

# Chapter 16

## Reinforcement of Logical and Mathematical Competences Using a Didactic Aid Based on the Theory of Constructivism



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### 16.1 Introduction

Mathematics, like other products of culture, is a creation of the human mind. Logic and mathematics is the basis of the learning society's pyramid; without this competence, the participant is not able to consciously and creatively participate in the learning society. From the evolutionary point of view, its development is legitimized by the fact that it was more practical for one to formalize certain matters by means of a universal system of recording quantity, volume or other measures that were to serve this purpose. In order to function in the world, one needs a certain set of mathematical skills to be able to distinguish one object from several others. What is more, the ability to manipulate them by means of arithmetic calculation not only provides one with the ability to classify objects in a logical way, but above all, to solve specific problems in an abstract manner.

The aim of the article is to present the results of a study utilizing the EduMata didactic aid, which has been developed on the basis of Jean Piaget's constructivism theory and enables a problem-based approach to working on numerals/objects. The aid may also be used to carry out similar work in fields other than mathematics.

Currently, three main concepts are used to try to explain the processes of teaching and using mathematical operations: developmental and cognitive concepts related to the biological development of a child (Piaget 1966; Snow et al. 1989), neuropsychological and cognitive concepts based on the neuroimaging of brain structures and functions (Piazza et al. 2007; Piazza 2011) and linguistic concepts explained in relation to social interactions (Spelke 2005; Pinker 1995). As describ-

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ing all three concepts would significantly extend and go beyond the framework of this article, the author will venture to focus only on presenting the developmental and cognitive concept employed by him and then proceed to make a methodological description of the presented research.

## 16.2 Theoretical Background for Selecting the Subject

There are four large studies being conducted in Poland to help diagnose mathematical education in the country. Two deal with measuring overall class competences of primary school students. The third is focussed on teachers while the last one on the general characteristics of the school and activities (e.g. extracurricular activities). The first three are carried out by the Educational Research Institute (IBE), which include the OBUT/K3 nationwide survey of the skills of grade three students (2015 Report), the DUMa diagnosis of mathematical skills of primary school students (2014 Report) and a survey of the needs of teachers of early childhood education and mathematics in the scope of professional development (2015 Report). The fourth survey is carried out by the Central Statistical Office (GUS) (2016/2017 Report) entitled “Education” and consists of regularly published reports on the general characteristics of educational institutions in the country. All of them provide a very concise characterization of the teaching process. The conclusion drawn from the research is that the level of mathematical competence in Poland is unsatisfactory and, what is more, its quality has a significant influence on the later educational choices of students and their chances on the labour market. The most important conclusions from the above reports are as follows: too little emphasis on developing reasoning, lack of variety during math lessons, blocking of original solutions put forward by students, lack of teamwork in lessons and training for teachers that does not meet their practical expectations.

A large part of the above statements leads to the conclusion that there is a lack of universal educational support allowing for a practical presentation of the problem in the scope of mathematical tasks, providing variety in lessons, conducting lessons and tasks in a team manner and developing a coherent system of good practices between teachers and schools. Therefore, it was necessary to develop the aid, implement it and evaluate it in accordance with methodological standards.

The theoretical foundation adopted for the study is based on the developmental and cognitive approach created by such researchers as J. Piaget, J. S. Bruner and L. Wygotski (1966, 1987, 1989). According to them, mathematical knowledge is actively created by the learner. The learner’s knowledge and cognitive structures are developed in his or her mind. J. Piaget noticed that the child produces personal concepts of the reality being discovered. They are created by confronting concepts belonging to the system of knowledge of the individual with visions offered by the environment, especially by educational institutions. As a result, the child’s mental structures (schema) undergo constant transformation, which is done through assimilation (i.e. adaptation) and accommodation of meanings through the creation of new

structures (Piaget 1966; Bruner and Haste 1987; Wygotski 1989). Thus, the child's development must complete its particular cycles, i.e. certain functions must first mature before the school can begin to teach the child with certain knowledge and habits. Therefore, the child's development must somehow precede teaching; otherwise the transfer of knowledge will be ineffective or will be conveyed in an insufficiently comprehensible way. It must be conveyed in a way that matches the child's intellectual capacity (Piaget, Inhelder 1970).

