Chapter 16
Algodoo as a Microworld: Informally Linking Mathematics and Physics

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16.1 Introduction

This chapter uses two case studies of high school and undergraduate students interacting with a two-dimensional sandbox modelling software, *Algodoo*, to show how physics students can make use of the mathematical representations offered by the software in unconventional yet meaningful ways. We show how affordances of the technology-supported learning environment allow the emergence of student creative engagement at the intersection of mathematics and physics. In terms of learning, the activities studied here are relevant in two central ways: (1) they open up alternative conceptual learning pathways for students by allowing them to access and engage with the content in original, self-directed and creative ways; (2) in doing this, the studied activities carry significant potential to motivate students and support their intrinsic interests.

Much of the existing research focused on digital learning environments in physics education comprises investigations of *simulation* software, such as the studies which examine PhET simulations (Perkins et al. 2006; Wieman et al. 2008), GeoGebra simulations (Arnone et al. 2017) or Physlets (Dancy et al. 2002). For this chapter, we consider simulations as those digital learning environments which allow students to interact with pre-built models of real or hypothesized situations (National Research Council 2011). As such, simulations are typically designed around a specific phenomenon or set of phenomena so as to provide students with access to particular disciplinary concepts. Much of the research into the use of simulations has produced strong support for their benefit to learning in many different contexts (for a comprehensive review, see Plass and Schwartz 2014).

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By way of contrast, we choose in this chapter to investigate the unique learning opportunities afforded by a less phenomenon-specific digital learning environment, *Algodoo*. In doing so, we make use of the notion of Papertian *microworlds* (1980), a term which refers to digital environments which offer more opportunities for creativity and invention than what is typically offered by simulation software (Plass and Schwartz 2014). While simulations tend to allow users to explore the effects of a set of parameters within the given phenomenon, microworlds provide users with the freedom to build their own environments and phenomena, making possible a wider range of scenarios within the same software. As Laurillard (2002, p. 162) explains, people who use simulations are 'controlling a system that someone else has built', while those using microworlds are 'building their own runnable system'.

In our investigation of the learning afforded by software such as *Algodoo*, we examine two cases of students using *Algodoo* on an interactive whiteboard (IWB) to carry out physics tasks. In particular, we examine how the students in both of these cases make creative use of the mathematical representations available within Algodoo as they reason about physics phenomena. We assert that it is precisely the way in which *Algodoo* seems to function as a microworld, which we refer to as its 'microworldiness', which enables the students to utilize mathematical representations in their playful[1] yet meaningful exploration of physics phenomena. By this we mean that the interactional and representational affordances of *Algodoo* which align with the characteristics of a microworld seem to allow the students in our cases to create and manipulate mathematical representations in ways that are both unconventional and also productive from a physics education perspective.

With the two cases presented in this chapter, we also examine how *Algodoo* and similar digital learning environments might be a useful way for mathematical representations to become interesting and meaningful for students as they engage with physics. Our analysis shows that, while using *Algodoo*, students can interact with mathematical representations in spontaneous, playful ways. Digital learning environments like *Algodoo* may, thereby, provide potentially motivating alternatives for students to make connections between the physical world and the formalisms we use to describe it.

We begin by reflecting on what it means to informally learn physics, followed by a brief review of the instructional philosophy of microworlds advocated by Seymour Papert in his book, *Mindstorms* (1980). Thereafter, we detail some of the features of *Algodoo*, highlighting some of the options the software offers for generating mathematical representations. Finally, we present the two cases of students using *Algodoo* to show that, when used in an appropriate manner, *Algodoo* appears to function as a microworld by supporting students in their creative implementation of mathematical representations.

[1] *Playful* is used in this chapter to mean voluntary, intrinsically motivating (pleasurable for its own sake) and/or creativity-driven (inspired by Rieber 1996).

16.2 Learning Formal Ideas in Informal Ways

By mastering the many mathematical representations used in physics (Van Heuvelen 1991), physicists can employ a diverse range mathematical tools such as force vectors, motion diagrams and graphs to conceptualize phenomena in terms of formal physics models and to appropriately solve problems (Hestenes 1992). Through their commitment to internalizing how nature is described by their discipline, physicists cultivate, among other things, a mathematically enhanced perspective towards the phenomena that they encounter. However, and perhaps not unexpectedly, this is not necessarily the case for most students while they learn physics. For students who are not adequately familiar with – or at least not confident in – the formal, mathematically intensive concepts of physics, the techniques used by physicists to describe the world are often not readily compatible with the students' daily experience of phenomena. There exists for such students a significant difference between how they perceive the world and the way in which physics canonically represents it using formal mathematics. Indeed, students' difficulties with navigating this difference are a common interest for physics education researchers, as evidenced by this very book or, found for example, in McDermott et al.'s (1987) famous discussion of students' difficulties when attempting to interpret kinematics graphs and relating them to their real-world counterparts.

In response to the sometimes-unnavigable disparity between the physical world and the mathematics which physicists use to describe it, many students gain access to the implications of formal physics concepts by other means than an explicit application of mathematics. This can be seen in students' informal cultural exposure to speed and speedometers from cars. Today, the notion of a speedometer can be called upon by physics students as they make sense of velocity and acceleration, something which was impossible for either Galileo or Newton to do in the time before speedometers were invented. Students who grow up in a culture where the enforcement of speed limits is a common occurrence, where a car's top speed is listed in advertisements and where they can ride in a car with an omnipresent visual display of their speed have a corpus of informal experiences which they can and, certainly do, involve in their reasoning with physics concepts such as velocity and acceleration.

In his book *Mindstorms*, Papert (1980) argued that the informal learning culture surrounding students is what provides them with the necessary *materials* with which they can construct their understanding of the world and incorporate them into their understanding of formal physics models. Thus, when the topic of velocity is discussed in a physics context, students from a speedometer-rich culture need not first conceptualize the idea of 'speed-in-general' to begin to become familiar with the concept in the formal physics sense. Such students are able to come to the physics classroom already equipped with the materials from their culture (in this example, their experiences around speedometers) with which they can build new understanding. Surely it should be noted that, as with any previously constructed understanding that students bring to a physics classroom, an everyday experience with speedometers neither certifies that students will automatically intuit physics, nor does it ensure that students will contextualize their understanding of kinematic quantities in the manner consistent with the discipline of physics (Trowbridge and McDermott 1980, 1981).

Nonetheless, in this chapter we explore how, as an environment rich in mathematical representations, *Algodoo* can provide resources to students which might act in a similar manner to the speedometer, providing them with access to materials which they can recruit in the construction of their own understanding of physics. We suggest that when combined with appropriate instructional approaches, *Algodoo* can not only expose students to mathematical ideas as they are used in physics but can also provide an environment for students where they are able to engage in playful inquiry and draw on mathematical representations in a spontaneous and nonthreatening way. Similar to how speedometers can be used as materials for conceptualizing velocity and acceleration in a physics context, the carefully crafted mathematical representations provided within *Algodoo* can be spontaneously recruited as rich materials in students' inquiry into physical phenomena.

16.3 Papert and Microworlds

After observing how young students tended to struggle with reasoning in terms of systematic procedures (necessary for tasks such as ordering beads in all possible combinations along a string), Seymour Papert argued in favour of creating environments rich in the necessary materials for students 'to build powerful, concrete ways to think about problems involving systematicity' (1980, p. 22). In *Mindstorms*, he presented a family of computer languages called LOGO systems (typically involving small programmable robots) as an example of an educational programming language that could enrich the learning environment to promote logical and systematic thinking skills in young students. Papert argued that LOGO systems could provide students with a sufficiently enticing environment for them to develop, in a relatively intuitive and spontaneous way, a mathematical language to communicate with computers. Just as learners of French might immerse themselves in the French language by visiting France, he suggests learners of mathematics could immerse themselves in the 'Mathland' (p. 6) cultivated in the LOGO systems.

Papert intended to provide students with an arena where they could explore formal topics in informal ways. By including what he characterized as *microworlds*, Papert aimed to make computer programming and even the formalisms of Newtonian mechanics accessible to students. In contrast to what he considered the often ineffective and ingenuine approaches taken by much of traditional education, Papert believed that the use of microworlds would result in 'Piagetian learning' or informal 'learning without being taught' (p. 7). He believed that this could be done by providing arenas which were rich in the building blocks needed for students to explore, create and experience formal concepts for themselves. In order to motivate and facilitate the students' learning process, Papert argued that microworlds needed to allow students to become active *builders* in the environment and support them in taking the initiative to engage creatively with the provided materials. The role of a microworld was thus twofold: it needed to (1) offer the correct *materials* for students to recruit and (2) provide a *space* where students could be inspired to create with these materials.

