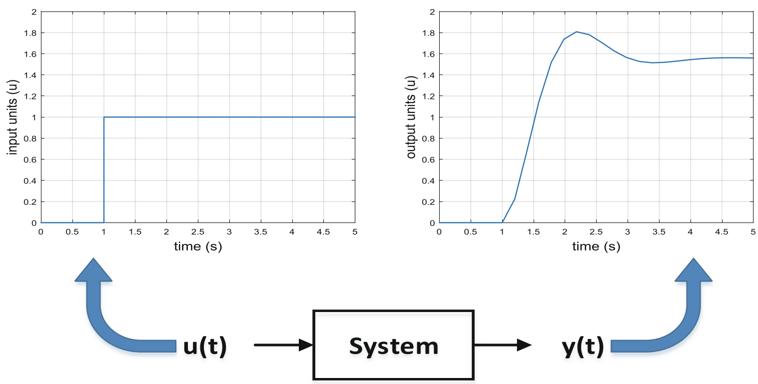

Fig. 5. Schematic of input-output data of a system

The scheme of our proposed approach is shown in Fig. 6. Once the input and output data of the real system has been acquired, one of the algorithms (GA and PSO) is executed, considering an initial structure (in this case, the simplest one, corresponding to a FO model). Once the optimization has been completed, it is verified that the ISE is lower than a given threshold (TH), a value that must be calibrated heuristically, and depends on factors such as, the maximum value of the output, the noise in the data, etc. If the condition is not met, then the model does not correspond to a FO model. So, the algorithm is executed for a FOPDT system estimation. If the ISE at the end of the optimization is below the threshold, then it is determined that the model obtained corresponds to this type of system. However, if this condition is not met, then the same procedure is performed for a SO model and an IR model. In case where it is not found an ISE lower than the threshold in any of the models, the algorithm will give the parameters of the estimated model with the lowest ISE, that is, the model closest to the real one.

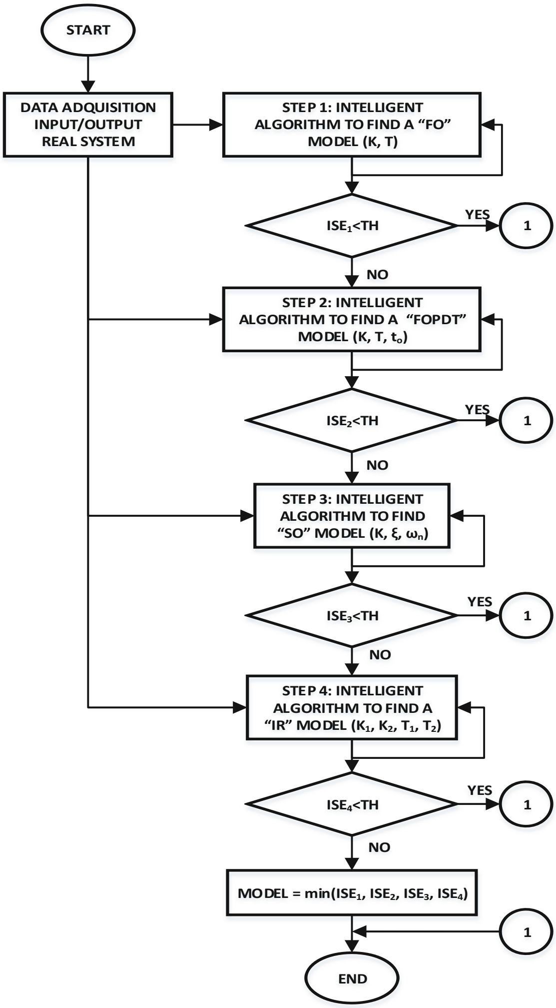

Fig. 6. Block diagram for the proposed algorithm

In this way, the algorithm makes an automatic estimation starting from the simplest model with fewer variables, to the most complex model with more variables. This allows to decrease the search time of the best estimate, since if it finds a good estimation represented by the simplest model, then it is not necessary to keep running the algorithm to find other more complex models.

