



# Application of Decision Logical Trees and Predominant Logical Variables in Analysis of Automatic Transmissions Gearboxes

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**Abstract.** In the article was discussed the possibility of using decisive logical trees and predominant variables for the analysis of automatic gearboxes. The purpose of modeling an automatic gearbox with graphs can be versatile, namely: determining the transmission ratio of individual gears, analyzing the speed and acceleration of individual rotating elements. In a further step, logic tree decision methods can be used to analyze functional schemes of selected transmission gears. Instead, for graphs that are models of transmission, parametrically acting tree structures can be used.

**Keywords:** Decision logical trees · Optimization · Hsu graph · Computer analysis · Automatic transmission gearboxes · Predominant variables

## 1 Introduction

Machine system as a system is a set of objects (blocks, elements), each of which is described by means of an appropriate mathematical model with indications of all connections existing between objects. System objects are usually individual devices: heat exchangers, extractors, compressors, etc. Engineering practice requires a correct assessment of the mathematical model describing a given system by means of variables. Models describe a given system with different accuracy. Graphs and structural numbers play a role as models of mechanical systems [1, 2] and are still systematically developed [3–9]. Power flow graphs (graphs of bonds) in layout modeling have been presented, among others by [10] and graphs used in hydraulic systems by [11]. In addition, there are special stream graphs, e.g. in chemical and process engineering. Unlike graphs, dendrite-tree structures do not have cycles, but there may be a different number of initial vertices. Therefore, such structures are applicable to variant searching and optimizing the solutions of the designed system [12].

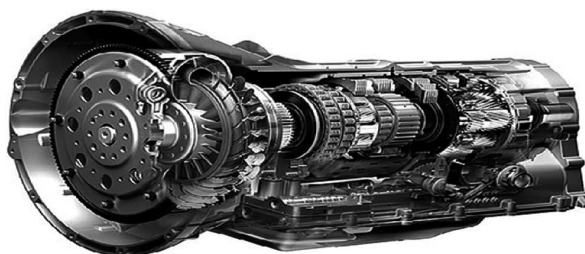
## 2 Theory-Graphic Models of Transmissions

An important advantage of modeling mechanical systems with graphs is, among others, that some considerations can be carried out in parallel in the field of mechanics and graph theory [13]. The relevance of the results is based on the transformation of knowledge between these two areas. At present, there is considerable interest in graphical methods in optimization, and especially in modeling of gears, hydraulic systems, all mechanisms, trusses and frames [13].

Among the methods of analysis of planetary gears, one can distinguish among others: Hsu [14], Freudenstein [15] and Marghit [16]. In the case of Hsu rules, the graph is built according to the following rules: geometrical dimensions are omitted and kinematic pairs are considered: rotary, “planet - yoke” and meshing. The contour graph method used for the analysis of mechanical systems was discussed, among others in [7–9, 12, 13]. It is particularly useful for considering mechanisms of various types (so-called planar, crossheads, etc.). In particular, it can be used in the analysis of planetary gears. The idea of this method is based on distinguishing a series of subsequent rigid links of mechanisms that form a closed loop - the so-called contour. Unlike graphs, dendrite-tree structures do not have cycles, but there may be a different number of initial vertices. Therefore, a different approach has been developed as a translation of a directed graph of dependence, among others for parametrically acting structures [5, 6]. For example, in [12], the structures that parametrically used for the contour graph were used as a further step in the analysis of planetary gears. In turn, in [9], a complex complexity index was used for parametrically acting structures as a further stage of analysis.

## 3 Analysis of the Automatic Transmission Gearboxes with Logical Decision Trees

The analysis of automatic gearboxes is similar to the analysis of a single planetary gear [13] (Fig. 1).

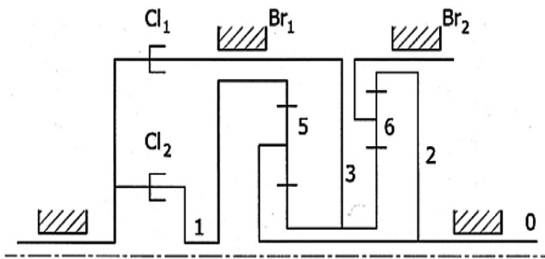


**Fig. 1.** Model drawing of an exemplary automatic gearbox

The analysis is carried out for each run separately by introducing some transformations of the respective graphs. A novelty proposed in [13] is the modification of the Hsu graph by introducing a path from the entrance to the exit. This path is formed by the corresponding edges of the gear graph. Input and output are marked additionally. This path allows the analysis of the sequence of transmission of rotational motion by subsequent elements of the transmission. In addition, it allows the detection of so-called redundant elements for a given gear currently under consideration. The consequence of this approach is the idea of transforming the graph proposed in the work. Determining the importance of the sequence of individual settings can be carried out taking into account logical decision trees, as shown in [17].

### 3.1 Analysis of an Exemplary Automatic Gearboxes Including Multi-Valued Logical Trees

Figure 2 shows an example of an automatic transmission gearbox performing four gears [13, 17].