In arguing for builder-focused microworlds, Papert developed what is referred to as a *constructionist* perspective on learning. This constructionist approach places explicit emphasis on the students' act of building – or *constructing* – as a means of learning. In this way, the constructionist perspective can be seen as a special case of the broader, more commonly known perspective of constructivism.

In the time since *Mindstorms*, a body of research has amassed examining the function of microworlds. Abelson and diSessa (1980) quickly adopted the LOGO systems in the teaching of advanced mathematics in the Logo Group of the MIT Artificial Intelligence Laboratory, and the term 'microworld' has persisted in the education research community in the many years since (e.g. diSessa 1988; Jimoyiannis and Komis 2001; Mayer et al. 2003; Miller et al. 1999). However, somewhat contrary to Papert's optimistic view of microworlds, many of the modern reports suggest that more is needed than an environment that simply provides opportunities for exploratory learning if achievement of specific learning goals is desired. Many researchers claim there is a need for some imposed structure of activities or curriculum around a microworld for the environment to become educationally useful (Rieber 2005; White 1984). For example, research has shown that, while using LOGO systems, many students do not spontaneously generate the powerful ideas that Papert had intended unless the microworld is used within a context that is 'well engineered and targeted at well-defined learning objectives' (Miller et al. 1999; referring to work such as Pea and Kurland 1984; Clements 1986, 1990; Klahr and Carver 1988; Lehrer et al. 1989). Even the definition of a microworld, and how the concept of microworlds compares to *simulations* or *games*, is not without contention in the literature. Rieber (1996) discusses the classification of microworlds, suggesting that a learning environment can be regarded as a microworld if it acts as such for a particular learner:

> In a sense, then, it is the learner who determines whether a learning environment should be considered a microworld since successful microworlds rely and build on an individual's own natural tendencies toward learning. It is possible for a learning environment to be a microworld for one person but not for another. (p. 46)

For Rieber, a learning environment should be considered as a microworld in its specific use within a particular context. It is precisely this user-subjective perspective on microworlds that we use in this chapter: whether or not *Algodoo* can be unanimously identified as a microworld for all students, we illustrate how the software *acts* as a microworld for certain students as they use it on an IWB, particularly when dealing with mathematical concepts in a physics context.

In this chapter we present two cases where, aligned with Papert's criteria for a digital microworld, *Algodoo* seems to (1) offer a diversity of mathematical materials – especially in the form of dynamic mathematical representations – and (2) provide an arena within which students are inspired to explore and create with these materials as they engage with physics phenomena.

16.4 Algodoo

Algodoo (www.algodoo.com) is a two-dimensional sandbox software which was inspired, at least in part, by Papert's constructionist approach to learning (Gregorcic and Bodin 2017). At first glance, *Algodoo* resembles other digital drawing software such as *Microsoft Paint*, *Corel Draw* or *Adobe Illustrator* in that it contains various toolbars for creating objects of different geometrical shapes, colours and sizes. However, unlike these other digital drawing platforms, *Algodoo* allows users to press play and has the user-drawn objects dynamically interact. These objects will bounce off each other, roll around, swing from ropes, etc. Thus, users are able to create *scenes* by using a diverse set of available construction elements within *Algodoo*, which include physics-relevant elements such as springs, axles, motors, thrusters, ropes and fastening tools. These scenes typically contain constructions ranging from simple systems (e.g. spring-mass pendula, balls rolling down the slopes or two-body gravitational systems) to more elaborate ones (e.g. suspension bridges, cars and engine transmission systems). When users create systems of objects within *Algodoo* and press the play button, the scenes they have built then evolve in accordance with Newtonian mechanics in two dimensions.

Unlike mathematics modelling tools such as *Modellus* and *Matlab,* which feature an exposed, editable architecture, *Algodoo* is not designed for users to easily change every aspect of the rules governing the virtual world. For example, while users can turn gravity or air resistance off, the underlying mechanics of object interaction cannot be altered from a two-dimensional Newtonian system. Indeed, some researchers might see this algorithmic opacity as a hindrance to students' learning of how to model (Hestenes 1995); however, others argue that *Algodoo* retains a level of *semi-transparency* for students that allows them the opportunity to create and manipulate a virtual world without requiring the prior knowledge of programming (Gregorcic and Bodin 2017). In doing so, *Algodoo* can facilitate new and potentially beneficial experiences in a digital learning environment for those users without fluency in coding languages (Euler and Gregorcic 2018; Gregorcic 2016). In fact, *Algodoo* appears to be an intuitive program for students at both high schools and universities, so much so that these students can, in a matter of minutes, start engaging in creative activities even when they use the software for the first time (Gregorcic et al. 2017a). *Algodoo*'s ease-of-access is a key component in our consideration of it functioning as a microworld.

The other important characteristic of *Algodoo* is that it provides, through visual and interactive means, a range of dynamic representations which have been shown by research to contribute to effective physics learning (e.g. Rosengrant et al. 2009). In what follows, we discuss *Algodoo*'s capability for representing mathematical concepts. Specifically, we emphasize the utility of combining *Algodoo* and an IWB, which can provide students with a collaborative space for engaging with mathematical and physics concepts.

16.4.1 Representations Afforded by Algodoo

Algodoo, like any other computer-based model of phenomena or modelling system, is built up from formal mathematical relationships in its source code. The software can track the motion of the objects created within it, therefore allowing it to display quantities such as momentum, force, velocity and position. This is due to the fact that these quantities are part of the internal structure that manifests in the external user interface (Plass and Schwartz 2014).

Algodoo dynamically updates visual representations in real time (i.e. while the microworld runs), which allows users to access and manipulate physical quantities describing virtual objects in ways that would be impossible to achieve in a traditional physics laboratory, in a classroom or in everyday life. Nonetheless, while including these mathematical aspects, the *Algodoo* environment still retains many characteristics of the world students experience every day. In the software, users can grab, move and even throw virtual objects, which can then be observed to bounce off each other, slide, tumble and generally behave in ways that most people can relate to their everyday experiences with real-world objects.

By including mathematical representations (e.g. Fig. 16.1) of quantities like dynamic vector arrows (e.g. velocity, momentum and force), numbers and sliders representing values of physics-relevant quantities (e.g. density, restitution, coeffi-

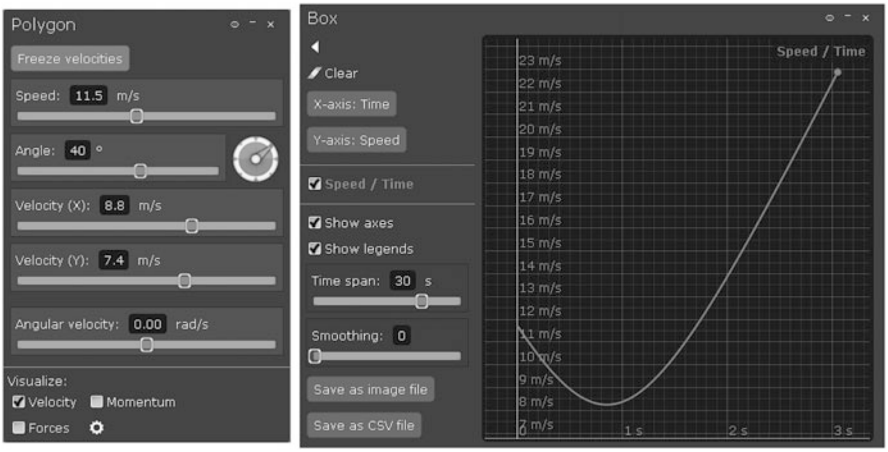

Fig. 16.1 Two examples of the representations provided by *Algodoo*, namely, the 'Velocities' tab (left) and a graph from the 'Show Plot' function (right) (In the Velocity tab, sliders for changing the speed, angle, velocity (x), velocity (y) and angular velocity are provided along with a wheel which displays the angle of velocity and checkboxes for displaying vectors (for velocity, momentum and forces) on the selected object(s). In the graph, various quantities can be assigned to the axes, and the options are provided to display the title ('speed/time' in this case), the axes and the legends. The slider-labelled 'time span' allows the user to select the length of time to include (from the most recent 'run' of the simulation), while the slider-labelled 'Smoothing' allows the user to smooth the graph of the data.)

cient of friction) and plots of quantities (e.g. kinetic energy vs. time, x-position vs. y-position) alongside the visually accessible virtual world, *Algodoo* superimposes formal physics and mathematical ideas onto a more familiar world of physical, albeit simulated, interactions. *Algodoo* provides opportunities for students to explore and engage in open-ended and creative tasks where they can experience physics-relevant, mathematical ideas *in action* and interact with physics content in new pedagogical ways which are not typically available. For example, students can observe the forces acting within a suspension bridge, which they may have built themselves, by selecting to display *Algodoo*'s overlay of dynamically changing force vectors on top of the bridge itself.