For the identification of the model, our approach minimizes an objective function based on the Integral Square Error (ISE). In Fig. 7 is shown how is obtained this metric, whose mathematical expression is:

$$ISE = \int_0^T (y(t) - \hat{y}(t))^2 dt = \int_0^T e(t)^2 dt \quad (13)$$

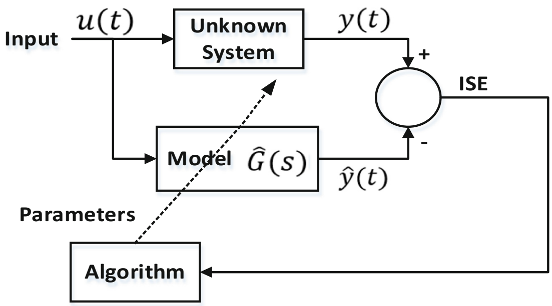

Fig. 7. Block diagram for obtaining the ISE metric

We have used GA and PSO because these two techniques have been well studied and have an excellent performance in optimization problems [8]. In our case, it is appropriate to find optimal parameters of the models, minimizing ISE.

4 Experiments

In this section, we present different case studies taken from different references, to verify the accuracy of our approach for the identification of some simulated and real systems, contrasting the results with the ones obtained by analytical and heuristic methods, in order to compare the performance of our proposal. In the case of GA, an initial population of 50 individuals and 300 generations has been used; in the case of PSO, it starts with 30 agents with parameters that give the best results.

4.1 Case Study 1: FOPDT System Identification

The simulation is performed for the identification of a FOPDT model where there is a random variation of the input (see Fig. 8). The model is easily estimated by our approach in the first stages, determining the parameters shown in the Table 1.

Table 1. Results of the FOPDT system identification

$G(s) = \frac{-2.47}{3.7s+1}e^{-1.3}$

$K = -2.47; \tau = 3.7[s]; t_0 = 1.3[s]$

Parameters	K	$\tau[s]$	$t_0[s]$	ISE
Real	−2.47	3.70	1.30	
GA	−2.46	3.66	1.28	2.57
Relative error GA	0.40%	1.08%	1.5%	
PSO	−2.46	3.73	1.29	2.34
Relative error PSO	0.40%	0.81%	0.76%	

According to Fig. 8, it is evident that the two algorithms provide values close to the real values; however, PSO is the most accurate, with a lower relative error in all its parameters. It is also observed that estimation is correct, even though the input signal is very variable.

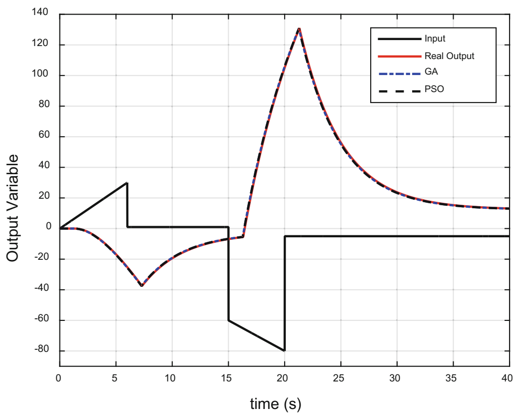

Fig. 8. Comparison of the real FOPDT and the estimated models

4.2 Case Study 2: Higher Order System Identification

Our approach is tested in a higher order system, taken from the example 6.4 presented in [17]. In that work, its approximated models are based on two analytical approaches, which we use to compare them with the results obtained by our proposed. These results are shown in the Table 2.

The results of PSO are the best, followed by the analytical method of Skogestad. In the case of GA, the ISE is not the best because it needs a calibration of its initial parameters (initial population and number of generations) to avoid falling into a local minimum in models of this type; however, its results are better than Taylor Series analytical method, with less time and without the need of mathematical calculations.

Figure 9 shows the response of the real system and of all the estimated models.

Table 2. Results of the higher order system identification

$G(s) = \frac{1.5(-0.1s+1)}{(5s+1)(3s+1)(0.5s+1)}(real\ system)$		
Method	Best estimation	ISE
GA	$\mathrm{G(s)} = \frac{1.51}{4.97\mathrm{S}+1}\mathrm{e}^{-3.13\mathrm{s}}$	0.106
PSO	$\mathrm{G(s)} = \frac{1.51}{6.89\mathrm{S}+1}\mathrm{e}^{-2.25\mathrm{s}}$	0.025
Analytical 1 (Skogestad)	$\mathrm{G(s)} = \frac{1.50}{6.50\mathrm{S}+1}\mathrm{e}^{-2.10\mathrm{s}}$	0.037
Analytical 2 (Taylor Series)	$\mathrm{G(s)} = \frac{1.50}{5.00\mathrm{S}+1}\mathrm{e}^{-3.60\mathrm{s}}$	0.130