**Fig. 2.** Functional diagram of an exemplary automatic gear box, where: *Cl*- clutch, *Br*-brake

The automatic clutch and brake control system makes it possible to achieve next gears, hence Table 1 lists the corresponding sequences of control settings. In the work of the gear unit, it is assumed that the clutch *Cl* and the brake *Br* can take two states *I* and *O* (1- active, 0- passive). For the gears from Fig. 2 there are 4 decision variables: *Cl*<sub>1</sub>, *Cl*<sub>2</sub>, *Br*<sub>1</sub>, *Br*<sub>2</sub> - divalent [17].

**Table 1.** Sequences of control elements in the considered transmission

Control/Drive	<i>Cl</i> <sub>1</sub>	<i>Cl</i> <sub>2</sub>	<i>Br</i> <sub>1</sub>	<i>Br</i> <sub>2</sub>
I	0	1	0	1
II	0	1	1	0
III	1	1	0	0
Rev	1	0	0	1

### 3.2 Decision Logical Trees

The canonical alternative normal form of KAPN of a two- or multi-valued logical function describes all variants or real (realizable) solutions of a given problem, obtained according to the principles of the morphological table, because the full array of logical variables combinations describes all the theoretical variants. As a result of minimization (e.g. following the application of the Quine-McCluskey algorithm), we obtain from the solutions realizable real solutions as an abbreviated alternative normal form of the SAPN logic function. After further binary or multivalent logic transformations, the minimal alternative normal form of the MAPN logical function is finally obtained, which means the most important real solutions. To reduce the computational complexity, i.e. permutational analysis of logical decision trees with possible floor conversions, the search for true sub-solutions with real-code cutting of full branches on contractual code entries using the Quine- McCluskey algorithm minimizes complex alternative forms of ZAPN, which ultimately leads to the minimal complex alternative form of normal MZAPN logic function. In the case of multi-valued logic functions, as in Boolean functions, the basic role in searching for the first implicants is played by concepts of incomplete sticking and elemental absorption, which apply to the APN of a given logic function.

The bonding operation is called transformation [18]:

$$A_{j_o}(x_r) + \dots + A_{j_{m_r-1}}(x_r) = A \tag{1}$$

The operation of incomplete sticking is called transformation:

$$A_{j_o}(x_r) + \dots + A_{j_{m_r-1}}(x_r) = A + A_{j_o}(x_r) + \dots + A_{j_{m_r-1}}(x_r) \tag{2}$$

where:  $r = 1, \dots, n$  and  $A$  - a partial elementary product whose variables of individual literals belong to the set  $\{x_1, \dots, x_{r-i}, x_{r+i}, \dots, x_n\}$ .

The elementary absorption operation is called transformation:

$$A_{j_u}(x_r) + A = A \tag{3}$$

where:  $0 \leq u \leq m_r - 1$ ,  $1 \leq r \leq n$  and  $A$ - a partial elementary product whose variables of individual literals belong to the set  $\{x_1, \dots, x_{r-i}, x_{r+i}, \dots, x_n\}$ . If the above equality occurs, then  $A_{j_u}$  absorbs  $(x_r)$ .

All transformations concern the so-called Quine- McCluskey algorithm to minimize individual partial multi-valued logic functions.

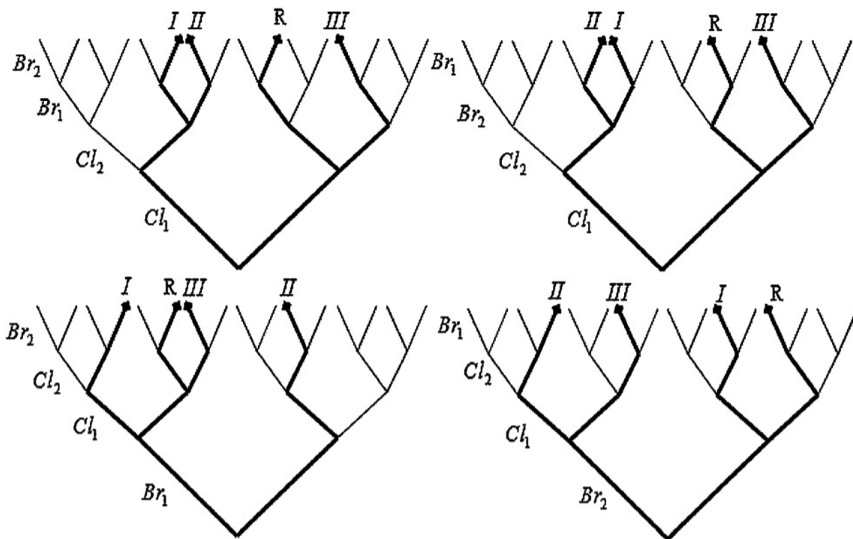
In the combinatorial sense there are 16 combinations (states): 000, 0001, 0010, 0011, 0100, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111. In the transmission of Fig. 1 there are 3 forward gears (I-III) and reverse gear (Rev). For example, for the first gear we have:  $Cl_2$  clutch and  $Br_2$  brake are active (0101). All possible sequences of gear control elements from the figure are shown in Table 2. These are realizable gear states representing simultaneously the KAPN (Canonical Alternative Normal Form) of a given logic function.

