Modelling and Simulating Extreme Opinion Diffusion

Enzo Battistella and Laurence Cholvy(✉)

ONERA, Toulouse, France
cholvy@onera.fr

Abstract. This paper focuses on modelling and simulating diffusion of extreme opinions among agents. In this work, opinions are modelled as formulas of the propositional logic. Moreover, agents influence each other and any agent changes its current opinion by merging the opinions of its influencers, taking into account the strength of their influence. We propose several definitions of extreme opinions and extremism. Formal studies of these definitions are made as well as some simulations.

1 Introduction

Understanding the dynamics of opinion diffusion and especially of extremism is a tremendous question in Multi-Agent System and Artificial Intelligence communities. See for instance [1–8] for the study of opinion diffusion and [9–11] for the study of extremism diffusion.

Opinions are usually represented by a single real value between 0 (or -1) and 1 corresponding to the position of an agent regarding a given question. The closer to 1 an agent's opinion is, the more this agent positively answers the question. For instance, if the question is "Do you think that the cafeteria serves GMO food ?", an agent whose opinion is 0.9 strongly believes that GMO food is served; an agent whose opinion is 0.2 rather thinks that GMO food is not served. If the question is now "Do you agree with serving GMO food at the cafeteria ?", an agent whose opinion is 0.9 strongly agrees in serving GMO food while an agent whose opinion is 0.2 is rather against serving GMO food. As for extremism, it is obviously defined there by having an opinion which is close to 0 (or -1) or to 1. Moreover, in such models, agents can easily be classified from most to least extremist. Recently, some works in Artificial Intelligence community [4,8] have adopted a different way of modelling opinions and represent an opinion by a single binary vector whose values correspond to answers to several questions. For instance, if the two questions are "Do you think that the cafeteria must be open until 4pm?" and "Do you think that vegan food should be served at the cafeteria ?" then the vector $(0, 1)$ represents the opinion of an agent which thinks that the cafeteria must not to open until 4pm but has to serve vegan food.

Following [12], we have recently adopted an even more general approach [3,13] and we consider that an opinion is modelled by a set of binary vectors, or equivalently, by a propositional formula. For instance, in such a model, the set of

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binary vectors $\{(1,1),(0,1),(0,0)\}$, which is equivalent to the formula $F \rightarrow V$, represents the opinion of an agent who thinks that, if the cafeteria is open until 4 then it has to serve vegan food. The set of binary vectors $\{(1,1),(0,1)\}$, which is equivalent to the formula V represents the opinion of an agent who thinks that, whatever the open hours are, the cafeteria has to serve vegan food. As for the diffusion process, we consider a model in which any agent is influenced by some other agents called its influencers. There, an agent regularly updates its opinion by merging the opinions of its influencers according to the strenght of their influence. For this, we introduced the notion of Importance-Based Merging Opinion Structures (IODS).

In [14], we have started studying extremism diffusion in IODS. We proposed a definition of extreme opinions which could be called "precise opinions" and we studied their diffusion in IODS.

The present paper extends this work by proposing several definitions and studying their diffusion in IODS. We recall precise opinions definition but we also define extreme opinions based on selected topics and extreme opinions based on selected agents. Moreover we study and compare their diffusion in IODS.

This paper is organized as follows. Section 2 recalls the notion of Importance-Based Opinion Diffusion Structures (IODS). The different definitions of extreme opinions are given in Sect. 3 and their diffusion in IODS is studied in Sect. 4. Sections 5 and 6 focus on experiments. Section 5 shows how to generate graphs corresponding to real social networks. Section 6 presents experiments for the diffusion od some extreme opinions. Section 7 concludes this paper. The proofs of the different propositions given in the paper are gathered in Sect. 8.

2 Importance-Based Opinion Diffusion Structures

This section presents Importance-Based Opinion Diffusion Structures.

We consider a finite propositional langage L. The set of interpretations of L is $Mod(L)$ with $\mid Mod(L) \mid = 2^{|L|}$. An element w of $Mod(L)$ is denoted $\{p_1, ..., p_n, \neg q_1, ..., \neg q_m\}$ where $p_1...p_n$ are the propositional letters satisfied in w and $q_1...q_m$ are the propositional letters which are not satisfied in w. If φ is a propositional formula of language L, $Mod(\varphi)$ is the set of its models (i.e., the set of the interpretation which satisfy it). A multi-set of formulas is a set with possible repeted occurrences of formulas. An ordered multi-set of formulas is a multi-set of formulas in which formulas are ranked with a total ranking. It is denoted $\varphi_1 \prec .. \prec \varphi_n$. The distance between an interpretation w and a formula φ is defined by: $D(w, \varphi) = min_{w' \in Mod(\varphi)} d(w, w')$, where d is a pseudo-distance between interpretations (i.e., $\forall w \forall w' \; d(w, w') = d(w', w)$ and $d(w, w') = 0 \implies w = w'$). Some simple pseudo-distances d are d_D, the drastic pseudo-distance, ($d_D(w, w') = 0$ iff $w = w', 1$ otherwise); d_H, the Hamming pseudo-distance ($d_H(w, w') = m$ iff w and w' differ on m propositional letters).

Definition 1. *An Importance-Based Merging Operator is a function Δ which associates a formula μ and a non-empty ordered multi-set of consistent formulas*

$\varphi_1 \prec ... \prec \varphi_n$ *with a formula denoted* $\Delta_\mu(\varphi_1 \prec ... \prec \varphi_n)$ *so that:* $Mod(\Delta_\mu(\varphi_1 \prec ... \prec \varphi_n)) = Min^d_{\leq_{\varphi_1 \prec ... \prec \varphi_n}} Mod(\mu)$ *with:*

- $w \leq^d_{\varphi_1 \prec ... \prec \varphi_n} w'$ *iff* $[D(w, \varphi_1), ..., D(w, \varphi_n)] \leq_{lex} [D(w', \varphi_1), ..., D(w', \varphi_n)]$
- $[D(w, \varphi_1), ..., D(w, \varphi_n)]$ *is a vector which* k^{th} *element is* $D(w, \varphi_k)$
- $\leq_{lex}$ *is a lexicographic comparison of vectors of reals defined by:* $[v_1, ..., v_n] \leq_{lex} [v'_1, ..., v'_n]$ *iff (i)* $\forall k\ v_k = v'_k$ *or (ii)* $\exists k\ v_k < v'_k$ *and* $\forall j < k\ v_j = v'_j$

Definition 2. *An Importance-Based Opinion Diffusion Structure (IODS) is a quadruplet* $DS = (A, \mu, B, Inf)$ *where:* $A = \{1, ..., n\}$ *is a finite set of agents.* μ *is a consistent formula of* L*.* B *is a function which associates any agent* i *of* A *with a consistent formula of* L *denoted for short* B_i *such that* $B_i \models \mu$*.* Inf *is a function which associates any agent* i *of* A *with a non-empty set of agents* $\{i_1, ..., i_{n_i}\}$ *equipped with a total order* $\prec_i$ *s.t.* $i_k \prec_i i_{k+1}$ *for* $k = 1...(n_i - 1)$*. The agents of* $Inf(i)$ *will be called influencers of* i*. The influencer* i_1 *will be called the main influencer of* i*. For short, we denote* $Inf(i) = \{i_1 \prec_i ... \prec_i i_{n_i}\}$*.*

In an IODS any agent updates its opinion by applying an Importance-based Merging Operator on the opinions of its influencers as shown below.

Definition 3 (Opinion Sequence). *Let* $DS = (A, \mu, B, Inf)$ *be an IODS and* $i \in A$ *with* $Inf(i) = \{i_1 \prec_i ... \prec_i i_{n_i}\}$*. The Opinion Sequence of* i *in* DS *is denoted* $(B^s_i)_{s \in \mathbb{N}}$ *and is defined by* $(B^s_i)_{s \in \mathbb{N}}$*, is defined by:* $B^0_i = B_i$ *and* $\forall s > 0$*,* $B^s_i = \Delta_\mu(B^{s-1}_{i_1} \prec ... \prec B^{s-1}_{i_{n_i}})$

Example 1. Consider a language with propositional letters a and b. Let $S = (A, \mu, B, Inf)$ be an IODS with: $A = \{1, 2, 3\}$, μ is a tautology, $B_1 = \neg a$, $B_2 = a \vee b$, $B_3 = \neg b$, $Inf_1 = \{1\}$, $Inf_2 = \{2 \prec_2 1\}$, $Inf_3 = \{3 \prec_3 2\}$. The graph of influence is represented in Fig. 1. Moreover, Table 1 shows the evolution of the agents opinions.

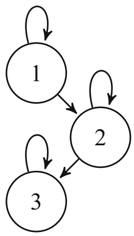


Fig. 1. Graph of influence of Example 1.

Let us finally introduce some interesting definitions.

Table 1. Opinion evolution in Example 1 (extracted from [14]).

	$s = 0$	$s = 1$	$s \geq 2$
$i = 1$	$\neg a$	$\neg a$	$\neg a$
$i = 2$	$a \vee b$	$\neg a \wedge b$	$\neg a \wedge b$
$i = 3$	$\neg b$	$a \wedge \neg b$	$a \wedge \neg b$

Definition 4 (dogmatic and self-confident agents). *Let $DS = (A, \mu, B, Inf)$ be an IODS, and $i \in A$.*

i dogmatic iff $Inf(i) = \{i\}$.

i is self-confident iff $Inf(i) = \{i \prec_i i_2 \prec_i ... \prec_i i_{n_i}\}$ with $n_i \geq 1$.