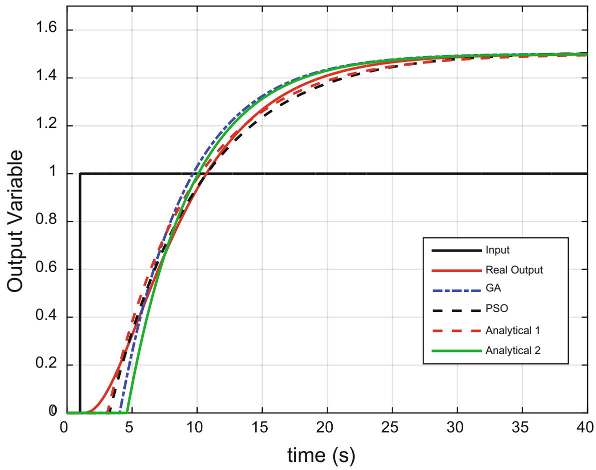

Fig. 9. Comparison of the real higher order system and the estimated models

4.3 Case Study 3: Inverse Response System Identification

For the next test is used the case study presented in [10], where an isothermal continuous stirred tank reactor (CSTR) is considered, from which an analytical model is proposed. In this system, a change of 10% in the manipulated variable (flow through the reactor) occurs, and the response to the output or controlled variable (concentration) is observed, as is shown in the Fig. 10. Results obtained with our approach and analytically, are presented in the Table 3.

In general, PSO and GA propose a very good approximation with respect to the analytical model. This last requires an analytical-graphic analysis considering at least three points of the response curve, and then it performs several mathematical calculations, and a parameter adjustment, which involve a lot of time with respect to the runtime of our approach.

Table 3. Results of the inverse response system identification

Method	Best estimation	ISE
GA	$\boldsymbol{G(s)} = \frac{\mathbf{0.78}}{\mathbf{0.64s+1}} - \frac{\mathbf{0.46}}{\mathbf{0.24s+1}}$	0.0061
PSO	$\boldsymbol{G(s)} = \frac{\mathbf{0.95}}{\mathbf{0.59s+1}} - \frac{\mathbf{0.63}}{\mathbf{0.27s+1}}$	0.0032
Analytical (Balaguer Method)	$\boldsymbol{G(s)} = \frac{\mathbf{0.32(-0.35s+1)}}{\mathbf{(0.56s+1)(0.31s+1)}}$	0.0022

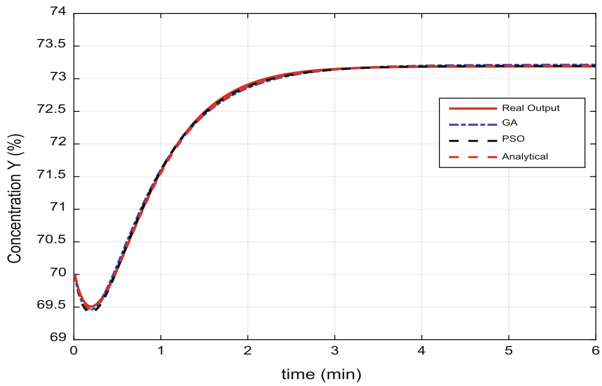

Fig. 10. Comparison of the real inverse response system and the estimated models

4.4 Case Study 4: Identification of a Real System

For this case study, the exercise 10-2 presented in [1] is used, which corresponds to the reactor of the Fig. 11. Once the system is stable at the temperature of 1463 °F, a change of 5% is made in the opening of the fuel valve (input variable with negative step), taking the data (temperature) shown in the Table 4.

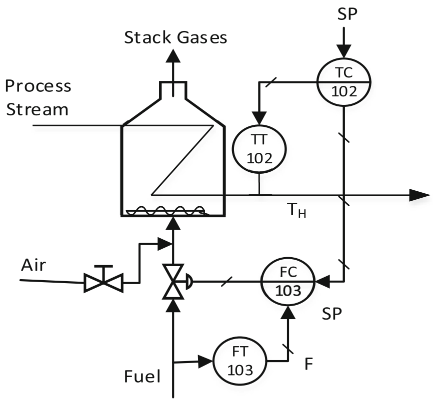

Fig. 11. Schematic of the real process of a heater system (Reactor)

Table 4. Real data of the process of a heater system

Time [min]	T [°F]	Time [min]	T [°F]	Time [min]	T [°F]	Time [min]	T [°F]
0	1463	16	1435	32	1351	48	1287
2	1463	18	1426	34	1341	50	1281
4	1463	20	1415	36	1332	52	1275
6	1463	22	1405	38	1324	54	1275
8	1461	24	1393	40	1316	56	1263
10	1457	26	1382	42	1308	58	1258
12	1452	28	1372	44	1301	94	1235
14	1444	30	1361	46	1293		

Table 5, shows the results obtained through the execution of our approach and an analytical model. The system responses are shown in the Fig. 12.