The close interplay of the mathematical representations within an intuitively manipulable virtual world gives students and instructors access to a rich collection of meaning-making resources. These resources can be employed to help students develop a better understanding of the meanings embedded in mathematical representations that are used in physics and may even encourage them to make use of these representations in their communication of physics ideas.

The creative potential of *Algodoo* appears to be significantly enhanced when used in combination with a large touch screen, such as an interactive whiteboard (IWB) (Gregorcic 2015a). The IWB provides students with common perceptual ground which they can visually appreciate in small groups (Roth and Lawless 2002) and which they can refer to using environmentally coupled hand gestures (Goodwin 2007; Gregorcic et al. 2017b). This allows students to engage with *Algodoo* in collaborative exploration and communication (Mellingsæter and Bungum 2015). The affordances of the *Algodoo*-IWB setup for multimodal communication allow students to address conceptually interesting ideas even when their knowledge of corresponding vocabulary is limited. Where they struggle to find words to express meaning, they can resort to gestures, such as pointing to patterns and values on the screen (Gregorcic et al. 2017b). The pronounced gestural and interactional components of student communication in front of the IWB can also provide researchers with a better insight into students' meaning-making than paying attention to their speech alone (Euler and Gregorcic 2018; Gregorcic et al. 2017b).

16.5 The Cases

In order to illustrate how the *Algodoo*-IWB setup can provide new opportunities for students to learn how to appropriately use mathematical representations in playful (yet useful) ways, we present two cases of open-ended physics activities which utilized the *Algodoo*-IWB setup: (1) one where students threw planets into orbits and (2) another where students rolled an object down a ramp. The data for both of these cases was video recorded in a small room with researchers (referred to as instructors) present to act as the facilitators of the activity, to push the students to further clarify their thinking out loud, and/or to aid the students with any technical difficulties with *Algodoo* or the IWB.

In order to present the data in a manner which captures both the speech of the students and also their gestural activity, we include a multimodal transcript (Bezemer and Mavers 2011) comprising written excerpts of talk[2] and line illustrations drawn from frames of the video data (which are occasionally augmented by close-ups of the relevant *Algodoo* menus). Each line of the transcript is numbered and labelled with the speaker or actor responsible for the speech or action contained in the line ('S1' to 'S5' for Student 1 to Student 5, respectively; and 'In' for the instructors). Actions such as gestures or manipulations of the IWB are included as *italicized* text in [brackets] and represented visually by illustration when useful. In the section that follows, each excerpt of transcript is followed by a summary of the what was said and done by the students to make explicit the things we wish to highlight from the students' interactions.

While the physics content varies between the two data sets presented here, we will show how in both instances, the presence of representational options within *Algodoo* led students to coordinate their discussion and creative inputs around complex mathematical representations in ways which we can appreciate as appropriate for the learning of physics. While exploiting the open, microworld-like nature of *Algodoo,* students were able to creatively link mathematics and physics through their informal use of mathematical representations.

16.5.1 Case 1: Vector-Sense in Orbital Motion with the 'Velocity' Tab

Our first case comes from a data set collected as part of a previous study on the use of IWBs in astronomy instruction (Gregorcic 2015a) where small groups of high school students used the *Algodoo*-IWB setup to explore celestial motion. The students were presented a scene in *Algodoo* which involved a central circular body with an attractive potential – representing a star or planet in an astronomical system. The students used the *Algodoo*-IWB setup to qualitatively investigate Kepler's laws of planetary motion (Gregorcic et al. 2015, 2017b), specifically following the prompt to explore how relatively smaller bodies behave in the vicinity of the larger central massive body. The students drew planet-like or moon-like objects and, by swiping on the IWB, threw these objects into orbit around the star-like object located in the centre of the scene. It was also possible for the students to send objects into orbit by pausing the simulation, placing the object at the desired radius away from the central circle, assigning a velocity to the object and then running the simulation. Some groups chose to display the force vectors or velocity vectors of the objects as these objects orbited the central object (referred to hereafter as the 'Sun'). Gregorcic designed the Kepler's laws scene in *Algodoo* to provide students with 'hands-on

[2]The data collection session for Case 1 originally took place in Slovenian, but we have translated the speech into English for the purposes of this chapter.

[access] to [the] otherwise experimentally inaccessible topic' of orbital mechanics (2015a, p. 515).

Analysis of the data collected in these sessions has previously focused on social and embodied aspects of students' learning of physics concepts with the IWB-*Algodoo* setup (Gregorcic 2015a, b; Gregorcic et al. 2017b). For the analysis presented in this chapter, we instead focus on the students' engagement with mathematical representations of velocity.

We begin by highlighting an excerpt from a session where a group of three students – who we refer to as Student 1 (S1), Student 2 (S2) and Student 3 (S3), along with the Instructor (In) – try to send an object into orbit by setting its initial velocity within the 'Velocities' tab in the drop-down menu (while the simulation is paused). They estimate the initial conditions (radius and velocity) necessary to send the object into orbit by comparing them to that of an already orbiting object from before. They press the play button and then watch as the newly launched object collides with another object that was already orbiting the Sun. The collision sends the new object out of the frame of view and pushes the original object into a new orbit around the Sun. While the new object is sent out of the frame of view, its Velocity menu remains open in *Algodoo*. We include sections of the transcript to illustrate the informal exploration that took place after the students first observe the collision.

1 **S2:** Okay . . .
2 **S1:** Aha!
3 **In:** What happened now?
4 **S1:** This one's trajectory changed, but it remained constant.
5 **S1:** And it's losing speed.
6 **S2:** No, it's not losing speed.
7 **S1:** [*points to the slider for speed*] (Fig. 16.2)
8 **S1:** One of them is losing speed.
9 **S2:** Yeah, yeah. That one.
10 **S1:** Yeah, that one, yeah. That one that is going away.
11 **In:** Ah, now you're looking at that one!

Excerpt Summary In this exchange, we see the students make sense of the behaviour of the two objects after the collision. They notice how the originally orbiting object has been pushed into a new, stable orbit – which Student 1 refers to as being 'constant' (line 4) and which we take to mean stable in time (self-repeating on a closed trajectory). Noticing how the Velocity tab is displaying a decreasing speed, the students quickly come to realize that the Velocity tab is still showing data for the runaway object, which is now out of sight, past the edge of the view in *Algodoo*.

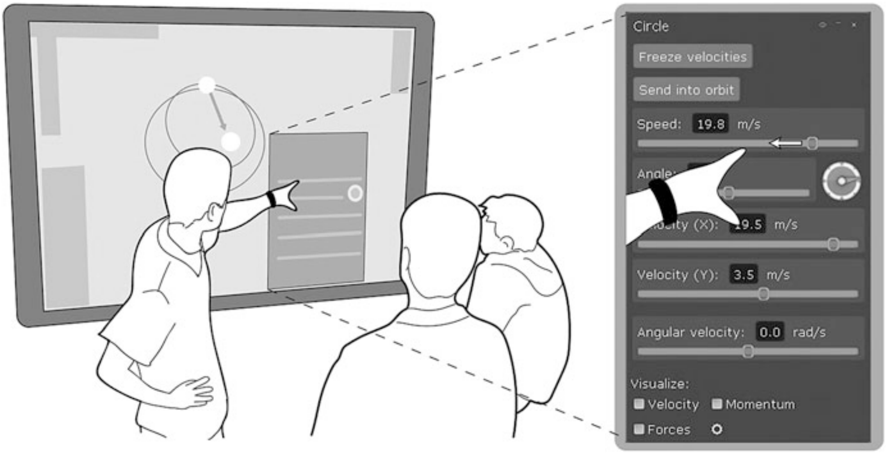

Fig. 16.2 Student 1 (left, with Student 2, middle, and Student 3, right) points to the moving slider labelled Speed within the Velocity tab as he emphasizes that one of the objects is 'losing speed' (line 6) (It should be noted that the values for Speed, Angle, etc. in the Velocity tab are an approximate recreation and do not necessarily reflect the exact values seen by the students during the session. These values are 'unrealistic' for objects on planetary scales, but their usefulness holds in their proportions to one another and their qualitative changes over time.)

(continued from above)

12	**S3:**	Turn its angle, so it will come back.
13	**S1:**	[*laughs*]
14	**S2:**	[*starts dragging the Angle slider to the right, changing the angle at which the runaway planet is travelling*]
15	**In:**	You can also turn the little wheel if you want to turn the angle. There, on the right side
16	**S1:**	And let's add some speed . . . Or not. It's already coming back! [*performs a U-turn gesture in front of the IWB*] (Fig. 16.3)
17	**In:**	So, you noticed something interesting.
18	**S1:**	So, now it's slowly coming back into orbit. Because it's becoming faster. [*points to the speed slider, where the value is increasing*]
19	**S3:**	Yes.
20	**S2:**	Yes.