The concept developed by constructivists sets a child's mathematical skills in the context of one's overall cognitive development. Knowledge of this subject is the result of numerous studies conducted in a single paradigm, thanks to which it is coherent and organized. Both supporters and opponents of Piaget's concept use a common language of concepts, which enables a constructive discussion in the field of the undertaken research.

### 16.3 Introduction to Methodology of Research

The collected data was subjected to statistical analysis in the SPSS Statistics 23.0 program in order to demonstrate the significance of the EduMata didactic aid in improving third grade primary school children's logical and mathematical proficiency. The theoretical foundation of the experiment is based on the cognitive theory of Jean Piaget, which speaks of the legitimacy of building mathematical concepts on the basis of particulars. The test for mathematical competence K3 2015 version M1 was used for both groups in the pretest, and for the post-test the mathematical competence test K3 2015 version M2 was tested. The lessons were taught by several designated teachers who had previously been trained on how they could teach the lesson (limiting the method, limiting the scope of the material) to exclude additional independent variables.

The said objective was reached using *ANOVA* two-factor variance analysis in a mixed scheme: 2 (time of measurement: initial vs. final) x 2 (use of the EduMata didactic aid: control vs. experimental). The dependant variable consisted of the total result of mathematical competence in the K3 (version M1 for the pretest and version M2 for the post-test) test and the result in three areas of competence, i.e. calculation skills, geometry skills and the ability to solve word problems. Care was taken to ensure uniformity of research by drawing attention to the most important disruptive factors. This was done thanks to the instruction for teachers including choosing a homogeneous teaching material, time devoted to its implementation (which results come from the plan of school hours) and strict transfer of procedures (methods and forms) to teachers in both groups.

The level of significance of differences (level of mathematical competence) that is received is much higher than the standard accepted as acceptable ( $p < 0.05$ ), which indicates a very small probability of the significance of random factors as the causes of the detected differences between the studied groups.

Lack of statistically significant differences between the dependent variable in the pretest in both randomly selected groups indicates the high reliability of the obtained results (Fig. 16.4).

The distribution was found to conform with normal distribution as demonstrated by skewness and kurtosis values that did not exceed the  $<-1, 1>$  range significantly. The homogeneity of variance of the groups compared was also confirmed on the basis of statistically insignificant Levene test results. The statistical significance of the results was established at  $p < 0.05$ .

## 16.4 Sample Characteristics

The study was conducted on a sample of 469 pupils from 9 third grade classes. The experimental group consisted of 239 students aged 7–11 years ( $M = 8.35$ ;  $SD = 0.63$ ), including 51.1% girls and 48.9% boys. The control group consisted of 230 students aged 7–11 years ( $M = 8.44$ ;  $SD = 0.64$ ), including 50.2% girls and 49.8% boys.

## 16.5 Descriptive Statistics

The main descriptive statistics measure values are presented, i.e. central tendency measures (average, median, mode), dispersion measures (minimum, maximum, standard deviation, dispersion) and dispersion shape measures (skewness, kurtosis).

Table 16.1 shows the descriptive statistics of the K3 M1 mathematical competence test results of the experimental group in the initial and final measurement.

In the overall result of the K3 M1 test, the experimental group scored between 0 and 16 points in the initial measurement, averaging 7.46 points ( $SD = 4.15$ ) out of 16 points possible, usually 5 points. In the final measurement, the group scored 0–16 points, averaging 10.33 points ( $SD = 3.87$ ), usually 11 points.