Table 5. Results of the identification of a real system

Method	Best estimation	ISE
GA	$G(s) = \frac{2.37}{26.02s+1}e^{-13.95s}$	3128
PSO	$G(s) = \frac{2.38}{25.97s+1}e^{-13.95s}$	3127
Analytical	$G(s) = \frac{2.28}{23.25s+1}e^{-14.75s}$	5241

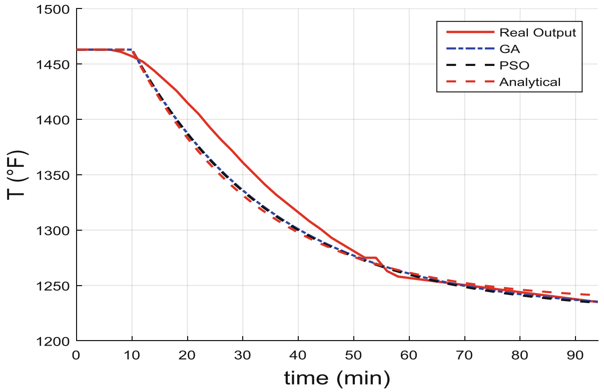

Fig. 12. Comparison of a real system with the estimated models

The results obtained show that our proposal presents the best approximation. It can also be observed that the values of the ISE metric are large compared to the other case studies, because this is related to the high values presented in the output of the system, which is in the order of 1250 °F to 1450 °F.

4.5 Case Study 5: System with Noisy Data

The following experiment is based on the example 6.2 shown in [9]. This is a system with realistic data set for the two stirred-tank heating. The data has noise, which may be due to imperfect mixing, noise in the sensors, among other causes. The estimations of our approach have been compared with the values obtained by means of two analytical strategies presented in [1], which are shown in Table 6.

Table 6. Results of the identification of system with noisy data

Method	Best estimation	ISE
GA	$G(s) = \frac{2.60}{5.55s+1}e^{-4.23s}$	13.95
PSO	$G(s) = \frac{2.74}{7.31s+1}e^{-3.27s}$	13.75
Analytical 1	$G(s) = \frac{2.60}{10.80s+1}e^{-2.40s}$	82.83
Analytical 2	$G(s) = \frac{2.60}{5.90s+1}e^{-3.70s}$	14.03

The PSO algorithm provides the most approximate model, despite the presence of noise in the data (see Fig. 13), showing that the approach performs well, even if a filter has not been placed at the input. One important remark is that our approach determines automatically that the best model is a SO, using GA or PSO.

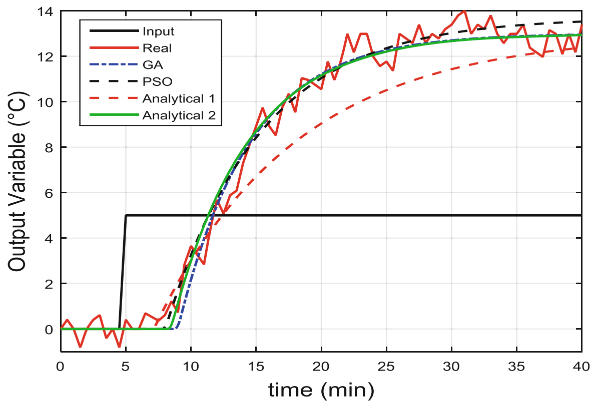

Fig. 13. Comparison of a real system with noisy data and the estimated models

5 Discussion

Based on the results obtained, it is important to differentiate the advantages from the quantitative and qualitative point of view of our approach.

Regarding the quantitative point of view, it can be observed that in most experiments, PSO is the technique that presents the best approximations of the test model, with lower ISE values, even comparing them with analytical models. In the case of inverse response systems, the approximation is quite good, but the Balaguer method [10] has the best estimation; however, it requires mathematical calculations that involve a lot of time.