Excerpt Summary Here, Student 3 suggests that they 'turn [the planet's] angle' (line 12) in order to bring it back into sight. Student 2 then drags the Angle slider to the right to change the angle at which the planet is travelling, and the instructor suggests that he can also use the Wheel to change the angle. After Student 2 changes the angle, Student 1 initially wants to alter the object's speed as well but changes his mind as he watches the angle spontaneously rotate with the motion of the planet. He interprets the changing angle as the planet reversing direction and he gestures with his hand in a U-turn motion. He also notices that the Speed slider is moving

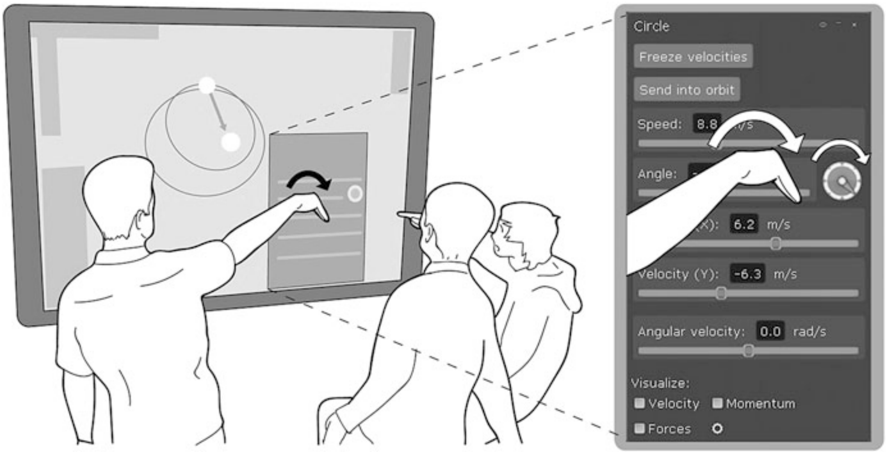

Fig. 16.3 Student 1 gestures in front of the IWB with a U-turn gesture (downwards) as he vocalizes that the runaway planet is 'already coming back' (line 16). Student 3 points towards the wheel of the velocity menu as it turns with the changing trajectory of the planet

to the right, which he interprets as meaning that the object's speed is increasing. He explains this as the object 'slowly coming back into orbit' (line 18), and the other two students agree.

(continued from above)

21 **In:** Coming into orbit, what does that mean?

22 **S3:** Closer . . .

23 **S1:** Closer to the [Sun].

24 **S2:** Actually, it is already kind of in orbit, unless it will crash into it. Because it . . . because it is attracting it. It means it will . . . [*starts gesturing a large curve in the air*]

25 **S1:** Just a moment. Considering it was travelling away from this object and it was losing speed . . .

26 **S2:** Yes, it was.

27 **S1:** And there was no resistance . . .

28 **S2:** It was in orbit from the beginning, but . . .

29 **In:** Okay. Okay. Interesting observation. It was flying away. It was losing speed.

30 **S2:** It was losing speed and it had no resistance.

31 **S2:** Yes, but that's normal. If you have a body out here and a gravitational force between them, and there is no other force, and you don't accelerate [the body out there], its speed will get smaller until it will turn around and travel the other way. [*mimics the motion of a planet moving away from the Sun and then back towards it with his hand*] (Fig. 16.4)

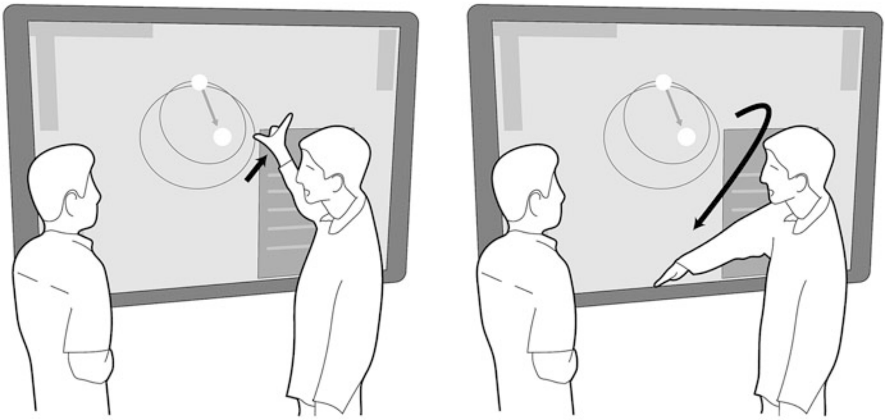

Fig. 16.4 Student 2 gestures to show the movement of a planet as it is accelerated by the Sun. We can interpret this explanation as one that uses a Newtonian model of Sun-planet interaction

32 **S2:** Which is interesting, but . . . I mean, it's interesting . . .
33 **S1:** Yeah, I get it.

Excerpt Summary Here, the students engage in a discussion about orbital motion and the underlying mechanisms that govern the changes in an object's velocity. While Student 1 first has an issue with the slowing down of a planet in a frictionless environment, Student 2 is able to explain how the object's behaviour makes sense in a system with gravitational force (line 31, which we interpret as a Newtonian perspective). Student 2 supports his argumentation with environmentally coupled hand gestures, symbolizing the movement of the planet and the direction of forces (Fig. 16.4).

(continued from above)

34 **S1:** Aha, okay, now its angle started changing, which means . . . [*starts repositioning himself in front of the IWB, pointing to the Velocity tab*] (Fig. 16.5)
35 **In:** Oh, yes, now you are observing that body just through [the Velocity tab].
36 **S2:** Yeah, um . . . Good point.
37 **S1:** [*laughs*]
38 **S2:** [*uses the Zoom tool to zoom out, revealing more of the space around the Sun*]
39 **S1:** Here it is. [*notices the runaway planet on the left side of the Sun, close to the edge of the screen*]

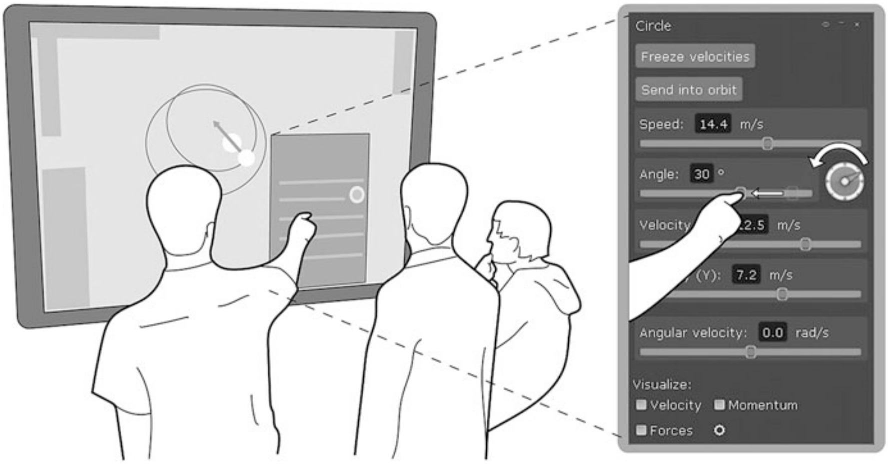

Fig. 16.5 Student 1 notices the changing velocity of the runaway object in the Velocity tab. He repositions himself in front of the IWB and points to the changing Angle slider

40 **S2:** It's here. [*pointing to the runaway planet*]
41 **S1:** Let's do it by hand.
42 **S2:** Let's zoom out more. Can we zoom out more?
43 **S1:** No.
44 **In:** This is the most zoomed out it can be.
45 **S1:** Quickly. [*turns the angle wheel CW, in the direction towards the Sun*]
46 **S2:** But now we are changing its things again.
47 **S1:** [*drags the speed slider to the right and the planet starts travelling faster towards the Sun*]
48 **S2:** It is going to crash directly into it.
49 **S1:** [*adjusting the direction using the angle wheel*] So now it is already growing. [*watches as the speed slider spontaneously moves to the right*]

Excerpt Summary Again, Student 1 notices an increased rate of change in the object's angle of velocity by watching the Velocity tab, all while the planet remains outside the field of view in the scene. The instructor points out that the students are interpreting the motion of the planet by just looking at the values in Velocity tab, to which the students respond by zooming out to find the object (now on the left side of the Sun) just as it is about to fly out of the field of view. Student 1 quickly manipulates the object's velocity by changing the angle (turning the wheel CW towards the Sun) and then increasing its speed (by dragging the Speed slider to the right). Finally, he watches the object and its Velocity tab simultaneously and notices that the Speed slider continues to move to the right as the object accelerates towards the Sun.