Definition 5 (Sphere of Influence of an Agent). *Let $DS = (A, \mu, B, Inf)$ be an IODS and $i \in A$. The sphere of influence of i is defined by: $Sphere(i) = \bigcup_{k \geq 1} Sphere^k(i)$ with*

$Sphere^1(i) = \{j_1 : Inf(j_1) = \{i \prec ...\}\}$

$Sphere^k(i) = \{j_k : Inf(j_k) = \{j_{k-1} \prec ...\}$ and $j_{k-1} \in Sphere^{k-1}(i)\}$

An agent is *dogmatic* when it is not influenced by other agents. As a consequence, a dogmatic agent i will never change its opinion i.e., $\forall s \geq 0 \; B_i^s = B_i^0$. An agent is *self-confident* when it is its main influencer. Notice that dogmatic agents in IODS are self-confident. Moreover if i is self-confident then $i \in Sphere(i)$.

3 Definitions of Extreme Opinions

This section presents several definitions of extreme opinions. Notice that precise opinions have been introduced in [14].

3.1 Extreme Opinions as Precise Opinions

Here we consider that extreme opinions are precise opinions i.e. formulas which have "few" models.

Definition 6 (Precise Opinions). *Let R be a given integer closer to 1 than to $2^{|L|}$. An opinion o is extreme iff $1 \leq | Mod(o) | \leq R$.*

The choice of the threshold R will depend on the application. But R has to be much smaller than the number of interpretations in the language. Moreover, inconsistent opinions are not considered as extreme. For instance, consider that the two letters of the language are a, b. If $R = 1$ then $a \wedge b$, $a \wedge \neg b$, $\neg a \wedge b$, $\neg a \wedge \neg b$ are the extreme opinions. The following proposition gives a description of extreme opinions from a syntactical point of view. More precisely, it shows that an extreme opinion is equivalent to a disjunction of less than R conjunctions of all the literals.

Proposition 1. *Assume that the propositional letters are $a_1, ..., a_n$. Opinion o is extreme iff $o \equiv \bigvee_{k=1}^{N} l_{1,k} \wedge ... \wedge l_{n,k}$ with $\forall k \in [1, N]$, $\forall i \in [1, n]$, $l_i^k = a_i$ or $l_i^k \neg a_i$ and $N \leq R$.*

Proof. Proofs are given in Sect. 8.

3.2 Extreme Opinions Based on Selected Topics

Definition 7 (Selected Topics). *A selected topic is a propositional letter of L. The set of selected topics is denoted S with $S \subseteq L$.*

The choice of "selected topics" depends on the application we consider. For instance, in the context of food served at the cafeteria, GMO food is a controversial subject according to which we may want to measure polarization. On the other side, expressing a position towards serving potatoes or serving fresh fruits is usually not. Thus here we will consider $L = \{GMO, potatoes, fruits\}$ with GMO meaning "I am ok with GMO food", $potatoes$ meaning "I am ok with serving potatoes", $fruits$ meaning "I am ok with serving fresh fruits" and $S = \{GMO\}$. But in winter, serving fresh fruits may become a controversial subject because of ecological reasons and expressing a positive position towards serving fresh fruits may become sensitive. In this context we will consider $S = \{GMO, fruits\}$.

In the following, we propose two definitions of extreme opinions. Each of them is based on a set of selected topics S and also on an integer $\alpha \in [1, 2^{|L|}]$, used to quantify the level of extremism. We will call these opinions $S^1\alpha$-extreme and $S^2\alpha$-extreme respectively.

Definition 8

- *An opinion o is $S^1\alpha$-extreme iff there is a subset of S, S_α, whose size is α and so that $Mod(o) \subseteq Mod(\bigwedge S_\alpha)$.*
- *An opinion o is $S^2\alpha$-extreme iff $Mod(o) \subseteq \{w \in Mod(L) : S(w) \geq \alpha\}$, $S(w)$ being the number of letters in S which are true in w.*

Thus an opinion is $S^1\alpha$-extreme iff there are α selected topics each model of o agrees on. An opinion is $S^2\alpha$-extreme iff its models agree on α selected topics at least. Moreover, we define S^1-not-extreme opinions and S^2-not-extreme opinions as follows.

Definition 9. *An opinion o is S^1-not-extreme (resp S^2-not-extreme) iff $\forall \alpha \in [1, 2^{|L|}]$, o is not $S^1\alpha$-extreme (resp $S^2\alpha$-extreme).*

The next proposition proves some results.

Proposition 2

- *If o is $S^1\alpha$-extreme (resp $S^2\alpha$-extreme) then for any β st $1 \leq \beta \leq \alpha$, o is $S^1\beta$-extreme (resp, $S^2\alpha$-extreme).*

- $S^1\alpha$*-extreme* $\subseteq S^2\alpha$*-extreme but in general* $S^2\alpha$*-extreme* $\not\subseteq S^1\alpha$*-extreme*
- S^2*-not-extreme* $\subseteq S^1$*-not-extreme*
- $S^1 \mid S \mid$*-extreme* $= S^2 \mid S \mid$*-extreme*

Proposition 3. *Assume* $S = \{s_1, ..., s_n\}$.

- o *is* S^1*-not-extreme iff for any* $S_\alpha = \{s_{i_1}, ...s_{i_\alpha}\} \subseteq S$, $\exists s_{i_j} \in S_\alpha$ *so that* $o \wedge \neg s_{i_j}$ *is consistent.*
- o *is* S^2*-not-extreme iff* $o \wedge \neg s_1 ... \wedge \neg s_n$ *is consistent.*
- *There is an* α *so that* o *is* $S^2\alpha$*-extreme iff* $o \models s_1 \vee ... \vee s_n$.
- α *is the highest value st* o *is* $S^2\alpha$*-extreme iff there are* $S_1 \subseteq S$ ... $S_k \subseteq S$ *with* $\mid S_1 \mid = ... = \mid S_k \mid = \alpha$ *st* o *equivalent to* $\bigvee_{i=1...k} (\bigwedge_{s \in S_i} s \bigwedge_{s \in S \setminus S_i} \neg s \bigwedge_{s \notin S} l_s)$, l_s *being* s *or* $\neg s$.

Let us give an example to illustrate these definitions.

Example 2. Suppose that $L = \{GMO, potatoes, fruits\}$ and $S = \{GMO, fruits\}$. Consider the following opinions: $GMO \wedge fruits$, GMO, $GMO \vee fruits$, $GMO \wedge potatoes$, $GMO \vee potatoes$, $potatoes$. Table 2 shows if they are extreme or not.

Table 2. Illustration of example 2.

	$S^1\alpha$-extreme	$S^2\alpha$-extreme
$GMO \wedge fruits$	$S^1$2-extreme with $S_2 = \{GMO, fruits\}$ $S^1$1-extreme with $S_1 = \{GMO\}$ or $S_1 = \{fruits\}$	$S^2$2-extreme $S^2$1-extreme
GMO	$S^1$1-extreme with $S_1 = \{GMO\}$	$S^2$1-extreme
$GMO \vee fruits$	S^1-not-extreme	$S^2$1-extreme
$GMO \wedge potatoes$	$S^1$1-extreme with $S_1 = \{GMO\}$	$S^2$1-extreme
$GMO \vee potatoes$	S^1-not-extreme	$S^2$1-extreme
$potatoes$	S^1-not-extreme	S^2-not-extreme

3.3 Extreme Opinions Based on Selected Agents

The two definitions presented in the previous section are dependent on S which is a subset of the language L. Here we propose a fourth definition which, assuming a set of agents A, depends on a subset SA of A. Elements of SA are called *selected agents* and are supposed to have extreme opinions. An opinion will then be considered extreme iff it is close to the current opinion of one of these selected agents.

In the following we consider δ, a function which measures how close two opinions are. We also consider a threshold $\epsilon \geq 0$.

Definition 10. *Let A be a set of agents. A selected agent is a particular agent in A. The set of selected agents is denoted SA with $SA \subseteq A$.*

Definition 11. *Let A be a set of agents and $SA \subseteq A$ a set of selected agents. An opinion o is $SA\epsilon$-extreme at step t iff $\exists i \in SA$ st $\delta(o, B_i^t) \leq \epsilon$ where B_i^t denotes the opinion of i at step t*

According to the definition of $SA\epsilon$-extreme opinions[1], an opinion is extreme iff it is close to the opinion of a selected agent. As written before, the choice of selected agents depends on the context. For instance, in the cafeteria context, we could select the secretaries of the different student unions, thus considering them as a reference for extremism. As for the threshold, it also depends on the application. The smaller it is, the less we get extreme opinions.

As for the measure δ, there are many options. We could for instance consider some pseudo-distances [15].

- Sum of minimum distances:
  $\delta_{summin}(o_1, o_2) = \frac{1}{2}(\sum_{w \in Mod(o_1)} D(w, o_2) + \sum_{w \in Mod(o_2)} D(w, o_1))$
- Hausdorff distance:
  $\delta_{Hau}(o_1, o_2) = max(max_{w \in Mod(o_1)} D(w, o_2), max_{w \in Mod(o_2)} D(w, o_1))$
- Minimum of distances
  $\delta_{min}(o_1, o_2) = min_{w_1 \in Mod(o_1), w_2 \in Mod(o_2)} d(w_1, w_2)$
- Sum of distances
  $\delta_{sum}(o_1, o_2) = \sum_{w_1 \in Mod(o_1), w_2 \in Mod(o_2)} d(w_1, w_2)$

We could also consider the following measure:

- Agreement-disagreement distance: Assume that the propositional letters are $p_1, ..., p_n$. $\delta_{AD}(o_1, o_2) = asc(\delta_{AD}^1, ..., \delta_{AD}^n)$ such that:
  - $\forall i = 1..n$, $\delta_{AD}^i = 0$ iff o_1 and o_2 agree on p_i (i.e., $o_1 \models p_i$ iff $o_2 \models p_i$ and $o_1 \models \neg p_i$ iff $o_2 \models \neg p_i$); $\delta_{AD}^i = 1$ iff o_1 and o_2 disagree on p_i (i.e., $o_1 \models p_i$ iff $o_2 \models \neg p_i$ and $o_1 \models \neg p_i$ iff $o_2 \models p_i$); $\delta_{AD}^i = 0.5$ in the other case.
  - *cresc* is the function with orders a sequence of integers in the ascending order.