**Table 2.** Realizable gearbox operating conditions KAPN data combinations

Control/Drive	$Cl_1$	$Cl_2$	$Br_1$	$Br_2$
I	0	1	0	1
II	0	1	1	0
III	1	1	0	0
Rev	1	0	0	1

If all the paths of a traditional logical tree mean a set of all the theoretical variants of the discrete optimization process, then real variants, i.e. realizable ones, should be distinguished. The problems of minimizing logic functions, in relation to the morphological table and decision table enable computer aided design process at the stage of searching for realizable solutions, examining the importance of fixed variables, detailed analysis of realizable solutions, etc. In the considered transmission, logical trees decide decisively the sequences of control elements. At the same time, they indicate the continuity of the automatic control system when achieving next gears with equal importance for parameters  $Cl$  and  $Br$ . There are automatic gearboxes with a large number of possible element sequences. Then determining the rank of importance, in what order should be changed individual elements to active, can allow the detection of so-called redundant or temporarily redundant components for a given gear currently under consideration.

In the permutation analysis 24 logical decision trees were generated. Figure 3 shows examples of logical trees with given bunk systems [19].



**Fig. 3.** Example logical trees with bunk systems:  $\{Cl_1, Cl_2, Br_1, Br_2\} / \{Cl_1, Cl_2, Br_2, Br_1\} / \{Br_1, Cl_1, Cl_2, Br_2\} / \{Br_2, Cl_1, Cl_2, Br_1\}$

In addition, determining the rank of the importance of elements allows you to create a simplified graph of the given transmission [19].

## 4 The Application of Predominant Logical Variables

Examples of logical trees from Fig. 3 have the coding according to Table 2. The next constructional and operational parameters  $Cl_1, Cl_2, Br_1, Br_2$  can be determined by the decision variables  $x_1, x_2, x_3, x_4 = 0,1$  respectively and then 20 optimal logical decision trees with the least number of twigs are obtained 13:

$$\begin{aligned} &f(x_1, x_2, x_3, x_4), f(x_1, x_2, x_4, x_3), f(x_1, x_3, x_2, x_4), f(x_1, x_3, x_4, x_2), \\ &f(x_2, x_1, x_3, x_4), f(x_2, x_1, x_4, x_3), f(x_2, x_3, x_1, x_4), f(x_2, x_3, x_4, x_1), \\ &f(x_2, x_4, x_1, x_3), f(x_2, x_4, x_3, x_1), f(x_3, x_1, x_2, x_4), f(x_3, x_1, x_4, x_2), \\ &f(x_3, x_2, x_1, x_4), f(x_3, x_2, x_4, x_1), f(x_3, x_4, x_1, x_2), \\ &f(x_3, x_4, x_2, x_1), f(x_4, x_2, x_1, x_3), f(x_4, x_2, x_3, x_1), f(x_4, x_3, x_1, x_2), \\ &f(x_4, x_3, x_2, x_1), \end{aligned}$$

Such logical decision trees correctly describe the importance of design and operation parameters from the most important at the bottom to the least important at the top. The other logical decision trees  $f(x_1, x_4, x_2, x_3), f(x_1, x_4, x_3, x_2), f(x_4, x_1, x_2, x_3), f(x_4, x_1, x_3, x_2)$  have the number of branches 14 and are therefore not optimal, which means an incorrect description of the importance of decision variables. Records of logical decision trees  $f(x_1, x_4, \dots), f(x_4, x_1, \dots)$  mean that if  $x_1$  (or  $x_4$ ) is the most important, then  $x_4$  (or  $x_1$ ) can not be only slightly less important, which means that  $x_1$  (or  $x_4$ ) is dominant to  $x_4$  (or  $x_1$ ). In the case of optimal trees, such domination does not take place, i.e. the order is acceptable  $(\dots, x_4, x_1, \dots)$  or  $(\dots, x_1, x_4, \dots)$ .

## 5 Conclusions

There is a necessity to take into account logical dominant variables just as substitutive and conditional variables have been taken into account in the study of the importance of construction and operation parameters [20, 21]. Unlike traditional graphs of dependencies and tree classifiers, the graph of dependencies with parametric trees has the advantage of having a relationship of importance to the vertices (states) with the height of a tree structure.

The paper presents the possibility of applying logical decision trees and predominant variables in the analysis of an exemplary gearbox. Here the analysis does not end. There is a possibility of introducing further generalizations and modifications, in particular the development of an optimization method for parametrically acting structures and direct generation of systems of equations with their solutions. In addition, parametric structures allow for future analyzes and syntheses, such as checking the isomorphism of designed gears, analyzing the range of transmission applications by generating the optimal set of ratios on individual gears.

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