In the excerpts of transcript presented above, we see that, although the students originally speculated that the runaway object was lost after the collision, they noticed that the velocity of the runaway object changed in a way that suggested it would return if they kept waiting (meaning that the runaway object was in some type of orbit). Despite the object being absent from the frame of view in *Algodoo*, the students were able to track the motion of the object through the Velocity tab still open from before the ‘play’ button was pressed. They watched the Speed slider move and the Angle wheel rotate, interpreting them to understand that the runaway object was slowing down and turning back towards the Sun. The students were then able to propose explanations (which we identify as consistent with a formal, Newtonian model) for the patterns of motion seen in the Velocity tab. In the end, they located the runaway object in a zoomed-out field of view and manipulated its velocity so that it started moving back directly towards the Sun.

16.5.1.1 Analysis of Case 1

The case included above is an example of how a group of students creatively used one of the representations within *Algodoo*, namely, the Velocity tab, in a playful, unconventional way, which we can see was still meaningful from a physics learning perspective. From this case, we discern two functions for which students used the Velocity tab: (1) as a *tool* for manipulating (or setting) the velocity of an object and (2) as a *representation* which was recruited in making sense of the motion of an object.

The first function of the Velocity tab, as a tool for manipulating the velocity of an object, can be seen initially when the students used the Velocity tab to put a newly created object into motion (giving the object an initial velocity before hitting play). Then, once the collision had sent the object far away from the Sun, the students used the Velocity tab to manipulate the object’s motion dynamically (with *Algodoo* running). This manipulation appeared in two instances: first as Student 1 changed the angle of the object’s velocity (line 14) and again when the same student redirected the object towards the Sun (lines 45–49). In all of these instances, the presence of *Algodoo*’s Velocity tab, which allowed the students to set and manipulate the velocity of the object with sliders and a wheel, provided an opportunity for the students to engage with the orbital task creatively. More traditional approaches to the learning of orbital motion often do not provide such a means for interacting with objects’ velocities as they relate to orbits. In this case, the students were able to test their own ideas of orbital mechanics, giving them ownership of the result, all while they utilized a mathematically rich interface. The manner in which the Velocity tab was used as a dynamic tool for the manipulation of velocity showcases our first concrete example of *Algodoo*’s microworldiness: the software seems to have provided the students with mathematically rich materials while also allowing the students to be creative and self-directed in their activities.

The second role that the Velocity tab played in the presented case was that of a representation recruited in making sense of the motion of an object. During most of the episode, the Velocity tab served as a monitoring device for the orbiting object outside the field of view of the scene. Formally, the velocity vector of an object in two dimensions can be expressed in terms of speed and angle (magnitude and radial direction) or as the sum of the *x*- and *y*-components of the velocity. Interestingly, in the *Algodoo* environment, the students sent an object into motion and *observed* its components, interpreting the motion of the runaway object intuitively as they tracked the changes in the angle and speed. Thus, even without being prompted to discuss vector magnitudes or components, the students were able to demonstrate fluency (at least, partially) of vector sense in relation to two-dimensional motion. The presence of the Velocity tab allowed the students to spontaneously move between a familiar, informal experience of motion (the visual movement of the object on the IWB surface) and a formal mathematized representation of motion (within the sliders and wheel of the Velocity tab). Indeed, the limited field of vision in *Algodoo*, which made the students unable to watch the object's motion as they would normally, along with the persistence of the Velocity tab, which provided them with a dynamically updated rendition of the runaway object's velocity data, encouraged the students to interpret and make creative use of the mathematical representation made available by the software.

Though the significance of the dynamically changing information on the Velocity tab was not initially appreciated by the students, as they began to make sense of what was happening, they were able to interpret the motion of the runaway planet from the controls in the tab, translating the information of the sliders and wheel into more familiar, everyday language of gesture and speech. We see this when the students noticed one of the objects 'losing speed' (line 5), after which Student 1 started making sense of the changing angle and slowly increasing velocity of the runaway planet with an explanatory gesture (line 16). Student 1 reinterpreted the information within the Velocity tab with a gesture, transforming the meaning carried in the software into a dynamic mode of expression.[3] He then engaged with the Velocity tab as a source of information about the motion of the runaway object until he is able to demonstrate his interpretation of what is going on in a more conceptual way (see Fig. 16.3).

Beyond functioning in the two ways described above, the *Algodoo*-IWB learning environment was successful in encouraging students to spontaneously produce an explanatory model for the patterns of motion. This can be seen when Student 1 questioned the motion of the runaway object (line 25). Student 2 responded by proposing an explanation for the patterns of motion consistent with a Newtonian model of orbital motion (line 31). Student 2's interpretation of the patterns seen on the Velocity tab gave rise to explanatory talk and gesture about the behaviour of orbiting objects in general. In this way, the Velocity tab within *Algodoo* appears to have behaved as a point of departure for further inquiry, providing some

[3]This process of transforming meaning from one mode of expression to another is sometimes referred to as *transduction* in multimodality circles (Jewitt et al. 2016). For a discussion of how transduction may be a key concept in physics learning, see Volkwyn et al. (2018).

mathematical materials which students were compelled to observe and explain in a science-like discussion (Etkina 2015; Gregorcic et al. 2017b).

This can be taken to demonstrate, in a slightly different manner, how *Algodoo* can act as a microworld for students. That is, the students were inspired by the setup and the activity to not only explore and create within the mathematically rich environment but to also begin taking science-like approaches to solving the problems they encountered (Gregorcic et al. 2017b). Consequently, a case might be made for how microworlds like *Algodoo* can offer alternative ways to promote both the learning of nuanced content knowledge at the intersection of mathematics and physics and also the adoption of the behavioural patterns used by scientists, all while promoting active engagement and creativity.

We see from Case 1 that, when using *Algodoo,* students can use mathematical representations in a creative way, therein becoming inspired to discover how a physical system works. The students' use of the mathematical materials provided by *Algodoo* was both playful – due to *Algodoo*'s open-ended, creativity-driven structure – and meaningful for their understanding of the physics formalisms that underpinned the activity. It is precisely this richness of the digital environment, the way in which *Algodoo* is an explorable sandbox populated by mathematically rigorous representations, which seems to have made possible the unique, meaningful interaction presented above.

Indeed, in the case presented here, the particular affordances of *Algodoo* that resulted in students' meaningful use of mathematical representations were paired with an instructional strategy of open-ended – but task-based – inquiry and exploration with some guidance from an instructor. Throughout the activity, the instructor engaged with the students to help direct them in their exploration. If the students had simply been given the Kepler's law scene without any instruction or guiding activity, it is unlikely that they would consistently end up manipulating the velocity in such fruitful ways. Nonetheless, the above case shows an instance where the microworldiness of *Algodoo* contributed to a group of students' creative inquiry while at the same time engaging them with formal representations of motion.

16.5.2 Case 2: Graphical Representations in Kinematics with 'Show Plot'

We now present the second case to illustrate the potential for *Algodoo* to promote creative and meaningful use of mathematical representations. This case focuses on pairs of students that used *Algodoo* in an activity alongside a physical ramp and a hockey puck on a table (hereafter referred to as the ramp-puck setup, see Fig. 16.6). The data collection comprised six students, all of whom were selected on a volunteer basis and observed pairwise on separate occasions (i.e. three separate groups of two students). Like the session in Case 1, the participating students were provided with an IWB running *Algodoo*; however, to foster a direct link between

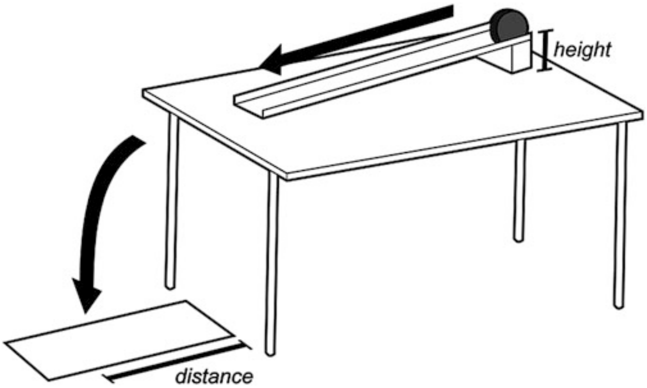

Fig. 16.6 The ramp-puck setup used by the students in Case 2 alongside *Algodoo* running on an IWB. The 'height' (above the table) and the 'distance' (horizontally along the floor from the edge of the table) are labelled

digital and physical learning environments, the students were asked to use *Algodoo* in parallel with the physical ramp-puck setup while answering a specific physics prompt. Specifically, the students were asked to convince the researchers of the relationship between (1) the height above the table from which the puck is released on the ramp and (2) the horizontal distance from the edge of the table which the puck travels before hitting the ground.