We could even define measure δ from an inconsistency measure as follows:

- $\delta_{Inc}(o_1, o_2) = Inc(\{o_1, o_2\})$ where Inc is a measure of inconsistency [16–18].

Or we could consider sweaker functions than pseudo-distances like:

- $\delta_{max}(o_1, o_2) = max_{w_1 \in Mod(o_1), w_2 \in Mod(o_2)} d(w_1, w_2)$

Finally, since the relation of influence is not symmetric, we could also drop the property of symmetry and consider:

[1] Notice that we should index this definition with δ but we omit it for readability reasons.

– $\delta_{maxmin}(o_1, o_2) = \max_{w_1 \in Mod(o_1)} min_{w_2 \in Mod(o_2)} d(w_1, w_2)$

Since there are many options, we propose to consider a set of requirements on which the different measures can be compared. They are given below. Notice that these requirements are not minimal since (R5) is subsumed by (R1).

– (R1) δ is a pseudo-distance, i.e. $\delta(\varphi, \psi)$ is minimal iff $\models \varphi \leftrightarrow \psi$ and $\delta(\varphi, \psi) = \delta(\psi, \varphi)$.
– (R2) The more propositional letters φ and ψ agree on, the smaller $\delta(\varphi, \psi)$ is.
– (R3) The more propositional letters φ and ψ disagree on, the higher $\delta(\varphi, \psi)$ is.
– (R4) If $\varphi_1 \wedge \psi_1$ is inconsistent and if $\varphi_2 \wedge \psi_2$ is consistent then $\delta(\varphi_2, \psi_2) < \delta(\varphi_1, \psi_1)$.
– (R5) If $\models \varphi \leftrightarrow \psi$ then $\delta(\varphi, \psi)$ is minimal.
– (R6) If $\models \varphi \leftrightarrow \varphi'$ and $\models \psi \leftrightarrow \psi'$ then $\delta(\varphi, \psi) = \delta(\varphi', \psi')$
– (R7) If $\varphi_1 \models \varphi_2$ then $\delta(\varphi_1, \varphi) \leq \delta(\varphi_2, \varphi)$

Proposition 4. *Table 3 shows which requirements the previous measures satisfy.*

Let us illustrate these definitions with an example.

Table 3. Measures versus requirements.

	δ_{summin}	δ_{Hau}	δ_{AD}	δ_{Inc}	δ_{max}	δ_{min}	δ_{sum}	δ_{maxmin}
(R1)	Yes	Yes	No	No	No	Yes	Yes	Yes
(R2)	No	No	Yes	No	No	No	No	No
(R3)	No	Yes	Yes	No	No	Yes	No	No
(R4)	No/yes	No	No	Yes	No	Yes	No	No
(R5)	Yes	Yes	No	Yes	No	Yes	Yes	Yes
(R6)	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
(R7)	Yes	Yes	No	No	Yes	Yes	Yes	Yes

Example 3. Take again $L = \{GMO, potatoes, fruits\}$. Consider one selected agent $John$ whose current opinion is $GMO \wedge \neg potatoes$. Consider the following opinions: GMO, $GMO \wedge fruits$, $potatoes$, $fruits$, $GMO \vee \neg potatoes$. Table 4 shows their distance to $John$'s opinion with δ_{summin}, δ_{Hau} and δ_{AD} when the distance between interpretations is d_H.

Consider δ_{summin} and assume that $\epsilon = 3$. Then, the extreme opinions are GMO, $GMO \wedge fruits$ and $GMO \vee \neg potatoes$.

Consider δ_{Hau} and assume that $\epsilon = 3$. Then, the extreme opinions are GMO and $GMO \wedge fruits$.

Consider δ_{AD} and assume that $\epsilon = [0, 0.5, 0.5]$. Then, the extreme opinions are GMO and $GMO \wedge fruits$.

This example shows that depending on the distances used, the set of extreme opinions may vary.

Table 4. Illustration of example 3.

	δ_{summin}	δ_{Hau}	δ_{AD}
GMO	1	1	[0, 0.5, 0.5]
$GMO \wedge fruits$	1	1	[0, 0.5, 0.5]
$potatoes$	4	6	[0, 0.5, 1]
$fruits$	4	6	[0.5, 0.5, 0.5]
$GMO \vee \neg potatoes$	2.5	5	[0.5, 0.5, 0.5]

4 Diffusion of Extreme Opinions in IODS

4.1 Diffusion of Precise Opinions

We first define extremist agents as agents whose opinions are extreme. Moreover, agent are moderate when they are not extremist.

Definition 12 (Extremist, Moderate). *An agent i is extremist at step s iff B_i^s is an extreme opinion. Otherwise it is moderate.*

Example 4. Consider two propositional letters a, b and assume that at a given step s agents opinions are: $B_i^s = a \vee b$, $B_j^s = a$, $B_k^s = a \wedge b$. If $R = 1$ then only k is extremist. If $R = 2$ then j and k are extremist.

By definition of extreme opinions, it is true that extremist agents are more certain of their opinions than moderate ones. Indeed, let $DS = (A, \mu, B, Inf)$ be an Opinion Diffusion Structure, then $\forall i \in A$, $\forall j \in A$, $\forall s \in \mathbb{N}$, if i is extremist at s and j is moderate at s then $\mid Mod(B_i^s) \mid < \mid Mod(B_j^s) \mid$. Moreover, due to the definition of dogmatic agents, it is true that in an Opinion Diffusion Structure, a dogmatic agent who initially is extremist will remain extremist.

In the following, we list some properties of diffusion of extreme opinions in $IODS$.

First we can show that an agent whose main influencer is extremist at some step will be extremist at the next step.

Proposition 5. *In an IODS $S = (A, \mu, B, Inf)$, for $i \in A$ with $Inf(i) = \{j \prec_i ...\}$, for $t \in \mathbb{N}$, if j is extremist at step s, then i is extremist at step $s + 1$.*

As a consequence, an agent whose all influencers are extremist will become extremist. Another consequence is that a self-confident agent which is extremist at some step will remain extremist ever after:

We can also show that even if all its influencers are moderate, an agent may become extremist. For instance, take $R = 1$ and consider an agent who is influenced by two agents whose opinions are respectively a and b. The agent's opinion, got after merging these two opinions, is $a \wedge b$. That is, the agent's opinion is extreme, while the opinions of all of its influencers are not.

The following proposition states that an agent which k-th influencer has an opinion consistent with the merging of the ones of the previous influencers at step s and which k-th influencer is extremist at step s will be extremist at step $s+1$.

Proposition 6. *In an IODS* $S = (A, \mu, B, Inf)$*, for* $i \in A$ *with* $Inf(i) = \{j_1 \prec_i ... \prec_i j_k \prec_i ... \prec_i j_n\}$*, for* $s \in \mathbb{N}$*, if* $\Delta_\mu(B^s_{j_1} \prec_i ... \prec_i B^s_{j_{k-1}}) \wedge B^s_{j_k}$ *is consistent and* j_k *is extremist at step* s*, then* i *is extremist at step* $s+1$*.*

More generally, an agent will be extremist at step $s+1$ iff for some k, the merging of the k first influencers' opinions at step s has less than R models.

Proposition 7. *In an IODS* $S = (A, \mu, B, Inf)$*, for* $i \in A$ *with* $Inf(i) = \{j_1 \prec_i ... \prec_i j_k \prec_i ...\}$*. Let* $s \in \mathbb{N}$*. If* $\exists k \in \mathbb{N}$*, such that* $\mid Mod(\Delta_\mu(B^s_{j_1} \prec ... \prec B^s_{j_k})) \mid \leq R$ *then,* i *is extremist at* $s+1$*. Otherwise, it is moderate at step* $s+1$*.*

The following proposition states that a self-confident extremist agent spreads extremism in its sphere of influence.

Proposition 8. *Let* $S = (A, \mu, B, Inf)$ *an IODS and* $i \in A$ *extremist at step* s *with* $Inf(i) = \{i \prec_i ...\}$*.* $\exists s' \geq s$*,* $\forall s \geq s'$*,* $\forall j \in Sphere(i)$ j *is extremist at step* s*.*

4.2 Diffusion of $S^1\alpha$-extreme and $S^2\alpha$-extreme Opinions

Definition 13. *We consider an IODS* $DS = (A, \mu, B, Inf)$*, a set of selected topics* S *and a value* α*. Let* $i \in A$ *and* $t > 0$*.* i *is* $S^1\alpha$*-extremist (resp* $S^2\alpha$*-extremist) at step* t *iff* B^t_i *is a* $S^1\alpha$*-extreme (resp* $S^2\alpha$*-extreme) opinion.* i *is* S^1*-not-extremist (resp* S^2*-not-extremist) at step* t *iff* B^t_i *is a* S^1*-not-extreme opinion (resp,* S^2*-not-extreme opinion).*

We first show how important is the main influncer in the spreading of extremism.

Proposition 9. *Consider an IODS* $DS = (A, \mu, B, Inf)$*, a set of selected topics* S *and a value* α*. Let* $i \in A$ *with* $Inf(i) = \{i_1 \prec_i ... \prec_i i_n\}$ *and let* $t \geq 0$*. If* i_1 *is* $S^1\alpha$*-extremist (resp,* $S^2\alpha$*-extremist) at step* t *then* i *is* $S^1\alpha$*-extremist (resp,* $S^2\alpha$*-extremist) at step* $t+1$*.*

As a consequence, given a definition of extremism, a self-confident agent which is extremist at a given step remains extremist. Moreover, a self-confident agent which is extremist spreads extremism in its sphere of influence as shown in the proposition below.