In the field of calculation skills, the experimental group achieved scores in a range of 0–4 points in the initial measurement, averaging 2.08 points ( $SD = 1.15$ ) out of 4 points possible, usually 3 points. In the final measurement, the group scored 0–4 points, averaging 2.87 points ( $SD = 1.06$ ), usually 3 points.

In the area of geometry skills, in the initial measurement, the experimental group achieved scores in the range of 0–6 points, averaging 2.46 points ( $SD = 1.67$ ) out of 6 points possible, usually 2 points. In the final measurement, the group scored 0–6 points, averaging 3.55 points ( $SD = 1.80$ ), usually 3 points.

In the initial measurement of the word problem-solving skills area, the experimental group achieved scores in the range of 0–6 points, averaging 2.94 points ( $SD = 2.08$ ) out of 6 possible points, usually 1 point. In the final measurement, the group scored 0–6 points, averaging 3.93 points ( $SD = 1.73$ ), usually 5 points.

**Table 16.1** Main descriptive statistics of the K3 M1 mathematical competence test results of the experimental group in the initial and final measurement

	Measurement	<i>N</i>	<i>M</i>	<i>Me</i>	<i>D</i>	<i>SD</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Min</i>	<i>Max</i>	<i>V</i>
Total result	Initial	239	7.46	8	5	4.15	0.05	-0.89	0	16	56%
	Final	239	10.33	11	11	3.87	-0.64	-0.38	0	16	37%
Calculation skills	Initial	239	2.08	2	3	1.15	-0.06	-0.94	0	4	55%
	Final	238	2.87	3	3	1.06	-1.03	0.72	0	4	37%
Geometry skills	Initial	239	2.46	2	2	1.67	0.53	-0.55	0	6	68%
	Final	239	3.55	4	3	1.80	-0.28	-0.89	0	6	51%
Word problem-solving skills	Initial	239	2.94	3	1	2.08	0.03	-1.09	0	6	71%
	Final	239	3.93	4	5	1.73	-0.56	-0.67	0	6	44%

Legend: *N* size of group, *M* average, *Me* median (middle value), *D* mode (most frequent value), *SD* standard deviation, *Skewness* skewness value, *Kurtosis* kurtosis value, *Min* minimum value, *Max* maximum value, *V* dispersion coefficient

**Table 16.2** Main descriptive statistics of the K3 M1 mathematical competence test results of the control group in the initial and final measurement

	Measurement	<i>N</i>	<i>M</i>	<i>Me</i>	<i>D</i>	<i>SD</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Min</i>	<i>Max</i>	<i>V</i>
Total result	Initial	230	6.88	6	5	4.41	0.28	-1.09	0	16	64%
	Final	230	8.40	9	13	4.19	-0.12	-1.10	0	16	50%
Calculation skills	Initial	230	2.00	2	1	1.29	0.00	-1.14	0	4	65%
	Final	230	2.53	3	3	1.20	-0.63	-0.51	0	4	47%
Geometry skills	Initial	230	2.20	2	1	1.65	0.49	-0.56	0	6	75%
	Final	230	2.74	2	2	1.88	0.24	-1.07	0	6	69%
Word problem-solving skills	Initial	230	2.66	2	0	2.14	0.28	-1.02	0	6	80%
	Final	228	3.18	3	3	1.81	-0.06	-1.12	0	6	57%

Legend: *N* size of group, *M* average, *Me* median (middle value), *D* mode (most frequent value), *SD* standard deviation, *Skewness* skewness value, *Kurtosis* kurtosis value, *Min* minimum value, *Max* maximum value, *V* dispersion coefficient

For all areas of mathematical competence, skewness and kurtosis values did not significantly exceed  $<-1.1 >$  in the initial and final measurement. The dispersion of the results was greater in the initial than in the final measurement, which indicates a reduction of results differentiation in the experimental group in the final measurement.

Table 16.2 shows the descriptive statistics of the K3 M1 mathematical competence test results of the control group in the initial and final measurement.