Each of the pairs of students were part of a larger, three-part session where they (1) familiarized themselves with the functions of the *Algodoo* (a duration of approximately 1 h), (2) completed the ramp activity (1 h) and then (3) concluded with a short discussion about their impression of *Algodoo* and the activity as a whole (30 min). As in Case 1, these sessions were all video recorded and transcribed for analysis.[4]

The original aim of the study was to examine how students use *Algodoo* in combination with real experiments when faced with a physics task (Euler and Gregorcic 2018). However, as in Case 1, we have found examples within the data of students engaging with a variety of mathematical representations. We can see some of the students coordinating physical observations and mathematical ideas within *Algodoo* in a manner that suggests the digital environment encourages the meaningful use of mathematical representations. The particular excerpt that we present here shows how one pair of students, referred to as Student 4 (S4) and Student 5 (S5), used the 'Show Plot' tool to quantify aspects of the puck's motion in a virtual model of the ramp-puck setup they had created. The excerpt we present here illustrates how the students can recruit and interpret graphical representations in *Algodoo*, as they attempt to quantify a physics phenomenon.

[4]For Case 2, the sessions were conducted in English, though the native language of both of the students was Swedish.

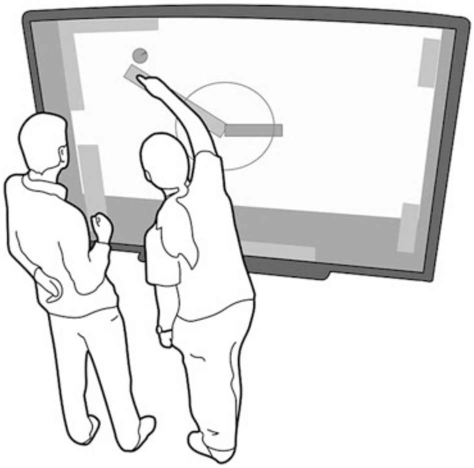

Fig. 16.7 Student 5 (right, with Student 4, left) rotating the ramp portion of the ramp-puck model to the desired angle. Here, the horizontal rectangle functions as a virtual table, the tilted rectangle functions as a virtual ramp and the circle functions as a virtual puck. In this scene, as opposed to the scene in Case 1, the ground is represented by a horizontal plane towards the bottom of the screen and gravity acts vertically downwards

Case 2 begins as the students finish setting up the virtual model of the ramp-puck experiment in *Algodoo*. They place two rectangular objects (representing the ramp and the table) and the circular object (the puck) in such a spatial arrangement that when they press the play button, the puck rolls down the ramp, continues off the table and then hits the ground below (Fig. 16.7). The students then try to address the prompt by finding a way in which they can measure the distance the puck travels horizontally from the edge of the table before hitting the ground.

After constructing the virtual ramp-puck setup, the students run the scene to check the function of their model. The circle successfully rolls down and off the rectangles before hitting the ground. The students immediately wish to measure the distance that the puck travels from the edge of the horizontal rectangle, but *Algodoo* does not include a purpose-built distance measuring tool. Student 4 stumbles upon the Show Plot tool. He opens the Show Plot tool and explores its possibilities for representing plots of various physical quantities for the selected object (the virtual puck in this case) in the form of a two-dimensional graph. He discovers that *Algodoo* allows you to plot different quantities on the horizontal and vertical axis of the displayed coordinate system.

50 **S4:** [*sets the vertical axis to 'Position (y) and then the horizontal axis to Position (x)*]

51 **S5:** [*drags the corner of the graph window to make it smaller and then moves the window to the left so they can watch the circle's motion as it rolls down the ramp*]

52 **S5:** Something like that.

53 **S4:** And start?

54 **S5:** Yeah.

Excerpt Summary In the first part in of the excerpt, the students look for a way to quantify the movement of the puck, in particular, to put a numerical value on the distance the puck travels off the edge of the table. By exploring the options provided by *Algodoo*, the students discover the Show Plot tool. Student 4 then interacts with the plotting tool to select the appropriate axes labels (the x-position and y-position of the virtual puck), and Student 5 positions the graph window in such a way that they can simultaneously observe both the virtual experiment and the plot.

(continued from above)

55 **S4:** [*presses the play button and they watch the puck move with the data being drawn in the graph window simultaneously*] (Fig. 16.8)

56 **S5:** And let's see. If we look closer at this . . . [*leans in to examine the graph*]

57 **S4:** Here. [*points to the point on the graph corresponding to where he thinks the circle hit the ground*]

58 **S5:** Yeah there. [*pointing to the same point as S4*]

59 **S5:** We can see that we have to look at it from here. [*touches the point on the graph which he interprets as where the circle left the table*] to there. [*touching the point on the graph corresponding to where they agreed the circle hit the ground*]

60 **S4:** Hits the ground there. That's what we need to get.

61 **S5:** Yeah, we want to know the distance here? [*gestures to show the length from the end of the physical table in the room and looks to the interviewers for confirmation*]

62 **In:** Mhm.

63 **S5:** Yeah. Uh . . . [*pauses for a long time to examine the graph*]

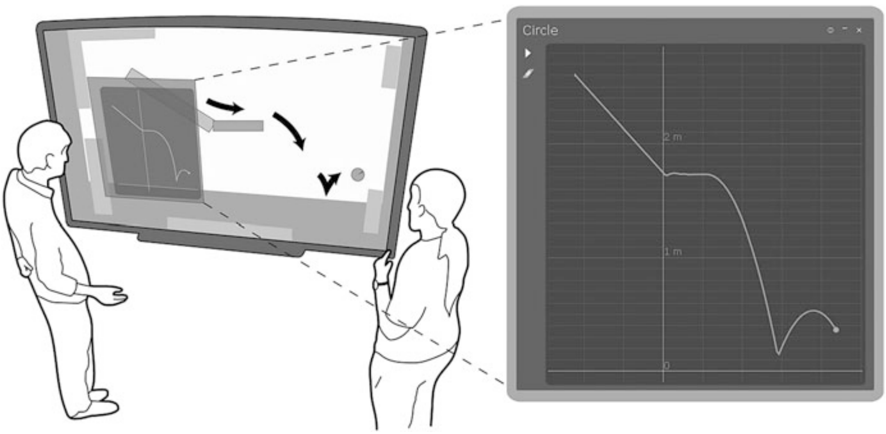

Fig. 16.8 The students examine the scene after watching the circle roll down the ramp and off the table. The graph displays a plot of the circle's motion

Excerpt Summary In the second part of the episode, the students have just run the simulation and noted its outcome by observing the movement of the puck, as well as the self-drawing graph in parallel. They continue by interpreting the graph. They start to relate characteristic points on the graph to spatial locations in the *Algodoo* scene, as well as in the physical experiment that is set up in the room next to the IWB. They identify the distance of interest and point out what they interpret as the corresponding distance on the graph.

(continued from above)

64	**S5:**	I'm trying to figure out why is there a zero here? [*points along the y-axis of the graph*] 'Cause we started way up here [*points to the upper left corner of the graph*], and where does this graph place the zero? How does this software determine where the origin is?
65	**In:**	Mhm. Is there a question?
66	**S5:**	Uh, I think so, I'm not . . . [*drags the corner of the graph window to make it larger*], I don't really know how to look at this graph to determine . . . I mean here it says 10 meters, there. [*points to the rightmost label of the x-axis*]
67	**In:**	So, what is this graph displaying really?
68	**S5:**	The *y*-position [*gestures up and down the IWB*] and the *x*-position. [*gestures left and right along the IWB*] (Fig. 16.9)
69	**In:**	Mhm.