Proposition 10. *Consider an IODS* $DS = (A, \mu, B, Inf)$*, a set of selected topics* S*, a value* α *and let* $t \geq 0$*. Let* $i \in A$ *a self-confident agent which is* $S^1\alpha$*-extremist (resp,* $S^2\alpha$*-extremist) at* t*. Then* $\forall j \in Sphere(i)$ $\exists s \geq t$ $\forall s' \geq s$ *st* j *is* $S^1\alpha$*-extremist (resp,* $S^2\alpha$*-extremist) at* s'*.*

So the two types of extremism defined previously, spread with influence. However, the corresponding non-extremism generally do not: an agent may become $S^1\alpha$-extremist (resp, $S^2\alpha$-extremist) even if its main influencer is S^1-not-extremist (resp, S^2-not-extremist) or worst, even if its influencers are $S^1\alpha$-not-extremist (resp, $S^2\alpha$-not-extremist) as shown in the following example.

Example 5. Consider a language with letters a, b, c and $S = \{a\}$. Take $\alpha = 1$. Assume $SD = (A, \mu, B, Inf)$ with $A = \{1, 2, 3\}$, μ being a tautology, $Inf(1) = \{1\}, Inf(2) = \{2\}, Inf(3) = \{1 \prec 2 \prec 3\}$ and $B_1^0 = b, B_2^0 = a \vee \neg b, B_3^0 = a \vee a$. We can show that $B_3^1 = a \wedge b$. This proves that, even 1 and 2 are S^1-not-extremist nor S^2-not-extremist, 3 becomes $S^1 1$-extremist and $S^2 1$-extremist.

In the following, we study a case when non-extremism spreads under influence. But before we introduce the notion of opposition, as a particular case of non-extremism.

Definition 14. *Consider an IODS $DS = (A, \mu, B, Inf)$, $i \in A$ and S a set of n selected topics. i is S-opponent at t iff B_i^t is $\overline{S}^1 n$-extreme (or equivalently, $\overline{S}^2 n$-extreme).*

Thus i is S-opponent at t iff $B_i^t \models \bigwedge \overline{S}$, iff $B_i^t \models \bigwedge_{s_i \in S} \neg s_i$. Obviously, S-opposition is a particular case of non-extremism. i.e., if i is S-opponent at t then i is S^1-not-extreme and S^2-not-extreme at t.

The following proposition shows that S-opposition spreads under influence.

Proposition 11. *Consider an IODS $DS = (A, \mu, B, Inf)$, a set of selected topics S. Let $i \in A$ with $Inf(i) = \{i_1 \prec_i ... \prec_i i_n\}$ and $t \geq 0$. If i_1 is S-opponent at t then i is S-opponent at $t + 1$.*

Again this shows the importance of the main influencer.

Example 6. In Example 5, agent 1 is not S-opponent since $Mod(B_1^0) = \{\{a, b\}, \{\neg a, b\}\}$. Moreover in this case, the merging operator selectes the model $\{a, b\}$ which ensures that 3 becomes extremist. Consider now a modified version of Example 5 and supose now that 1 is S-opponent by assuming $B_1^0 = \neg a \wedge b$. Then $B_3^1 = \neg a \wedge b$ i.e. 3 is S-opponent.

4.3 Diffusion of $SA\epsilon$-extreme Opinions

In this section, we study the diffusion of extremism when extreme opinions are defined as $SA\epsilon$ extreme opinions. Thus we consider the following definition.

Definition 15. *Let A be a set of agents and $SA \subseteq A$ a set of selected agents. An agent $i \in A$ is $SA\epsilon$-extremist at step t iff B_i^t is an $SA\epsilon$-extreme opinion.*

Moreover, we assume that the selected agents are dogmatic i.e., they are not influenced by others and thus they don't change their opinions. Under this assumption, we can prove the following propositions.

Proposition 12. *Consider an IODS* $DS = (A, \mu, B, Inf)$*, a set of selected agents* SA*, a value* ϵ*. Let* $i \in A$ *so that* $Inf(i) = \{i_1 \prec ....\}$*. Suppose that the distance used to characterize the* $SA\epsilon$ *extremism satisfies (R6) and (R7). Then: If* i_1 *is* $SA\epsilon$*-extremist at step* t *then* i *is* $SA\epsilon$*-extremist at step* $t+1$

The following is a corollary.

Proposition 13. *Consider an IODS* $DS = (A, \mu, B, Inf)$*, a set of selected agents* SA*, a value* ϵ*. Suppose that the distance used to characterize the* $SA\epsilon$ *extremism satisfies (R6) and (R7). Let* $i \in A$ *be a self-confident who becomes* $SA\epsilon$*-extremist at time* t*. Then:*

(1) $\forall t' \geq t$ i *is* $SA\epsilon$*-extremist at time* t'*.*
(2) $\forall j \in Sphere(i)$ $\forall t' \geq t$ j *is* $SA\epsilon$*-extremist at time* t'*.*

5 Generating Graphs for Experiments

In this section and the following, we focus on simulating with NetLogo the diffusion of some extreme opinions. More precisely, in this section, we address the question of generating graphs corresponding to real social networks. For that, we review some propositions made in graph theory during the last decades. Then, we adapt them to our context.

5.1 Graph Theory Bases

One of the most used models of graph is the one of Erdös-Rényi. It is a model of random graph (see [19]).

Definition 16 (Erdös-Rényi Graph). *Given a number of nodes* n *and an integer* m*. An Erdös-Rényi Graph is any graph obtained by selecting randomly* m *edges among the* 2^n *possible ones.*

Another model of graph that is widely used is the model of Watts-Strogatz. This model has been made to describe the phenomenon of Small-World or "six degrees of separation" highlighted by Milgram [20]. This psychologist established through an experiment the theory that a message can be transmitted from one person to one another by passing by an average of six friends. The Small-World theory is commonly formalized [19,21,22] as follows:

Definition 17 (Small-World). *A graph* G *is said Small-World if it satisfies:*

1. *G is connected.*
2. *G is sparse: the average degree of the nodes k is low compared to the number of nodes n, $k \ll n$.*
3. *G is decentralized: the maximal degree of the nodes k_{max} is low compared to the number of nodes n, $k_{max} \ll n$.*

4. *The characteristic path L (the average number of nodes traversed by a short path between two nodes) is close to the one of a random graph with the same number of nodes n and the same average degree k,* $L \approx L_{random} \sim \frac{ln(n)}{ln(k)}$.
5. *The clustering coefficient C (the probability that two nodes i and j are connected given that they share a common neighbor) is high compared to the one of a random graph with the same number of nodes n and the same average degree k,* $C \gg C_{random} \sim \frac{k}{n}$.

One can notice that Erdös-Rényi graphs as random graphs have low characteristic paths by definition.

The following model, from [19] and adapted from a model generally attributed to Watts and Strogatz, define Small-World graphs:

Definition 18 (Rank-Based Friendship Graph). *Given a number of nodes n, a threshold r, an exponent q and a dimension d, the nodes are randomly distributed in a space of dimension d. Rank-Based Friendship Graph is obtained by going as follows:*

For each node i, we rank the other nodes according to their distances to i and we break ties with a chosen method. There will be an edge from a node j to the node i with probability $\frac{1}{Z.rank_i(j)^q}$, $rank_i(j)$ *being the rank of j in i's neighbors and Z a coefficient of normalization,* $Z = \sum_{i=1}^{n} \frac{1}{rank_i(j)^q} = \sum_{i=1}^{n} \frac{1}{i^q}$.

5.2 Models

Here we adapt the previous models of graphs to IODS and explain how we construct them for the simulations. In the following we take an integrity constraint being a tautology.

The first model we adapt is the one of the random graph defined by Erdös and Rényi. The following definition shows how we construct Erdös and Rényi IODS. Notice that we add a parameter, the number of self-confident agents, which is an interesting variable to study.

Definition 19 (Erdös-Rényi-Based IODS). *Given the parameters num-letters, num-nodes, num-links and num-self-confident, the IODS is constructed as follows:*

We begin by creating num-nodes agents, each of them has a random opinion in a language of num-letters letters. Then, we create num-links relations of influence by choosing randomly an influencer and an influenced agent (potentially the same). The influencers are ordered according to the order of creation of the relation of influence, the sooner a relation of influence would have been created the more influencing it is. Finally, each agent with no influencers will become dogmatic and, if necessary, we add relations of self-influence until we have num-self-confident self-confident agents (dogmatic agents included). We pick randomly an agent and if it does not already influence himself we make it self-confident by putting it as its main influencer (the order of the other influencers remains unchanged).

This second model adapts the model of Rank-Based Friendship by considering a distance between opinions instead of a physical distance as for the graph model.

Definition 20 (Rank-Based Influenceship IODS). *Given the parameters num-letters, num-nodes, opinions-distance, q and num-self-confident, the IODS is constructed as follows:*

We begin by creating num-nodes agents, each of them has a random opinion in a language of num-letters letters. Then, we fill a matrix with the distances between every couple of agents according to the distance between their opinions and computed with the distance opinions-distance. For each agent i we have a list l_i of all the agents (i included) sorted according to their distances to i. If two agents j_1 and j_2 are at the same distance of i, then the tie will be randomly solved. Each agent j will be an influencer of i with probability $\frac{1}{Z.rank_i(j)^q}$, $rank_i(j)$ being the rank of j in l_i and Z being a coefficient of normalization, $Z = \sum_{i=1}^{num-nodes} \frac{1}{rank_i(j)^q} = \sum_{i=1}^{num-nodes} \frac{1}{i^q}$. The influencers of i are ordered as in l_i. Finally, if necessary, we add relations of self-influence such as we have num-self-confident self-confident agents (dogmatic agents included). We pick randomly an agent and if it is not already self-confident we make it so by putting it as its main influencer (the order of the other influencers remains unchanged).

The third model is a variant of the previous one, here an agent will be influenced by the m agents that have the closest opinions from its own one for a given integer m.