The overall result of the K3 M1 test showed that the control group scored between 0 and 16 points in the initial measurement, averaging 6,88 points ( $SD = 4.41$ ), usually 5 points. In the final measurement, the group scored 0–16 points, averaging 8.40 points ( $SD = 4.19$ ), usually 13 points.

In the field of calculation skills, in the initial results, the control group achieved scores in a range of 0–4 points, averaging 2 points (SD = 1.29), usually 1 point. In the final measurement, the group scored 0–4 points, averaging 2.53 points (SD = 1.20), usually 3 points.

In the area of geometry skills, the control group achieved scores in a range of 0–6 points in the initial measurement, averaging 2.20 points (SD = 1.65), usually 1 point. In the final measurement, the group scored 0–6 points, averaging 2.74 points (SD = 1.88), usually 2 points.

In the word problem-solving skills area, the control group achieved scores in the range of 0–6 points in the initial measurement, averaging 2.66 points (SD = 2.14), usually 0 points. In the final measurement, the group scored 0–6 points, averaging 3.18 points (SD = 1.81), usually 3 points.

For all areas of mathematical competence, skewness and kurtosis values did not significantly exceed  $<-1.1 >$  in the initial and final measurement. The dispersion of the results was greater in the initial than in the final measurement, which indicates a reduction of results differentiation in the control group in the final measurement.

## 16.6 Results

Table 16.3 presents ANOVA variation analysis statistics for the influence of the interaction between the measurement time and EduMata didactic aid use on the results of the K3 mathematical competence test.

A statistically significant influence of the interaction between the time of measurement and application of the EduMata didactic aid on the overall result of the K3 mathematical competence test as well as on the results in the areas of calculation, geometry and word problem-solving skills was found. In order to learn the structure of the said influence of interaction, pairwise comparisons were made. Table 16.4 presents pairwise comparison statistics between the initial and final measurement in the control and experimental group.

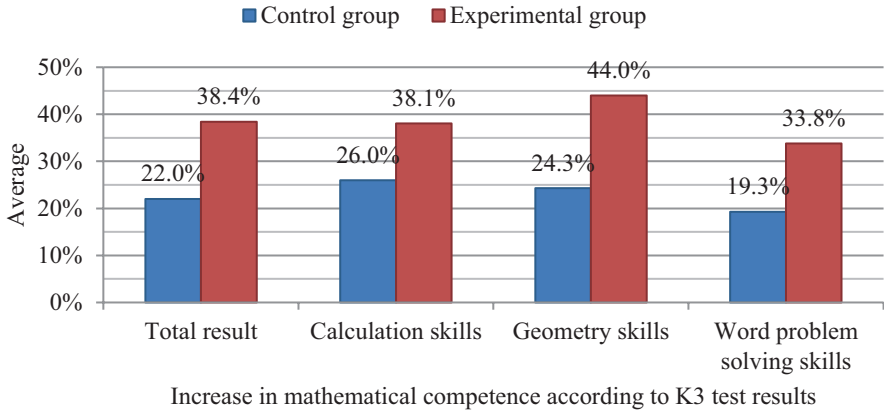
In the experimental group, a statistically significant increase in mathematical competence was found between the initial and final measurement in the areas of general mathematical competences (by 38.4%), calculation skills (38.1%), geometry skills (44%) and word problem-solving skills (33.8%) (Fig. 16.1).