From the qualitative point of view, it has been possible to observe several advantages with respect to other similar works, such as:

– It is not necessary to know a priori the model to be estimated, because the algorithm automatically finds the reduced order model closest to the real one.
– In the case of data with noise, the algorithm finds the closest model, without the need for additional filters.
– The runtime of the algorithm is low and can be considered as a very good alternative to analytical methods.
– In previous works, we have used genetic programming in identification problems, but the expressions they give are very complex, which are not useful to build later controllers [11, 12]. Our approach based on an optimization problem allows us reusing the classic control models to solve the identification problem.

6 Conclusions

In this paper, an evolutionary intelligent approach for the identification of reduced order models, through GA and PSO, is proposed as an alternative to conventional modeling approaches. The optimization of parameters of the estimated model is done by minimizing the ISE.

From the results obtained, it can be determined that PSO is the most suitable algorithm for the model identification, since it presents the results with lower ISE and reduces the estimation time of the parameters, with respect to GA and the analytical methods. The simulation and comparative analysis of these techniques has been carried out, observing the best estimate of the model obtained, without the need for the user to know a priori the model of the real system. The approach can be used for adaptive control systems, avoiding performing mathematical calculations or graphic estimation that requires considerable time, and does not guarantee the lowest ISE.

As future work, the extension of our approach to nonlinear systems will be considered to determine more exact approximations. This will allow designing more robust controllers, with better characteristics that can handle the intrinsic properties of these systems, which are generally found in real industrial processes. At the same time, the extension of this proposal to discrete models will be of great help, since most systems are measured with digital instruments. This will allow a better modeling of real plants.

References

1. Smith, C., Corripio, A.: Principles and Practice of Automatic Process Control, 3rd edn. Wiley, New York (2006)
2. Johnson, M., Moradi, M.: PID Control - New Identification and Design Methods. Springer, London (2005). https://doi.org/10.1007/1-84628-148-2
3. Kristinsson, K., Dumont, G.A.: System identification and control using genetic algorithms. IEEE Trans. Syst. Man. Cybern. **22**(5), 1033–1046 (1992)
4. Johnson, T., Husbands, P.: System identification using genetic algorithms. In: Parallel Problem Solving from Nature, no. 1, pp. 85–89. Springer, Heidelberg (1991)
5. Zhang, R., Tao, J.: A nonlinear fuzzy neural network modeling approach using an improved genetic algorithm. IEEE Trans. Ind. Electron. **65**(7), 5882–5892 (2018)
6. Alfi, A., Modares, H.: System identification and control using adaptive particle swarm optimization. Appl. Math. Model. **35**(3), 1210–1221 (2011)
7. Dub, M., Stefek, A.: Mechatronics 2013. Springer, Cham (2014)
8. Hassan, R., Cohanim, B., de Weck, O., Venter, G.: A comparison of particle swarm optimization and the genetic algorithm. In: 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, no. April, pp. 1–13 (2005)
9. Marlin, T.: Process Control. Design Processes and Control System for Dynamic Performance. McGraw Hill, New York (1995)
10. Balaguer, P., Alfaro, V., Arrieta, O.: Second order inverse response process identification from transient step response. ISA Trans. **50**(2), 231–238 (2011)
11. Aguilar, J., Cerrada, M.: Genetic programming-based approach for system identification. Adv. Fuzzy Syst. Evol. Comput. Artif. Intell. 329–324 (2001)
12. Carabalí, C.A., Tituaña, L., Aguilar, J., Camacho, O., Chavez, D.: Inverse response systems identification using genetic programming. In: Proceedings of the 14th International Conference on Informatics in Control, Automation and Robotics, vol. 1, no. Icinco, pp. 238–245 (2017)
13. Tang, H., Xue, S., Fan, C.: Differential evolution strategy for structural system identification. Comput. Struct. **86**(21–22), 2004–2012 (2008)
14. Venter, G., Sobieszczanski-Sobieski, J.: Particle swarm optimization. AIAA J. **41**(8), 1583–1589 (2003)
15. Aguilar, J.: The evolutionary programming in the identification of discreet events dynamic systems. IEEE Lat. Am. Trans. **5**(5), 301–310 (2007)
16. Garnier, H., Mensler, M., Richard, A.: Continuous-time model identification from sampled data: implementation issues and performance evaluation. Int. J. Control **76**(13), 1337–1357 (2003)
17. Seborg, D., Edgar, T., Mellichamp, D., Doyle, F.: Process Dynamics and Control, 3rd edn. Wiley, New York (2011)