Definition 21 (Deterministic Rank-Based Influenceship IODS). *Given the parameters num-letters, num-nodes, opinions-distance, m and num-self-confident, the IODS is constructed as follows:*

We begin by creating num-nodes agents, each of them has a random opinion in a language of num-letters letters. Then, we fill a matrix with the distances between every couple of agents according to the distance between their opinions and computed with the distance opinions-distance. For each agent i, we conserve the m closest agents to i to be its influencers. If two agents j_1 and j_2 are at the same distance of i, then the tie will be randomly solved. The influencers of i are ordered according to their distances to i. Finally, we add relations of self-influence such as we have num-self-confident self-confident agents (dogmatic agents included). We pick randomly an agent and if it is not already self-confident we make it so by putting it as its main influencer (the order of the other influencers remains unchanged).

The fourth model is a generalization of the Rank-Based Influenceship in which we have in addition to the distance between opinions a physical distance along a circle. The influencers of an agent i are ordered according to the distance between their opinions and the one of i.

Definition 22 (Opinions and Physical Rank-Based Influenceship IODS). *Given the parameters num-letters, num-nodes, opinions-distance, r, q and num-self-confident, the IODS is constructed as follows:*

We begin by creating num-nodes agents, each of them has a random opinion in a language of num-letters letters. We fill a matrix with the distances between every couple of agents according to the distance between their opinions and computed with the distance opinions-distance. For each agent i we have a list l_i of all the agents (i included) sorted according to their distances to i. If two agents j_1 and j_2 are at the same distance of i, then the tie will be randomly solved. Each agent j will be an influencer of i with probability $\frac{1}{Z.rank_i(j)^q}$, $rank_i(j)$ being the rank of j in l_i and Z a coefficient of normalization $Z = \sum_{i=1}^{num-nodes} \frac{1}{rank_i(j)^q} = \sum_{i=1}^{num-nodes} \frac{1}{i^q}$. At the previous influencers we add influencers that are physically close. Indeed, all the agents will be placed on a circle. The agents that are separated on the circle from an agent i by less than r agents will influence i. The influencers of i are ordered as in l_i (according to the distance between opinions). Finally, if necessary, we add relations of self-influence such as we have num-self-confident self-confident agents (dogmatic agents included). We pick randomly an agent and if it is not already self-confident we make it so by putting it as its main influencer (the order of the other influencers remains unchanged).

6 Some Experiments

In this section, we present simulations of precise opinion diffusion in the different graphs previously presented. Moreover, as for the distances between opinions, we focus on δ_{summin}, δ_{Hau}, δ_{max}, δ_{min}, δ_{maxmin}, δ_{sum} with the drastic pseudo-distance between interpretations d_D.

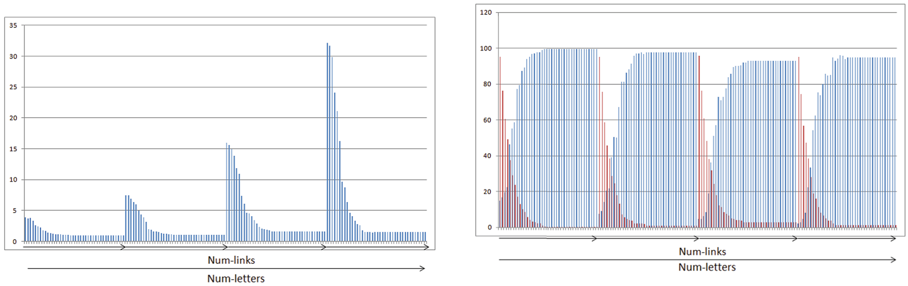
To study and compare the results between the different models and distances, we carried out several simulations with the same settings. Furthermore, in order to do comparable and reproducible experiments we chose some values of seeds for the random operations in Netlogo. Seeds allow to have the same results in the same order for random operations when we repeat the simulations. The values we study are the number of extremist agents for $R = 1$, the average number of models per agents and the number of dogmatic agents.

In the simulations we present here, we have taken the following values: seed $\{0, 100, 200\}$, *num-letters* $\{3, 4, 5, 6\}$, *num-nodes* $\{10, 60, 110, 160, 210\}$, *num-self-confident* $\{0, 50, 100, 150, 200, 210\}$. For the three models using ranks we tested δ_{summin}, δ_{Hau}, δ_{max}, δ_{min}, δ_{maxmin}, δ_{sum} with the drastic pseudo-distance between interpretations. For the Erdös-Rényi-Based model we took *num-links* varying from 10 to 2000 with an increment of 50. For the Rank-Based Influenceship and the Opinions and Physical Rank-Based Influenceship models we took the values $\{1, 2, 3, 4, 5\}$ for q, for the Deterministic Rank-Based Influenceship model we took m in $\{1, 2, 3, 4\}$ and for the Opinions and Physical Rank-Based Influenceship model we took r in $\{1, 2, 3, 4, 5\}$. The pseudo-distance between interpretations used to compute the Importance-based Merging operator is the drastic one.

First of all, *with the drastic pseudo-distance* d_H, we can notice that the different distances we used have particular behaviors. $\delta_{summin}(o_1, o_2) \in [0, 2^{num-letters}]$ and $\delta_{summin}(o_1, o_2) = 0$ iff $o_1 \equiv o_2$. δ_{summin} favors relations of influence between agents which opinions have models very close according to D, in average. $\delta_{Hau}(o_1, o_2)$ is 0 or 1 and $\delta_{Hau}(o_1, o_2) = 0$ iff $o_1 \equiv o_2$. δ_{Hau} favors relations of influence between agents which opinions have no models very far from one another. So, in the case of the drastic pseudo-distance, it favors relations between agents that have the same opinion. So, for the number of letters and agents we will consider, as such a case is unlikely the relations of influence will be mostly random. $\delta_{max}(o_1, o_2)$ is 0 or 1 and $\delta_{max}(o_1, o_2) = 0$ iff $\exists w$, $Mod(o_1) = Mod(o_2) = \{w\}$. So, as having two agents with only one model and the same model is very unlikely for the number of letters and agents we consider, δ_{max} favors random relations of influence and it will be interesting to compare the results obtained with this distance and the ones obtained with the other distances. $\delta_{min}(o_1, o_2)$ is 0 or 1 and $\delta_{min}(o_1, o_2) = 0$ iff $\exists w \in Mod(o_1) \cap Mod(o_2)$. Then, for a given agent i, δ_{min} favors relations of influence that are from agents that share a model with i's opinion but that are otherwise random. $\delta_{maxmin}(o_1, o_2)$ is 0 or 1 and $\delta_{maxmin}(o_1, o_2) = 0$ iff $o_1 \models o_2$. So, δ_{maxmin} favors relations of influence from an agent i to an agent j such that all the models of B_j are models of B_i. $\delta_{sum}(o_1, o_2) \in [0, 2^{num-letters}]$ and $\delta_{sum}(o_1, o_2) = 0$ iff $Mod(o_1) = Mod(o_2) = \{w\}$. δ_{sum} favors relations of influence from agents which opinions have the less models.

For the Erdös-Rényi-Based model (see Fig. 2), we have several peaks of the average number of models and of the number of dogmatic agents, corresponding to having low *num-links*. Indeed, in these cases, there are potentially more agents that are not influenced by other agents and that keep their initial opinions. Furthermore, we can see that the dogmatic agents are almost the only agents that are not extremist and thus contribute the more to the average number of models. So, the diffusion of extremism depends a lot on the ratio between *num-links* and *num-nodes*, the more there are relations of influence the more the agents will become extremist. We can only notice that the peaks of average number of models are higher and higher according to the increasing of the number of letters. Another experiment in which we took 200 agents and much more relations of influence (up to 7000) showed that for more than 2000 there are very few simulations with non-extremist agents.

For the Rank-Based Influenceship model (see Fig. 3), we have several plateaus higher and higher according to the increasing of the number of letters. Furthermore, there are cases with very low numbers of extremist agents and without very much dogmatic agents. Then, we can notice that there are big differences according to the distance we use. Indeed, a thorougher analysis highlights that the biggest peaks are with δ_{summin} and then with δ_{Haus} and δ_{maxmin}. δ_{max} and δ_{min} cause some lesser peaks when q gets bigger (more than 3) and δ_{sum} causes very small peaks for $q = 5$. For $q = 1$ almost all the agents are extremist

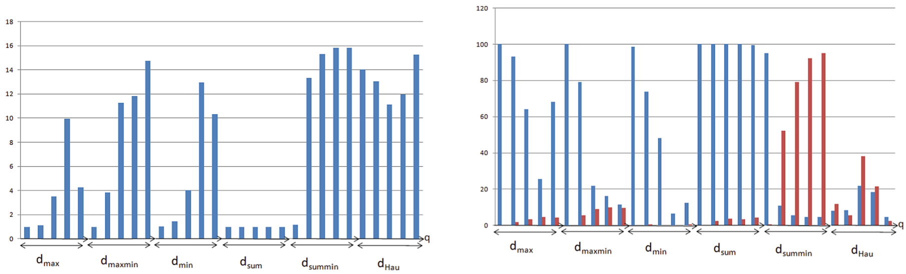


Average Number of Models per Agents

Proportion of extremist agents (blue) and dogmatic agents (red)

Fig. 2. Example: Erdös-Rényi-Based model *seed = 0, num-nodes = 210, num-self-confident = 0* (extracted from [14]).

whatever the distance we use. It can be explained by the fact that the lesser q is the more likely relations of influence are to be created, furthermore for q high enough the distance used matter less then even δ_{sum} that in the other cases spread extremism may be used to create an IODS where they may remain some moderate agents. But, according to [19] in the case of graphs, the Rank-Based Friendship generates graphs the closest of reality for $q = 1$. Furthermore, when the number of agents increases, the average number of models decreases because more relations of influence may be created. One can notice that with this model δ_{summin} and δ_{Haus} particularly favor moderation. So, having influencers with opinions for which each model is close of one of us model or for which each model is not far of any of our model favor moderation. But, we can notice that with δ_{Haus} agents are much less dogmatic than with δ_{summin}.