**Table 16.3** ANOVA variation analysis statistics for the influence of the interaction between measurement time and EduMata didactic aid use on the results of the K3 mathematical competence test

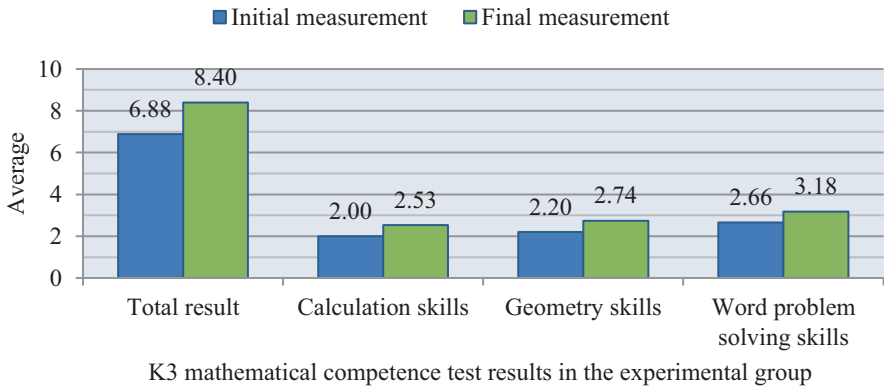
	Type III sum of squares	Average square	$F(1, 464)$	$p$	$\eta^2$
Total result	100.16	100.16	9.66	<b>0.001</b>	0.02
Calculation skills	4.18	4.18	3.85	<b>0.05</b>	0.01
Geometry skills	17.25	17.25	9.12	<b>0.001</b>	0.02
Word problem-solving skills	12.11	12.11	4.60	<b>0.03</b>	0.01

**Table 16.4** Pairwise comparison statistics between the initial and final measurements of the K3 M1 mathematical competence test results in the control and experimental group

	Group	Initial measurement		Final measurement		Difference in average values	<i>p</i> of the difference
		<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>		
Total result	Control	6.88	4.41	8.40	4.19	22.0%	<b>0.001</b>
	Experimental	7.46	4.15	10.33	3.87	38.4%	<b>0.001</b>
Calculation skills	Control	2.00	1.29	2.53	1.20	26.0%	<b>0.001</b>
	Experimental	2.08	1.15	2.87	1.06	38.1%	<b>0.001</b>
Geometry skills	Control	2.20	1.65	2.74	1.88	24.3%	<b>0.001</b>
	Experimental	2.46	1.67	3.55	1.80	44.0%	<b>0.001</b>
Word problem-solving skills	Control	2.66	2.14	3.18	1.81	19.3%	<b>0.001</b>
	Experimental	2.94	2.08	3.93	1.73	33.8%	<b>0.001</b>



**Fig. 16.1** Comparison of the K3 mathematical competence test results of the experimental group between the initial and final measurement



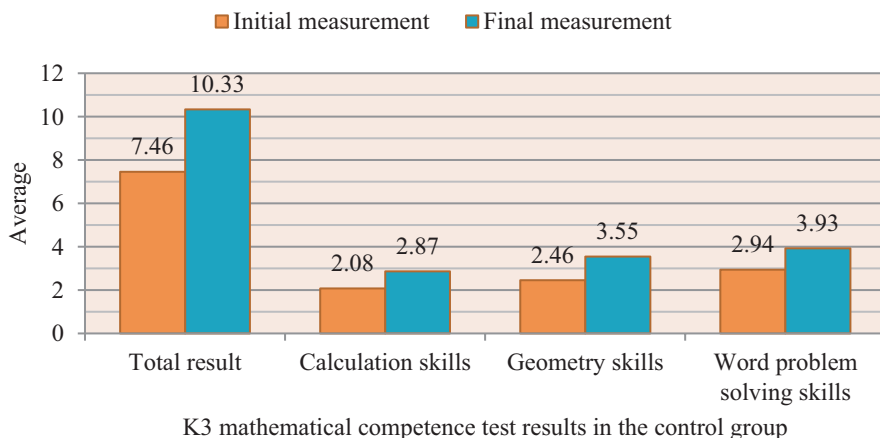
**Fig. 16.2** Comparison of the K3 mathematical competence test results of the control group between the initial and final measurement

A statistically significant increase in mathematical competences was also found in the control group between the initial and final measurement in the area of general mathematical competences (by 22%), calculation skills (26%), geometry skills (24.3%) and word problem-solving skills (19.3%) (Fig. 16.2).