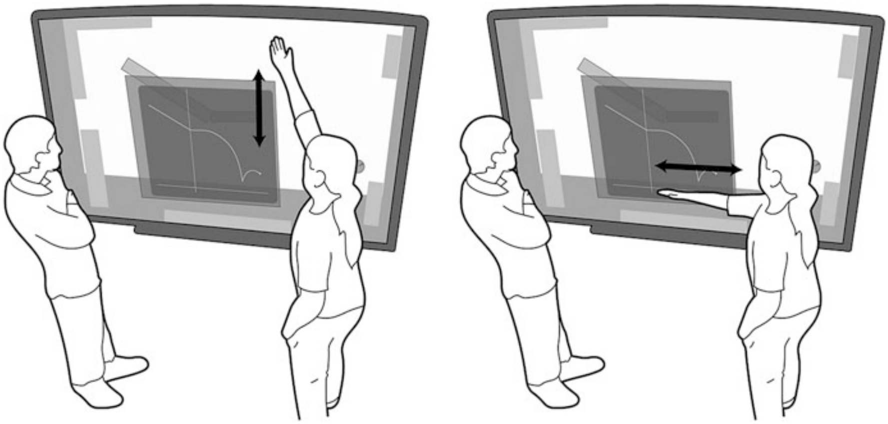

Fig. 16.9 Student 5 gestures to describe what each of the axes is displaying. He describes that the *y*-axis displays the *y*-position of the circle (gesturing up and down), while the *x*-axis displays the *x*-position of the circle (gesturing left and right)

70 **S5:** But what I can't really see is where the *x*-position zero point is. That should be there. [*points to the origin in the graph window*] But it doesn't show much more [*taps around in the graph space to see what selecting the axes does then traces the graphed path of the ball in the plot to select various data points*]

71 **In:** Can you say from the graph where the *x*-position zero is? [*pauses*] So, this graph, what does this graph represent? Like in other words, what would you say this graph represents? 'Cause you can have velocity versus time graphs. You can have *x* versus time graphs, but this is a *y* versus *x* graph

72 **S5:** Yeah it describes exactly where the ball has been. It shows the path of the ball.

73 **In:** Mhm! So, in space, right?

74 **S5:** In space, yes.

75 **In:** So, I think you can actually see where the *x*-zero is then.

76 **S5:** Yeah when it starts rolling on the other one . . . [*grabs the graph window and drags it out of the way of the ramp*] When it starts rolling on that one. [*points to the intersection of the ramp rectangle and the table rectangle*] (Fig. 16.10)

77 **In:** And where would you like it to be?

78 **S4:** Here. [*points to the top right corner of the tilted rectangle*] (Fig. 16.11)

79 **S5:** No [*drags the graph window out of the way*]. We want it on the end there [*points to the end of the horizontal rectangle*]

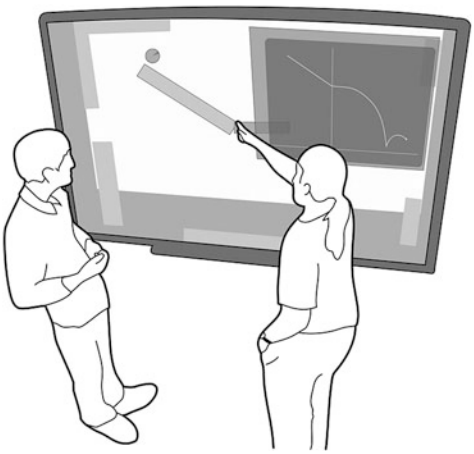

Fig. 16.10 Student 5 points to the intersection of the ramp rectangle (the tipped rectangle) and the table rectangle (the horizontal rectangle) to indicate the location in the scene which he interprets as the position of the $x = 0$ line of the graph

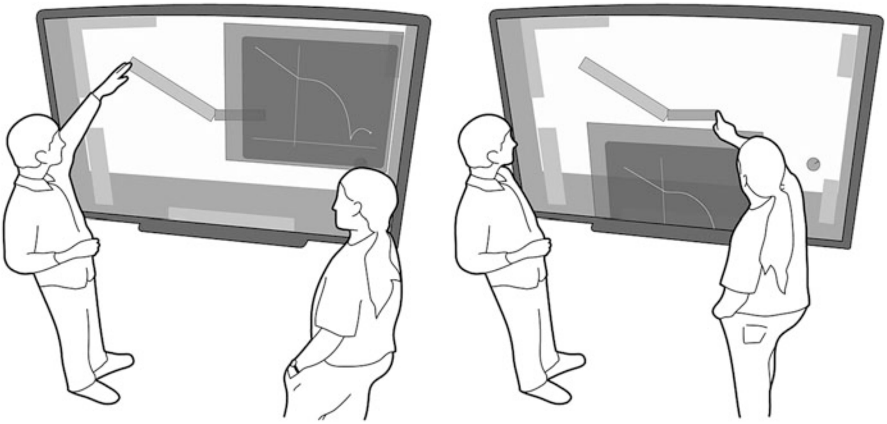

Fig. 16.11 Student 4 points to the position he thinks would be best for the $x = 0$ position at the top of the tilted rectangle (left image). Student 5 disagrees and points to the point he thinks they should place the $x = 0$, at the end of the horizontal rectangle (right image)

Excerpt Summary In this exchange, the students try to make sense of the position of the origin of the coordinate system used to describe the position of the puck. The instructors encourage them to interpret from the existing plot of the puck's motion, where the origin (zero) is currently placed and where they would like it to be. Student 4 proposes that the desired placement of zero for the *x*-coordinate would be the edge of the table (due to the convenience of reading off the distance from the edge of the table at which the puck first hits the ground). After line 79, with some technical help from the instructors, the students reposition the objects in *Algodoo* so that the right edge of the horizontal rectangle (the virtual table) is positioned at $x = 0$. This is done since *Algodoo* does not allow the user to move the origin of the built-in reference frame, which is fixed to the background of the scene.

(after positioning the virtual setup as desired)

80 **S4:** [*presses start and watches as the ball rolls down again, tracing a path on the graph similar to the one before, but with the axes reposition as they wanted*]

81 **S5:** [*presses pause*] Then we can find . . . [*traces finger along the data in the graph from top left to bottom right, stopping where the circle hit the ground*] the *x*-position! Point 75 meters (Fig. 16.12)

Excerpt Summary In this last excerpt from Case 2, the students manage to assign a numeric value to the horizontal distance the rolling puck travelled before it first hits the ground. They do this by touching the location in the graph where the tracked object (the virtual puck) appears to have first bounced and then reading the *x*-value of its position from the built-in graph examining tool (Fig. 16.12).

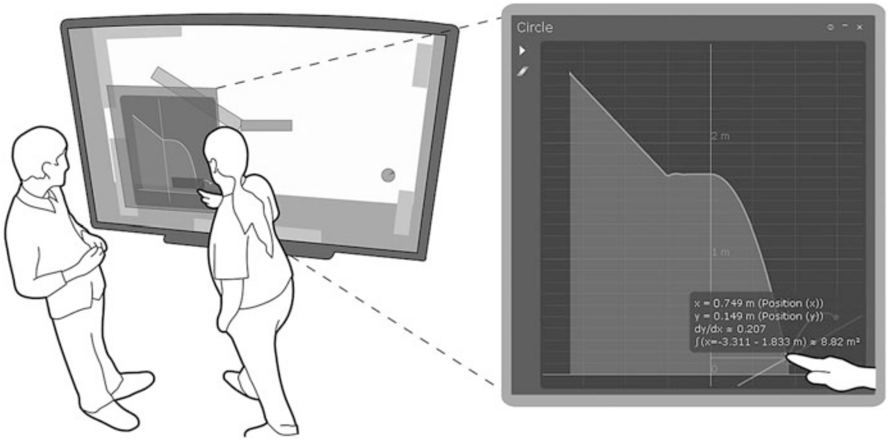

Fig. 16.12 shows Student 5 tracing the data in the graph (of the shifted setup where the y-axis is more conveniently placed) with his finger until he finds the point where the circle hits the ground. He is then able to read off the value for the horizontal distance from the x-coordinate of the dynamic graph label

In the student dialogue from Case 2, we see that the students stumbled upon the Show Plot tool in *Algodoo* and then tried to figure out how to place the origin of their graph in a useful position for their measurement purposes. In order to figure out how to move the axes to where they wanted, the students first had to interpret what the graph was showing so that they could understand how *Algodoo* had placed the origin for them (the origin is fixed by default to the background in *Algodoo*, and they had to move their virtual setup so that axes were aligned with the desired part of their ramp-puck model).

16.5.2.1 Analysis of Case 2

In Case 2, the students engaged with the *Algodoo*-IWB setup to mathematize the motion of a puck in a graph. Despite the physics content being different from that in Case 1, we use the students' interaction in Case 2 to highlight how *Algodoo* appeared to act as a microworld for the students by providing them with mathematical material to draw upon in a meaningful, if slightly unconventional, exploration of a physics phenomenon.

With the Show Plot tool in *Algodoo*, the two students in Case 2 made use of a graph in a somewhat atypical manner: that is, to *measure* the horizontal distance travelled by the puck after leaving the table within their *Algodoo* scene. As they might have used a meter stick to measure the physical distance that the puck travels in the non-virtual ramp-puck setup, the students used a graph within *Algodoo* to plot the position of their virtual puck (the circle) and read off the *x*-value from this graph as the *x*-component of its plotted motion. This implementation of the graphical representation is interesting in that the students measured a quantity *with* the graph rather than populating the graph with data measured by another tool. This is made possible in digital environments like *Algodoo* due to the fact that these programs are necessarily built up from mathematics. *Algodoo* was already tracking the position of the circle in relation to the background of the scene, so, for the students in Case 2, it was simply a matter of finding a way to display the position of the circle in a graph for their use.