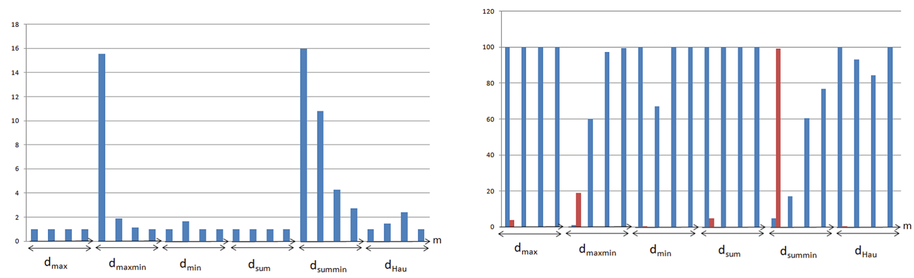


Average Number of Models per Agents

Proportion of extremist agents (blue) and dogmatic agents (red)

Fig. 3. Example: Rank-Based Influenceship model *seed = 0, num-letters = 5, num-nodes = 210, num-self-confident = 0* (extracted from [14]).

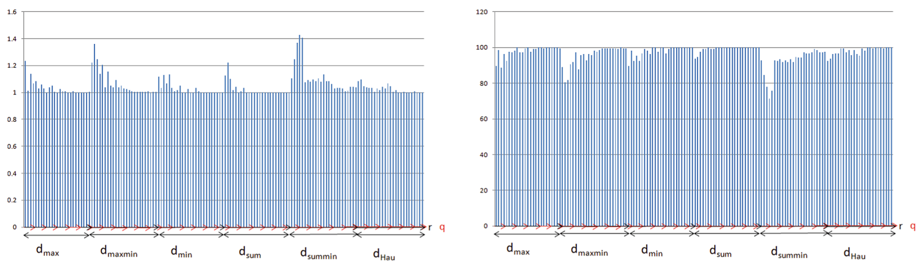
For the Deterministic Rank-Based Influenceship model (see Fig. 4), extremism spreads more and more when m gets bigger. Moreover, this time there is much more differences according to the distance we used because the ranking is more important in the choice of the influencers than before. Then, only δ_{summin} keeps many non-extremist agents when m is at its highest. Indeed, this distance characterizes the best the similarity between opinions, the first agents in the ranking of an agent i actually have opinions that share many models with the one of i and it often is i itself. Thus, when $m = 1$, we have almost only dogmatic agents with δ_{summin}. When m gets higher than 3 only models with δ_{summin} keep moderate agents.



Average Number of Models per Agents

Proportion of extremist agents (blue) and dogmatic agents (red)

Fig. 4. Example: Deterministic Rank-Based Influenceship model *seed = 0, num-nodes = 210, num-self-confident = 0* (extracted from [14]).



Average Number of Models per Agents

Proportion of extremist agents

Fig. 5. Example: Opinions and Physical Rank-Based Influenceship model *seed = 0, num-letters = 5, num-nodes = 210, num-self-confident = 0* (extracted from [14]).

For the Opinions and Physical Rank-Based Influenceship model (see Fig. 5), we have very few non-extremist agents even with δ_{summin} and even less when r increases. It is due to the fact that here there cannot be any dogmatic agent (contrary to the case of the Rank-Based Influenceship) and that an agent may have influencers with very different opinions (contrary to the case of the Deterministic Rank-Based Influenceship).

In all the simulations, the number of letters does not affect the proportion of extremist agents. The number of nodes affects the proportion of extremist agents for the Erdös-Rényi-Based model because of our definition of the model, in fact it is the ratio between the number of agents and the number of relations of influence that truly matters. It also has an influence for Rank-Based Influenceship model and the Opinions and Physical Rank-Based Influenceship model because it increases the average number of influencers.

For summarizing, among the different models of IODS, the ones which spread extremism the less are the Rank-Based Influenceship when q is very high and the Erdös-Rényi-Based when *num-links* is much lower than *num-nodes*. But, those models have many dogmatic agents, on the other hand, the Deterministic Rank-Based Influenceship spreads extremism very little with δ_{summin} and a small m and without many dogmatic agents. For the distances, it is δ_{summin} that spreads extremism the less because it favors relations of influence from agents with opinions sharing many models and it spreads extremism less than δ_{max} (the random one). At the opposite, δ_{sum} spreads extremism very well by creating hubs, agents with very few models that influence a lot of agents. δ_{min} spreads extremism a little less because it is less random, there is a constraint on one model. So, with the Importance-Based Merging Operator, the extremism spreads very well when the most extremist agents are very influential and much less when agents are influenced by agents with opinions similar to its own in the sense of they share many models. So, what makes that an agent remains moderate is the fact that he is influenced by agents which opinions share many models between them and that he does not have too many influencers. Having many self-confident agents favor extremism spreading with the Erdös-Rényi-Based model as it increases the average number of influencers but in the other models it favors moderation. Indeed, in this case the agents keep opinions close to their initial ones and so agents' influencers keep close opinions.

We can notice that, in every simulation, we reached the convergence very quickly in general in less than 5 updates.

It would have been interesting to test the models for much larger numbers of agents to increase the probabilities we have deemed negligible in our study of the distances for instance. Indeed, the Small-World phenomenon is considered interesting for very large number of nodes i.e. billions of nodes (see [23]) but the computation time that would be needed only for models of thousands of agents is very important.

Furthermore, other simulations with Hamming pseudo distance both for the computation of the distances between opinions and the update of the opinions gave similar results. Notwithstanding, extremism spreads slightly much, in average 0.8 less models per agents and 9% less extremist agents. This can mainly be explain by the fact that $Min_{\leq_{\delta_H,\varphi_1 \prec \varphi_2}} Mod(\mu)$ contains generally less models than $Min_{\leq_{\delta_D,\varphi_1 \prec \varphi_2}} Mod(\mu)$ as the second one keeps all the models of φ_1 if φ_1 and φ_2 are inconsistent. The only type of IODS that spreads less extremism in this case is the Rank-Based Influenceship model, in average there are 2 more models per agents and 10% less extremist agents. But, it can be explained by the fact that there are twice more (15% more) dogmatic agents, the hamming pseudo-distance allows a more accurate ranking of the agents and thus, it is less likely that agents with very different opinions influence an agent. It appears that this accuracy is all the more significant that the number of letters is important. However, the first agents in the rankings do not change a lot, so the Deterministic Rank-Based Influenceship model spreads more extremism. Another noticeable difference are for δ_{Hau} and δ_{summin} which spread extremism much less in the three Rank-Based Influenceship models.

What we can notice is that the more relations of influence there are, the more extremism spreads. And, the more influencers of agents have close opinions, the less extremism spreads. This result can be interpreted as follows: When someone makes its own mind by taking into account the opinions of many people it considers as reliable or experts on the matter and with different opinions then, it will be very sure of its new opinion as it is a compromised between the opinions of many experts. And so, this person will become extremist according to our definition.

7 Conclusion

This paper focused on extremism diffusion in IODS. Its main contributions are: a proposal of different definitions of extreme opinions and extremist agents; a formal study of diffusion of these different kinds of extreme opinions in IODS; a simulation in NetLogo of the diffusion of the first kind of extreme opinions.

This work could be continued according to several directions. Let us mention three of them. First, experiments for the other kinds of extreme opinions are to be made in a near future. Besides, we want to extend this study in the case when the procedure for updating opinions is not Importance-Based Merging but Majority-Based Merging Operator [24]. Finally, we plan to add a dynamic aspect by changing the relations of influence through time as it is often done in the usual models [1,2,9,10]. It will be especially interesting with the rank-based models where the ranking of the influencers is based on the distances between opinions. Since opinions change through time these distances also change and computing new rankings could be done.

8 Proofs

Proof of Proposition 1

Suppose that o is extreme. Let $\{m_1, ...m_N\}$ (with $N \leq R$) be its models. By definition, each model m_k of o is of the form $\{l_1^k, ...l_n^k\}$ where l_i^k is a_i or $\neg a_i$. As a consequence, o is equivalent to $\bigvee_{k=1}^{N} l_1^k \wedge ... \wedge l_n^k$ with l_i^k being a_i or $\neg a_i$.

Proof of Proposition 2

– If o is $S^1\alpha$-extreme then there is S_α st $Mod(o) \subseteq Mod(\bigwedge S_\alpha)$. Consider $1 \leq \beta \leq \alpha$ and take $S_\beta \subseteq S_\alpha$ of size β. $S_\beta \subseteq S_\alpha$ implies $Mod(\bigwedge S_\alpha) \subseteq$

$Mod(\bigwedge S_\beta)$. As a consequence, o is $S^1\beta$-extreme
Suppose now that o is $S^2\alpha$-extreme. Then $Mod(o) \subseteq \{w : S(w) \geq \alpha\} \subseteq \{w : S(w) \geq \beta\}$ thus o is $S^2\beta$-extreme.