It is worth noting that the increase in the K3 mathematical competence test results between the initial and final measurement was greater in the experimental group than in the control group in the area of general mathematical competences, as well as calculation skills, geometry skills and word problem-solving skills (Fig. 16.3).

Table 16.5 presents pairwise comparison statistics between the K3 mathematical competence test results of the control and experimental group in the initial and final measurements.





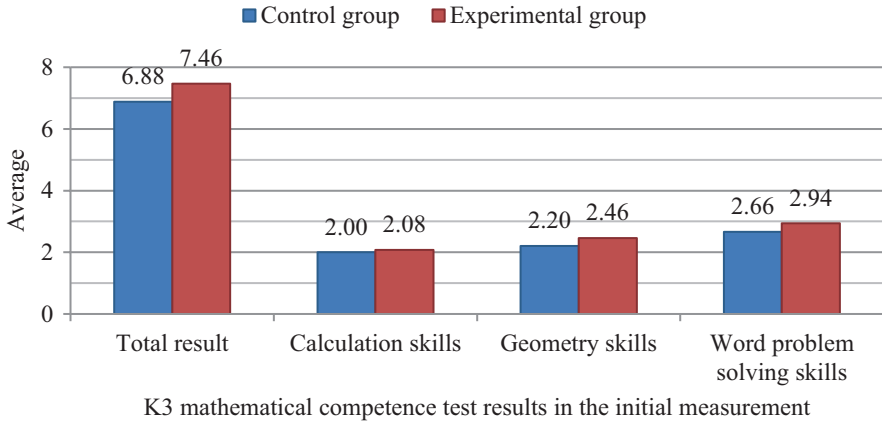
**Fig. 16.3** Comparison of the K3 mathematical competence test results of the experimental and control group between the initial and final measurement

**Table 16.5** Pairwise comparison statistics between the K3 mathematical competence test results of the control and experimental group in the initial and final measurements

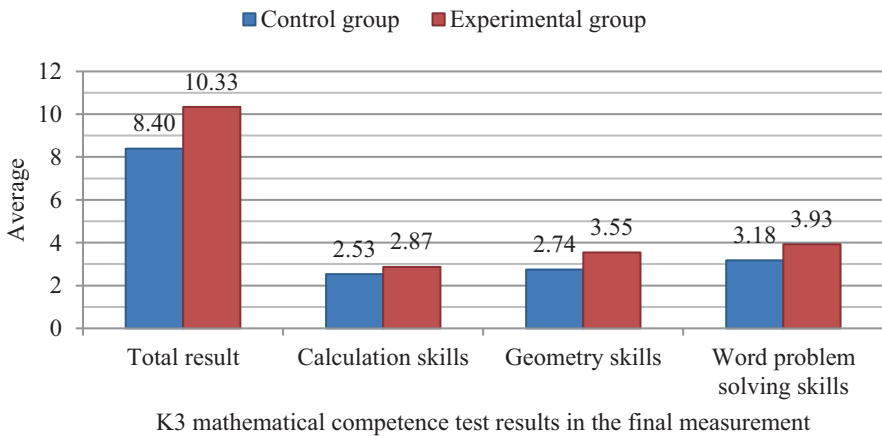
	Measurement	Control group		Experimental group		Difference in average values	p of the difference
		<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>		
Total result	Initial	6.88	4.41	7.46	4.15	7.8%	0.13
	Final	8.40	4.19	10.33	3.87	18.7%	<b>0.001</b>
Calculation skills	Initial	2.00	1.29	2.08	1.15	3.4%	0.48
	Final	2.53	1.20	2.87	1.06	11.8%	<b>0.001</b>
Geometry skills	Initial	2.20	1.65	2.46	1.67	10.6%	0.09
	Final	2.74	1.88	3.55	1.80	22.8%	<b>0.001</b>
Word problem-solving skills	Initial	2.66	2.14	2.94	2.08	9.4%	0.13
	Final	3.18	1.81	3.93	1.73	19.2%	<b>0.001</b>

In the initial measurement, there were no statistically significant differences between the control and experimental group in terms of general mathematical competences, calculation skills, geometry skills and word problem-solving skills (Fig. 16.4).