However, their imaginative use of the Show Plot tool still required them to employ the mathematical representation *correctly*. In order for their graph to display the position of the circle, the students first had to select the appropriate quantities for each of the axes. Student 4 chose axes labels of Position (y) and Position (x), changing them from the default labels of Speed and Time. Even though *Algodoo* generated an option for graphical mathematical representation for the students, they were still required to engage with the representation enough to responsibly select an appropriate version of the graph for their given situation. The students had to tailor the mathematical representation so that it could be used in their unconventional implementation. This is our first example from Case 2 of how the microworldiness of *Algodoo* allowed the students to use mathematical representations in a creative yet meaningful manner: the software provided the students with mathematical materials in the form of a graphical tool, which they implemented in their own creative problem-solving.

The other way in which *Algodoo*'s microworld-like behaviour appears to have afforded unique opportunities to the students is in how it constrained their actual construction a model of the ramp-puck setup. While the transcript above focuses on the students' use of the Show Plot tool, the students' mathematization within *Algodoo* began even before the excerpts of line 50, when the students *geometrized* the ramp-puck setup into the virtual space. The students first had to interpret the parts of the physical experiment (the ramp, the table and the puck) as simple geometrical entities, spatially organized in the *Algodoo* scene so as to result in a simple geometrical model of the experiment. This meant that the students needed to make creative, physicist-like decisions about how to simplify the three-dimensional problem into a two-dimensional collection of simple shapes.

Furthermore, as the students overlaid the graph of the circle's motion in the *Algodoo* scene, they then needed to *interpret* the interactions of the objects within their model in terms of how they related to the mathematical representation. In his choice to plot the horizontal and vertical position of the circle in a graph, Student 4 effectively produced an abstract, mathematized version of the puck's trajectory. However, since the graph did not display some of the main visual features of the scene itself (i.e. the ramp rectangle, the table rectangle, the circle or the ground), the students were presented with the challenge of interpreting how the plotted data related to the virtual ramp-puck model. For example, the location of the edge of the table, which was particularly important for determining the distance of interest, was not explicitly represented in the graph itself. This led the students to explore the connection between the mathematical representation and the phenomenon which it represented. They do this by first running the simulation and then realizing that the axes of their plot were not where they wanted. Eventually, the students were likely able to relate specific points of the graph to places in the virtual setup in part due to the proximity and simultaneity of the representations (topics discussed in depth in work such as Ainsworth 2006).

We see in Case 2 how the Show Plot tool, while being used as a quantification tool for measuring horizontal distance, also involved the students in a purposeful coordination of a geometrical representation (the virtual ramp-puck model) and mathematical representation (the graph) of a physical experiment (the real ramp-puck setup). As we saw with Case 1, the student activity in Case 2 around the given prompt showcases how users of *Algodoo* can make creative yet meaningful use of the representations within the digital environment. The students were creatively engaged not only as they explored a novel physics phenomenon but also as they generated a geometrical model of a real experiment. They were involved in the tailoring of a mathematical representation of motion, and, by creatively leveraging the affordances of the *Algodoo*-IWB setup, they were able to determine the desired distance and continue with their task. This suggests that such *Algodoo*-IWB setups might be used for a variety of tasks, by a variety of students, to support student creativity and fluency in formal and mathematical representations of physics phenomena.

16.6 Discussion and Implications for Instruction

By appropriately encouraging and guiding students in environments such as *Algodoo*, those software that are rich in the mathematical materials with which users can build and have experiences, it may be possible for instructors to help students attain a better conceptual understanding of physics and to help them relate those conceptual understandings to mathematical formalisms. The cases presented in this chapter are used to show how the open structure of *Algodoo* inspired students to informally create and explore with formal mathematical representations.

16.6.1 Algodoo as a Tool for Conceptual Learning

We recognize *Algodoo* as a potentially valuable tool for expanding the possible ways in which students can engage with mathematics in physics contexts. The software allows the object of learning to be presented to students as something around which they can safely and inventively build an understanding of physics phenomena. Especially when paired with large touchscreen displays such as an IWB, students using *Algodoo* may be able to experience physics phenomena through mathematical representations in much the same way that they can begin to experience velocity and acceleration in our speedometer-rich culture. By bringing mathematical representations to life within the dynamic system of a virtual world, digital learning environments like *Algodoo* might better construe representations as part of – and intrinsically related to – observable phenomena, thereby also making representations and the phenomena they represent available to students as objects of inquiry. In a way, students using *Algodoo* can observe how mathematical representations behave much like one observes an experiment.

Put in another way, *Algodoo* seems to function as conceptual stepping stone between physical phenomena and mathematical formalisms. This is an idea that we have explored in previous research (Euler and Gregorcic 2018), wherein we specifically examined how students transitioned between using a physical laboratory setup, a digital model they created in *Algodoo*, and mathematical representations related to both the physical and digital environments. In this work, while building on Hestenes's (1992) mathematical modelling games and diSessa's (1988) discussion of the functions of educational technology, we interpret the role of *Algodoo* as one of a *semi-formalism*: that is, a conceptual intermediary between the experiences of the physical world and the formal models used in physics.

Furthermore, while much of Papert's work – and the well-known work of his colleague, Piaget – focused on learning in young children, we argue that *Algodoo* and other open-ended software have the potential to be a learning tool for a wide variety of students spanning many age groups. By providing a creative arena that adapts to the exploration and creativity of each user, *Algodoo* not only provides novice learners with alternative means for accessing physics but also allows more experienced learners to further develop, assess and/or verify their understanding of the interplay of physics and mathematics concepts. We suggest that *Algodoo* can be useful for physics learners from primary school through university.

16.6.2 Student Motivation and Interest

The processes discussed in this chapter are relevant not only from a conceptual learning perspective but also as a way of providing students with nonthreatening opportunities to approach problems in self-directed ways. The studied setup seems to have fostered exploratory behaviour even in novice users. This suggests that Algodoo and similar software could have potential for engaging learners in the early stages of mathematization through novel and less threatening ways than traditional instruction. In both cases presented here, we see that by giving students control to create and choose among the many available mathematical representations within *Algodoo*, as opposed to insisting that they use the 'most appropriate' representation for the task, the activity that results can be student-directed and playful in nature while at the same time meaningful from the perspective of conceptual learning.

This aligns well with prior research on the use of *Algodoo* in physics education, which has shown that it can be used in ways that promote the engagement and interest of students who do not consider themselves particularly savvy with physics (Gregorcic et al. 2017a). There is also a growing body of examples of *Algodoo* use wherein students from various backgrounds seem to consistently explore conceptual physics content in playful ways (Euler and Gregorcic 2018; Gregorcic et al. 2017b; Rådahl 2017). In addition to these published reports, our extensive anecdotal experience with the use of the software with a geographically diverse population of preservice and in-service physics teachers suggests that the visual clarity, user-friendliness and open-endedness of the software are usually met with great enthusiasm, particularly by preservice teachers.

16.6.3 Concluding Remarks

It is sometimes easy to be impressed with a new technology to the point of overestimating its utility in the classroom. It is worth noting that there exists much debate around the usefulness of open-ended technologies which align with the constructionist ideas of microworlds. This is especially the case among cognitivists who claim that exploratory learning places too much of a load on the cognitive processes of students (see such arguments as Hmelo-Silver et al. 2007; Kirschner et al. 2006; Sweller et al. 2007). Nonetheless, we hold that if the learning activities are appropriately framed (e.g. as playful inquiry with instructor guidance and specified tasks), the microworldiness of *Algodoo* seems to provide students with meaningful opportunities to engage with mathematical concepts, which they might have found as prohibitively challenging or uninspiring in traditional classroom or laboratory circumstances. As we stressed earlier, an open-ended software may not be sufficient on its own to ensure that powerful ideas are learned. However, as the cases in this chapter illustrate, when software such as *Algodoo* is paired with some intentional structure in the activity, students can still be creative – and self-directed – in their activities to the extent that they make meaningful use of mathematical representations. While further research is needed to find the optimal use of technologies, such as *Algodoo*, which may function as microworlds for students in the learning of physics, these digital tools seem to be potentially valuable in the way that they can (1) help students to coordinate physics concepts with mathematical representations and (2) foster student motivation in inviting avenues for playful exploration. As such, it represents a unique and exciting class of digital resources for the teaching and learning of physics across many levels of education.

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