- If o is $S^1\alpha$-extreme then $\exists S_\alpha \subseteq S$ st$Mod(o) \subseteq Mod(\bigwedge S_\alpha)$. Thus any model of o satisfies exactly α selected topics. Thus $Mod(o) \subseteq \{w \in Mod(L) : S(w) \geq \alpha\}$ (i.e. o is $S^2\alpha$-extreme).
  Moreover, the fifth line of Table 2 proves that $S^2\alpha$-extreme $\not\subseteq S^1\alpha$-extreme
- This is a corollary of the previous point.
- We notice that $\{w \in Mod(L) : S(w) \geq \mid S \mid\} = \{w : S(w) = \mid S \mid\} = Mod(S)$. Now suppose that o is $S^1 \mid S \mid$-extreme. This means that $Mod(o) \subseteq Mod(\bigwedge S)$, i.e. $Mod(o) \subseteq \{w \in Mod(L) : S(w) \geq \mid S \mid\}$ i.e., o is $S^2 \mid S \mid$-extreme

Proof of Proposition 3

- o is S^1-not-extreme iff for all α o is not $S^1\alpha$-extreme i.e., for all α, for any $S_\alpha = \{s_{i_1}, ..., s_{i_\alpha}\} \subseteq S$, $Mod(o) \not\subseteq Mod(S_\alpha)$ i.e., $\exists w \in Mod(o)$ such that $w \models \neg s_{i_1} \vee ... \vee \neg s_{i_\alpha}$ i.e. $\exists w \in Mod(o)$ and $\exists j$ such that $w \models \neg s_{i_j}$ i.e. $o \wedge \neg s_{i_j}$ is consistent.
- o is S^2-not-extreme iff $\exists w \in Mod(o)$ st $S(w) = 0$ i.e., $\exists w \in Mod(o)$ st $w \models \neg s_1$ and... and $w \models \neg s_n$, i.e., $o \wedge \neg s_1 ... \wedge \neg s_n$ is consistent.
- There is an α so that o is $S^2\alpha$-extreme iff o is not not-extreme. I.e., $o \wedge \neg s_1 ... \wedge \neg s_n$ is inconsistent, i.e., $o \models s_1 \vee ... \vee s_n$.
- α is the highest value st o is $S^2\alpha$-extreme iff each model of o satisfies exactly α letters of S. Suppose that $Mod(o) = \{w_1, ...w_k\}$. Then for any w_i, there exists $S_i \subseteq S$ with $\mid S_i \mid = \alpha$ st $w_i \models \bigwedge_{s \in S_i} s \bigwedge_{s \in S \setminus S_i} \neg s \bigwedge_{s \notin S_i} l_s$, l_s being s ou $\neg s$.
  Thus o is equivalent to $\bigvee_{i=1...k} (\bigwedge_{s \in S_i} s \bigwedge_{s \in S \setminus S_i} \neg s \bigwedge_{s \notin S} l_s)$.

Proof of Proposition 4

- δ_{summin} satisfies (R1). δ_{summin} does not satisfy (R2). Indeed if the distance between interpretations being d_H Then $\delta_{summin}(c, a \wedge b) = 5/2$ and $\delta_{summin}(a \wedge c, a \wedge b) = 1$; if it is d_D, then $\delta_{summin}(c, a \wedge b) = 0$ and $\delta_{summin}(a \wedge c, a \wedge b) = 0$. δ_{summin} does not satisfy (R3). Indeed, if the distance between interpretations being d_H then $\delta_{summin}(a, b \wedge c) = 2$ and $\delta_{summin}(a \wedge b \wedge c, \neg \wedge b \wedge c) = 1$; if it is d_D then $\delta_{summin}(a, \neg a) = 1$ and $\delta_{summin}(a \wedge b, \neg a \wedge \neg b) = 1$. δ_{summin} does not satisfy (R4) when the distance between interpretations is d_H. Indeed $\delta_{summin}(a \wedge b \wedge c, \neg \wedge b \wedge c) = 1$ is not strictly less than $\delta_{summin}(a \wedge c, a \wedge b) = 1$. But it does when the distance between interpretations is d_D. Indeed, in this case, the distance between two consistent formulas is 0 while the distance between two inconsistent formulas is 1. δ_{summin} satisfies (R5) since it satisfies (R1). δ_{summin} satisfies (R6) since it is model-based. δ_{summin} satisfies (R7) because if $\varphi_1 \models \varphi_2$ then $Mod(\varphi_1) \subseteq Mod(\varphi_2)$ thus $\sum_{w \in Mod(\varphi_1)} D(w, \varphi) \leq \sum_{w \in Mod(\varphi_2)} D(w, \varphi)$.

- δ_{Hau} does not satisfy (R2). Indeed take d_H. $\delta_{Hau}(a \wedge b, a) = 1$ while $\delta_{Hau}(a \wedge b, a \wedge (a \rightarrow b)) = 0$. Now take d_D. $\delta_{Hau}(a, \neg a) = 0$ and $\delta_{Hau}(a \wedge b, \neg a \wedge b) = 0$ also. δ_{Hau} satisfies (R3). Indeed, if the number of letters o_1 and o_2 disagree on is less than the number of letters o'_1 and o_2 disagree on, then $max_{w \models o_1} D(w, o_2) \leq max_{w \models o'_1} D(w, o_2)$ thus $\delta_{Hau}(o_1, o_2) \leq \delta_{Hau}(o'_1, o_2)$. δ_{Hau} does not satisfy (R4). Take d_H, then $\delta_{Hau}(a \wedge b, c) = 2$ and $\delta_{Hau}(a \wedge b \wedge c, \neg a \wedge b \wedge c) = 1$. Take d_D, $\delta_{Hau}(a \wedge b, c) = 1$ and $\delta_{Hau}(a \wedge b, \neg a \wedge b) = 1$. Since δ_{Hau} satisfies (R1) it also satisfies (R5) δ_{Hau} satisfies (R6) since it is model-based. δ_{Hau} satisfies (R7). Indeed, if $\emptyset_1 \models \emptyset'_1$ then $Mod(\emptyset_1) \subseteq Mod(\emptyset'_1)$ thus $max_{w \models \emptyset_1} D(w, o_2) \leq max_{w \models \emptyset'_1} D(w, o_2)$ and $max_{w \models \emptyset_2} D(w, o_1) \leq max_{w \models \emptyset_2} D(w, o'_1)$. Thus $\delta_{Hau}(o_1, o_2) \leq \delta_{Hau}(o'_1, o_2)$.
- δ_{AD} does not satisfy (R1). Indeed, δ_{AD} is a less than a pseudo-distance. First, notice that $\delta_{AD}(\varphi, \varphi') = (0, ..., 0) \not\Longleftarrow \varphi \leftrightarrow \varphi'$. We only have $\delta_{AD}(\varphi, \varphi') = (0, ..., 0)$ iff they totally agree; However we have $\delta_{AD}(\varphi, \varphi') = \delta_{AD}(\varphi', \varphi)$. δ_{AD} satisfy (R2) and (R3). Indeed, if φ and ψ_1 agree on more letters than φ and psi_2 then $\delta_{AD}(\varphi, \psi_1) \leq_v \delta_{AD}(\varphi, \psi_2)$. If φ and ψ_1 disagree on more letters than φ and ψ_2 then $\delta_{AD}(\varphi, \psi_2) \leq_v \delta_{AD}(\varphi, \psi_1)$. δ_{AD} does not satisfy (R4). Take a and b on one side and $a \wedge b$ and $a \wedge \neg b$ on the other. The first two formulas are consistent while the second two are inconsistent. However $\delta_{AD}(a, b) = (0.5, 0.5)$ and $\delta_{AD}(a \wedge b, \neg a \wedge \neg b) = (0, 1)$. δ_{AD} does not satisfy (R5). Indeed, suppose that the language is $L = \{a, b, c\}$. Then $\delta_{AD}(a \wedge (b \vee c), a \wedge (a \rightarrow (b \vee c))) = (0, 0.5, 0.5)$. This proves that two equivalent formulas may not have the minimal δ_{AD}-distance. δ_{AD} satisfy (R6). Indeed, if $\models \varphi \leftrightarrow \varphi'$ and $\models \psi \leftrightarrow \psi'$ then φ and ψ agree (resp disagree) on p iff φ' and ψ' agree (resp disagree) on p. Thus $\delta(\varphi, \psi) = \delta(\varphi', \psi')$. δ_{AD} does not satisfy (R7). Take for instance $a \wedge \neg b \wedge \neg c \models a \wedge \neg b$ but $\delta_{AD}(a \wedge \neg b \wedge \neg c, a \wedge b \wedge c) = (0, 1, 1)$ and $\delta_{AD}(a \wedge \neg b, a \wedge b \wedge c) = (0, 0.5, 1)$.
- δ_{Inc} does not satisfy (R1). Take a and b. $\delta_{Inc}(a, b) = 0$ even if $\not\models a \leftrightarrow b$. δ_{Inc} does not satisfy (R2). Indeed, we will have $\delta_{Inc}(\varphi, \psi) = 0$ as soon as $\varphi \wedge \psi$ is consistent. Take a and b on one side and a and $a \wedge b$ on the other. Whatever the measure Inc is, $\delta_{Inc}(a, b) = \delta_{Inc}(a, a \wedge b) = 0$, even if a and $a \wedge b$ agree on more letter than a and b. δ_{Inc} generally does not satisfy (R3). For instance δ_{Inc} where Inc is the drastic inconsistency measure (defined by $Inc(S) = 1$ iff S is inconsistent and $Inc(S) = 0$ iff S is consistent) does not satisfy (R3). Indeed, even if $a \wedge b$ and $\neg a \wedge \neg b$ disagree on more letters than $a \wedge b$ and $\neg a$, $\delta_{Inc}(a \wedge b, \neg a) = \delta_{Inc}(a \wedge b, \neg a \wedge \neg b)$. δ_{Inc} satisfies (R4). Indeed, an inconsistency measure associates a set of formulas with a non negative real so that this real is 0 if and only if the set is consistent. As a consequence, if $\varphi_1 \wedge \psi_1$ is inconsistent, then $Inc(\{\varphi_1, \psi_1\}) > 0$ and if $\varphi_2 \wedge \psi_2$ is consistent, then $Inc(\{\varphi_2, \psi_2\}) = 0$. Thus $\delta_{Inc}(\varphi_2, \psi_2) < \delta_{Inc}(\varphi_1, \psi_1)$. δ_{Inc} satisfies (R5). Indeed, we only consider consistent formulas. Then, if $\models \varphi \leftrightarrow \psi$, $\varphi \wedge \psi$ is consistent thus $\delta_{Inc}(\varphi, \psi) = 0$. If δ_{Inc} is defined from a syntactical measure (such as the one which counts the minimal inconsistent subsets) then (R6) is not satisfied. δ_{Inc} does not satisfy (R7). For instance, indeed if $a \wedge \neg a \models b$, we have $\delta_{Inc}(a \wedge \neg a, a) > \delta_{Inc}(b, a)$. More generally, according to [16] any inconsistency measure Inc satisfies the property of dominance i.e., $\varphi_1 \models \varphi_2$

implies $Inc(S \cup \{\varphi_1\}) \geq Inc(S \cup \{\varphi_2\})$. Thus $\varphi_1 \models \varphi_2$ implies $\delta_{Inc}(\varphi_1, \varphi) \geq \delta_{Inc}(\varphi_2, \varphi)$ i.e., δ_{Inc} never satisfies (R7).