In contrast, in the final measurement, statistically significant differences were found in the K3 M2 mathematical competence test results between the control and experimental groups. The experimental group, compared to the control group, achieved a higher score for general mathematical competences (by 18.7%), calculation skills (by 11.8%), geometry skills (by 22.8%) and word problem-solving skills (by 19.2%) (Fig. 16.5).



**Fig. 16.4** Comparison between the K3 M1 mathematical competence test results of the control and experimental group in the initial measurement



**Fig. 16.5** Comparison between the K3 M2 mathematical competence test results of the control and experimental group in the final measurement

## 16.7 Conclusion

Research carried out by researchers (Semadeni 2016; Majewska 2014; Gruszczyn-Kolczyńska 2015, 2012; Siwek 2011; Raszka 2011) in the use of additional forms of verbalization of content have shown that using them causes a better understanding and remembering content that is related to both the learning environment and sensory materials. Extensive research in this area was carried out in 2007–2010 by Gruszczyn-Kolczyńska, showing a positive relationship between the verbalization of mathematical operations carried out and their lack in the form of negative effects. Lack of verbalization in the process of understanding the tasks of the mathematician

was called “paper math” because it did not involve the role of verbal learning of children. Gruszczyk-Kolczyńska and Semadeni in their research refer to a more broad and general context of the constructivist approach in teaching mathematics; however, they do not provide any specific didactic help that could be the implementation of these tasks and give a ready solution to teachers wanting to teach in this way. Therefore, the authors of this article deepened the research issues of the constructivist approach by introducing a specific solution in the form of using EduMat’s didactic help.

Researchers in this area (Semadeni 2016; Majewska 2014; Gruszczyk-Kolczyńska 2015, 2012; Siwek 2011; Raszka 2011) write about creating and organizing an educational space in the field of observing, manipulating and collecting experiences. However, in the final conclusion they do not give a name or a specific didactic tool that would modify the educational environment.

The authors of the article see a multitude of various didactic aids that could support a constructivist approach. However, without distinguishing a given element through which this concept is practiced, it is difficult to conclude on the validity of a given concept.

It is difficult to conclude on effectiveness in teaching based only on a given method without providing specific tools. It is true that Piaget in his research introduced a very wide spectrum of various types of help, puzzles, levers and vessels that were verbal in nature.

However, in the educational reality of at least Polish, this element (a constructivist approach) is rarely introduced in the teaching of mathematics. This is evidenced by numerous studies recalled at the beginning of the article.

Researcher Diallio Sessoms (2008) says that the teaching process is more often discussed in the context of a broad spectrum of concepts, theories and methodologies. On the other hand, research on specific didactic tools is rarely undertaken (multimedia boards, educational works, educational mats, counters, etc.). Therefore, the authors of this article made efforts to explore the last link of almost every educational concept.

The presented results show a significant statistical significance of the EduMata didactic aid on the results achieved in the area of logical-mathematical thinking among third grade primary school children. There was a significant increase in general mathematical competences and a significant increase in calculation skills, geometry skills and word problem-solving skills in the final measurement. The increase occurred both in the experimental group and in the control group. The improvement in mathematical competence, however, was greater in the experimental group.

There was no significant difference in mathematical competence between the control group and the experimental group in the initial measurement. However, in the final measurement, the experimental group achieved significantly higher general mathematical competence results and significantly improved calculation skills, geometry skills and word problem-solving skills than the control group. In conclusion, the use of the EduMata didactic aid in improving logical-mathematical thinking among grade 3 primary school children was proven to be effective.

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