- δ_{max} does not satisfy (R1). For instance if the letters are a and b, $\delta_{max}(a, a) = 1$ (with d_H and with d_D) which is not the minila value. δ_{max} does not satisfy (R2). Indeed, with d_H we have: $\delta_{max}(a, a \wedge b) = 1$ and $\delta_{max}(a \wedge b, a \wedge b \wedge c) = 1$. With d_D we have: $\delta_{max}(a, b) = 1$ and $\delta_{max}(a \wedge b, a \wedge b) = 0$. δ_{max} does not satisfy (R3). Indeed, with d_H we have: $\delta_{max}(a, b \wedge c) = 3$ while $\delta_{max}(a \wedge b \wedge c, \neg a \wedge b \wedge c) = 1$. With d_D we have: $\delta_{max}(a, b \wedge c) = 1$ while $\delta_{max}(a \wedge b \wedge c, \neg a \wedge b \wedge c) = 1$. δ_{max} does not satisfy (R4).Indeed, with d_H or d_D we have: $\delta_{max}(a \wedge b \wedge c, \neg a \wedge b \wedge c) = \delta_{max}(a \wedge b \wedge c, a \wedge b) = 1$. δ_{max} does not satisfy (R5). For instance,if the letters are a and b then with d_H $\delta_{max}(a, a) = 2$ is not minimal; with d_D $\delta_{max}(a, a) = 1$ is not minimal. δ_{max} satisfies (R6). δ_{max} satisfies (R7) since $\varphi_1 \models \varphi_2$ implies $Mod(\varphi_1) \subseteq Mod(\varphi_2)$.
- δ_{min} satisfies (R1). δ_{min} does not satisfy (R2). Indeed, $\delta_{min}(a, b) = 0$ (with d_H or d_D) and $\delta_{min}(a \wedge b, \neg a \wedge b) = 1$
  δ_{min} satisfies (R3). Indeed $\delta_{min}(\varphi, \psi)$ is the number of letters on which φ and ψ disagree on. δ_{min} satisfies (R4). Indeed, if $\varphi_2 \wedge \psi_2$ is consistent then $\delta_{min}(\varphi_2, \psi_2) = 0$ and if $\varphi_1 \wedge \psi_1$ is inconsistent then $\delta_{min}(\varphi_2, \psi_2) > 0$ with d_H and d_D. δ_{min} satisfies (R5), (R6) and (R7).
- δ_{sum} satisfies (R1). δ_{sum} does not satisfies (R2). Indeed, with d_H (resp, d_D) $\delta_{sum}(a \wedge b \wedge c, a \vee b) = 7$ (resp, 5) and $\delta_{sum}((a \vee b) \wedge c, c) = 16$ (resp, 12). δ_{sum} does not satisfies (R3). Indeed, with d_H (resp, d_D) $\delta_{sum}((a \vee b) \wedge c, c) = 16$ (resp, 12) while $\delta_{sum}(a \wedge b \wedge c, \neg a \wedge b \wedge \neg c) = 3$ (resp 1). This example also shows that δ_{sum} does not satisfies (R4). δ_{sum} satisfies (R5), (R6) and (R7).
- δ_{maxmin} satisfies (R1). δ_{sum} does not satisfies (R2). Indeed, with d_H (resp, d_D) $\delta_{maxmin}(a, b) = 1$ (resp, 1) and $\delta_{maxmin}(a \wedge b \wedge c, a \wedge \neg b \wedge \neg c) = 2$ (resp, 1). δ_{sum} does not satisfies (R3). Indeed, with d_H and d_D, $\delta_{maxmin}(a \vee b, a) = 1$ and $\delta_{maxmin}(a \wedge b, a \wedge \neg b) = 1$. This example also shows that δ_{maxmin} does not satisfies (R4). δ_{maxmin} satisfies (R5), (R6) and (R7).

Proof of Proposition 5

By definition of Importance-Based Merging Operator, we have $Mod(B_i^{s+1}) \subseteq Mod(B_j^s)$. Thus if j is extremist at step s, then $\mid Mod(B_j^s) \mid \leq R$, is leading to $\mid Mod(B_i^{s+1}) \mid \leq R$ i.e., i is extremist at $s+1$.

Proof of Proposition 6

As $\Delta_\mu(B_{j_1}^s \prec_i \ldots \prec_i B_{j_{k-1}}^s) \wedge B_{j_k}^s$ is consistent, $\exists w \in Mod(\Delta_\mu(B_{j_1}^s \prec_i \ldots \prec_i B_{j_{k-1}}^s)) \cap Mod(B_{j_k}^s)$. So, by definition of the Importance-Based Merging Operator, w is such that $[D(w, B_{j_1}^s), \ldots, D(w, B_{j_{k-1}}^s)]$ is minimal according to $\leq_{lex}$. Furthermore, $D(w, B_{j_k}^s) = 0$. So, $[D(w, B_{j_1}^s), \ldots, D(w, B_{j_k}^s)]$ is minimal according to $\leq_{lex}$ and every $w \notin Mod(B_{j_k}^s)$ would not have such a property. By definition of $\leq_{lex}$, $\forall w' \in Mod(\Delta_\mu(B_{j_1}^s \prec_i \ldots \prec_i B_{j_n}^s))$, in particular, $[D(w', B_{j_1}^s), \ldots, D(w', B_{j_k}^s)]$ is minimal according to $\leq_{lex}$, thus $w' \in Mod(B_{j_k}^s)$. Then, as j_k is extremist at step s, then $\mid Mod(B_{j_k}^s) \mid \leq R$, is leading to $\mid Mod(B_i^{s+1}) \mid \leq R$ i.e., i is extremist at $s+1$.

Proof of Proposition 7

The proof is similar to the previous one. By definition of the Importance-Based Merging Operator, $\forall w \in Mod(B_i^{s+1})$, w is such that $[D(w, B_{j_1}^s), ..., D(w, B_{j_n}^s)]$ is minimal according to $\leq_{lex}$. And, in particular by definition of $\leq_{lex}$, $[D(w, B_{j_1}^s), ..., D(w, B_{j_k}^s)]$ is minimal according to $\leq_{lex}$, thus $w \in Mod(\Delta_\mu(B_{j_1}^s \prec ... \prec B_{j_k}^s))$. Then, $| Mod(\Delta_\mu(B_{j_1}^s \prec ... \prec B_{j_k}^s)) | \leq R$, is leading to $| Mod(B_i^{s+1}) | \leq R$ i.e., i is extremist at $s+1$.

Proof of Proposition 8

Let $j_k \in Sphere(i)$, then $\exists j_0 ... j_{k-1}$ $\forall m = 1...(k-1)$ $Inf(j_m) = \{j_{m-1} \prec_j ...\}$. We prove the proposition by induction on k. For $k = 1$, with $s' = s+1$, it comes from the Proposition 5. If we suppose the property for $k \in \mathbb{N}$, $\exists s' \geq t$, $\forall s \geq s'$, j_{k-1} is extremist at step s. So, by Proposition 5, the property is satisfied at $s'+1$.

Proof of Proposition 9

First, we notice that $Mod(B_i^{t+1}) \subseteq Mod(B_i^t)$.

Suppose that i_1 is $S^1\alpha$-extremist at step t. Then there a subset S_α of S st $Mod(B_{i_1}^t) \subseteq Mod(\bigwedge S_\alpha)$. Thus $Mod(B_i^{t+1}) \subseteq Mod(\bigwedge S_\alpha)$. Finally i is $S^1\alpha$-extremist at step $t+1$

Suppose now that i_1 is $S^2\alpha$-extremist at step t. Then any w in $Mod(B_{i_1}^t)$ satisfies $S(w) \geq \alpha$. Thus any w in $Mod(B_i^{t+1})$ satisfies $S(w) \geq \alpha$. I.e., i is $S^2\alpha$-extremist at step $t+1$.

Proof of Proposition 10

First we notice that if $j \in Sphere(i)$ then $\exists s \geq t$ $\forall s' \geq s$ $Mod(B_j^{s'}) \subseteq Mod(B_i^t)$.

Suppose that i is a self-confident agent which is $S^1\alpha$-extremist (resp, $S^2\alpha$-extremist) at step t. Thus there is S_α st $Mod(B_i^t) \subseteq Mod(\bigwedge S_\alpha)$ (resp, any $w \in Mod(B_i^t)$ satisfies $S(w) \geq \alpha$). Thus there is S_α st $Mod(B_j^{s'}) \subseteq Mod(\bigwedge S_\alpha)$ (resp, ny w in $Mod(B_j^{s'})$ satisfies $S(w) \geq \alpha$). This proves that j is $S^1\alpha$-extremist (resp, $S^2\alpha$-extremist) at step s'.

Proof of Proposition 11

This is a corollary of Proposition 9.

Proof of Proposition 12

First, if i_1 is $SA\epsilon$-extremist at step t, then there exists a selected agent $a \in SA$ st $\delta(B_{i_1}^t, B_a^t) \leq \epsilon$. Secondly, if i_1 is the most influential influencer of i then $B_i^{t+1} \models B_{i_1}^t$. Thirdly, if δ satisfies (R7) then $\delta(B_i^{t+1}, B_a^t) \leq \delta(B_{i_1}^t, B_a^t)$ thus $\delta(B_i^{t+1}, B_a^t) \leq \epsilon$. Moreover a being dogmatic, we have $\models B_a^t \leftrightarrow B_a^{t+1}$. Finally, δ satisfying (R6), we conclude $\delta(B_i^{t+1}, B_a^{t+1}) \leq \epsilon$ i.e., i is $SA\epsilon$-extremist at step $t+1$.

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